Observations in Astrophysics-2

Statistical description of processes

convolution of signal with transfer function, sampling etc

requires the concept of Fourier transforms via the Convolution Theorem & Cross Correlations

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Additional reading

Numerical Recipes

Press et al. 1992

Chapters 12-0,1, 13, 14

check : www.nr.com

Observational Astrophysics

Lena, P., Lebrun, F., Mignard, F.

Chapter 2: The observation and analysis of stellar photospheres:

Gray, D.F., C.U.P.,1992

Data reduction and error analysis

Bevington & Robinson 1992

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Data reduction and error analysis

Bevington & Robinson 1992

Chapter 1.1, 1.2, 1.3 & 2.1, 2.2 (not 2.2.2), 2.3 OAF-2 & Chapters 3 & 5 of OAF-1

Useful (observational) Astrophysics websites

http://xxx.soton.ac.uk/list/astro-ph/new

http://www.astronomerstelegram.org/

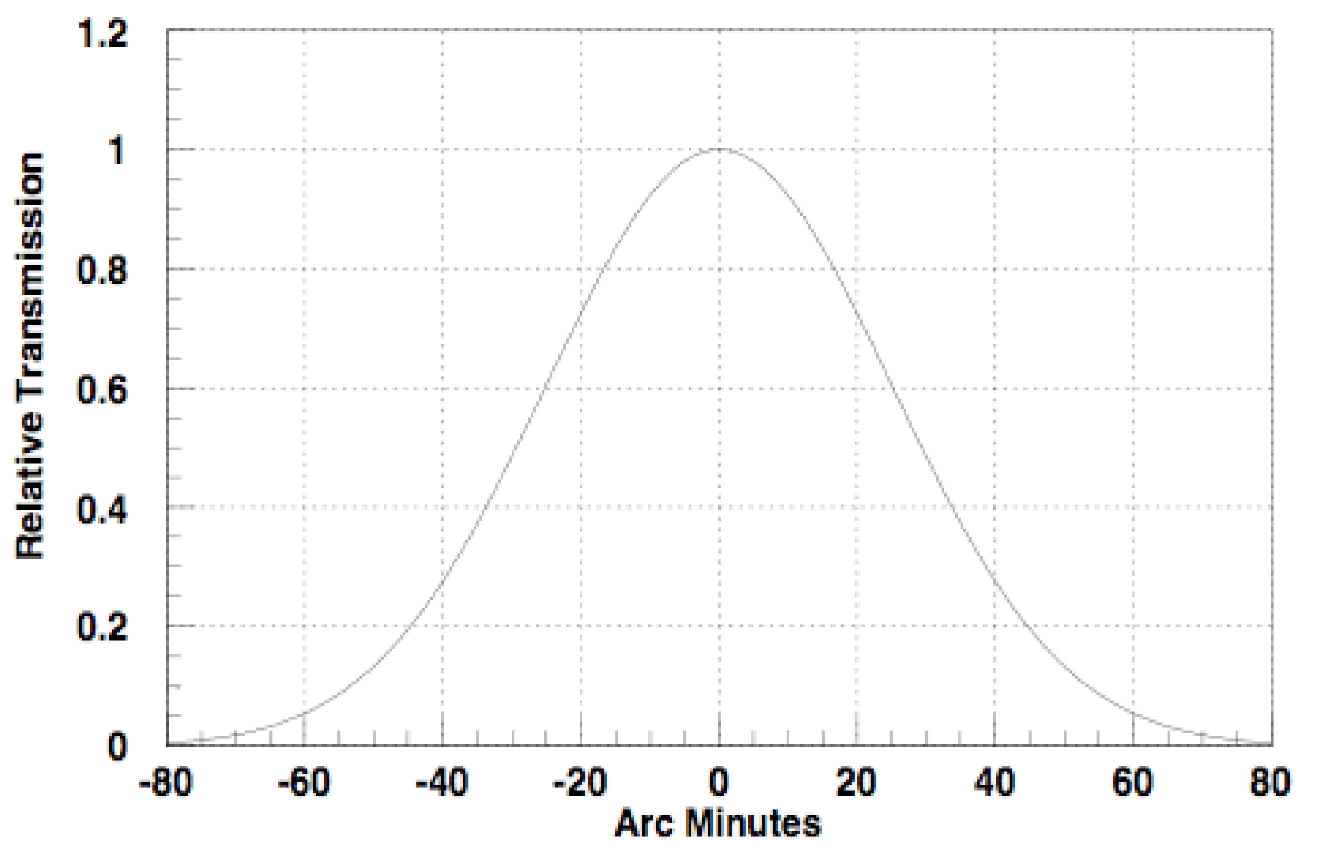
http://cdsads.u-strasbg.fr/abstract_service.html

http://simbad.u-strasbg.fr/simbad/sim-fid

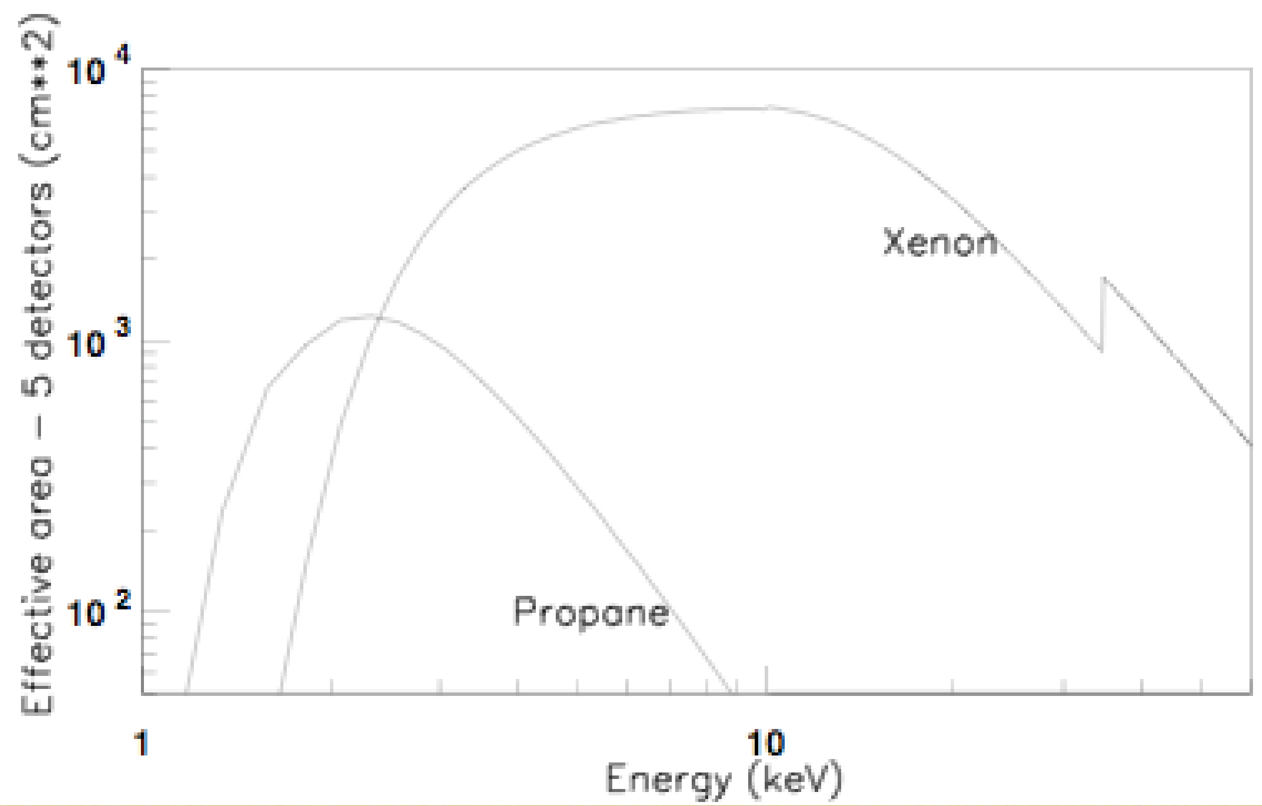
Detection of X-rays with the Rossi X-ray Timing Explorer

Three instruments: an ASM, the PCA, and HEXTE

Collimator response



Effective Area



Data sampling & Data binning

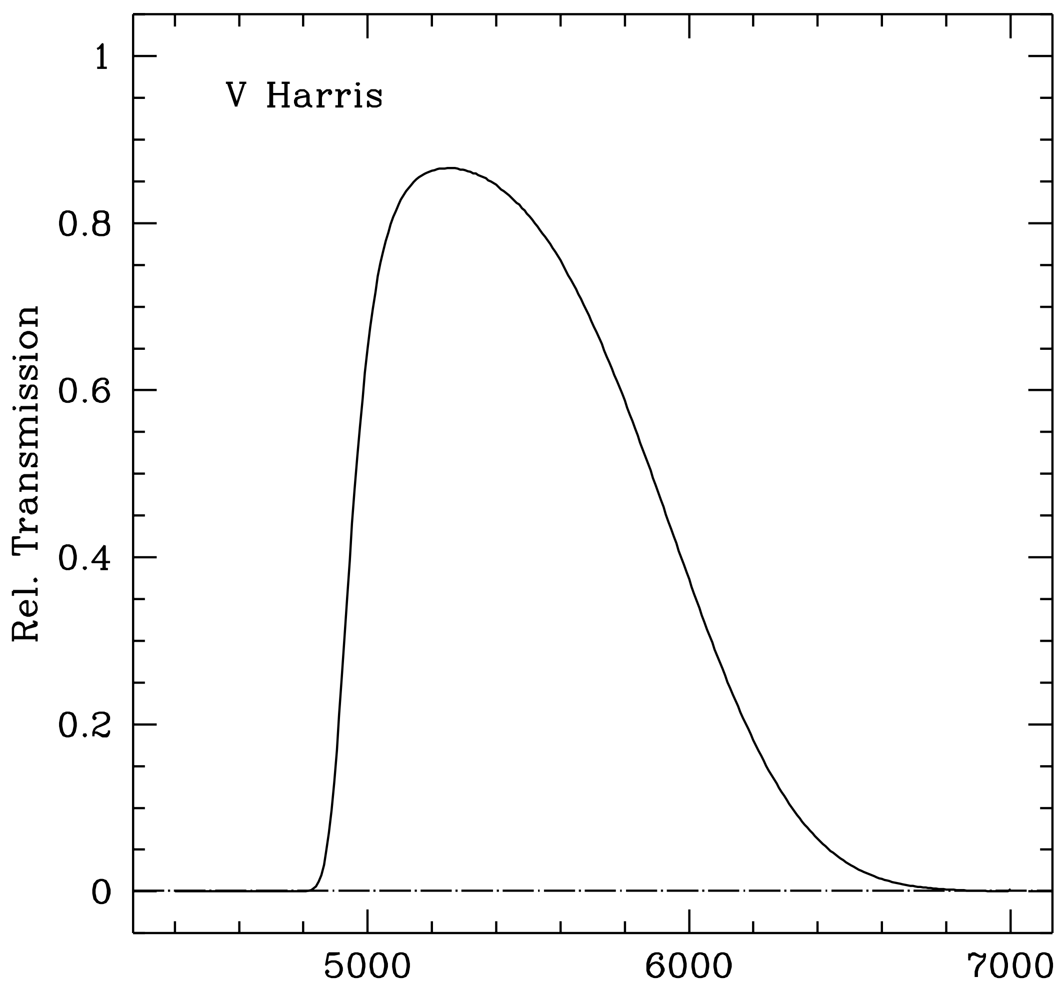
Examples: detection of optical light via a telescope

Palomar 200 inch Hale Telescope

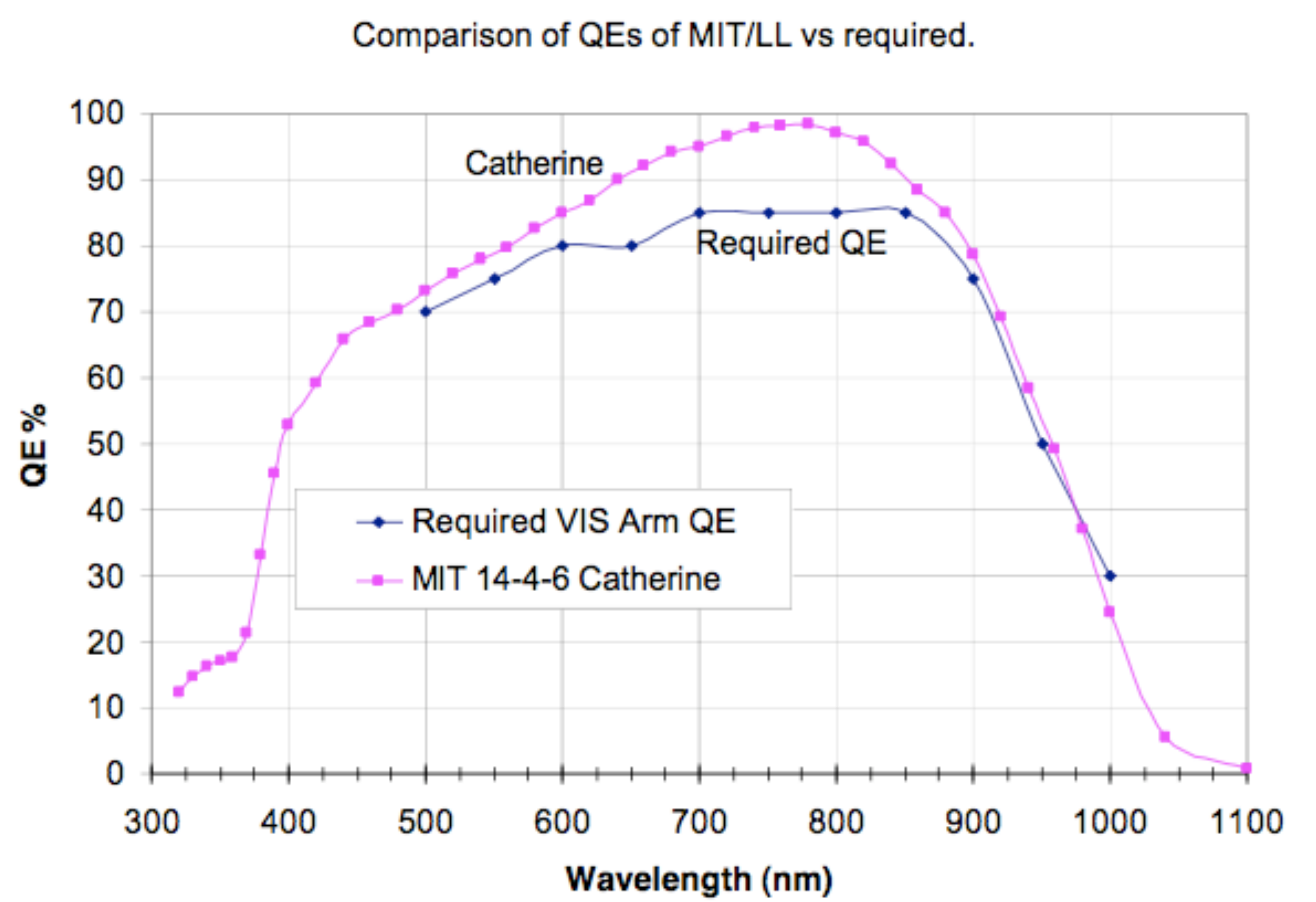
Examples: detection of optical light via a telescope

