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Exercise Set II

1 HI

I. **Population ratio** : The ratio of the populations in the upper, N_u, and lower, N_l, levels of the ground state of HI is given by the Boltzmann relation, where the statistical weights in these levels are 3 and 1, respectively: $N_u/N_l = 3\exp(-0.0682/T_s)$. We can assume that the spin temperature, T_s, equals the kinetic temperature, T_K. Calculate the population ratio for a temperature of 100 K. Repeat for a temperature of 3 K (the lowest temperature possible under local thermodynamic equilibrium), for 10^4 K (the warm interstellar medium), and 10^6 K. Compare the differences in populations.

II. **Emission or absorption :** Suppose an extended, uniform HI cloud has a physical temperature of $T_K = 2.73$ K.

(a) Assume the only background source to be the 2.73K microwave background radiation. Would you expect to observe the HI line in emission, absorption or not at all? Why?

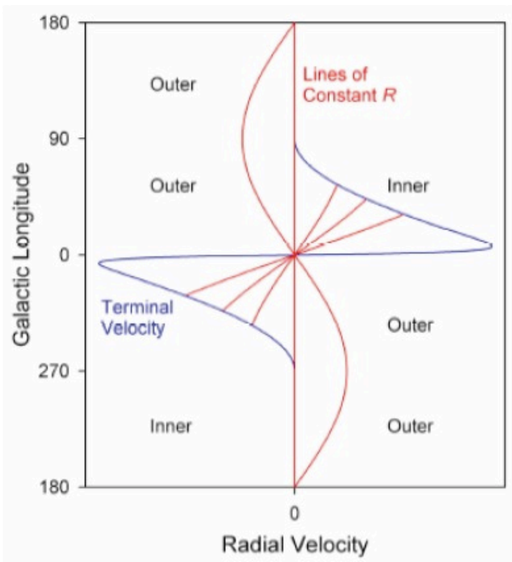
(b) Now assume there is a background source with main beam brightness temperature, $T_{\mathrm{MB}} = 3$ K. What would be the temperature of the absorption ΔT_{L} (in K) if $\tau = 1$?

(c) Repeat for $T_K = 3.5$ K.

2 Position-velocity diagram

As also mentioned in the lectures:

Exercise:
describe this diagram



3 J vs. C

Compare and contrast interstellar J and C shocks.

4 After a supernova explosion

A supernova explosion liberates 10^{44} J in interstellar gas of density $10^6 \mathrm{m}^{-3}$. Estimate what fraction of the initial energy has been radiated away by free-free radiation after 10000 yr (assume that the gas density and temperature in the hot bubble are uniform and equal to their post-shock values; energy is lost by free-free emission at a rate $1.4 \times 10^{-40} T_e^{1/2} n_e^2 \,\mathrm{J\,m^{-3}s^{-1}}$, where T_e and n_e are the electron temperature and density, respectively). Does it make a difference whether the SNR expands into a purely atomic ($\mu = 1.5$) or a molecular ($\mu = 2.7$) environment?

5 Hot gas – cooling time

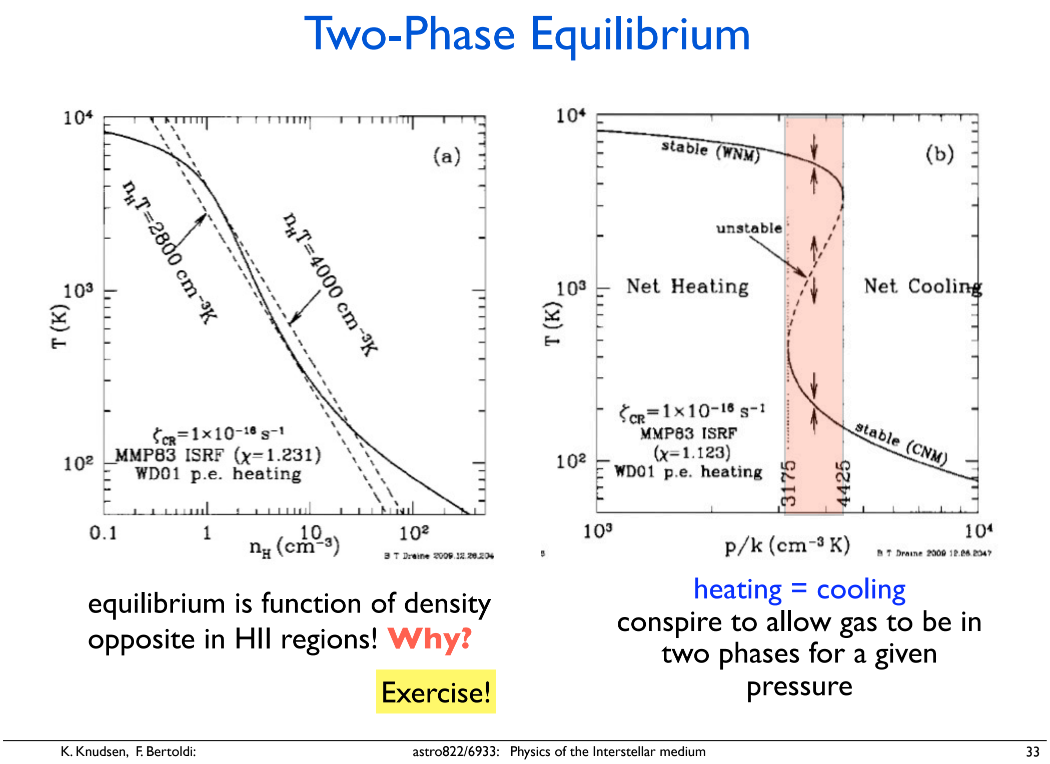
The cooling time of a hot plasma due to thermal bremsstrahlung is

