Adaptive Optics

ATI 2017 - Lecture 08
Kenworthy and Keller

Astronomers want as much spatial resolution as possible

Diffraction limited by the telescope's primary mirror: $\approx \frac{\lambda}{D_{tel}}$

for the Hubble Space Telescope

$$\approx \frac{0.5\mu m}{2.4m} = 0.2\mu rad$$

$$\approx 43 \text{ milliarcsec}$$

Hubble Space Telescope **Credit: NASA**

Exposure times in DL scale as D^4

$$F \propto D^2$$ **from the increase of the telescope mirror area**

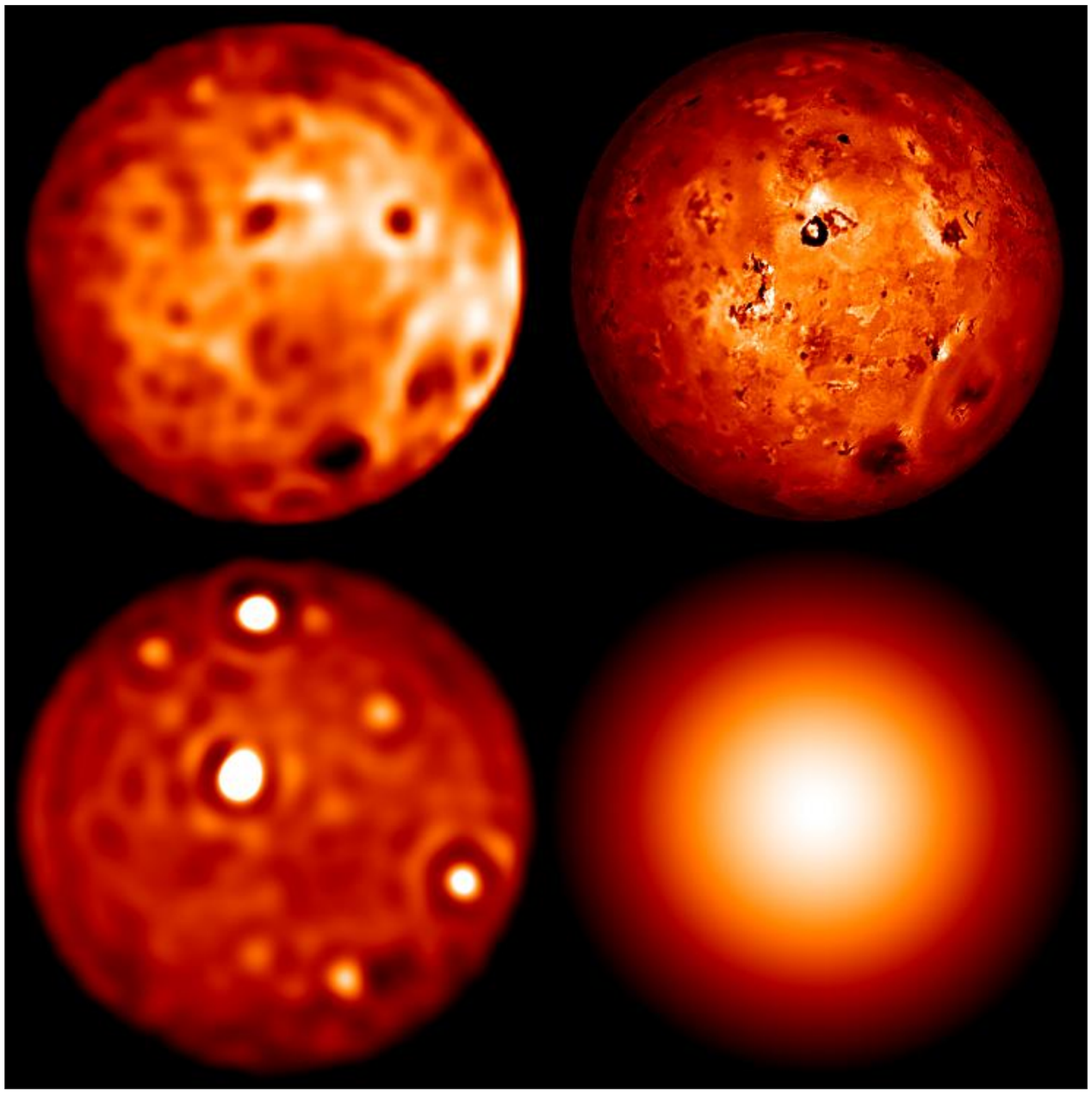
$$A_{PSF} = \pi d_{PSF} \propto \left(\frac{1}{D}\right)^2$$ **as sky background remains constant but Airy disk shrinks**

Double the telescope diameter, 4 times the flux and 4 times smaller Airy disk area

Io with and without Keck AO

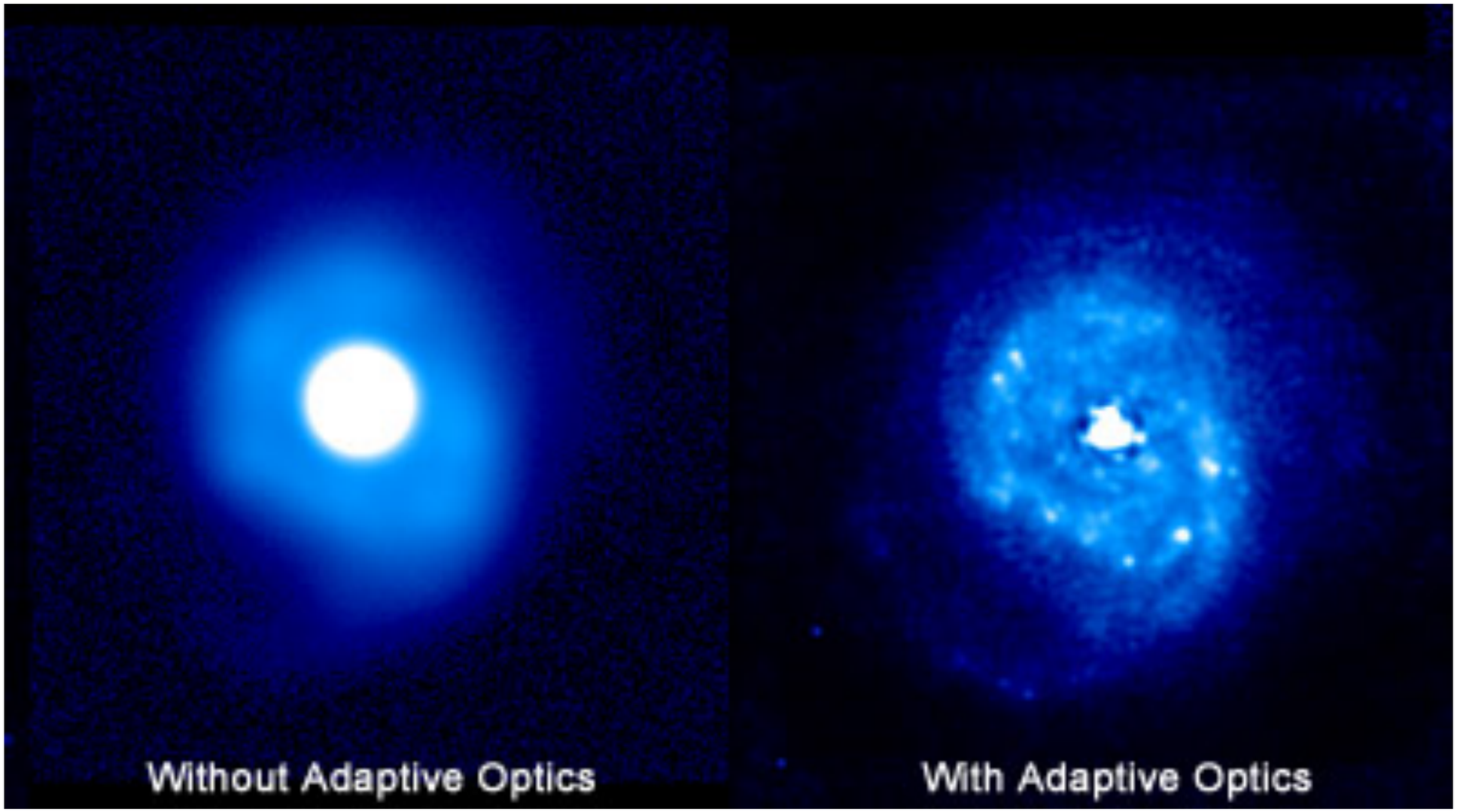
Io taken with Keck AO at 2.2 microns

Io from Galileo orbiter



Io from the ground without AO

Spiral arms and star forming structure seen in NGC 7469



cfao.ucolick.org/ao/why.php

Image motion with increasing D

Ground based telescopes do not reach the diffraction limit for diameters larger than 0.1m

Increasing Diameter

Atmospheric turbulence smears diffraction limited images into seeing limited images typically 1 arcsecond in diameter

Recap: The Atmosphere

Air heated next to the ground in the day starts to mix with cooler air, starting at large outer scales (30 to 100 m) and cascades down to an inertially damped inner scale (a few mm).

Temperature differences lead to refractive index differences in the air and to distortion of the incoming wavefronts

Several dominant boundary layers are responsible for most of the seeing introduced

Temperature differences in the atmosphere lead to changes in refractive index

Refractivity of air:

$$N \equiv (n-1) \times 10^6 = 77.6 \left(1 + \frac{7.52 \times 10^{-3}}{\lambda^2}\right) \times \left(\frac{P}{T}\right)$$

P = pressure in mbar
T = temperature in Kelvins
n = index of refraction
Wavelength in microns

NOTE: n is almost independent of wavelength!

Temperature fluctuations lead to index fluctuations..

Recap: The Atmosphere

Atmosphere is modelled with:

An outer and inner scale length, and a power spectrum of index fluctuations between them

Thin layers of frozen turbulence at 2 to 5 different altitudes

Each layer described with three parameters:

$$r_0, \tau_0 \text{ and } \theta_0$$

Fried length r_0

$$r_0(\lambda) = 0.185\lambda^{6/5}\left[\int_0^\infty C_n^2(z)dz\right]^{-3/5} \qquad r_0 \propto \lambda^{6/5}$$