Palomar 200 inch Hale Telescope

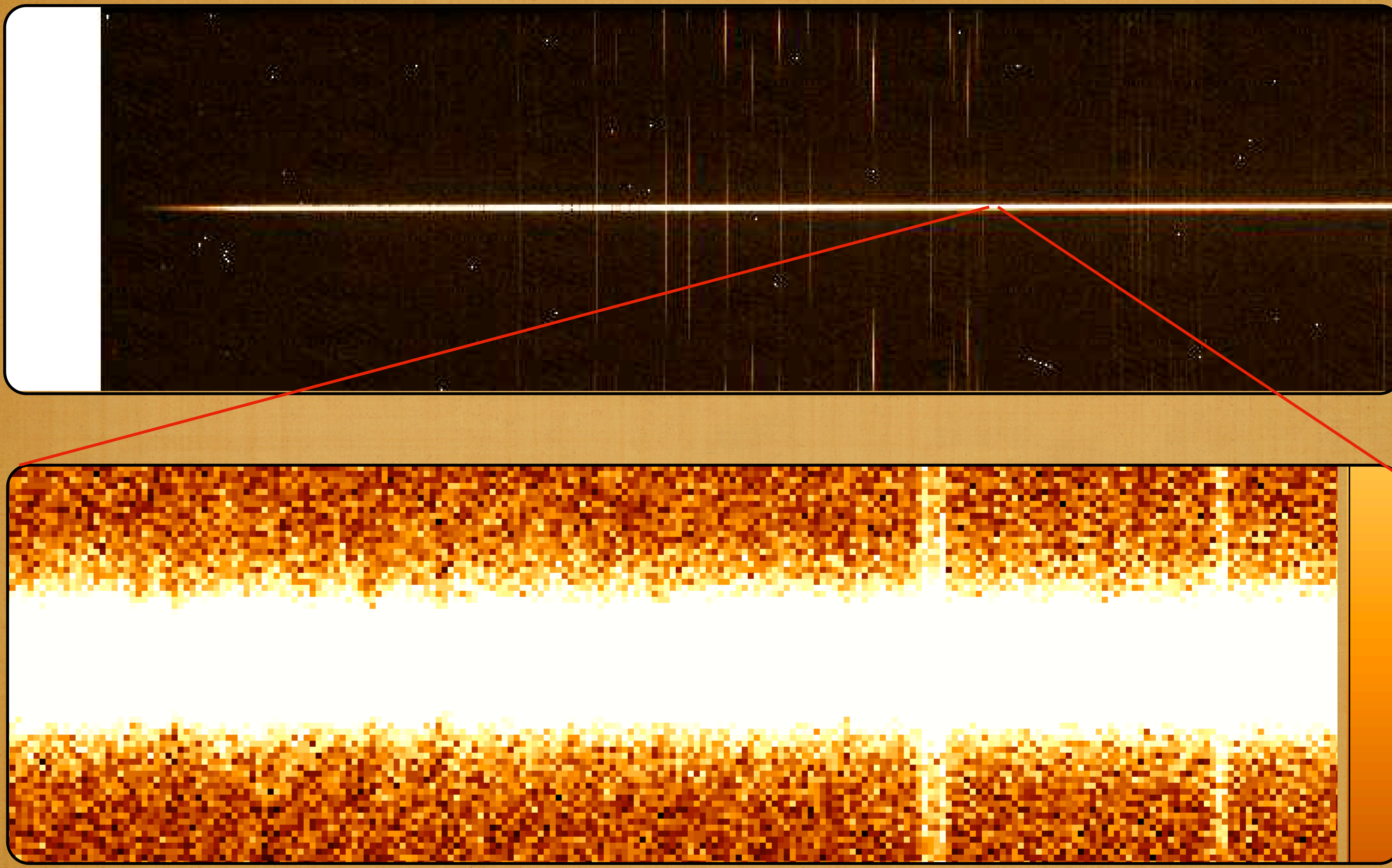
Broadband filter



Efficiency detector

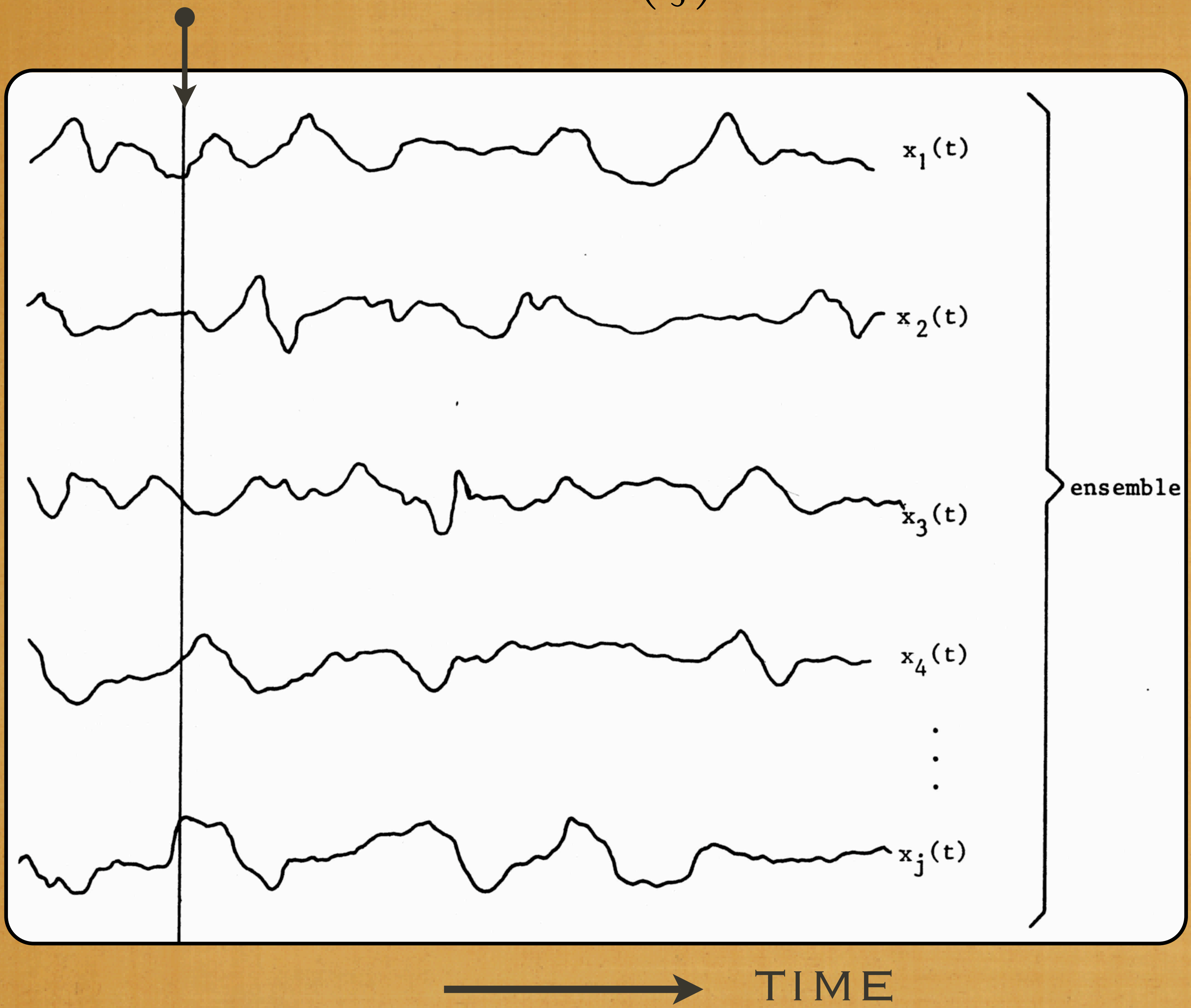


Data is discrete



Stochastic Processes

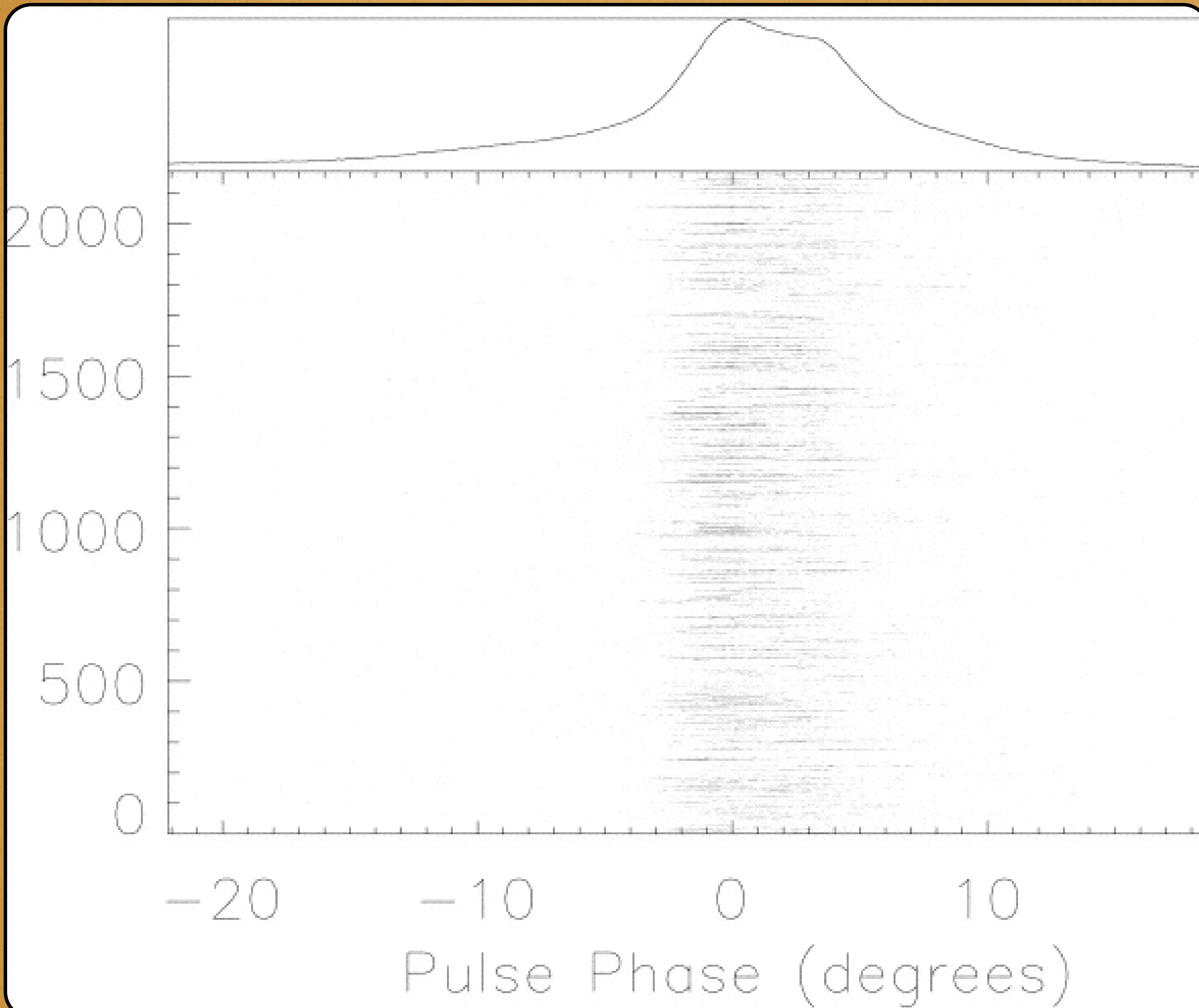
Random variable $x(\zeta)$



$x(\zeta, t)$

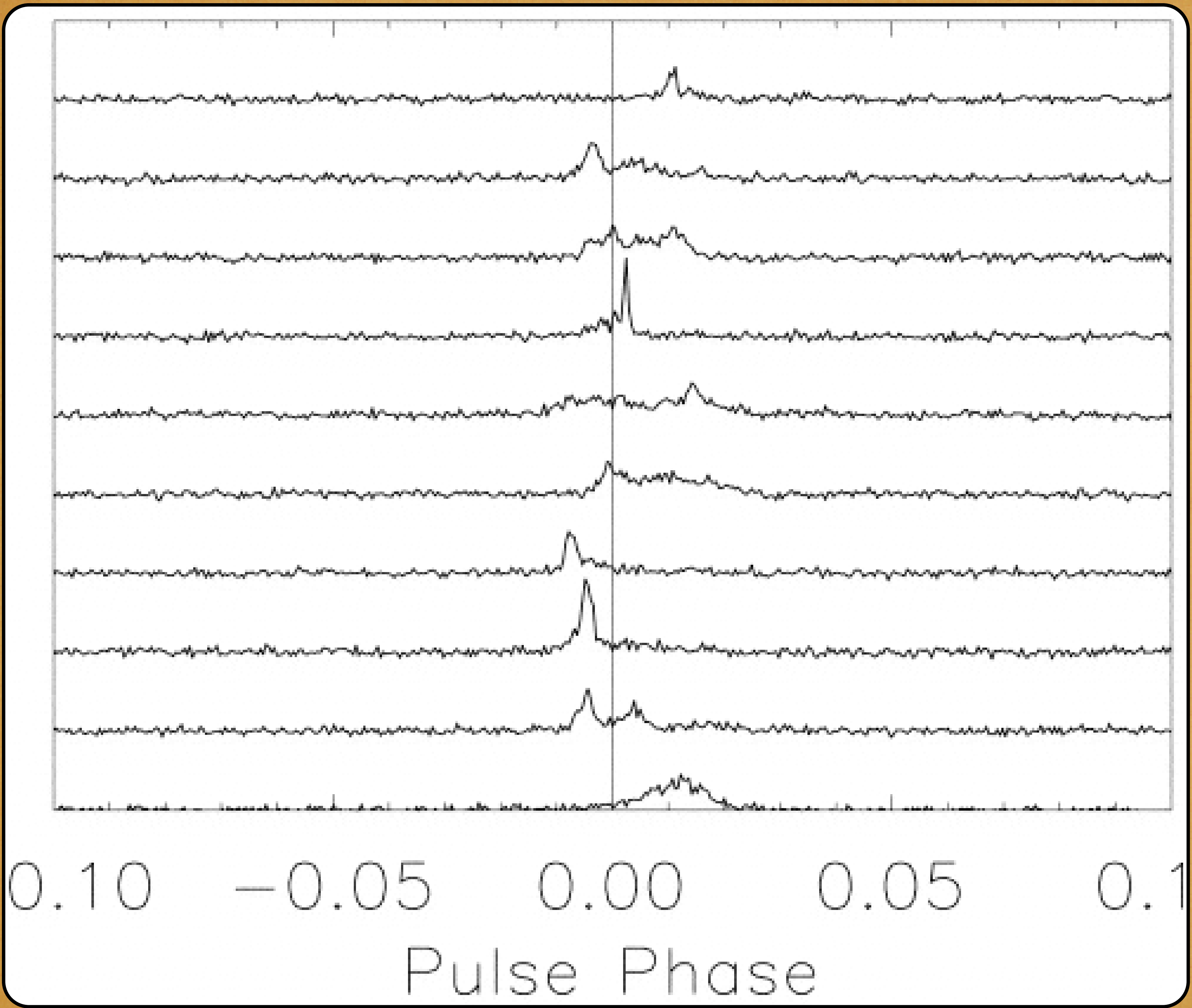
TIME

Average pulse profile & individual pulses



PSR J0437-4715 Janet et al. 1998

Average pulse profile & individual pulses



PSR J0437-4715 Janet et al. 1998

Stochastic Processes

4 different aspects of a s.p.

a: a family of functions depending on time (indexed by ζ)

b: a particular function of time (ζ fixed)

c: a random variable (at fixed t, for a set of trial outcomes ζ

d: a number (at fixed t and for fixed ζ)

$x(\zeta)$ DESCRIBES THE RELATION BETWEEN THE POSSIBLE OUTCOMES ζ AND THE RANDOM VARIABLE x

E.G.

DIE THROWING: ζ_1 OUTCOME IS FACE 1 OF DIE

$$x(\zeta_1) = 0$$

$$x(\zeta_2) = x(\zeta_3) = 10$$

$$x(\zeta_4) = x(\zeta_5) = 100$$

