Lecture 5: Linear Polarizers

Outline

- Jones and Mueller Matrices for Linear Polarizers
- Wire Grid Polarizers
- Polaroid-type Polarizers
- Crystal-based Polarizers
- Thin-Film Polarizers
- Polarizer Selection Guide

Jones and Mueller Matrices for Linear Polarizers

- A linear polarizer described by its transmittance of electrical field in two orthogonal directions.
- The Jones matrix for a linear polarizer:
  \[ J_\text{p} = \begin{pmatrix} p_x & 0 \\ 0 & p_y \end{pmatrix} \]
- Real values \(0 \leq p_x, p_y \leq 1\) are transmission factors for \(x\) and \(y\)-components of electric field
- \(E'_x = p_x E_x\), \(E'_y = p_y E_y\)
- \(p_x = 1\), \(p_y = 0\): linear polarizer in +\(Q\) direction
- \(p_x = 0\), \(p_y = 1\): linear polarizer in −\(Q\) direction
- \(p_x = p_y\): neutral density filter

Mueller Matrix for Linear Polarizer

- The Mueller matrix for a linear polarizer:
  \[ M_\text{p} = \frac{1}{2} \begin{pmatrix} p_x^2 + p_y^2 & p_x^2 - p_y^2 & 0 & 0 \\ p_x^2 - p_y^2 & p_y^2 + p_x^2 & 0 & 0 \\ 0 & 0 & 2p_xp_y & 0 \\ 0 & 0 & 0 & 2p_xp_y \end{pmatrix} \]
- An unpolarized incoming beam will always be linearly polarized.
- Emerging Stokes vector only completely polarized if \(p_x^2 + p_y^2 = |p_x^2 - p_y^2|\)
- Partial linear polarizer produces partially polarized beam from unpolarized light.
- Real polarizers always only partial polarizers.
- Polarized incoming beam \(\Rightarrow\) emerging beam is, in general, elliptically polarized because of non-zero diagonal terms \(2p_xp_y\).
- Totally polarized beam remains totally polarized even when passing partial linear polarizer \(\Rightarrow\) ideal partial polarizer does not depolarize.
Mueller Matrix for Ideal Linear Polarizer at Angle $\theta$

\[
M_{\text{pol}}(\theta) = \frac{1}{2} \begin{pmatrix}
1 & \cos 2\theta & \sin 2\theta & 0 \\
\cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\
\sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

Characterizing Linear Polarizers

- 2 parameters describe linear polarizer performance:
  - $k_1$: (intensity) transmittance of polarizer for fully linearly polarized beam at angle that maximize transmitted intensity
  - $k_2$: minimum transmittance for incoming linearly polarized beam

\[
k_1 = p_x^2, \quad k_2 = p_y^2 \quad \text{if} \quad p_x > p_y
\]

- ratio of $k_1$ to $k_2$ is called extinction ratio contrast

- $k_1$ and $k_2$ are functions of wavelength

- can be determined from transmittances for unpolarized light of parallel and crossed identical polarizers

\[
T_{\text{parallel}} = \frac{1}{2} \left( k_1^2 + k_2^2 \right)
\]

\[
T_{\text{crossed}} = \frac{k_1}{k_2}
\]

- also used: degree of polarizability or polarizance defined by

\[
P = \frac{k_1 - k_2}{k_1 + k_2}
\]

Poincaré Sphere

- polarizer is a point on the Poincaré sphere
- transmitted intensity: $\cos^2(l/2)$, $l$ is arch length of great circle between incoming polarization and polarizer on Poincaré sphere

Wire Grid Polarizers

- parallel conducting wires, spacing $d \lesssim \lambda$ act as polarizer
- plane of polarization perpendicular to wires is transmitted because electric field component parallel to wires induces electrical currents in wires, which strongly attenuates transmitted electric field parallel to wires
- induced electrical current such that polarization parallel to wires is reflected
- can make polarizing beam-splitter with wire grid polarizer, reflects, transmits orthogonal linear polarization states

Wire Grid Polarizers for the Infrared

- rule of thumb:
  - $d < \lambda/2 \Rightarrow$ strong polarization
  - $d \gg \lambda \Rightarrow$ high transmission of both polarization states (weak polarization)