Equal to diameter of 1rad² error variance in phase

$$r_0 \sim 10 - 20cm$$

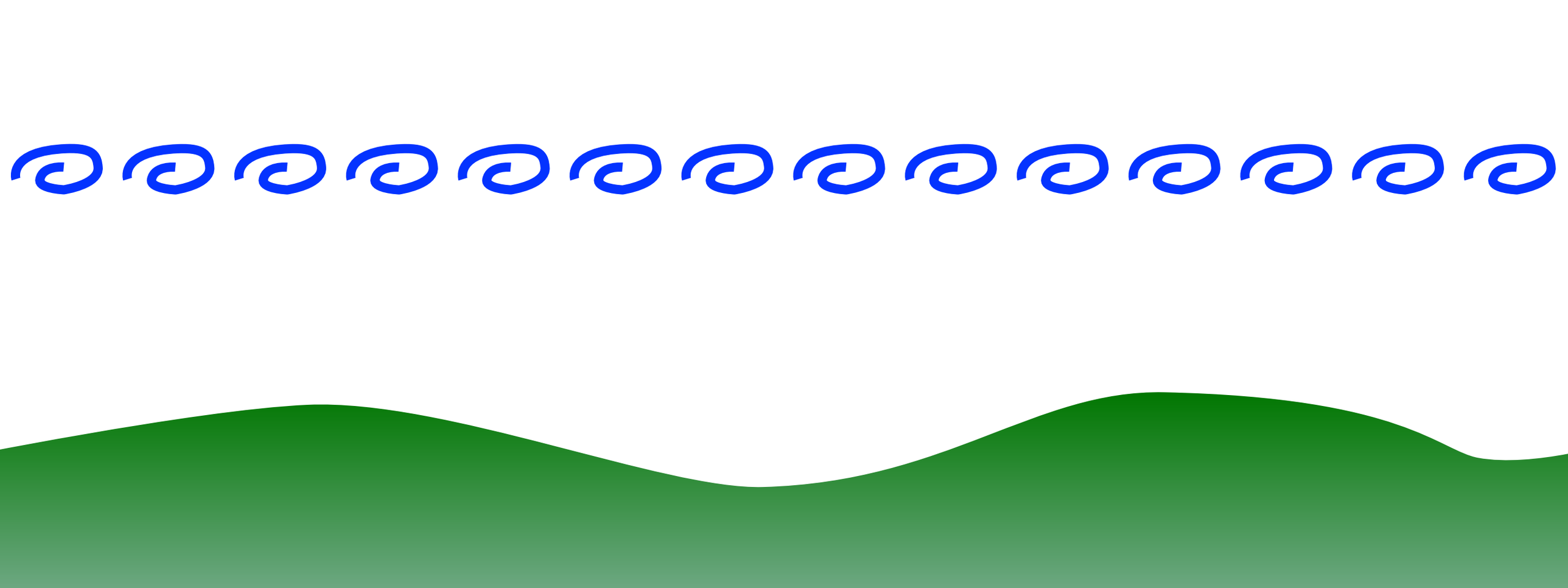
$v \sim 10\ \mathrm{m.s}^{-1}$

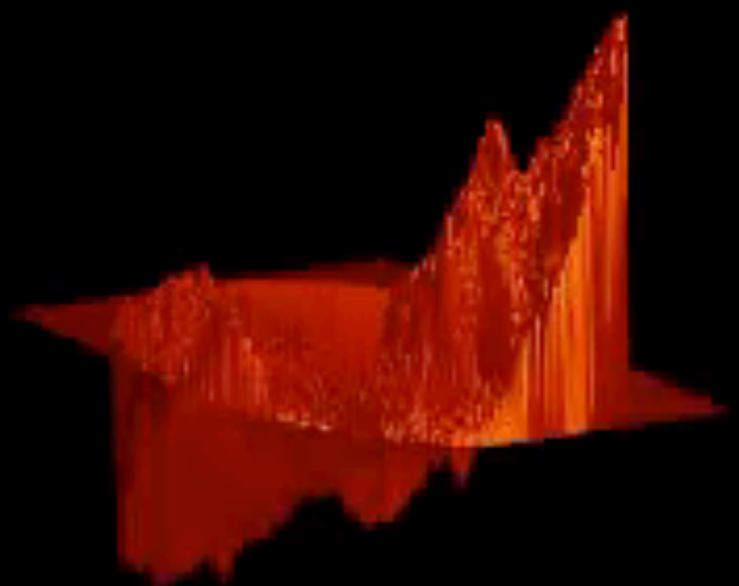
Atmospheric time constant $\tau_0 = 0.31\dfrac{r_0}{v}$

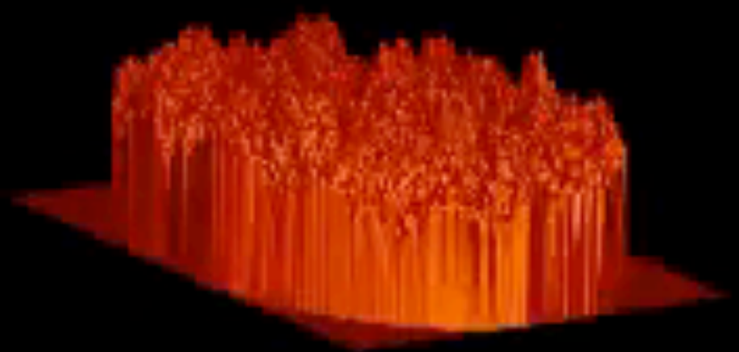
$$\tau_0 \sim 1 - 10\ \mathrm{ms}$$

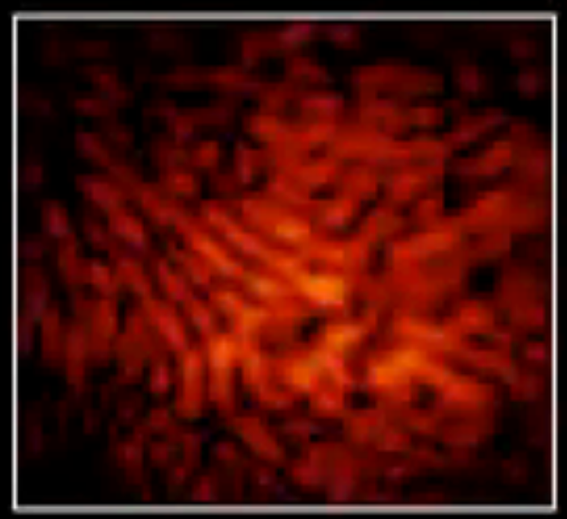
Seeing $\propto \frac{\lambda}{r_0} \sim \lambda^{-1/5}$

….typically quoted at 500nm

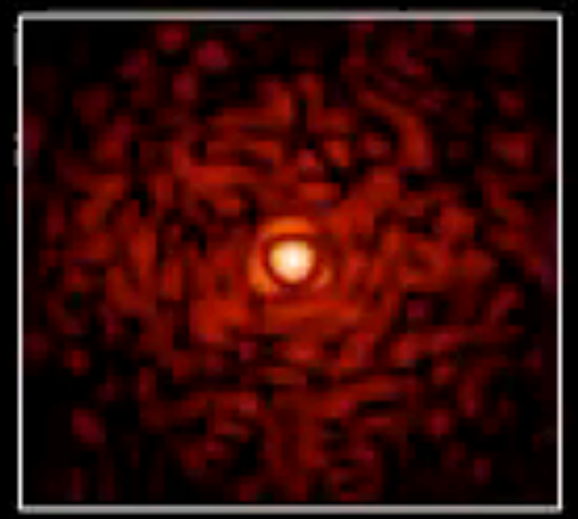








The Lyot Project http://lyot.org/



Zernike Polynomials

2D structure of circular aperture can be represented by an infinite series of orthogonal functions

One such basis set are the Zernike polynomials:

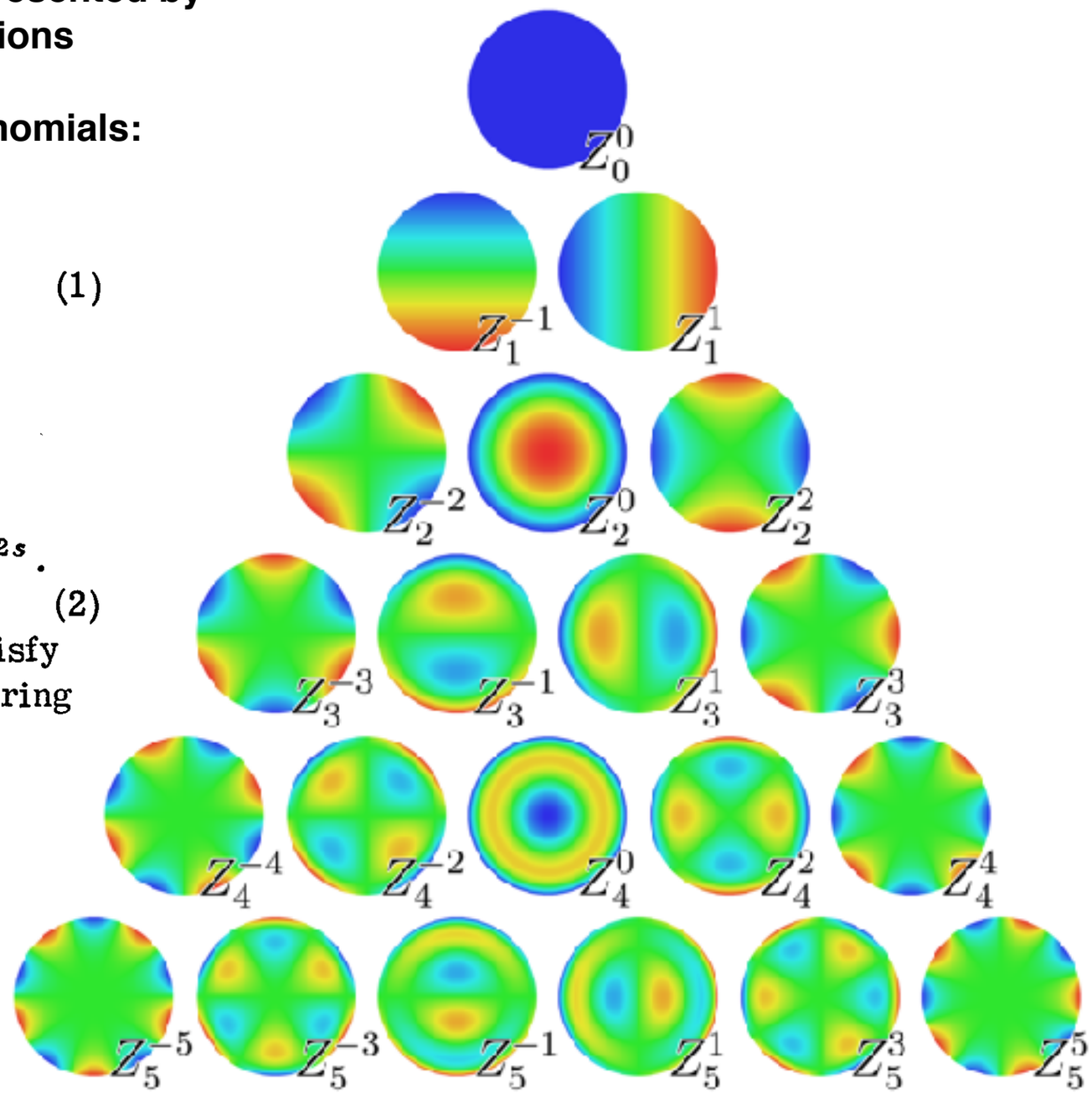
$$\left.\begin{aligned} Z_{\text{even } j} &= \sqrt{n+1}\, R_n^m(r)\sqrt{2}\cos m\theta \\ Z_{\text{odd } j} &= \sqrt{n+1}\, R_n^m(r)\sqrt{2}\sin m\theta \end{aligned}\right\} \quad m \neq 0 \qquad (1)$$

$$Z_j = \sqrt{n+1}\, R_n^o(r)\,, \qquad m = 0$$

where

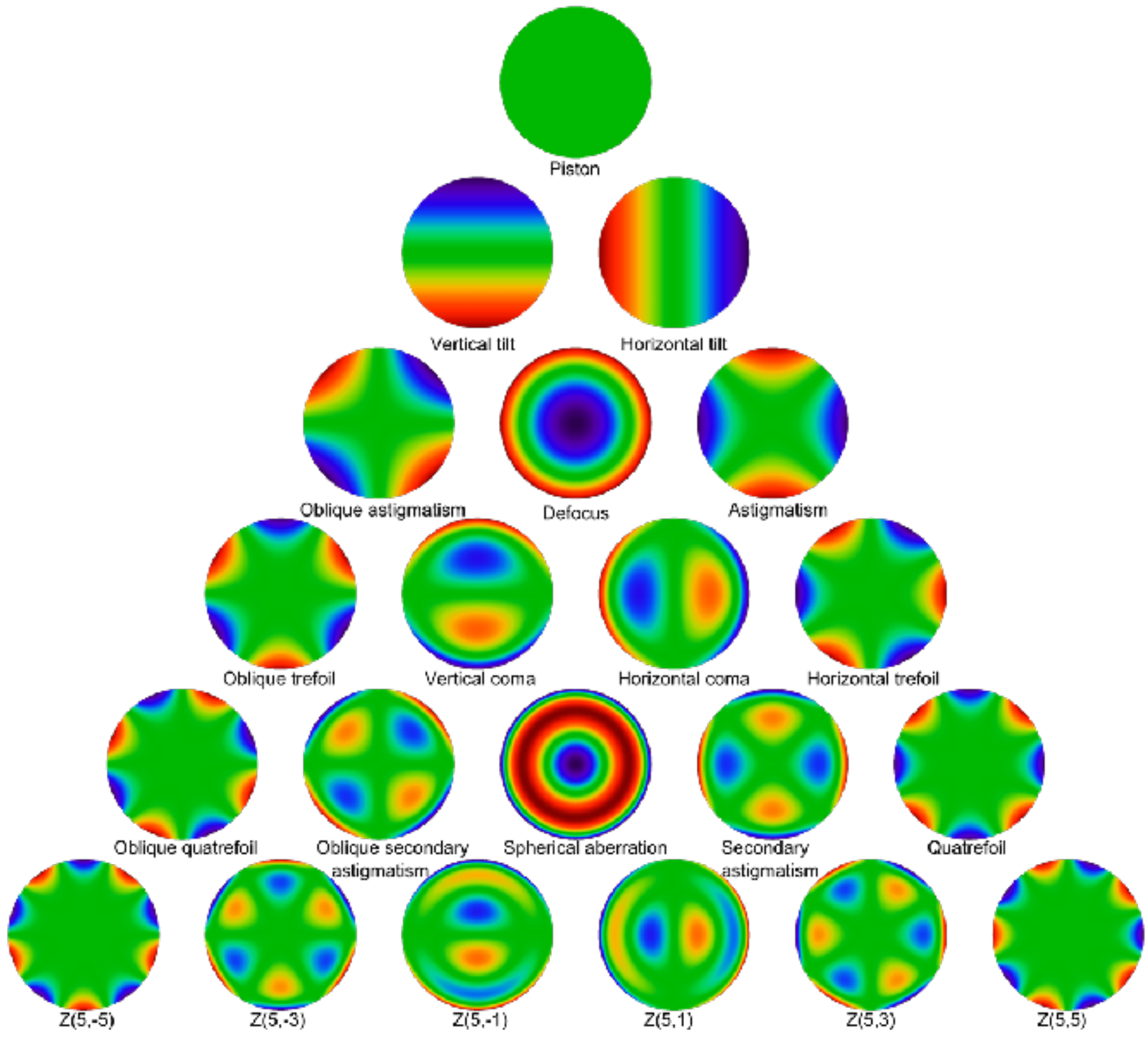
$$R_n^m(r) = \sum_{s=0}^{(n-m)/2} \frac{(-1)^s (n-s)!}{s![(n+m)/2-s]![(n-m)/2-s]!}\, r^{n-2s}. \qquad (2)$$

The values of n and m are always integral and satisfy $m \leq n$, $n - |m| = \text{even}$. The index j is a mode ordering number and is a function of n and m.