$$x(\zeta_6) = 1000$$

DISTRIBUTION FUNCTION $F(x) = P\{x \leq y\}$

PROBABILITY DENSITY FUNCTION $\dfrac{dF(x)}{dx} = f(x)$

$\hookrightarrow$ GAUSS, POISSON, χ^2 ETC

Expectation values

$$E\{\phi(x)\} = \int_{-\infty}^{\infty} \phi(x) f(x) dx$$

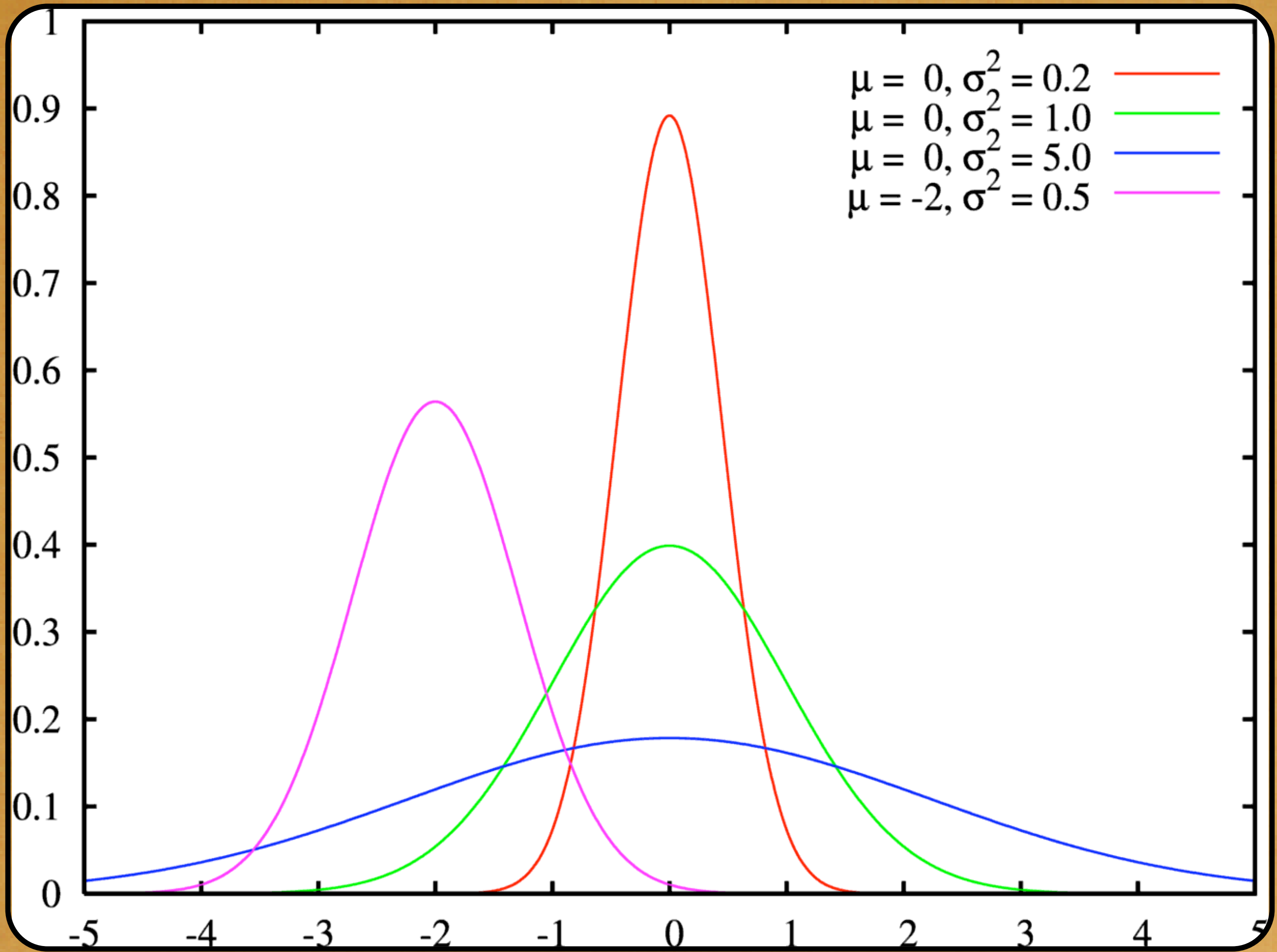
discrete version $$E\{\phi(x)\} = \sum_{n=-\infty}^{\infty} \phi(x_n) P_n$$

Gaussian or Normal Probability Density Distribution

$$f(x) = \frac{1}{\sigma\sqrt{(2\pi)}} exp(-\frac{1}{2}\frac{(x-\eta)^2}{\sigma^2})$$

Two parameters completely describe the distribution

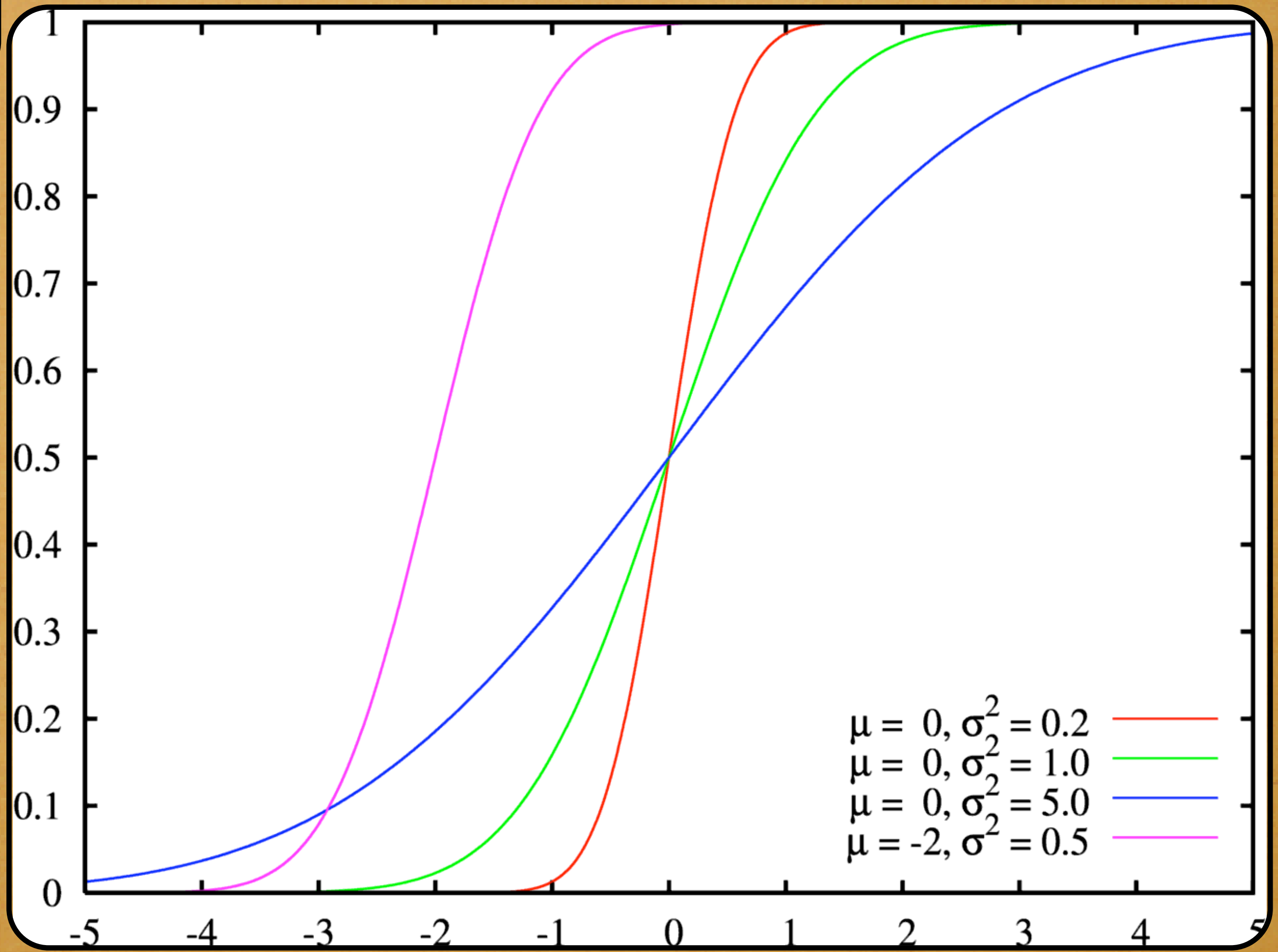
A Gaussian Distribution



Gaussian Cumulative Distribution function

$$F(x, \eta, \sigma) = 0.5 + erf\frac{x - \eta}{\sigma^2}$$

(see chapter 6 Num Res)



Moments of a distribution

moment $\mu'_k = E\{(x)^k\}$

central moment $\mu_k = E\{(x - E\{x\})^k\}$

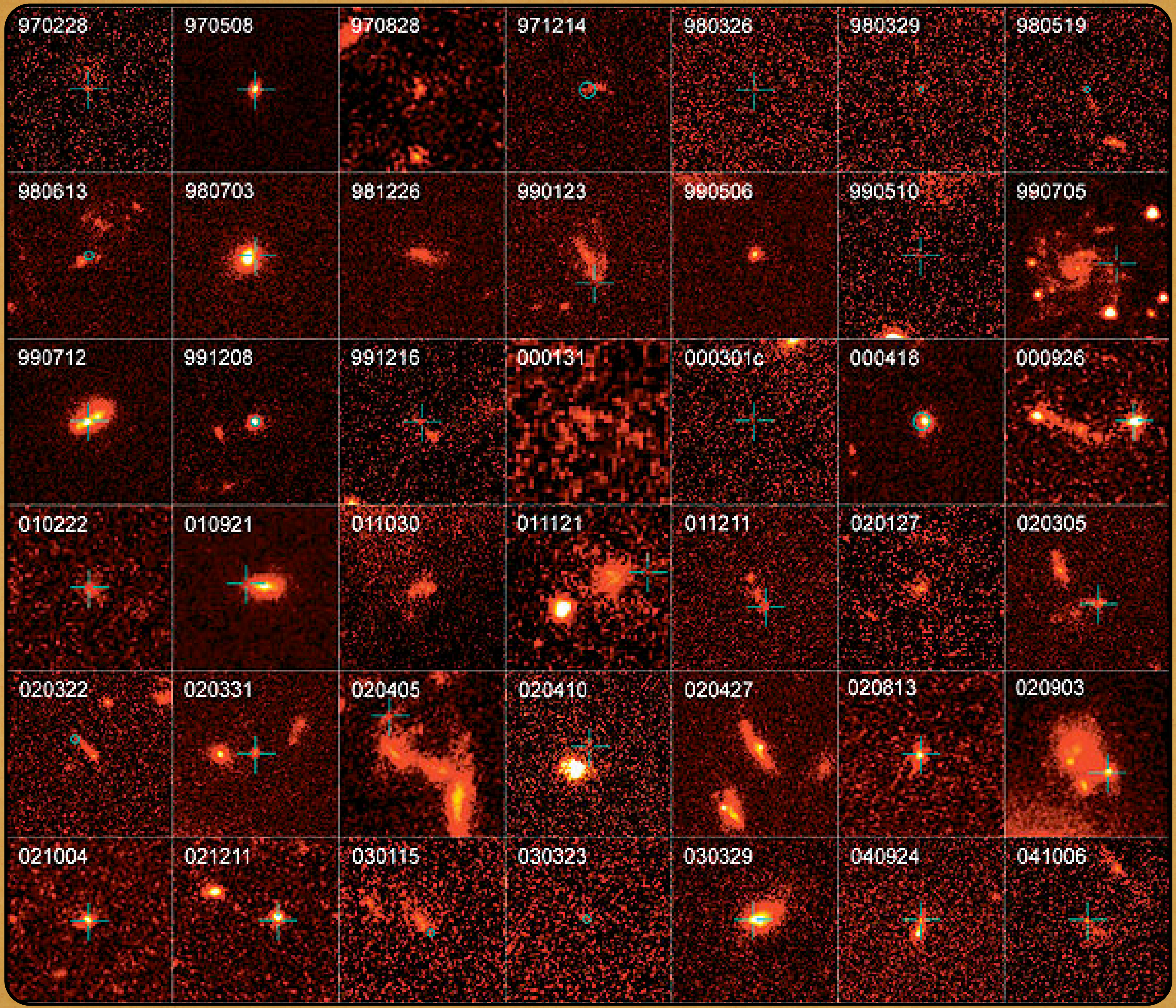
Mean $\eta = E\{x\} = \int_{-\infty}^{\infty} x f(x) dx$