$$\tau_{\rm cool} = 8.5 \times 10^{10} \left(\frac{n_e}{10^{-3}\ \mathrm{cm^{-3}}}\right)^{-1} \left(\frac{T_e}{10^8\ \mathrm{K}}\right)^{-1/2} [\mathrm{yr}] \ . \quad (1)$$

Calculate the cooling time of the hot coronal gas in the Milky Way, which has an average density of $n_e = 0.05\,\mathrm{cm^{-3}}$ and a mean temperature of $T = 10^6\,\mathrm{K}$. How does this compare to the synchrotron cooling time of relativistic electrons with energy $E = 3\,\mathrm{GeV}$ in a mean magnetic field of strength $B = 5\mu\mathrm{G}$? This cooling time is equivalent to the half-life time of the particles

$$t_{1/2} = 824 \times 10^9 \left(\frac{B}{\mu\mathrm{G}}\right)^{-2} \left(\frac{E}{\mathrm{GeV}}\right)^{-1} [\mathrm{yrs}] \ . \quad (2)$$

6 Short exercises from the lectures



7 HII regions and the 'energy balance' of the Milky Way

1. Adopt for the typical HII region in the Milky Way an average density of 30 $\mathrm{cm^{-3}}$ and an average ionizing photon energy of 20 eV, calculate the total cooling rate per atom.

2. Adopting the total mass in HII regions given in Table 1, calculate the total luminosity of ionized gas in the Milky Way.

Phase	n_0[a] (cm^{-3})	T[b] (K)	ϕ_v[c] (%)	M[d] (10^9 $M_\odot$)	$< n_0 >$[e] (cm^{-3})	H[f] (pc)	Σ[g] ($M_\odot pc^{-2}$)
Hot intercloud	0.003	10^6	$\sim$50.0	—	0.0015	3000	0.3
Warm neutral medium	0.5	8000	30.0	2.8	0.1[h]	220[h]	1.5
					0.06[h]	400[h]	1.4
Warm ionized medium	0.1	8000	25.0	1.0	0.025[i]	900[i]	1.1
Cold neutral medium[j]	50.0	80	1.0	2.2	0.4	94	2.3
Molecular clouds	>200.0	10	0.05	1.3	0.12	75	1.0
HII regions	1–10^5	10^4	—	0.05	0.015[k]	70[k]	0.05

[a] Typical gas density for each phase.
[b] Typical gas temperature for each phase.
[c] Volume filling factor (very uncertain and controversial!) of each phase.
[d] Total mass.
[e] Average mid-plane density.
[f] Gaussian scale height, $\sim \exp[-(z/H)^2/2]$, unless otherwise indicated.
[g] Surface density in the solar neighborhood.
[h] Best represented by a Gaussian and an exponential.
[i] WIM represented by an exponential.
[j] Diffuse clouds.
[k] HII regions represented by an exponential.

Figure 1: Table 1 (copied from Tielens book).

3. Adopting the physical characteristics of the WIM given in Table 1, calculate the total cooling rate per atom for this phase.

4. Adopting the total mass in the WIM given in Table 1, calculate the total cooling luminosity of this phase in the Galaxy.

5. Comparing these values with those derived for the phases of the ISM (as determined in the exercise above), explain why – on a galaxy-wide scale – ionized gas is so much more luminous than neutral gas.

6. Compare the derived total luminosity originating from ionized gas with the stellar radiative luminosities and the dust far-IR luminosity given in Table 2. What do you conclude?