Noll 1976 JOSA 66 3

Lowest orders match optical aberrations



https://www.comsol.com/release/5.2a/ray-optics-module

Atmospheric turbulence in Zernikes

Free atmosphere can be described as a Zernike expansion

Δ_J

...is the r.m.s. residual of the wavefront after correction of J Zernike terms

TABLE IV. Zernike-Kolmogoroff residual errors (Δ_J). (D is the aperture diameter.)

$\Delta_1 = 1.0299\ (D/r_0)^{5/3}$	$\Delta_{12} = 0.0352\ (D/r_0)^{5/3}$
$\Delta_2 = 0.582\ (D/r_0)^{5/3}$	$\Delta_{13} = 0.0328\ (D/r_0)^{5/3}$
$\Delta_3 = 0.134\ (D/r_0)^{5/3}$	$\Delta_{14} = 0.0304\ (D/r_0)^{5/3}$
$\Delta_4 = 0.111\ (D/r_0)^{5/3}$	$\Delta_{15} = 0.0279\ (D/r_0)^{5/3}$
$\Delta_5 = 0.0880\ (D/r_0)^{5/3}$	$\Delta_{16} = 0.0267\ (D/r_0)^{5/3}$
$\Delta_6 = 0.0648\ (D/r_0)^{5/3}$	$\Delta_{17} = 0.0255\ (D/r_0)^{5/3}$
$\Delta_7 = 0.0587\ (D/r_0)^{5/3}$	$\Delta_{18} = 0.0243\ (D/r_0)^{5/3}$
$\Delta_8 = 0.0525\ (D/r_0)^{5/3}$	$\Delta_{19} = 0.0232\ (D/r_0)^{5/3}$
$\Delta_9 = 0.0463\ (D/r_0)^{5/3}$	$\Delta_{20} = 0.0220\ (D/r_0)^{5/3}$
$\Delta_{10} = 0.0401\ (D/r_0)^{5/3}$	$\Delta_{21} = 0.0208\ (D/r_0)^{5/3}$
$\Delta_{11} = 0.0377\ (D/r_0)^{5/3}$	

$$\Delta_J \sim 0.2944\, J^{-\sqrt{3}/2}\ (D/r_0)^{5/3} \quad \text{(For large } J\text{)}$$

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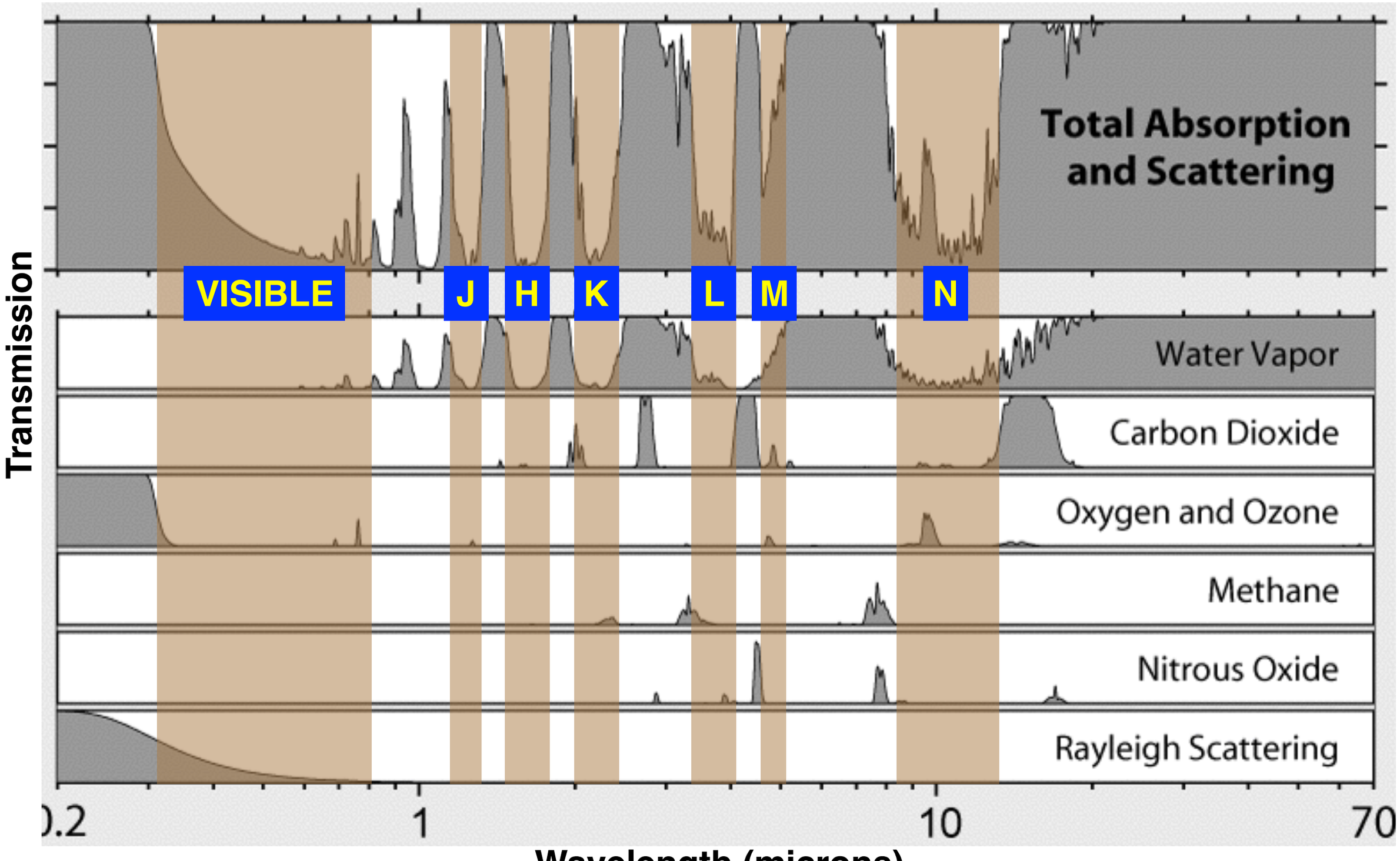
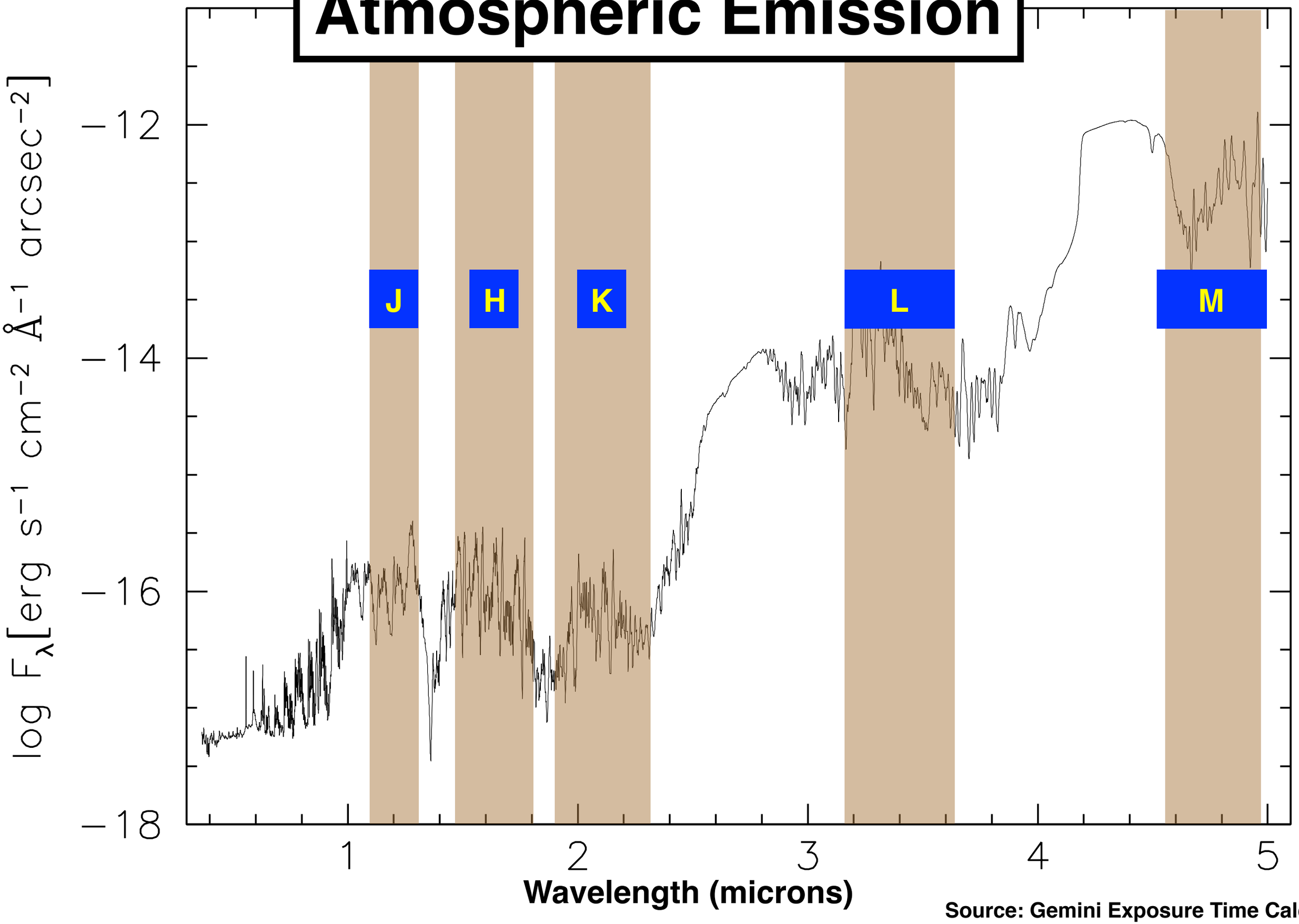
Atmospheric Transmission