Variance = central moment of 2nd order

$$\mu_2 = E\{(x-\eta)^2\} = \int_{-\infty}^{\infty} (x-\eta)^2 f(x) dx \equiv \sigma^2$$

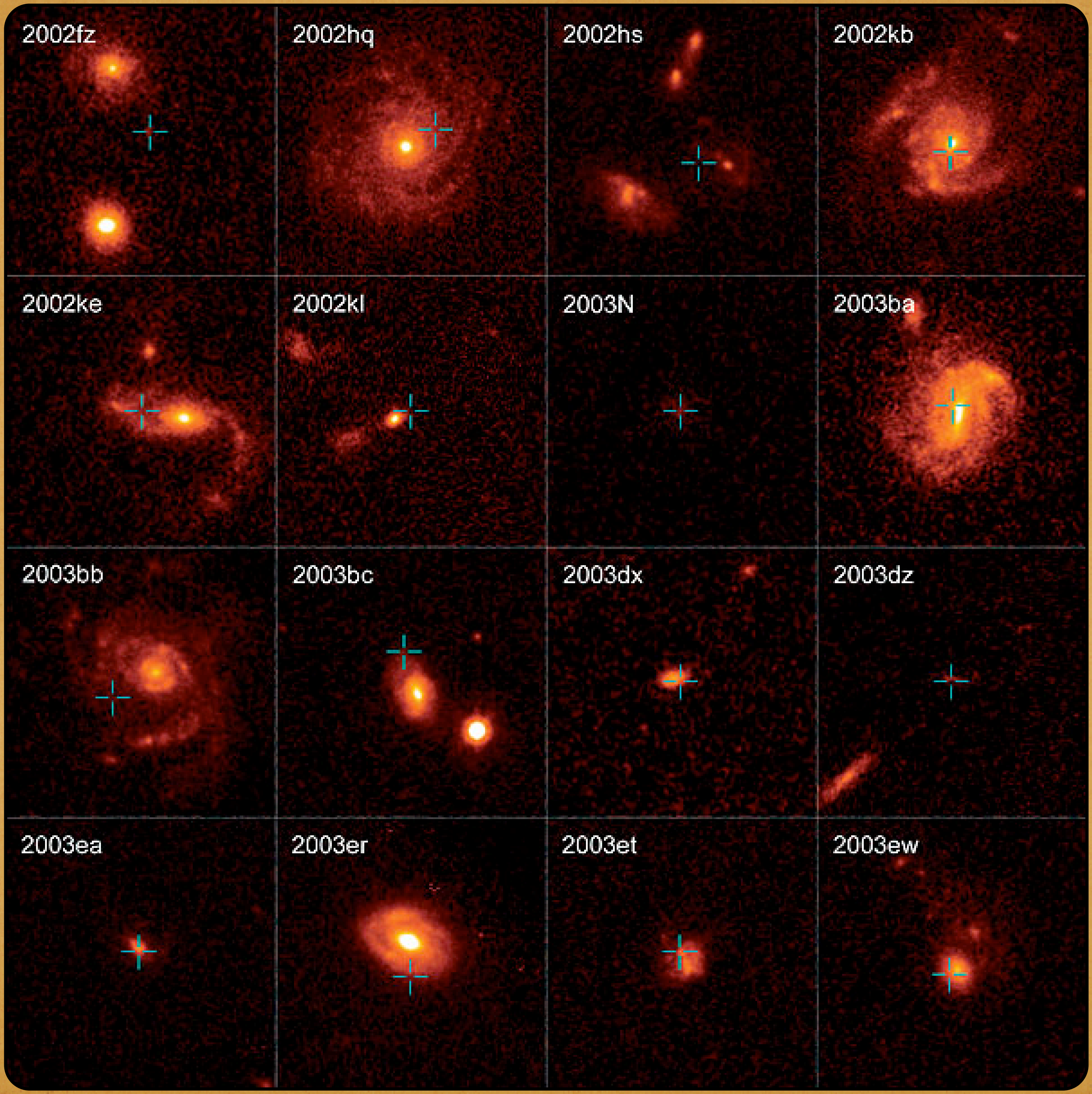
$$\sigma^2 = E\{x^2\} - \eta^2 = E\{x^2\} - (E\{x\})^2$$

Example of use of moments: GRB distribution

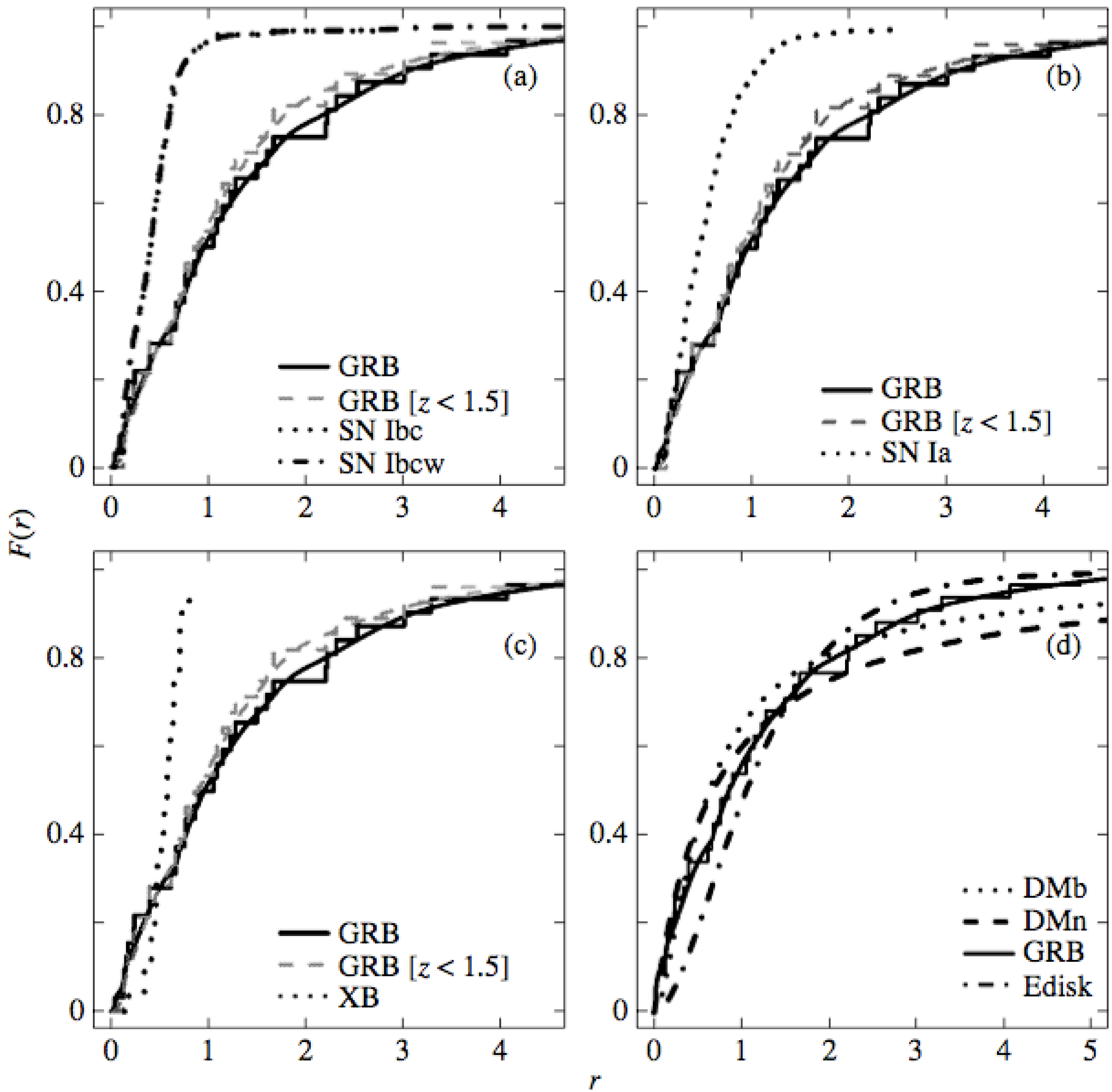


Bloom et al. 2002, Blinnikov et al. 2004, Fruchter et al. 2006

Core-collapse SN distribution



Distributions Blinnikov et al.2004



Correlation, auto-correlation

Correlation

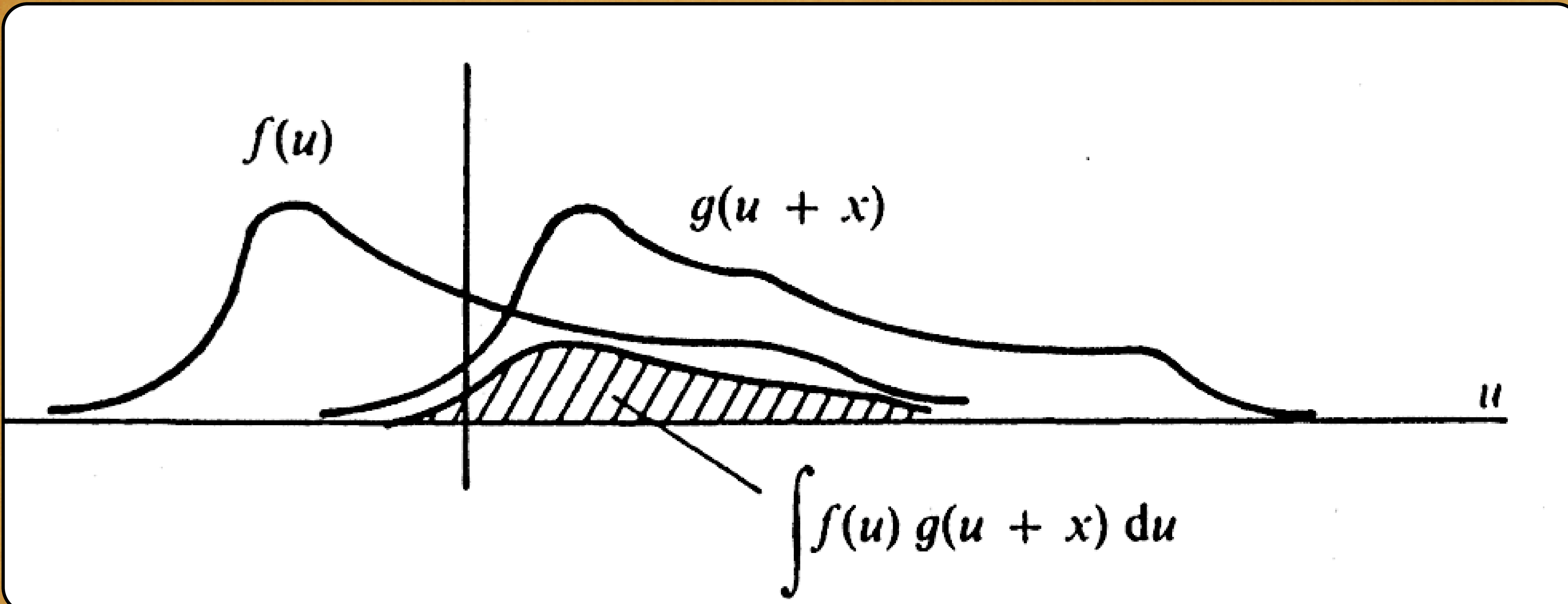
$$k(x) = f(x) \otimes g(x) \qquad k(x) = \int_{-\infty}^{\infty} f(u)g(u+x)du$$

If X and Y are two independent random variables with probability distributions f and g, respectively, then the probability distribution of the difference – X + Y is given by the cross-correlation f⊗g.