- mostly used in infrared because wire spacing becomes very small at visible wavelengths
- made by depositing thin-film metallic grid on substrate
- free-standing wire grid for longer wavelengths
Wire-Grid Polarizers for the Visible
- excellent performance at high light/energy levels
- 90% of energy reflected instead of absorbed
- good in high heat environment
- trade-off between transmission and contrast

MOXTEK Inc. ProFlux

Polarcor (Corning)
- glass polarizer with high performance for 600 to 2300 nm
- borosilicate glass containing aligned silver nano-particles in surface layers
- elongated, conducting silver particles act as small wires
- polarization occurs in 25 to 50 \( \mu \text{m} \) top layer
- maximum diameter currently limited to 20 mm
- contrast ratio > 10000

Corning Polarcor

Dichroic Materials
- dichroic materials preferentially absorb one polarization state
- absorption depends on wavelength \( \Rightarrow \) different colors depending on angles of illumination and viewing
- arises from anisotropy of complex index of refraction
- natural dichroic crystals: tourmaline, herapathite
- W.B. Herapath discovered in 1852 salt of quinine with polarizing properties; made first artificial crystals large enough to study under microscope
- difficult to produce uniform, large dichroic crystals

O-type and E-type Dichroic Polarizers
- O-type polarizers transmit ordinary ray \( (k_o = 0) \), attenuate extraordinary ray \( (k_e > 0) \)
- O-type polarizers transmit independent of angle of incidence because index of refraction of the ordinary beam is independent of the angle of incidence
- E-type polarizers transmit extraordinary ray and absorb ordinary ray
- E-type polarizers attenuate light in any direction except for waves propagating perpendicular to \( c \)-axis
- O-type dichroic polarizers much preferred over E-type polarizers
Polaroid-type Polarizers

- H-type sheet polarizers: stretched polyvinyl alcohol (PVA) sheet, laminated to sheet of cellulose acetate butyrate, treated with iodine
- different H-type polarizers have different amounts of iodine in PVA
- PVA-iodine complex analogous to short, conducting wire
- Polaroid names (e.g. HN-38) identify overall type (H) color (N=neutral), approximate transmittance for unpolarized light
- K-type similar to H-type, but environmentally more stable
- HR-type based on a PVA-polyvinylene-iodine complex, works well from 0.7 to 2.3 µm

Field-of-View of Dichroic Sheet Polarizers

- conducting, needle-like particles aligned on common axis perpendicular to surface normal
- model them with uniaxially anisotropic medium with complex indices of refraction
- transmitted polarization state sees real index of refraction
- absorbed polarization state sees complex index of refraction with imaginary part large enough to reduce intensity by orders of magnitude
- dichroic polarizers have limited field of view, largely a geometrical effect inherent to uniaxial medium

Crystal-Based Polarizers

- uniaxial crystals are basis of highest quality polarizers
- precise arrangement of atom/molecules and anisotropy separate incoming beam into two beams with precisely orthogonal polarization
- polarizing beam-splitters (both beams usable) and polarizing prisms (only one useful state)
- prisms can be cemented or air spaced
- air-spaced: good for short wavelengths, high power densities
- cemented: much better optical quality
- calcite is most often used in crystal-based polarizers because of very large birefringence, low absorption in visible
- many other suitable materials

Beam Displacer, Savart and Modified Savart Plates

Beam Displacer

- most simple crystal polarizer: polarizing beam displacer
- single uniaxial crystal block with optic axis at $\sim 45^\circ$
- ordinary ray passes without deflection
- extraordinary ray deflected by dispersion angle $\alpha$
- beam separation $d$ as function of block length $D$:
  \[ d = D \tan \alpha = D \left( \frac{n_e^2 - n_o^2}{n_e^2 + n_o^2 \tan^2 \theta} \right) \]

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Lecture 5: Linear Polarizers
Beam Displacer Problems

- ordinary and extraordinary beams have different path lengths
- extraordinary ray suffers from crystal astigmatism