Source: HITRAN model Wikipedia

Atmospheric Emission



Source: Gemini Exposure Time Calc

The atmosphere limits diffraction limited imaging

Diffraction limited by the turbulent atmosphere: $\approx \frac{\lambda}{r_0}$

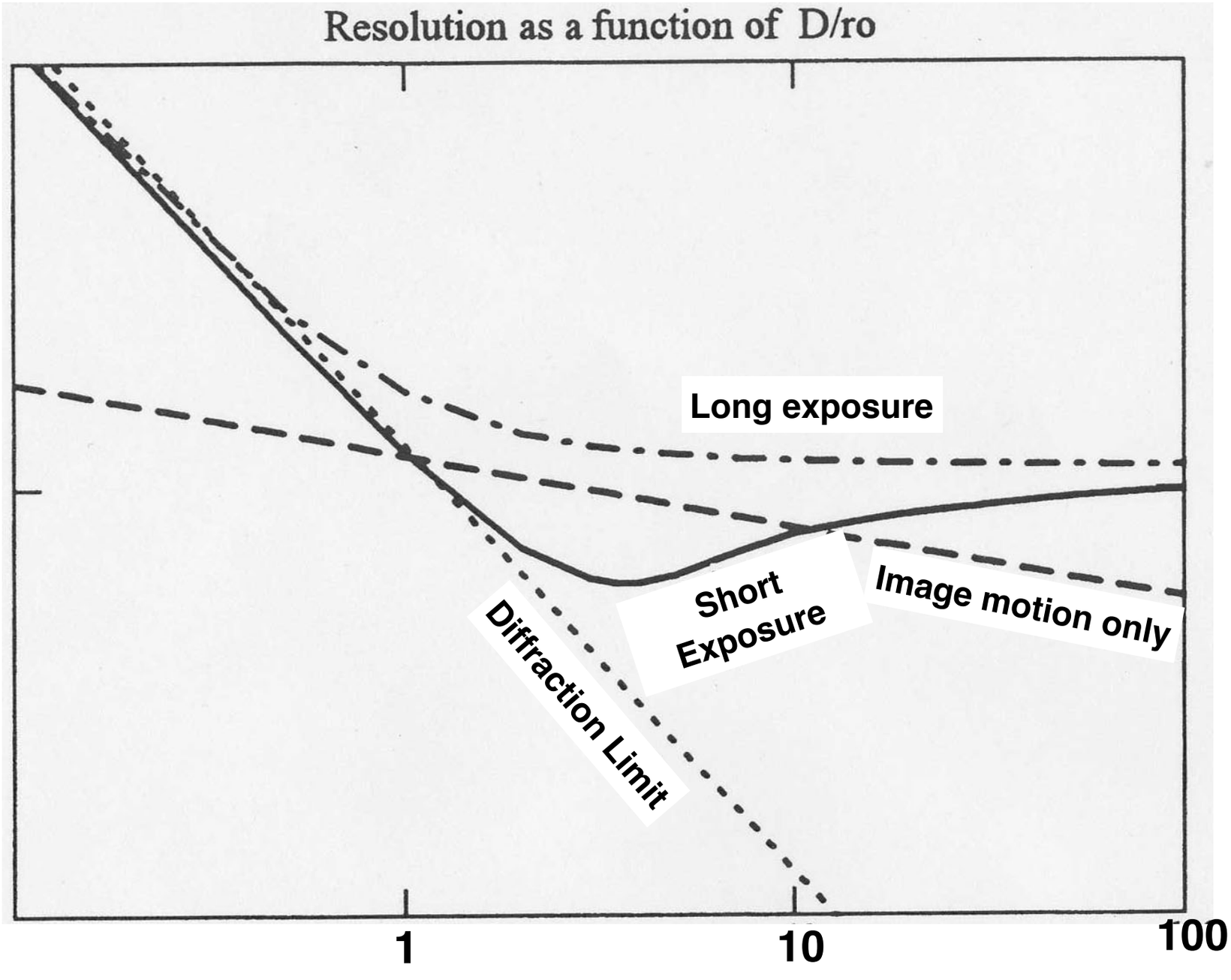
Typically for professional observatories:

$$\approx \frac{0.5\,\mu m}{10\,cm} = 5\mu rad$$

$$\approx 1 \text{ arcsec}$$

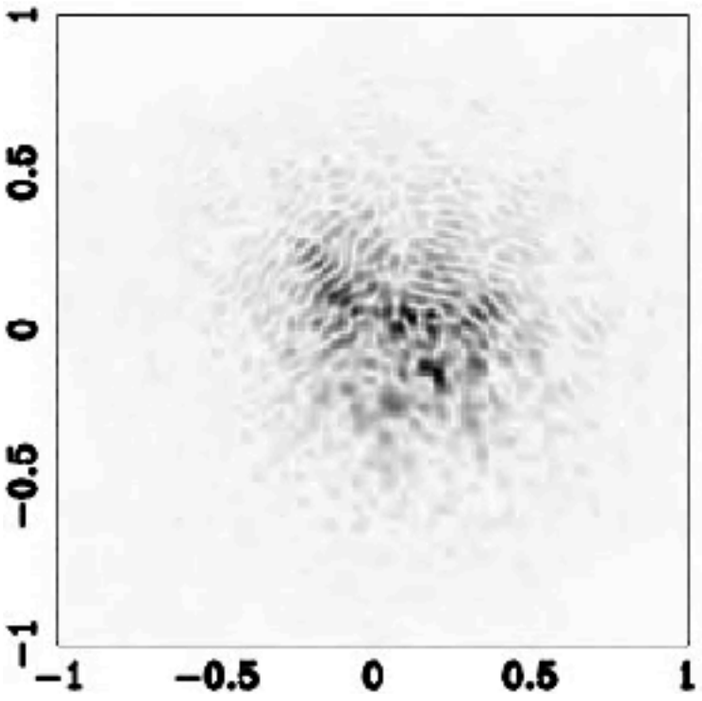
Hubble Space Telescope Credit: NASA

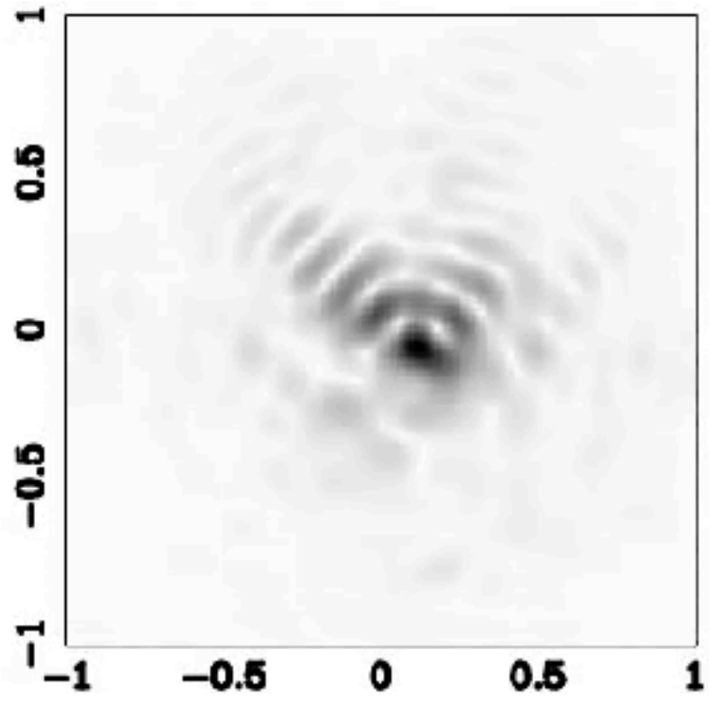
If telescope is similar to Fried length, cheap AO can be done with tip tilt removal

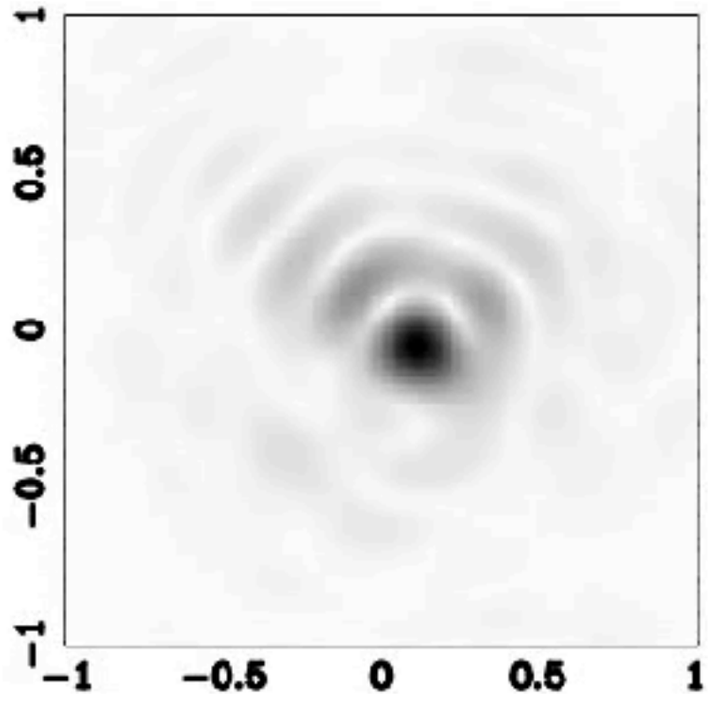


The achromaticity of the atmospheric OPD is exploited in AO

Measuring the wavefront at shorter wavelengths means that you can correct for the atmosphere at longer wavelengths





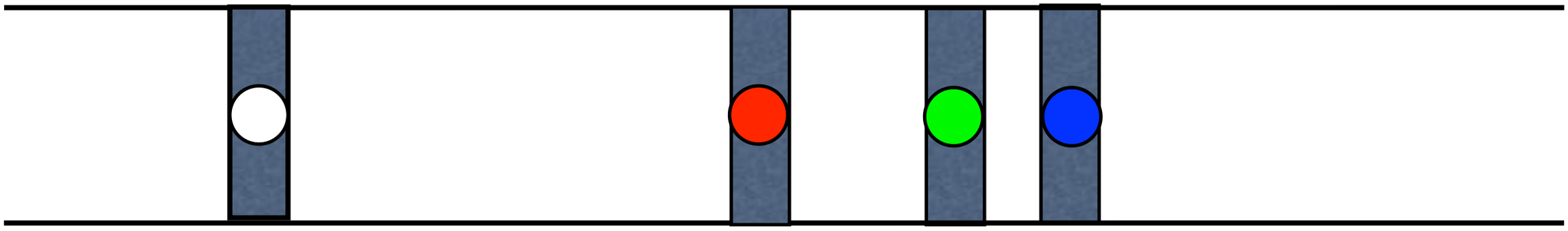


Measure in the blue → **Correction in the red**

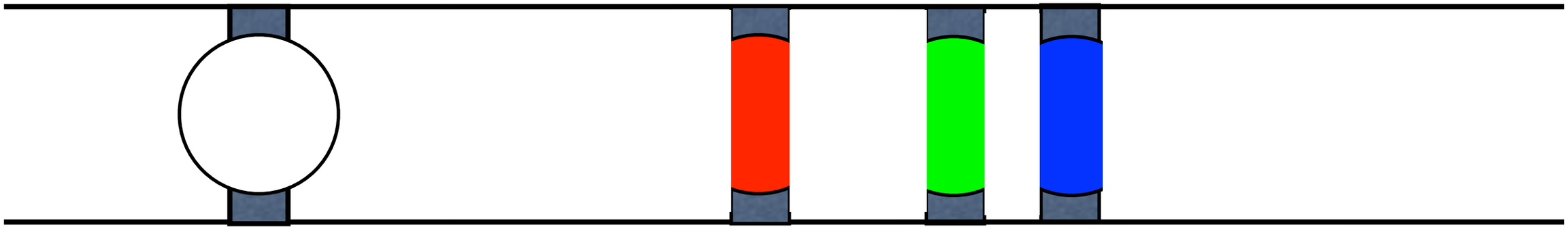
Many systems measure in the visible and provide correction for red and infra-red wavelengths

AO makes spectrographs smaller

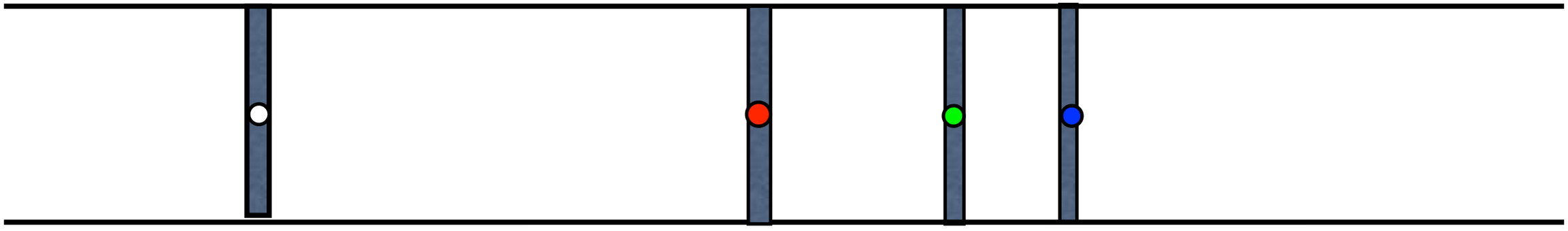
Spectrographs disperse the image of the slit...