the convolution f * g gives the probability distribution of the sum X + Y

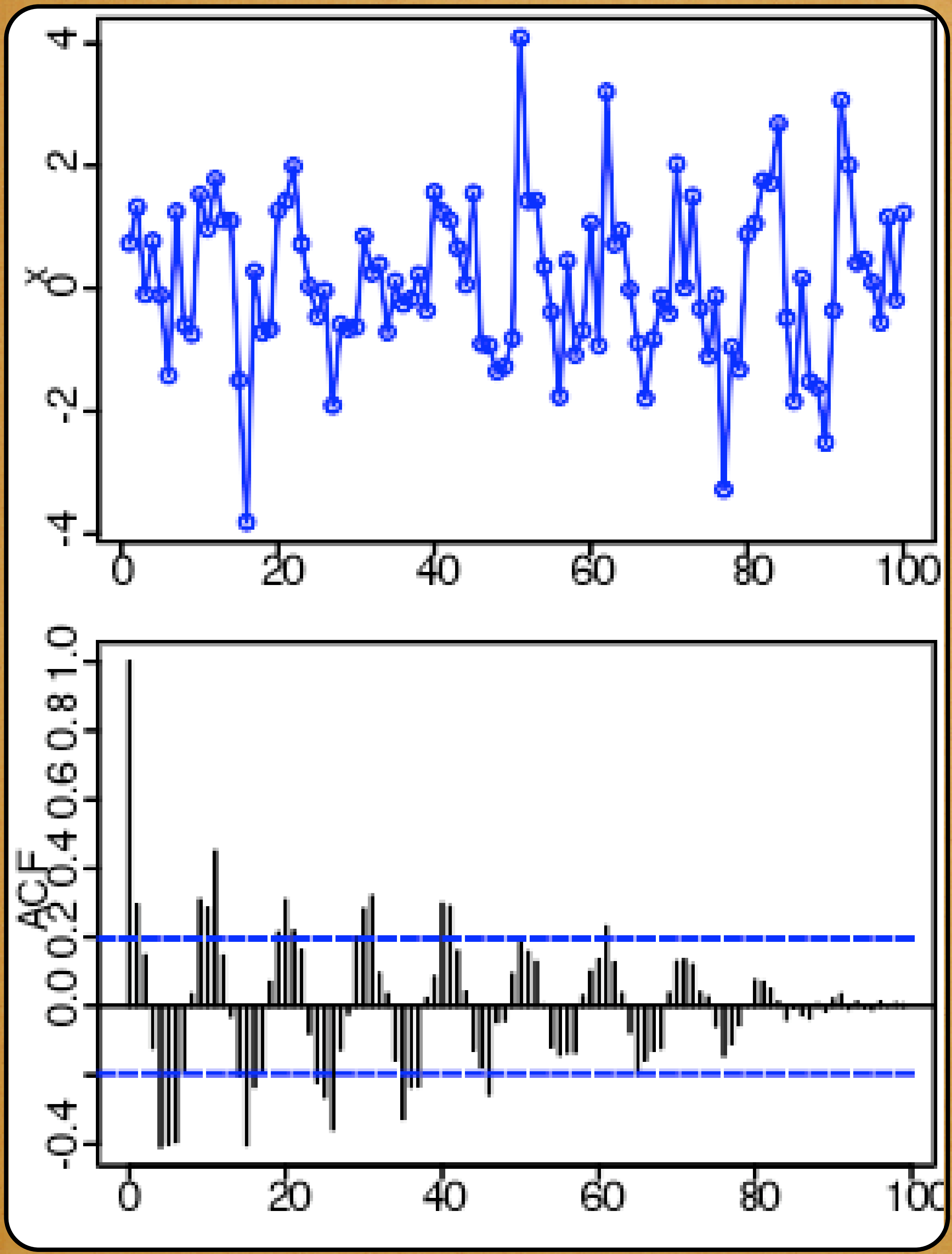
Correlation, auto-correlation

Correlation

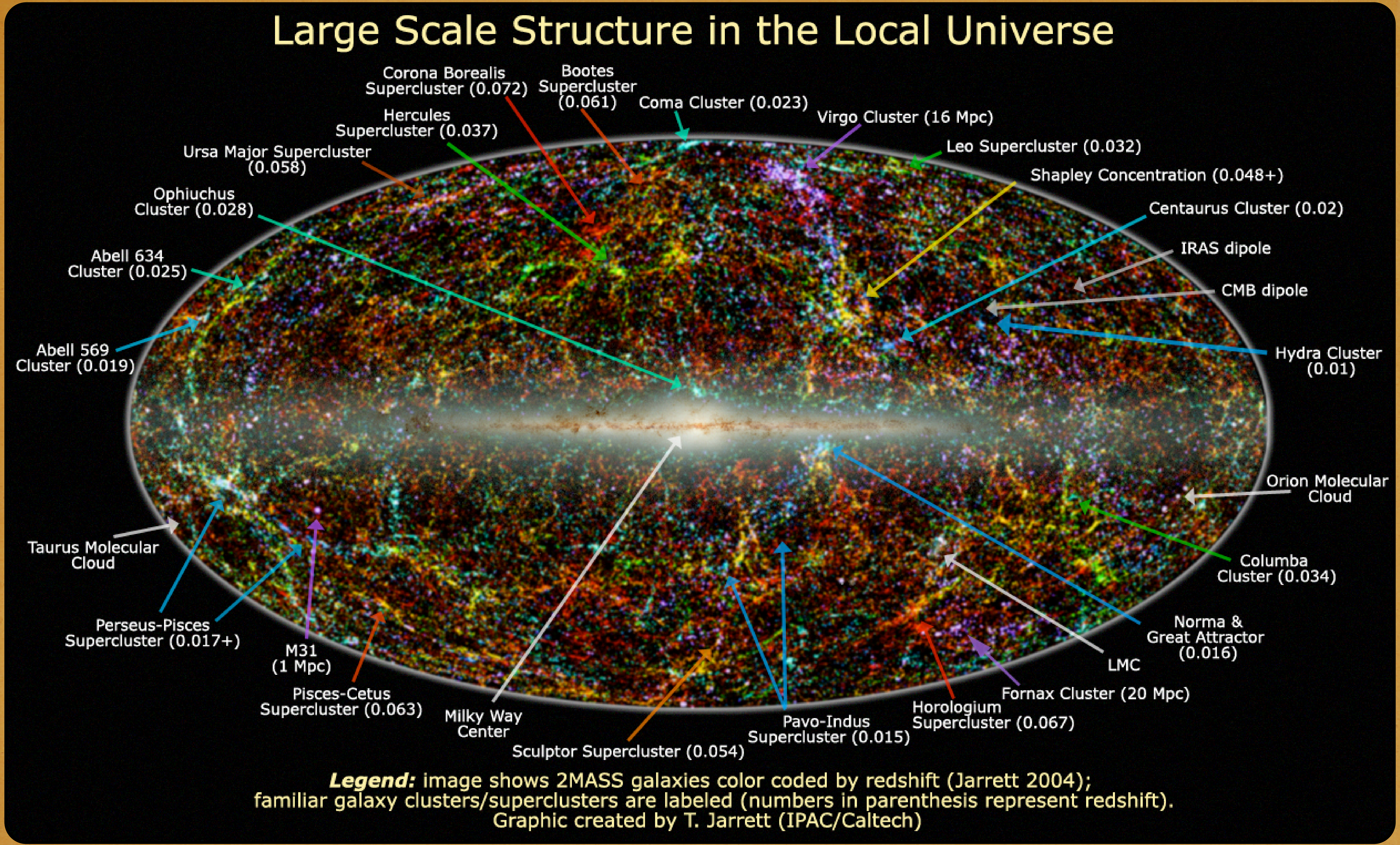


Auto-correlation

$$R(x) = f(x) \otimes f(x) = \int_{-\infty}^{\infty} f(u) f(u+x) du$$

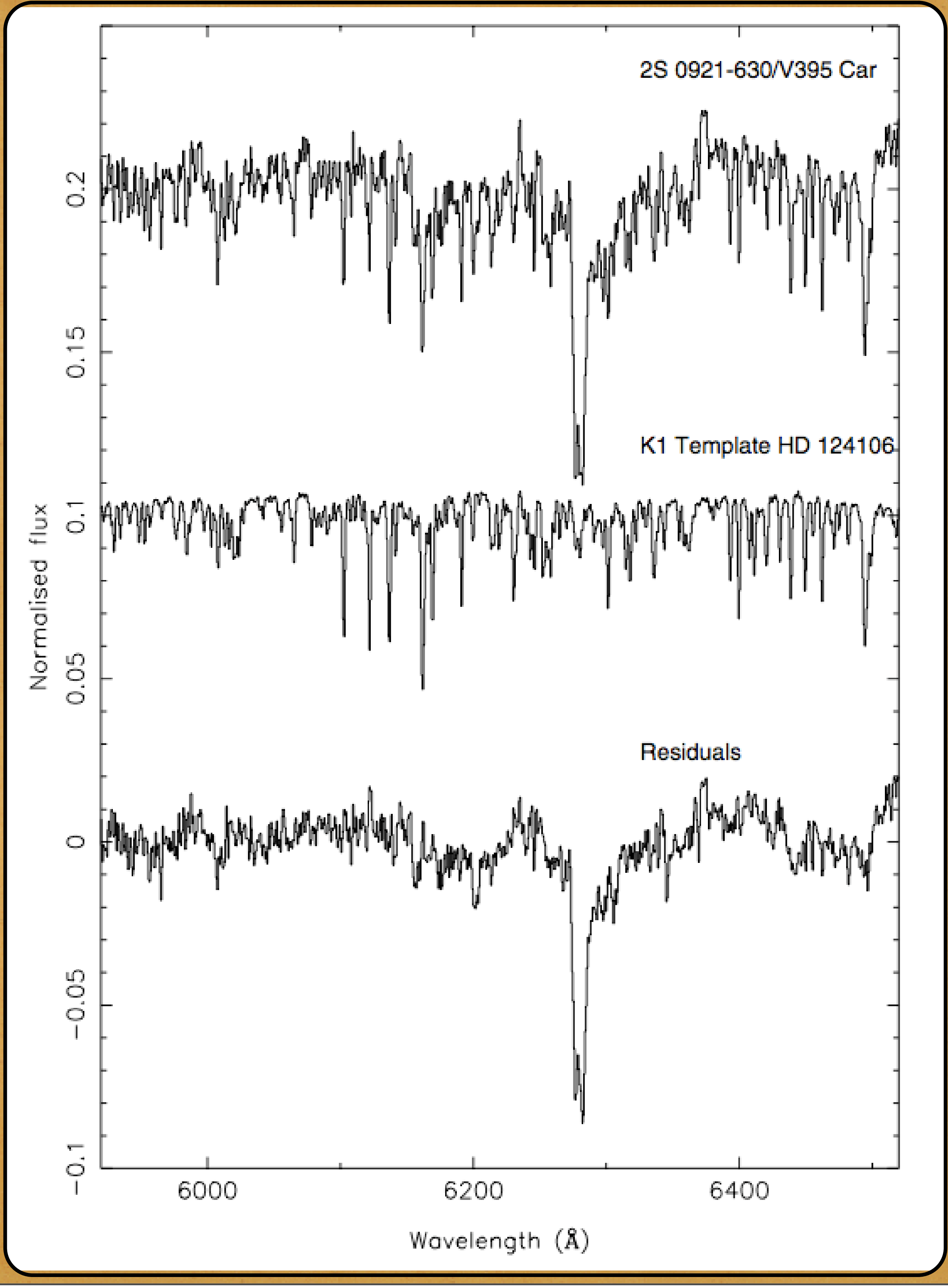


$$R(x) = E\{\underbrace{f(x)}_{f(x_1)}\underbrace{f(x+t)}_{f(x_2)}\}$$

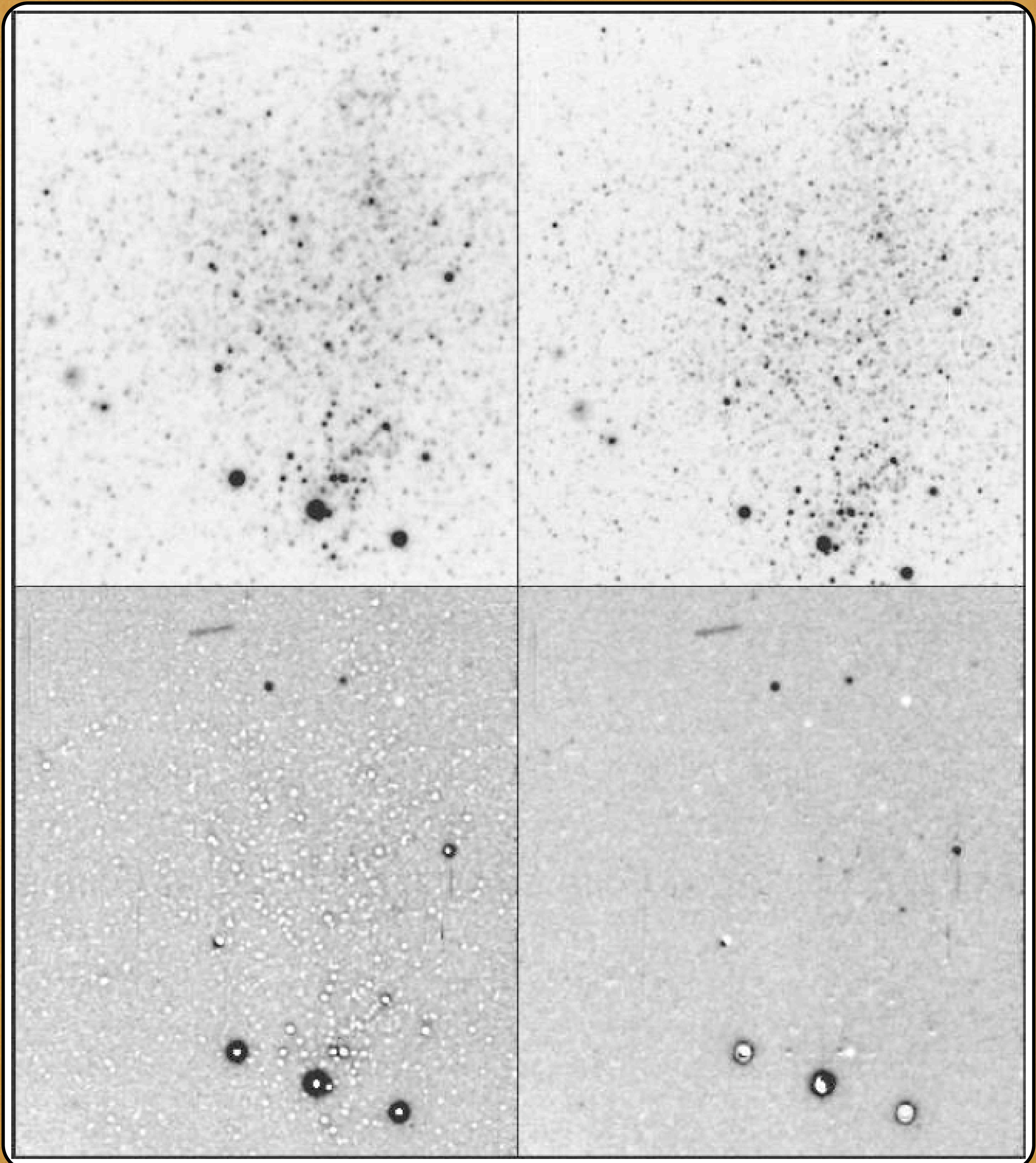


Given a random galaxy in a location the correlation function describes the probability that another galaxy will be found within a given distance (Peebles 1980)