Object	Mass ($M_\odot$)
Stars	1.8 (11)
Gas	4.5 (9)

Source	Luminosity ($L_\odot$)
Stellar luminosities	
All stars	4.0 (10)
OBA	8.0 (9)
Gas and dust	
[CII] 158 μm	5.0 (7)
FIR	1.7 (10)
Radio	1.5 (8)
γ-rays	3.0 (5)
Mechanical luminosities	
SN	2.0 (8)
WR	2.0 (7)
OBA	1.0 (7)
AGB	1.0 (4)

Figure 2: Table 2 (copied from Tielens book). The funny notation for the luminosity should probably have been: All stars 4.0×10^{10}, etc.

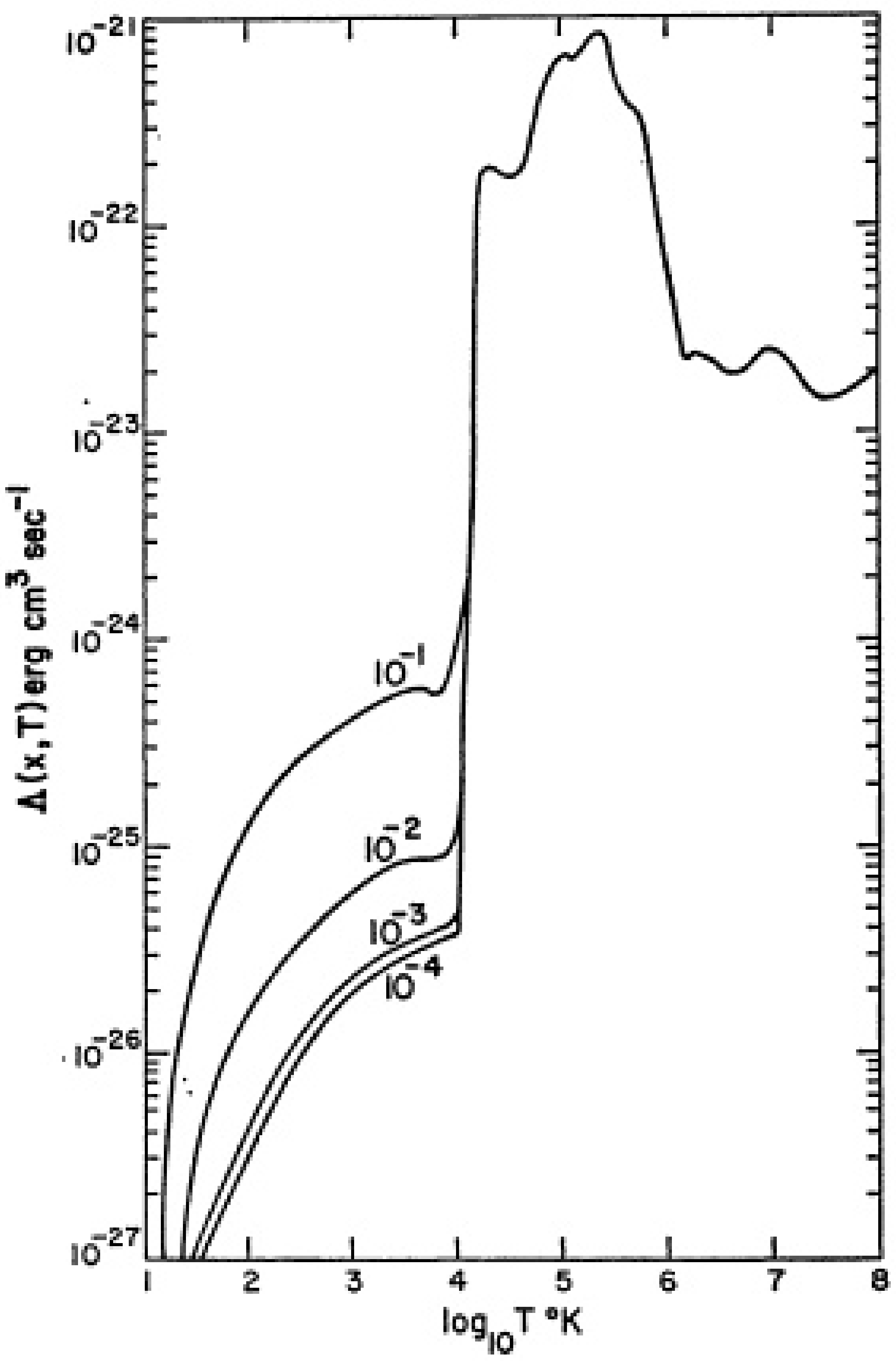


Figure 3: From Dalgarno & McCray, 1972: The (low-density) cooling rate for interstellar gas as function of temperature.