...but larger telescope means either larger spectrograph collimator or lower resolution

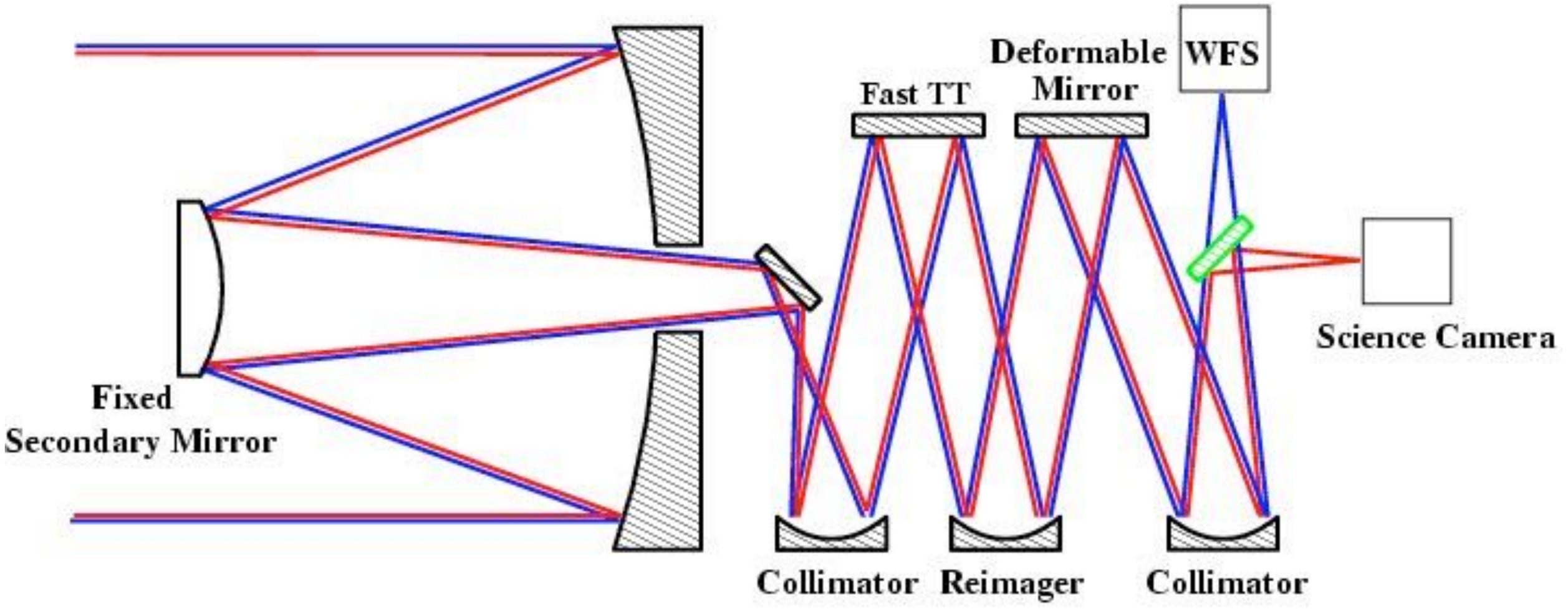


AO decouples image size from telescope!

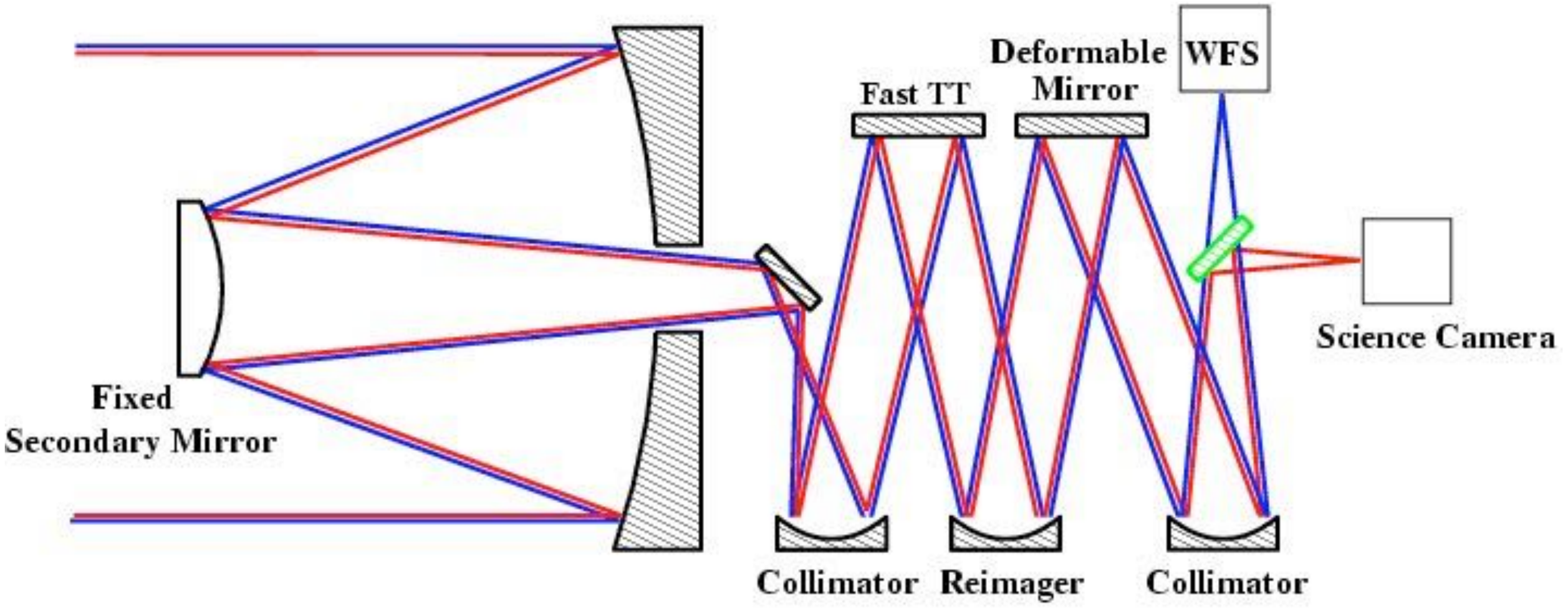


Natural Guide Stars

Layout of an AO System



WFS measures wavefront and commands the deformable mirror to compensate - but it's not perfect!



Several errors combine in quadrature to make imperfect correction

$$\sigma^2_{total} = \sigma^2_{fit} + \sigma^2_{tau} + \sigma^2_{phot} + \sigma^2_{iso} + \sigma^2_{focal} + \sigma^2_{other}$$

Fitting error

Temporal error

Photon error

Isoplanatic error

Focal isoplanatic

Error due to time lag

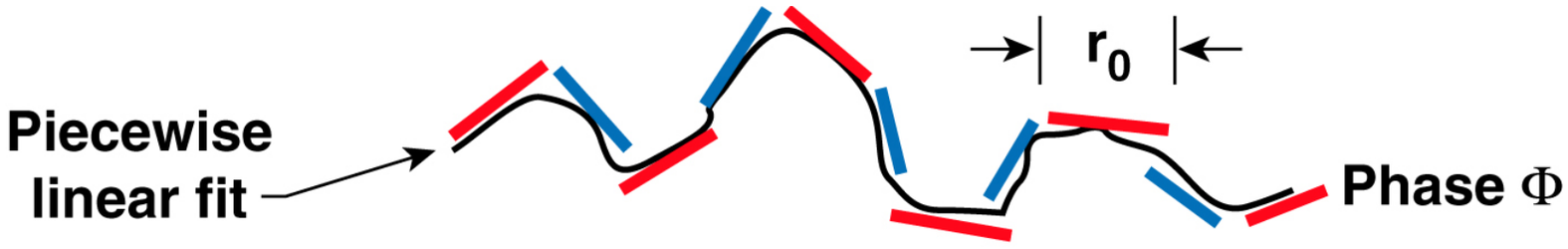
$$\sigma^2_{tau} = 28.4 \left(\frac{\tau}{\tau_0} \right)^{5/3}$$

You have to run your loop about 10x faster than τ_0

Credit: crowforsaken

$$\sigma^2_{tau} < 1, \tau < 0.13\tau_0$$

Error due to fitting



Subaperture diameter d

$$\sigma^2_{fit} = \mu \left(\frac{d}{r_0} \right)^{5/3}$$

Your deformable mirror cannot match perfectly the wavefront

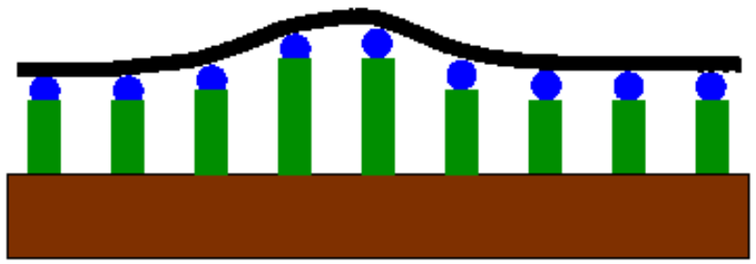
Segmented mirror with tip, tilt and piston:

$$\mu = 0.14$$

Piston + tilt

Continuous face sheet:

$$\mu = 0.28$$



Jet Stream

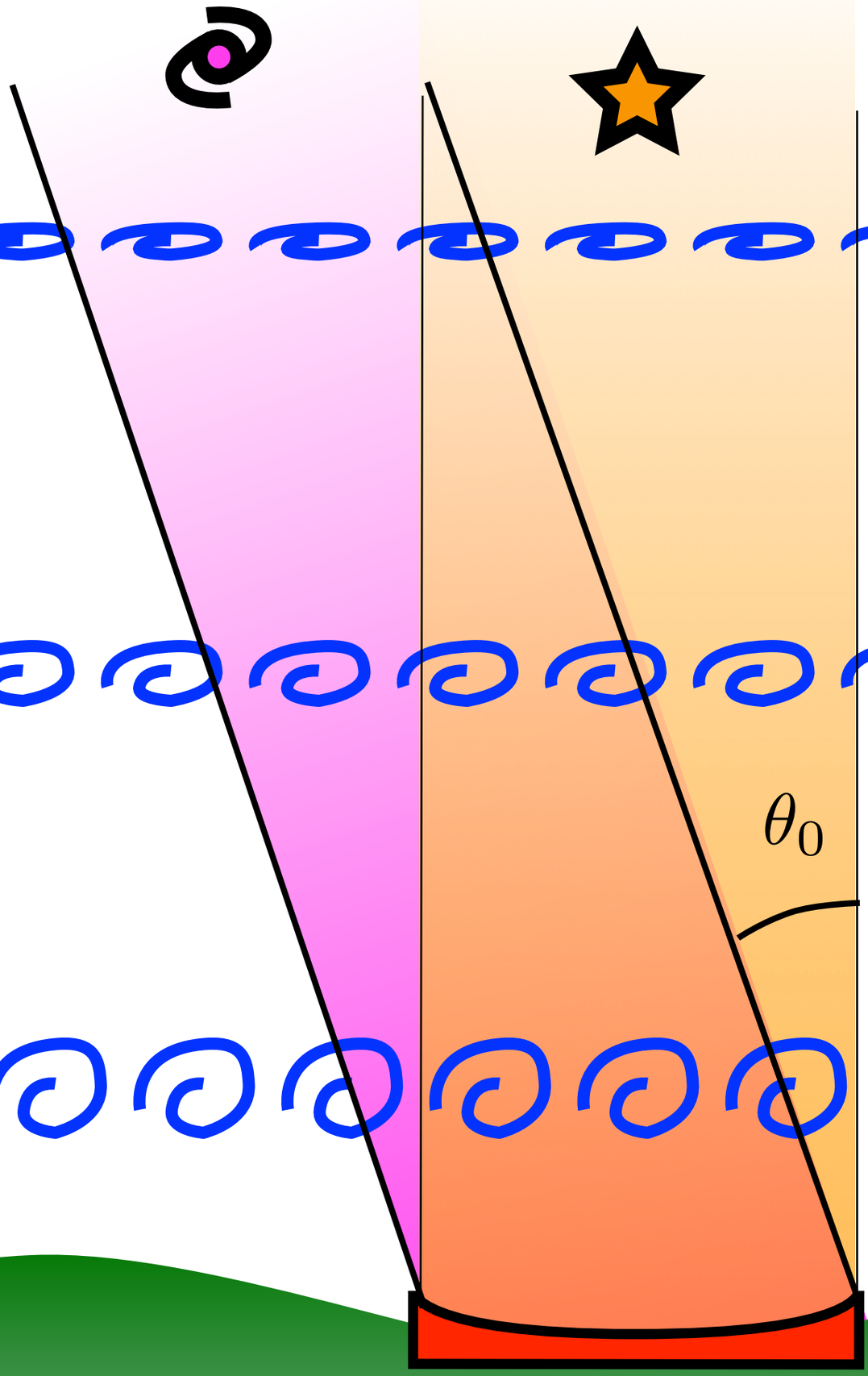
Mid-level

Ground Layer

Isoplanatic Angle

$$\theta_0 = 0.31 \frac{r_0}{h}$$

$h \sim 5$ km



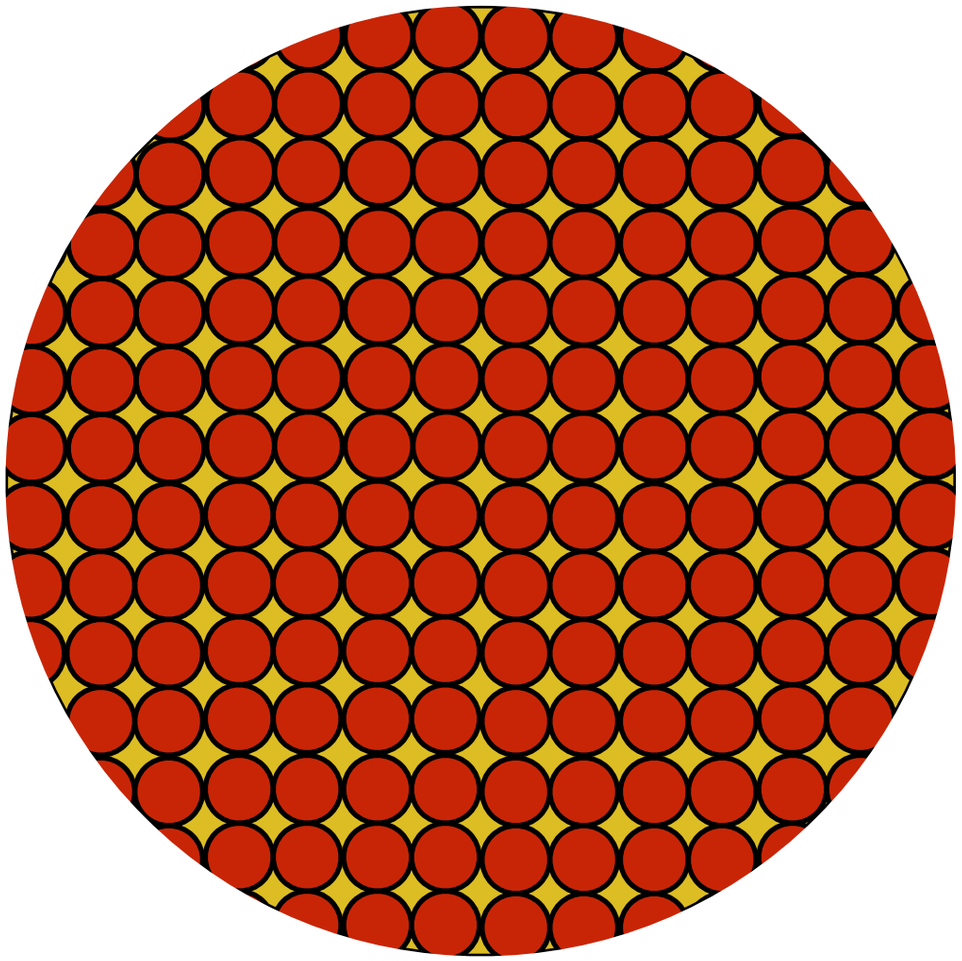
Error due to anisoplanatism

Your guide star doesn’t see the same atmosphere as the science target

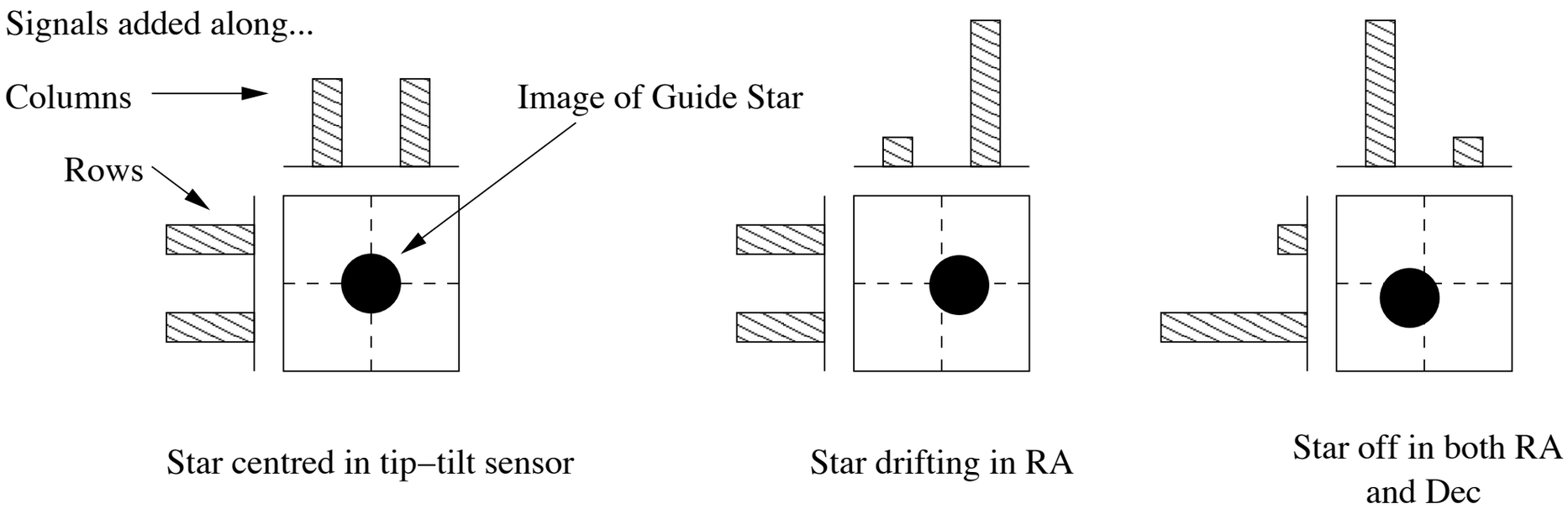
$$\sigma^2_{iso} = \left(\frac{\theta}{\theta_0}\right)^{5/3}$$

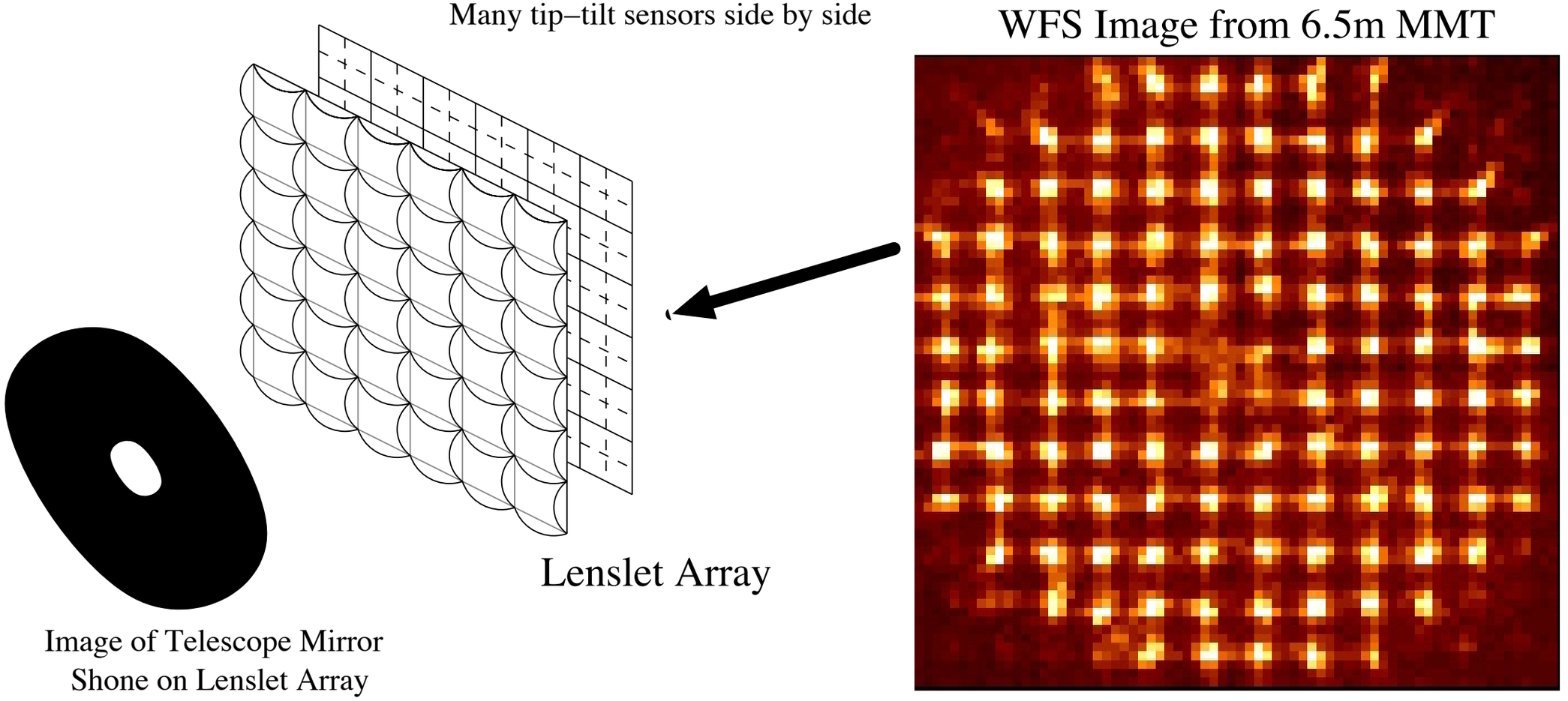
Theta is the angular distance between star and target

Split pupil into r_0 patches and measure tip tilt of each patch

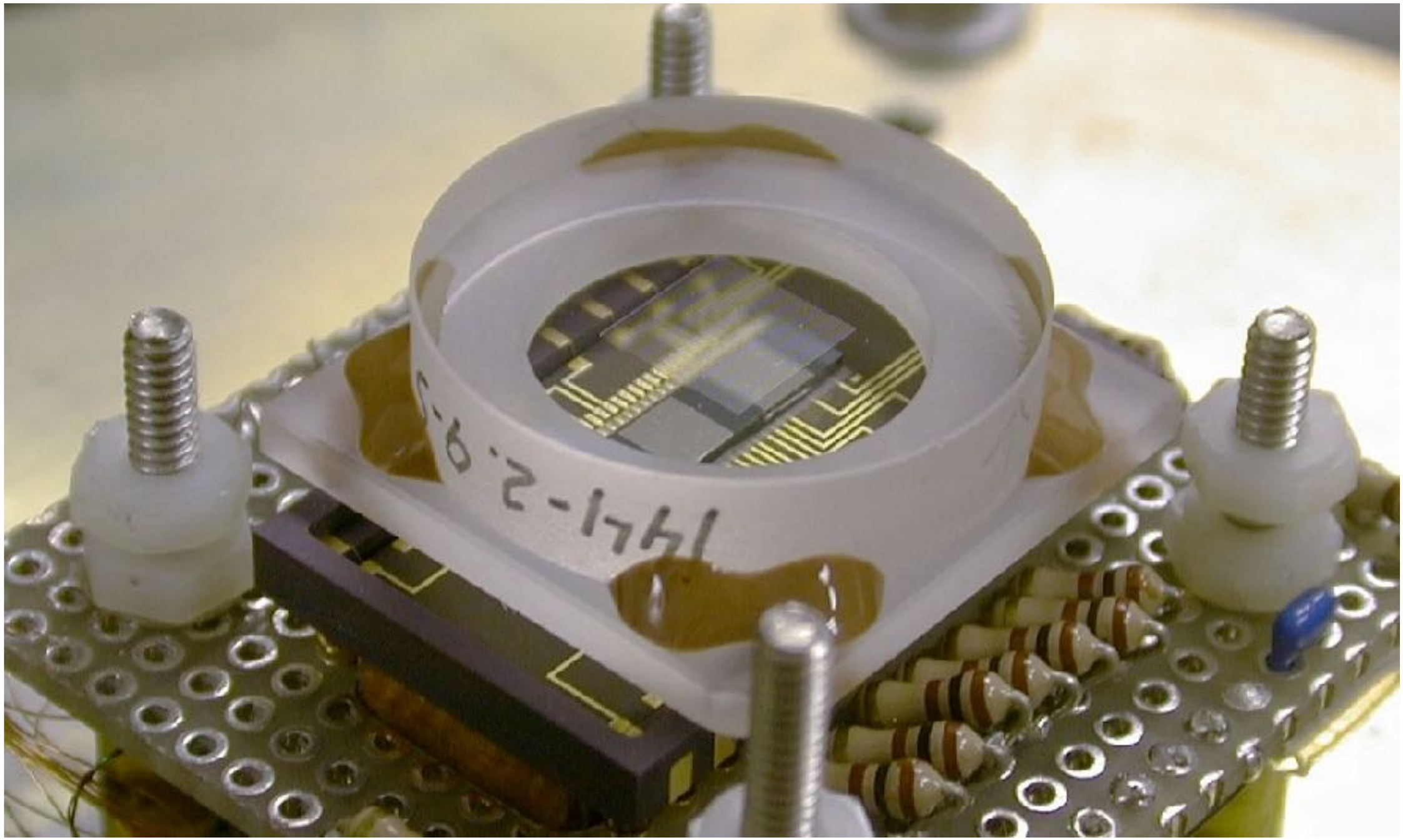


Wavefront Sensing





Wavefront Sensor (WFS)