Cross-correlating spectra



Cross-correlating images



$$R(x) = E\{\underbrace{f(x)}_{f(x_1)}\underbrace{f(x+t)}_{f(x_2)}\}$$

$$\text{if} \quad x_1 = x_2$$

$$R(x) = R(x,x) = \mathbf{E}\{f^2(x)\} = \mathbf{E}\{|f(x)|^2\}$$

AVERAGE GENERALLY NOT ZERO

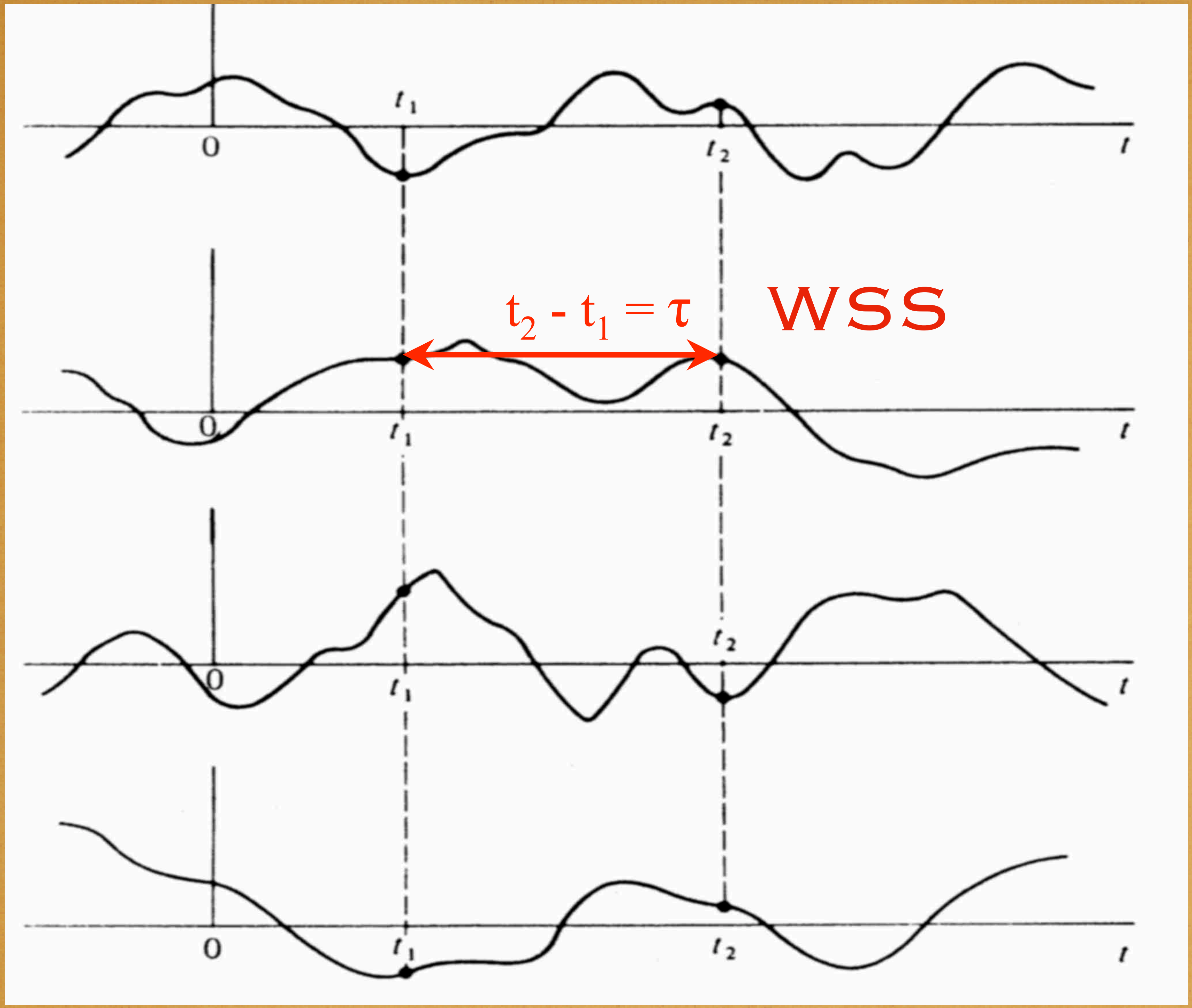
→ AUTOCOVARIANCE

$$C(x_1, x_2) = \mathbf{E}\{(f(x_1) - \eta(x_1))(f(x_2) - \eta(x_2))^*\}$$

$$C(x) = R(x) - |\eta(t)|^2 = \sigma^2(x)$$

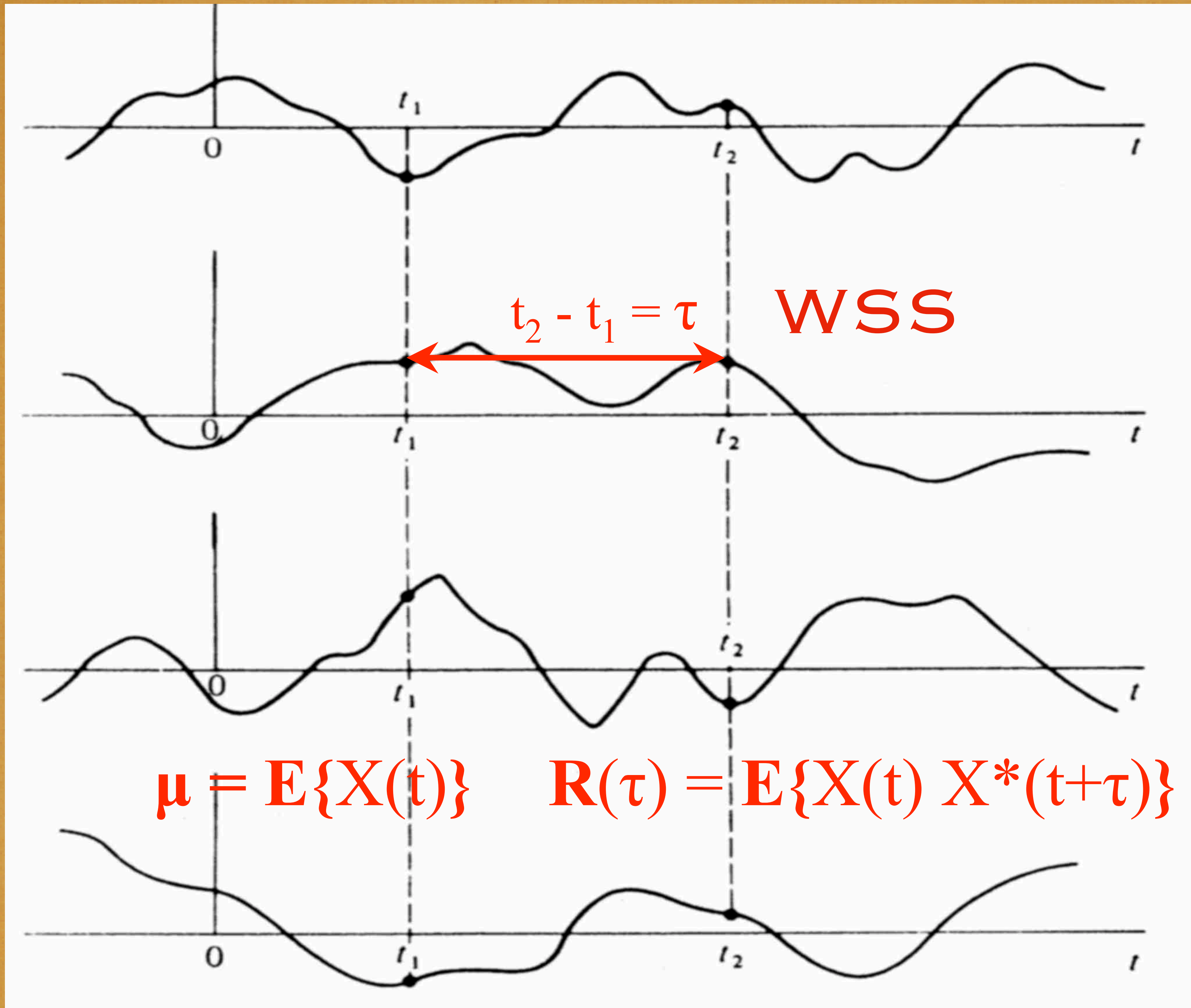
C(X) → AVERAGE POWER IN THE FLUCTUATIONS AROUND THE MEAN

Wide-sense stationary S.P.



WSS: mean time independent
& autocorrelation depends on
time difference

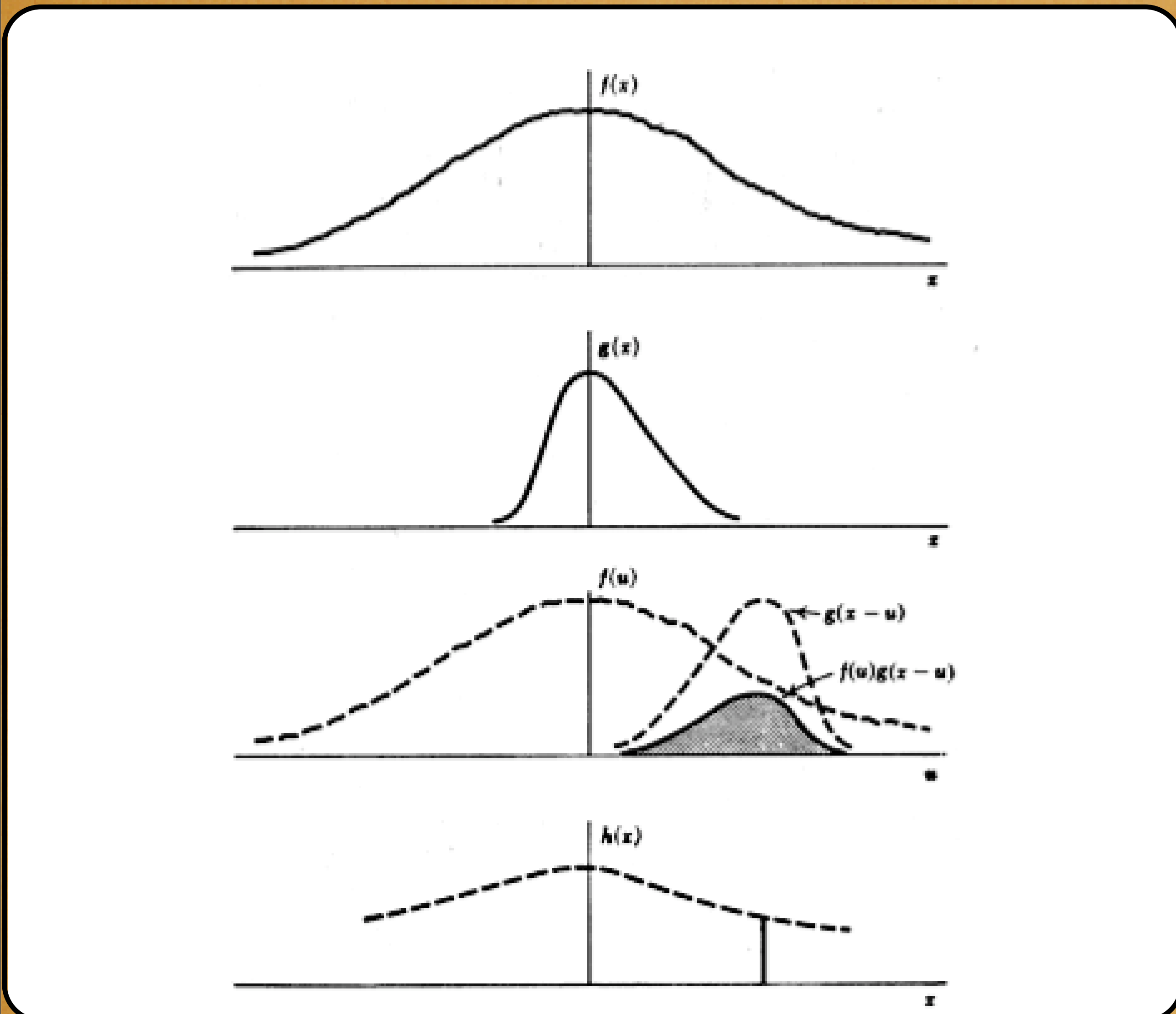
Wide-sense stationary S.P.