Focus Difference and Astigmatism

Converging beam focus difference $\epsilon$ between ordinary and extraordinary rays:

$$\epsilon = \frac{\sin 2\theta (n_e^2 - n_o^2)}{2 \left( n_o^2 \sin^2 \theta + n_e^2 \cos^2 \theta \right)}$$

$\theta$ is angle of optic axis with normal to interface
- astigmatism leads to two focus positions *longitudinal astigmatism*

$$\epsilon_l = \frac{D \tan \theta \tan \alpha n_o}{n_o \sqrt{n_o^2 \sin^2 \theta + n_e^2 \cos^2 \theta}}$$

for imaging system, ‘smearing’ of image given by *transverse astigmatism* $\epsilon_t = \frac{\epsilon_l}{F}$ where $F$ is the F-number of the beam
- rule of thumb for calcite: astigmatic focus difference $\epsilon_l \sim 5\%$ of thickness

Beam Separation and Astigmatism

- optic axis orientation of 45° close to optimum
- beam separation increases almost linearly for small optic axis angles
- astigmatism increases much more slowly
- possible to trade off beam separation versus astigmatism

Savart Plate

- avoid difference of focal point position problem by splitting calcite into two pieces crossed at 90°
- exchanges ordinary and extraordinary beams half-way ⇒ both beams have same optical path length
- splitting reduced by $\sqrt{2}$ for same total length of calcite
- both beams now show crystal astigmatism, oriented in opposite directions; amount of astigmatism half of single-piece beam displacer
**Modified Savart Plate**

- both calcite blocks in same orientation
- half-wave plate at 45° between them to exchange ordinary and extraordinary beams
- same direction of astigmatism in both beams

**Cylinder Lens Compensating Crystal Astigmatism**

- additional cylindrical lens can compensate astigmatism
- only works over a limited wavelength range because of wavelength dependence of half-wave retarder
- deviation from half-wave retardance or 45° orientation leads to ghost beam between oppositely polarized beams
- other aberrations such as spherical aberration occur also
- developed for SOLIS VSM, excellent performance

**Glan-Thompson Prism**

- two calcite prisms with cement (index \(n_c\)) in between
- surrounding dielectric medium with refractive index \(n\)
- ordinary beam undergoes TIR, absorbed by black paint on side
- critical cut angles for TIR for ordinary and extraordinary beams

\[
\sin \Omega_\circ = \frac{n_c}{n_o}, \quad \sin \Omega_e = \frac{n_c}{n_e}
\]

- cut angle \(\Omega\) such that acceptable range of angles of incidence is symmetric around normal incidence

**Glan-Thompson Properties**

- 2 versions differ in prism angles, index of cement
  - long version: acceptance angle of \(\sim 26°\)
  - short form: acceptance angle of \(\sim 15°\)
- typical extinction ratios \(10^5\) to \(10^7\)
- somewhat reduced extinction ratios when used from “wrong” side
- Glan-Thompson gives most uniform rotation of plane of polarization for conical beam of light
- plane of vibration of transmitted beam may vary if optic axis is not in the end face
- if prism is illuminated with convergent light, different parts of beam will have slightly different orientations of linear polarization \(\Rightarrow\) strong reduction in extinction ratio for convergent beam
Other Glan Prism Polarizers

- Glan or Glan-Foucault: Glan-Thompson with air gap
  - absorption of the deflected beam above 2 \( \mu \text{m} \)
  - field of view much reduced, between 13° and 7.5°
  - acceptance angle rotationally symmetric only at single wavelength
  - TIR of ordinary ray responsible for polarization
  - transmission is not as high as Glan-Thompson because of reflection losses at calcite-air interfaces
- Glan-Taylor: orientation of optic axis 90° rotated with respect to Glan-Thompson
  - of importance only air-spaced version
  - main advantage over Glan-Foucault type prisms is improved transmission and much smaller amount of multiple reflections between the two prism halves

Rochon Prism

- oldest polarizing beam-splitter (1783)
- first prism: beam parallel to optic axis \( \Rightarrow \) ordinary and extraordinary beams see same index \( n_o \)
- second prism: optic axis perpendicular to beam
- ordinary beam sees again same index, passes without deviation
- extraordinary beam sees extraordinary index \( \Rightarrow \) refracted according to Snell’s law
- deviation of extraordinary ray at exit face due to Snell’s law
- Sénarmont prism similar to Rochon, but ordinary and extraordinary beams have opposite polarization states