Measuring the influence matrix

There are N total spot positions from the SH WFS and actuator displacement/voltage a

slope of mirror surface *and* Shack-Hartmann star positions are proportional to actuator position

linear relationship between actuator *a* and star position *c*:

$$c_n = \sum_{k=1}^{N} a_k b_{nk}$$

combine equations for each spot position *n* into matrix equation:

$$C = BA$$

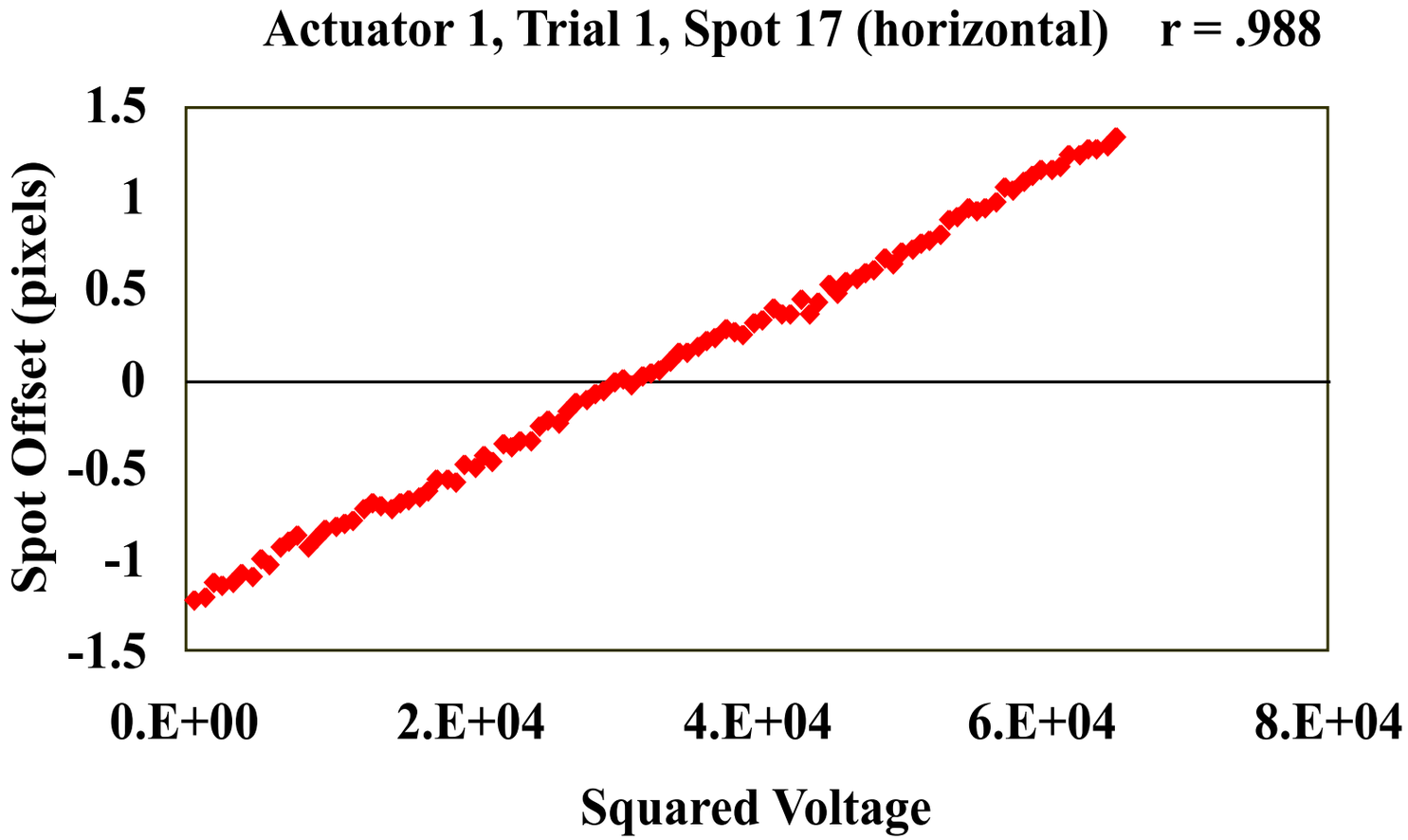
C = star positions

A = actuator positions

B = influence matrix describing influence of specific actuator position on star positions

Measuring the influence matrix

- measure centroid positions in subapertures for different settings of actuator *k*
- for actuator *k* and subaperture *n*, slope of best fit line is element (n, k) of influence matrix B



Determining the Control Vector

- Influence matrix B is known, C from wavefront sensor
- Find control vector A to correct for error in wavefront
- Matrix inversion of B?

$$A = B^{-1}C$$

This is an overdetermined system:
more centroid measurements than actuators
No exact solution for A exists
B is rectangular and noninvertible

Singular Value Decomposition can approximate the inverse of B

Natural Guide Star (NGS)

Jet Stream

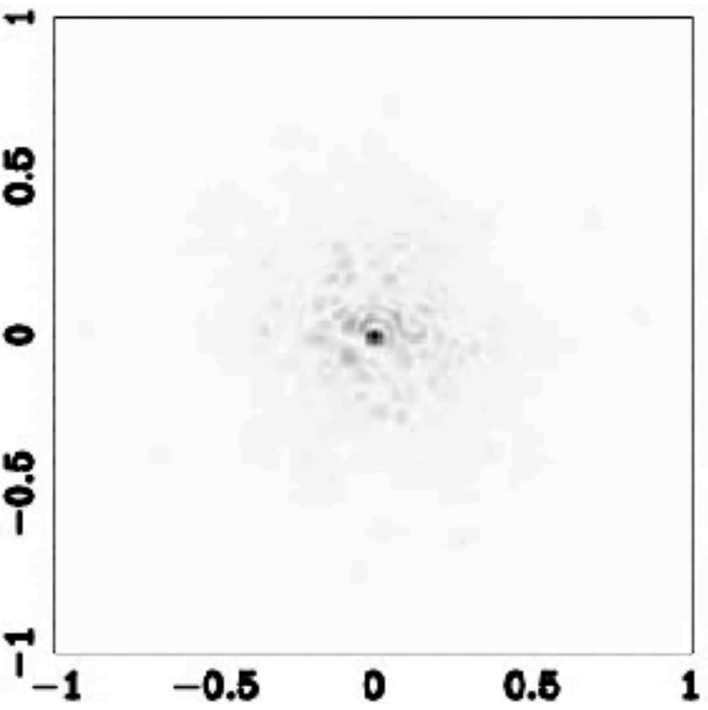
Larger telescope needs more WFS subapertures

Mid-level

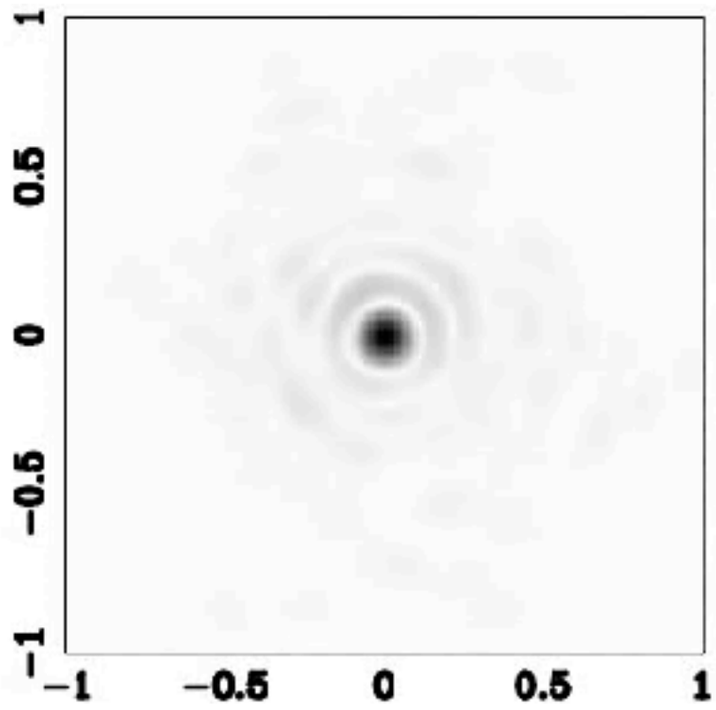
Ground Layer

Image quality is quoted in Strehl Ratio

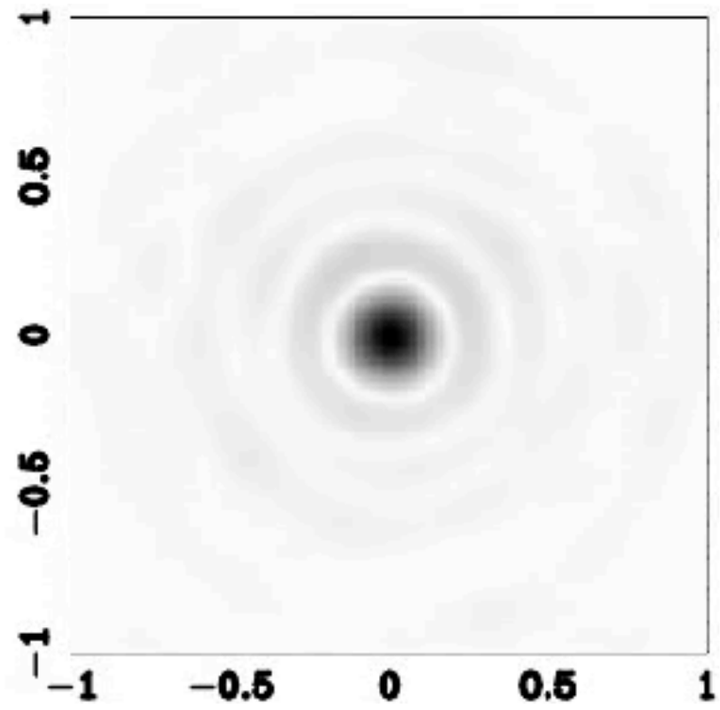
$$S = \text{Strehl ratio} = \frac{\text{peak of flux normalised measured PSF}}{\text{peak of flux normalised DL image PSF}}$$



1 micron: S=10%



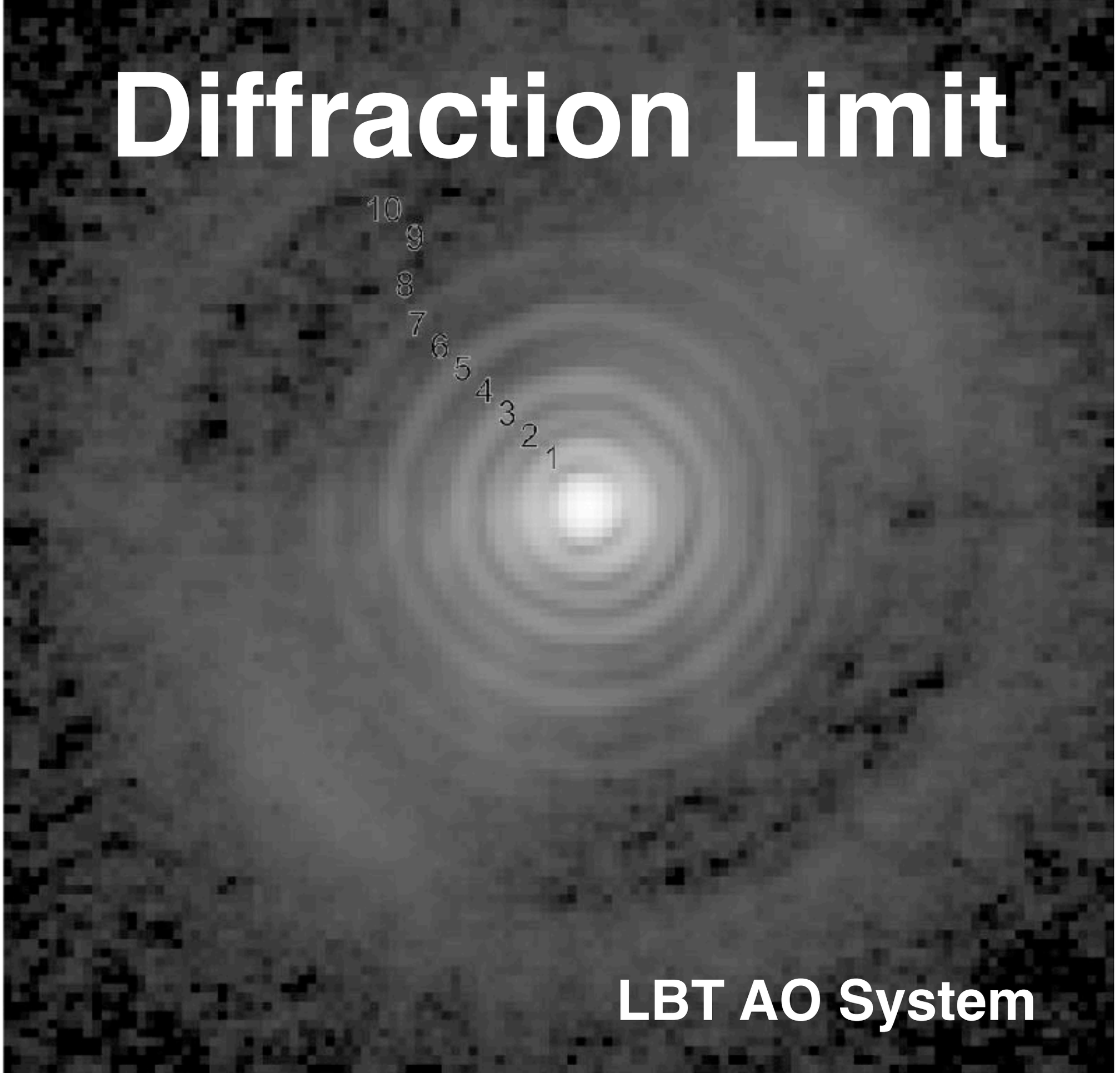
2 microns: S=40%



5 microns: S=90%

Strehl ratio increases with wavelength for a given AO system and gain

Diffraction Limit



Bigger telescopes see more turbulent cells....

