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Exercise Set II - Solutions

1 HI - Solution

I. Population ratio:

Using the given Boltzmann relation and $T_s = T_K$, we find for N_u/N_l:
$T_K = 3\text{K}$: 2.932
$T_K = 100\text{K}$: 2.998
$T_K = 10^4\text{K}$: 2.99998
$T_K = 10^6\text{K}$: 3.00
The difference in relative population between 3 K and 10^6 K is only 2.3%.

II. Emission or absorbtion:

(a) The basic equation of radiative transfer that is valid here (in the RJ-limit) is

$$T_L = T_{cont}e^{-\tau} + T_s(1 - e^{-\tau})$$

and

$$\Delta T_L = T_L - T_{cont} = (T_s - T_{cont})(1 - e^{-\tau}).$$

The only contribution to the continuum temperature is the 2.73 K cosmic background radiation, thus $T_{cont} = T_{BG} = 2.73$ K. We can assume that $T_s = T_K$. Thus, the temperatures cancel and we will see no line at all.

(b) $T_L = -0.17$ K, the line will appear in absorption.

(c) $T_L = 0.48$ K, the line will appear in emission.

2 Position-velocity diagram

See lecture notes.

3 J vs. C

See lecture notes.

4 After a supernova explosion

We need to compute the fraction ,x, of total energy radiated by free-free emissions, E_{ff}, during a period of 10^4 yr. The assumptions we need to do are :

- The electron density, n_e, is constant and is equal to the post shock value in the expanding bubble and it is equal to the gas density.

- As a first approximation, we assume $\mu = 1$ (we justify this later).

$$E_{ff} = \int_0^{10^4} \frac{4}{3}\pi R^3(t)\Lambda(t)dt \tag{1}$$

where

$$\Lambda(t) = 1.4 \times 10^{-40} T(t)^{-1/2} n_e^2 \; [J \; m^{-3} \; s^{-1}] \tag{2}$$

$$T(t) = (2300 \; K)\frac{\mu}{m_h}\left(\frac{v(t)}{10 \; km \; s^{-1}}\right)^2 \tag{3}$$

$$v(t) = (120 \; km \; s^{-1})\left(\frac{E_0}{10^{51} \; ergs}\right)\left(\frac{n_H}{cm^{-3}}\right)^{-1/5}\left(\frac{t}{10^5 yr}\right)^{2/5} \tag{4}$$

$$R(t) = (32 \; pc)\left(\frac{E_0}{10^{51} \; ergs}\right)\left(\frac{n_H}{cm^{-3}}\right)^{-1/5}\left(\frac{t}{10^5 yr}\right)^{2/5} \tag{5}$$

Since $E_0 = 10^{44} \; J = 10^{51} \; erg$ and $n_H = n_e = 10^6 \; m^{-3} = 1 \; cm^{-3}$, we can simplify and re-write the last two equations as :

$$\Lambda(t) = 1.4 \times 10^{-40} T^{-1/2} n_e^2 \; [J \; m^{-3} \; s^{-1}] \tag{6}$$

$$T(t) = 3.31 \times 10^5 \left(\left(\frac{t}{10^5 yr}\right)^{-3/5}\right)^2 \tag{7}$$

$$v(t) = (120 \; km \; s^{-1})\left(\frac{t}{10^5 yr}\right)^{-3/5} \tag{8}$$

$$R(t) = (32 \; pc)\left(\frac{t}{10^5 yr}\right)^{2/5} \tag{9}$$

Since we now have everything in terms of t only, we can replace them in Eq-1 and evaluate the integral. (BE CAREFUL WITH THE UNITS). After replacing all the numbers, we get $E_{ff} = 4 \times 10^{42} \; erg$, thus $x \sim 10^{-8}$. Changing the value of μ to 1.5 or 2.7 will still keep $x << 1$, which is still insignificant compared to the amount of the energy of the blast.

5 Hot gas – cooling time

Cooling timescale due to bremsstrahlung is given by τ_1 :

$$\tau_1 = 8.5 \times 10^{10}\left(\frac{n_e}{10^{-3} cm^{-3}}\right)^{-1}\left(\frac{T_e}{10^8 K}\right)^{-1/2} [yr] \tag{10}$$

Whereas the synchrotron cooling time (the half-life time of the particles) is :

$$\tau_2 = 824 \times 10^9 \left(\frac{B}{\mu G}\right)^{-2}\left(\frac{E}{GeV}\right)^{-1} [yr] \tag{11}$$

For the hot coronal gas of the Milky Way, $n_e = 0.05 \; cm^{-3}$ and $T = 10^6 K$. Replacing in Eq-10, we get $\tau_1 = 17 \; Gyr$. In computing τ_2, the magnetic field strength is $B = 5 \; \mu G$, and the energy of the electrons is $E = 3 \; GeV$, thus $\tau_2 = 11 \; Gyr$. Although both processes are of the same order of magnitude, since $\tau_1 > \tau_2$ we can say that the cooling due to synchrotron emissions is slightly faster.

6 Short exercises from the lectures

The equilibrium temperature is a function of the density, as one can see in the given figure for HI regions. For HII regions, the plot looks like the right figure (data points obtained from the left figure):