WSS: mean time independent & autocorrelation depends on time difference

CONVOLUTION:

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(x) g(x_1 - x) dx$$



Convolution Theorem

$$F(f(x) * g(x) = F(f(x))F(g(x))$$

$$f(x) * g(x) \Leftrightarrow F(s)G(s)$$

SIMILARLY FOR CROSS CORRELATIONS

$$F(f \otimes g) = F(f)F(g)$$

Fourier Transformations

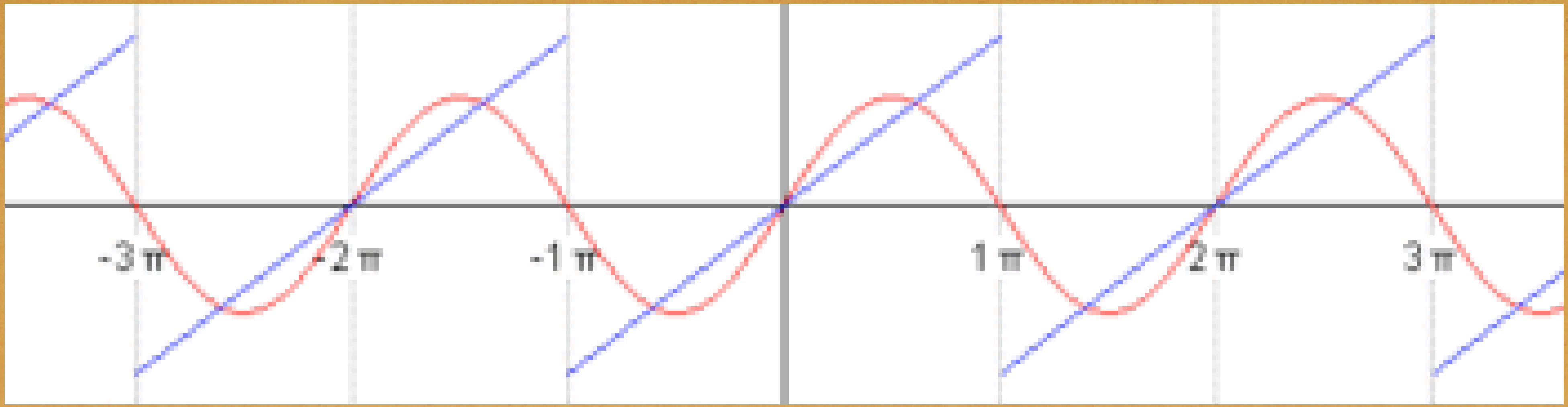


Figure from Wikipedia

$$F(t) \Leftrightarrow f(x)$$

$$F(t) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x t} dx$$

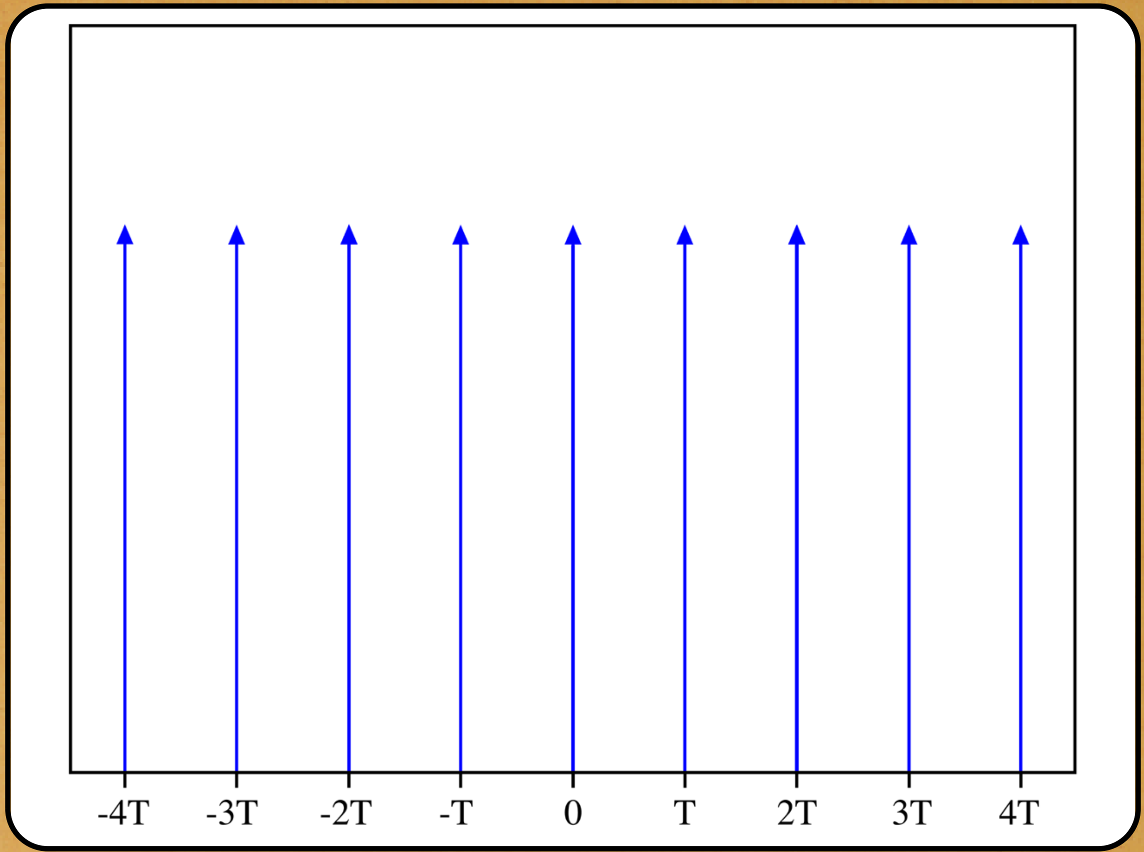
$$f(x) = \int_{-\infty}^{\infty} F(t) e^{2\pi i x t} dt$$

$$Euler's\ relation: \quad e^{ix} = cosx + isinx$$

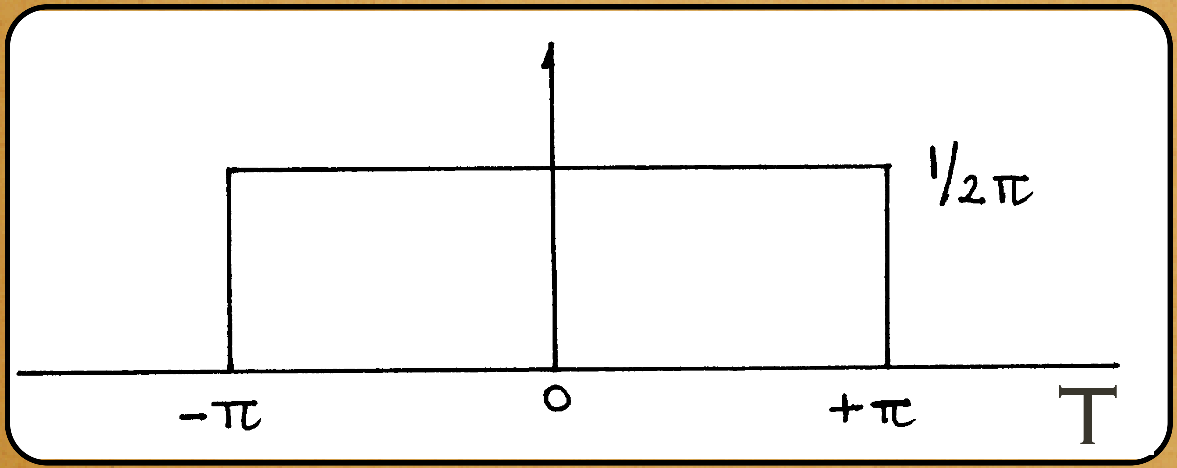
Some special functions:

Shah's function/Dirac comb

$$III(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$



Box/window function

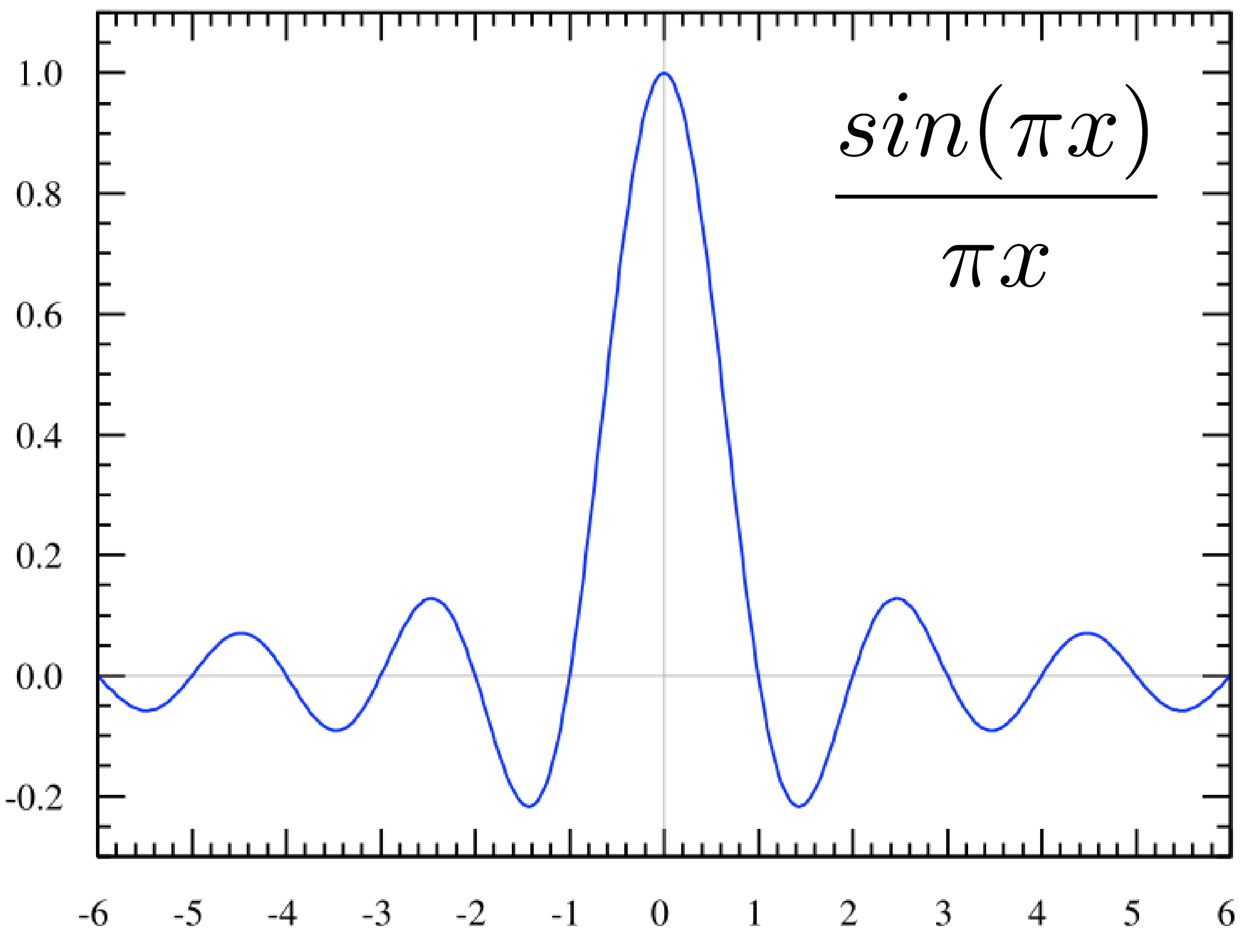


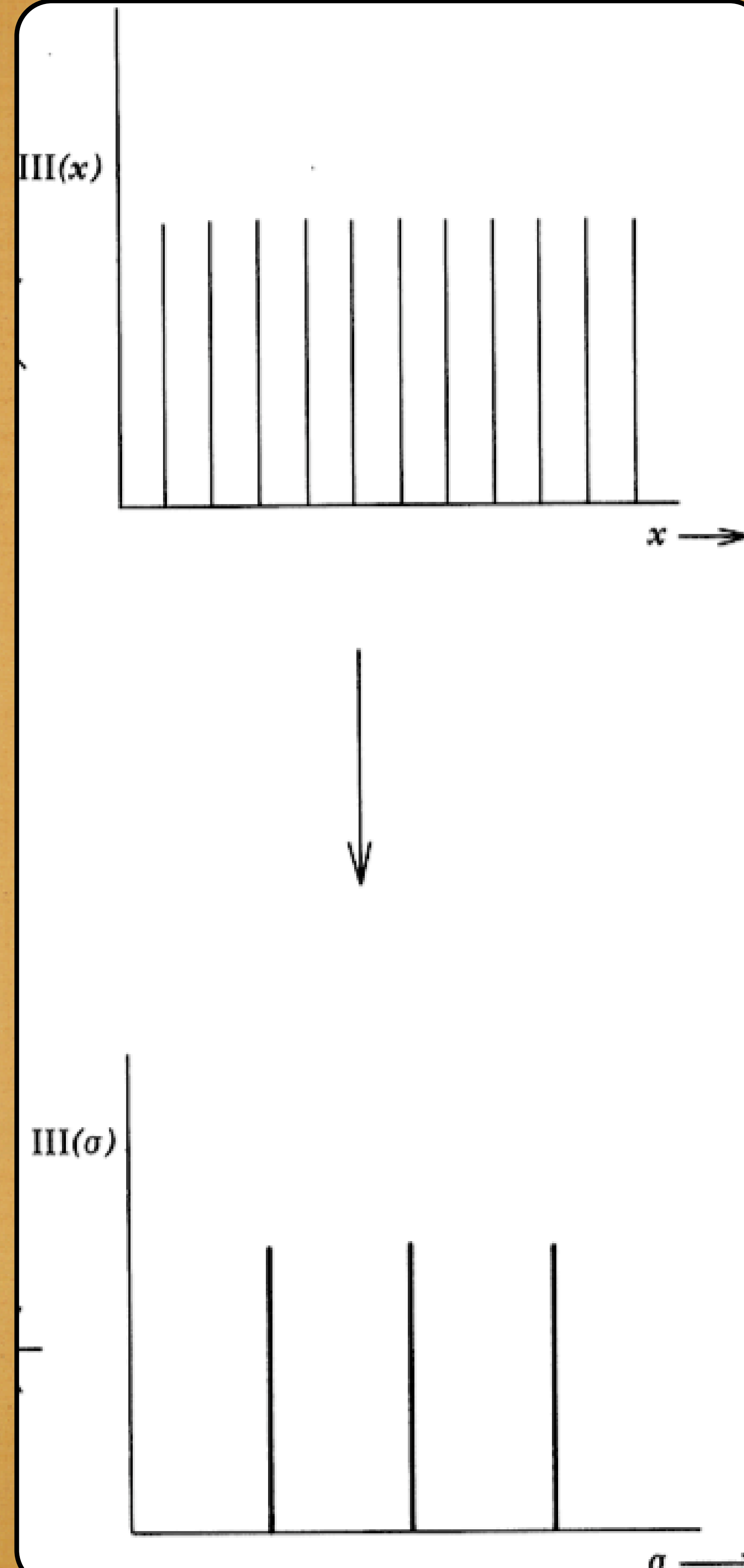
$$B(t) = 0 \ for \ -\frac{W}{2} > t > \frac{W}{2}$$