Wollaston Prism

- Wollaston prisms most common beam-deviating polarizers
- 2 beams with orthogonal linear polarizations, parallel and perpendicular to refracting edge
- both rays refracted away from normal of exit face
- first half: ordinary and extraordinary rays see their respective indices of refraction
- indices are reversed in second half of prism
- made of calcite or quartz prisms cemented together
- usable spectral range typically 300 nm to 2200 nm
- angle of divergence determined by wedge angle (\( \sim 15 - 45° \))

Wollaston Beam Deviation

- at refracting interface, Snell’s law for the ordinary and extraordinary beams
  \( n_o \sin \Omega = n_e \sin \theta_o \) and \( n_o \sin \Omega = n_o \sin \theta_e \)
- at exit face, two beams are refracted differently; with Snell’s law
  \( n_e \sin (\Omega - \theta_o) = n \sin \theta'_o \) and \( n_o \sin (\Omega - \theta_e) = n \sin \theta'_e \)
- \( n \) is index of external dielectric medium
- equations are not completely symmetrical \( \Rightarrow \) two beams will have slightly different absolute angles upon exiting
- beam deviation depends on wavelength-dependent birefringence
- in converging beam, two beams do not come to focus at the same distance behind the exit face
- each beam is astigmatic
Thin-Film Polarizers

- Thin-film polarizers mostly used in cube beam-splitters
- 2 orthogonally polarized beams emerge at right angles
- Thin-film stack between 2 cemented glass prisms
- Total internal reflection at Brewster angle within thin film
- Limited extinction ratio and wavelength range
- Extinction ratio of transmitted beam much better than reflected beam
- Produced cheaply even for large apertures (5–10 cm)
- Also possible on surface of oblique glass plate

### Polarizer Selection Guide

<table>
<thead>
<tr>
<th>Type</th>
<th>Extinction ratio</th>
<th>Transmission (polarized)</th>
<th>Wavelength range (nm)</th>
<th>Bandpass angle (°)</th>
<th>Acceptance angle (°)</th>
<th>Size (mm)</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glan</td>
<td>&gt; 10^7</td>
<td>&gt; 84%</td>
<td>300-2700</td>
<td>Full</td>
<td>8</td>
<td>&lt; 40</td>
<td>$$$</td>
</tr>
<tr>
<td>Glan-Thompson</td>
<td>&gt; 10^6</td>
<td>&gt; 92%</td>
<td>300-2700</td>
<td>Full</td>
<td>15-25</td>
<td>&lt; 30</td>
<td>$$$</td>
</tr>
<tr>
<td>Wollaston</td>
<td>&gt; 10^4</td>
<td>&gt; 92%</td>
<td>300-2200</td>
<td>Full</td>
<td>20</td>
<td>&lt; 50</td>
<td>$$</td>
</tr>
<tr>
<td>Polarcor</td>
<td>&gt; 10^4</td>
<td>&gt; 80%</td>
<td>630-2000</td>
<td>150</td>
<td>&gt; 20</td>
<td>&lt; 25</td>
<td>$$</td>
</tr>
<tr>
<td>Polaroid</td>
<td>150 – 10^4</td>
<td>&gt; 75%</td>
<td>310-2000</td>
<td>200</td>
<td>&gt; 20</td>
<td>&gt; 200</td>
<td>$</td>
</tr>
<tr>
<td>Polarizing cube</td>
<td>&gt; 500</td>
<td>&gt; 95%</td>
<td>400-1600</td>
<td>200-400</td>
<td>10</td>
<td>70</td>
<td>$$</td>
</tr>
<tr>
<td>Wire Grid</td>
<td>&gt; 100</td>
<td>&gt; 95%</td>
<td>4 · 10^7 – 10^8</td>
<td>∼ λ</td>
<td>&gt; 20</td>
<td>&gt; 70</td>
<td>$$$</td>
</tr>
</tbody>
</table>