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Exercise Set I - Solutions

1 Shell around an object - Solution

Along way A:
For the core, which is a blackbody, $I_\nu = B_\nu(T_c)$ holds, while for the thermal shell $S_\nu = B_\nu(T_s)$. For $T_s < T_c$, it follows that $B_\nu(T_s) < B_\nu(T_c)$ because of the monotonicity of the Planck function. Therefore, $S_\nu = B_\nu(T_s) < B_\nu(T_c) = I_\nu$. With that

$$\frac{dI}{d\tau} = -I + S \Rightarrow \frac{dI}{d\tau} < 0,$$

i.e. I_ν decreases along the way. For $T_c < T_s$, it follows analogous that $I_\nu < S_\nu$, and

$$\frac{dI}{d\tau} = -I + S \Rightarrow \frac{dI}{d\tau} > 0,$$

so I_ν increases along the way.
Along way B:
Here you have to consider only the shell, where $S_\nu = B_\nu(T_s)$ and Kirchhoffs law is valid:

$$j_\nu = \alpha_\nu B_\nu(T_s)$$

As B_ν is assumed not to change (much) with frequency, from $\alpha_{\nu_0} \gg \alpha_{\nu_1}$ follows $j_{\nu_0} \gg j_{\nu_1}$. Then

$$I_\nu = \int j_\nu ds \Rightarrow I_{\nu_0} \gg I_{\nu_1}.$$

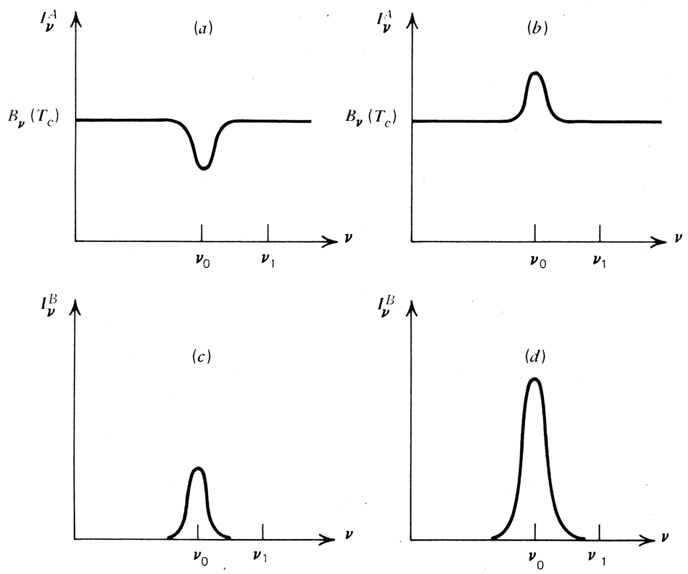

Figure S.1 Intensity from a blackbody surrounded by a thermal absorbing shell (a) along ray A when $T_s < T_c$, (b) along ray A when $T_s > T_c$, (c) along ray B when $T_s < T_c$, (d) along ray B when $T_s > T_c$.

2 Forbidden lines vs. permitted lines - Solution

The best vacua created in earths labs still have much higher gas densities than the interstellar medium. High densities lead to collisional deexcitations, which do not emit photons, where in low density enviroments the particles have enough time to decay radiatively.

3 Saha-Boltzmann equation - Solution

1.
$$\left(\frac{2\pi m_e k}{h^2}\right)^{3/2} = 2.414 \cdot 10^{21}\,\mathrm{m}^{-3}, \qquad e^{\chi_n/kT} = e^{-1.58\cdot 10^5/(n^2 T)}$$

2. Results for $\frac{n_e n_p}{n_n}$:

T[K]	n=1	n=100
5000	$1.669 \cdot 10^7$	$8.510 \cdot 10^{16}$
8000	$4.672 \cdot 10^{12}$	$1.724 \cdot 10^{17}$
10^4	$3.376 \cdot 10^{14}$	$2.411 \cdot 10^{17}$
$2 \cdot 10^4$	$2.554 \cdot 10^{18}$	$6.824 \cdot 10^{17}$

For temperatures $> 10^4$ K,the number of ions is very much larger than the level population of H in any level. Thus, an HII region like the Orion nebula is indeed expected to be fully ionized,

4 Strömgren sphere (I) - Solution

From
$$\dot{N}_{Ly-c} = \alpha(T)\frac{4}{3}\pi r_{H_{II}}^3 n_e^2$$
for same radii and temperature one can conclude
$$\frac{\dot{N}_{Ly-c,1}}{\dot{N}_{Ly-c,2}} = \frac{n_{e,1}^2}{n_{e,2}^2} \Rightarrow \frac{n_{e,1}}{n_{e,2}} = \sqrt{\frac{10^{47}}{10^{49}}} = 0.1$$

5 Strömgren sphere (II) - Solution

1. Assume that ionization rate = recombination rate $\dot{N}$. The number of ionized H atoms in the sphere is $n_e V = n_e \frac{4}{3}\pi R_S$, so the time to become neutral is
$$t = \frac{n_e \frac{4}{3}\pi R_S}{\dot{N}} = \frac{10^6\,\mathrm{m}^{-3}\frac{4}{3}\pi 10\,\mathrm{pc}}{10^{49}\,\mathrm{s}^{-1}} = 1.229 \cdot 10^{10}\,\mathrm{s} = 390\,\mathrm{yr}$$

2. Obtain the sound velocity from $\frac{1}{2}m_H v_s^2 = \frac{3}{2}kT$. Than the time to expand is
$$\frac{100\,\mathrm{pc}}{v_s} = 100\,\mathrm{pc}\left(\frac{m_H}{3kT}\right)^{1/2} = 1.96 \cdot 10^{14}\,\mathrm{s} = 6.2 \cdot 10^6\,\mathrm{yr}.$$

6 HII regions - Solution

1. See lecture...
2. Continuum (free-free, free-bound, two-photon radiation, scattered light, thermal emission of dust), optical and radio recombination lines, fine-structure forbidden lines
3.
   - Balmer lines or Balmer continuum ⇒ estimation of number of ionizations
   - Intensity of recombination lines ⇒ depends on temperature and is proportional to $n_e n_i$
   - Ratio of T_B of a radio recombination line with very high quantum number and the thermal free-free-continuum ⇒ T_e (ex. 6)
   - Intensity ratio between helium and hydrogen recombination lines ⇒ abundance of helium
   - Frequency of recombination lines ⇒ radial velocity of the HII region

   OIII as an example for information contained in **forbidden lines**:

   - Intensity ratio between forbidden lines of OIII (88.4 and 51.8 m lines) ⇒ n_e
   - Intensity of one of these lines ⇒ abundance of OIII
   - Nebular lines: 5007 $\mathring{A}$, 4959 $\mathring{A}$ of OIII ⇒ abundance of OIII with temperature and density known
   - Intensity ratio between the intensity of the nebular lines and the 4363 $\mathring{A}$ auroral line ⇒ T_e
   - Abundance determinations from fine-structure lines: e.g. Lequeux p. 108