$$B(t) = 1 \ for \ -\frac{W}{2} < t < \frac{W}{2}$$

Fourier transforms of these special FIE's

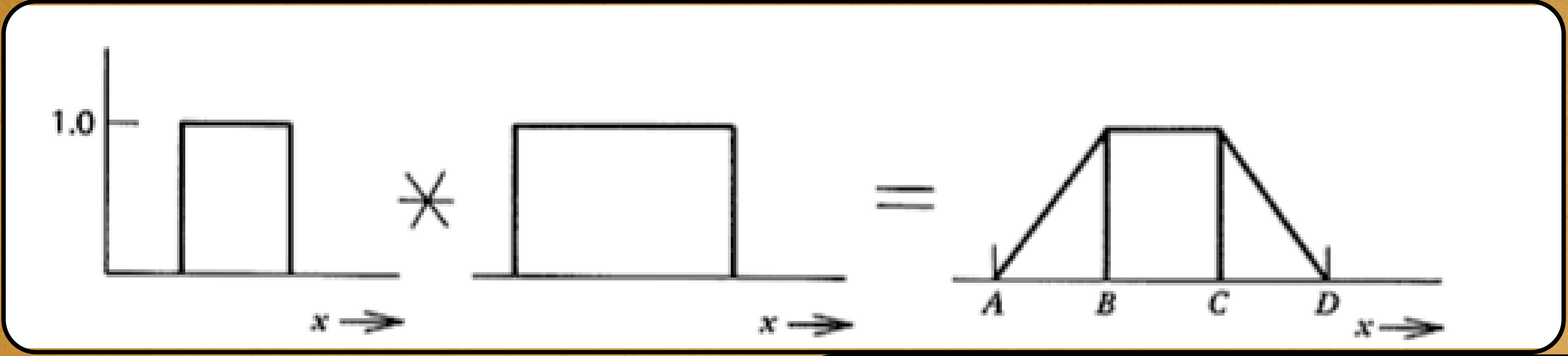
Sinc function



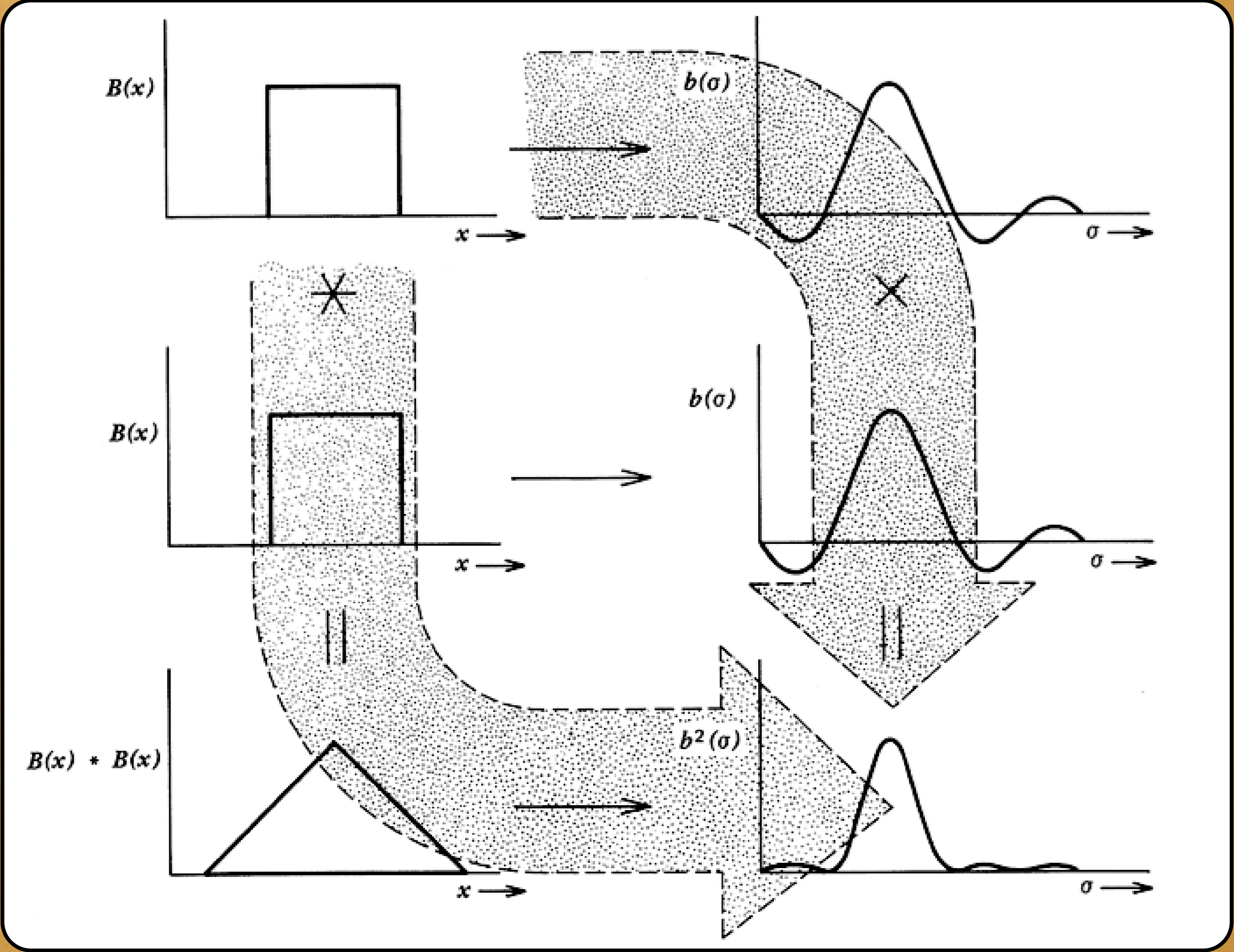


$$\sum_{n=-\infty}^{\infty} \delta(t - nT) \Leftrightarrow \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta(f - \frac{k}{T})$$

Convolution in practice

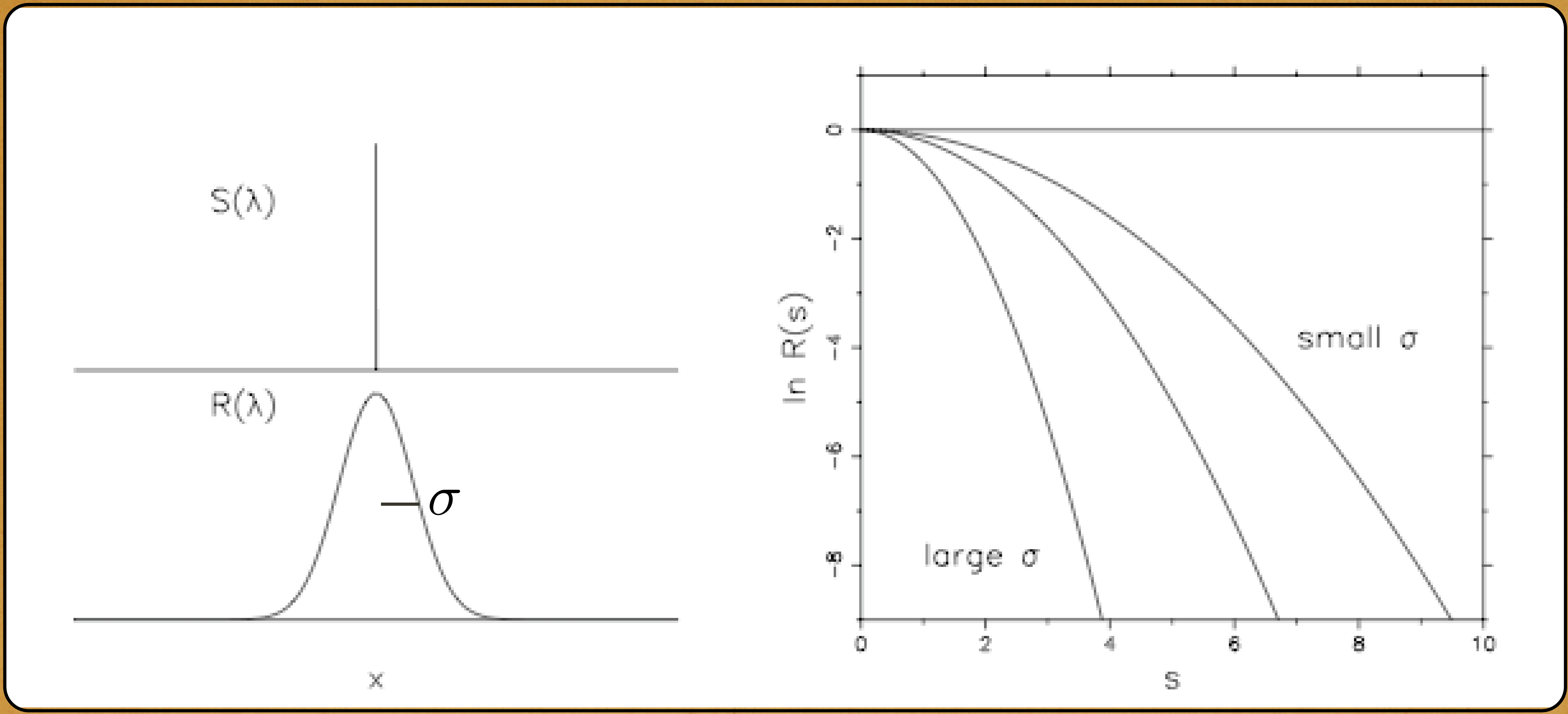


Always broadens the input function



Two figures from Gray page 28 & 29

Gaussian response function



$$R(\lambda)\frac{1}{\sqrt{(2\pi)}\sigma}exp-(\frac{\lambda^2}{2\sigma^2})$$

In ideal case find input spectrum back

$$M(\lambda) = \int_{-\infty}^{\infty} S(\lambda')R(\lambda - \lambda')d\lambda'$$

$$S(s) \Leftrightarrow S(\lambda)$$

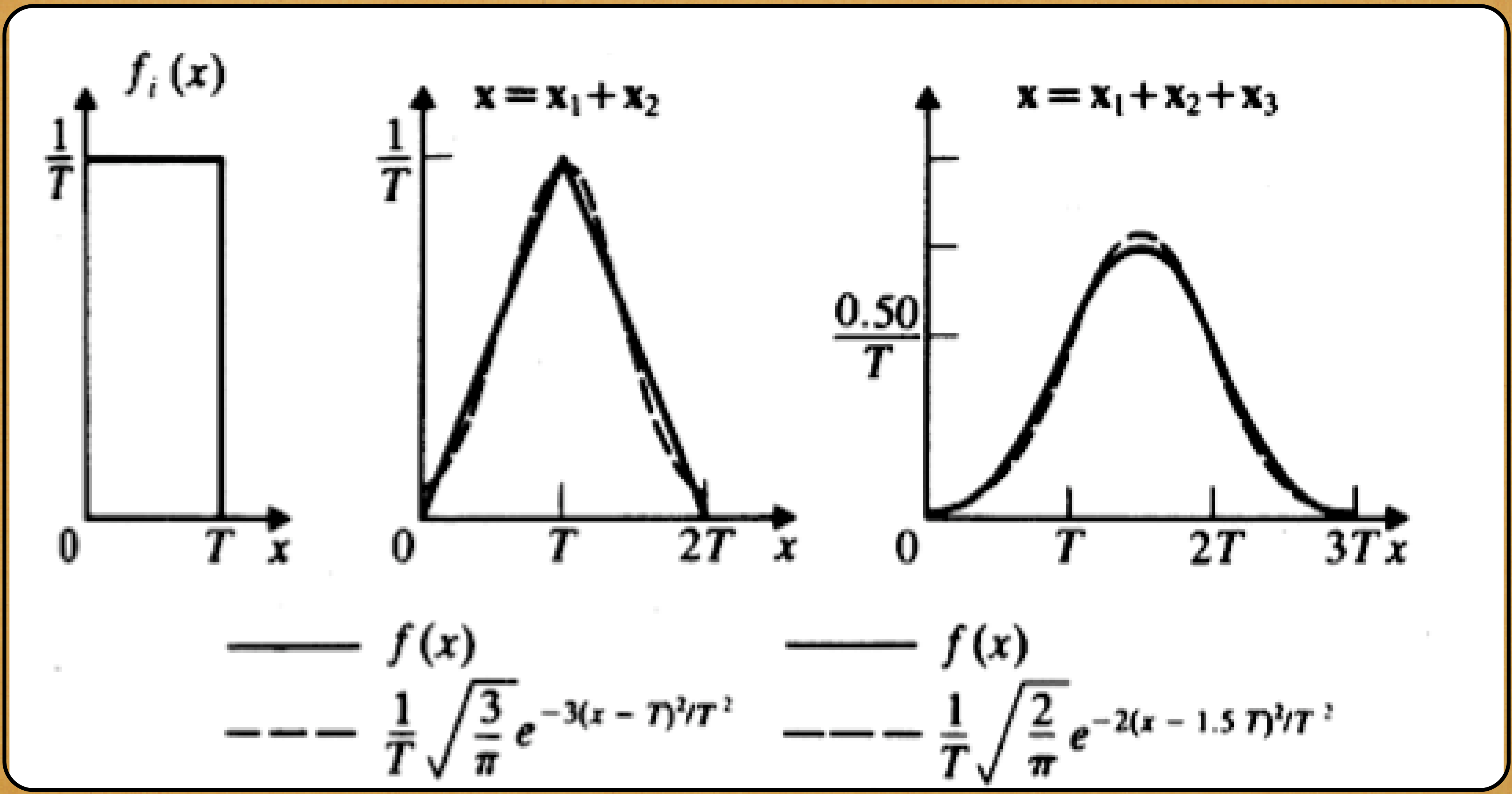
$$S(s) = \frac{M(s)}{R(s)}$$

$$S(\lambda) = F^{-1}[\frac{M(s)}{R(s)}]$$

Central limit theorem

many convolutions → smoothing

$$\lim_{n\to\infty} p_X(x) = \frac{1}{\sqrt{(2\pi)}\sigma} exp - \frac{(x-\eta)^2}{2\sigma^2}$$



many physical processes/measurements yield a Gaussian probability density function