...so that the limiting magnitude of many AO systems is the same (to an order of magnitude)

Better QE/read noise of cameras

More efficient optical train for AO system

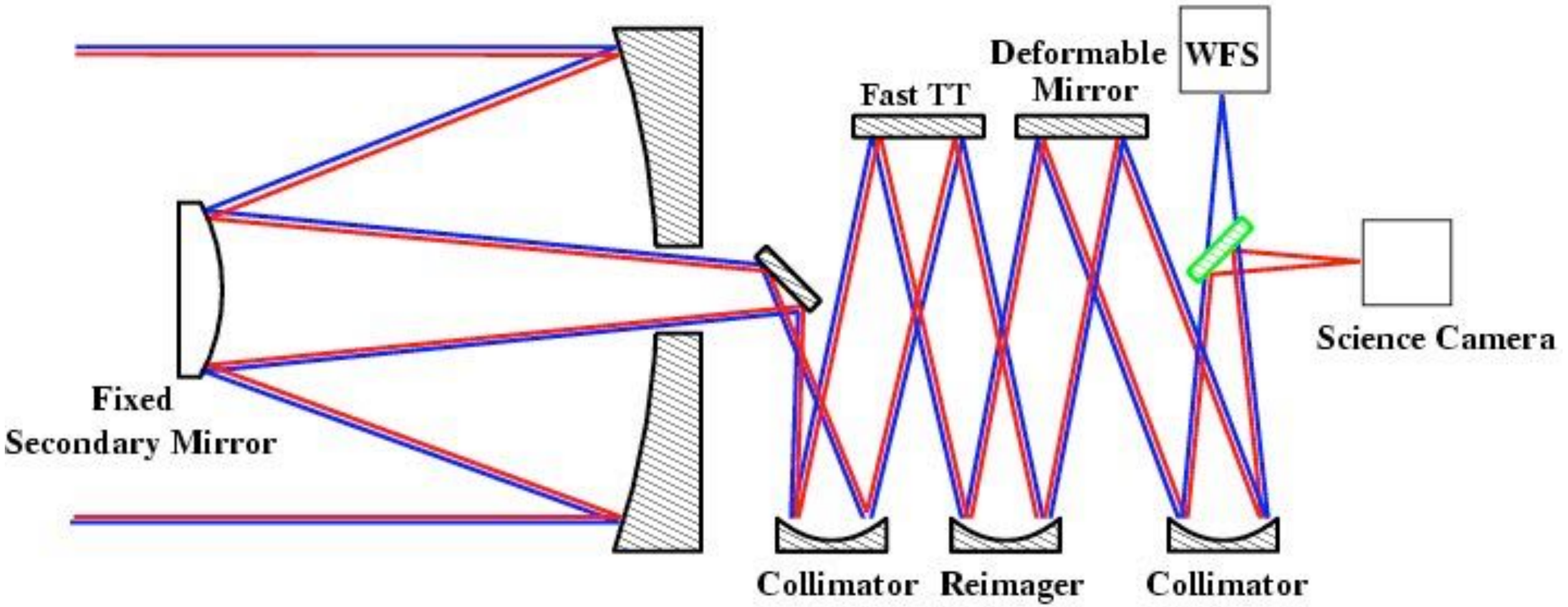
Better WFS designs - Pyramid, curvature....

Mostly at the largest telescopes, where there is the best payoff

- **Keck 10m LGS systems**
- **VLT 8.4m (LGS soon)**
- **Gemini 8.2m NGS and LGS**
- **Subaru 8.2m NGS and LGS**
- **MMT 6.5m NGS and LGS**

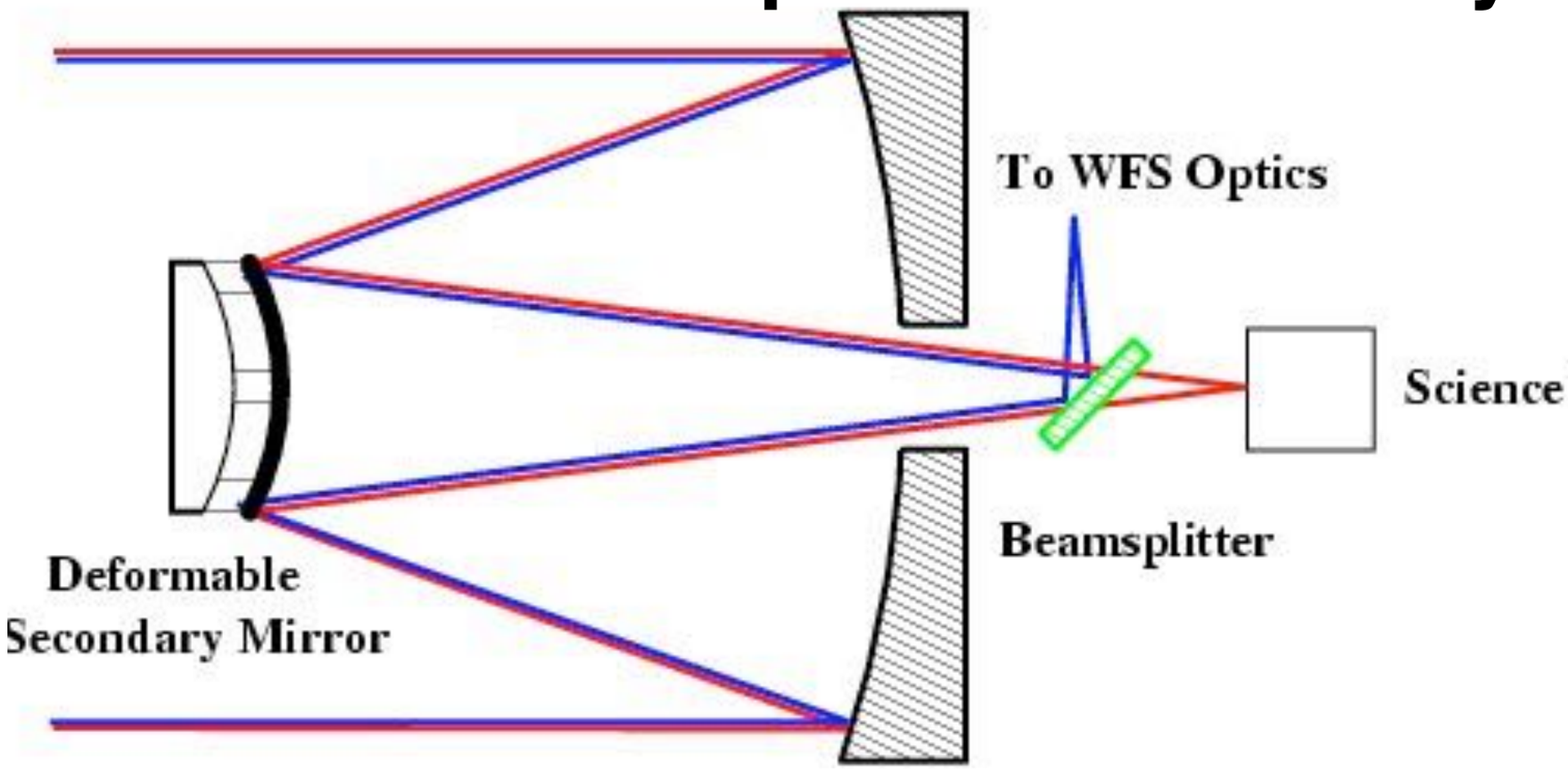
Deformable Secondary Mirrors

Most AO systems are added as an afterthought to classical telescopes



Leads to less than optimal paths and lower observing efficiency

Using a deformable secondary mirror (DSM) improves sensitivity



Two warm surfaces
Minimal thermal background

MMT 6.5m telescope with the world's first DSM

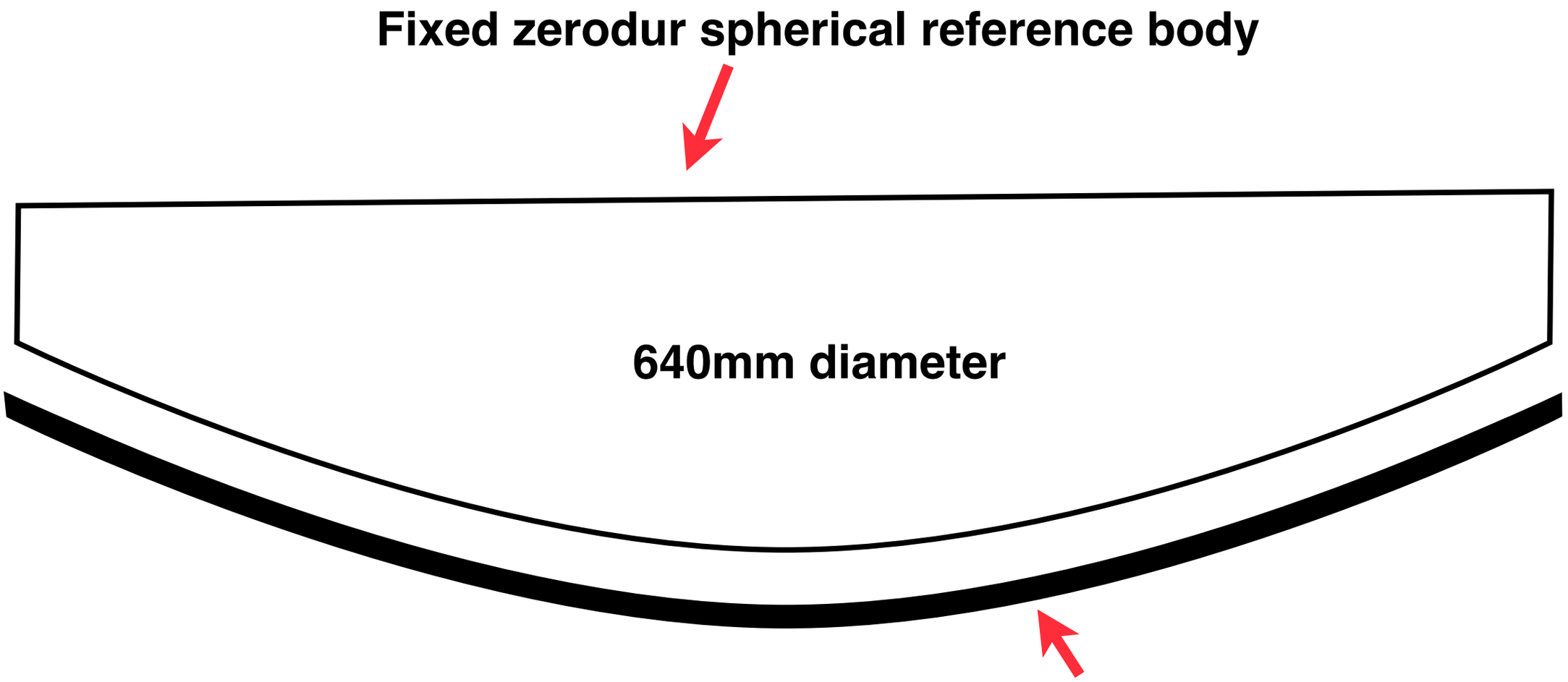
Deformable Secondary Mirror

2mm thick by 640 mm diameter

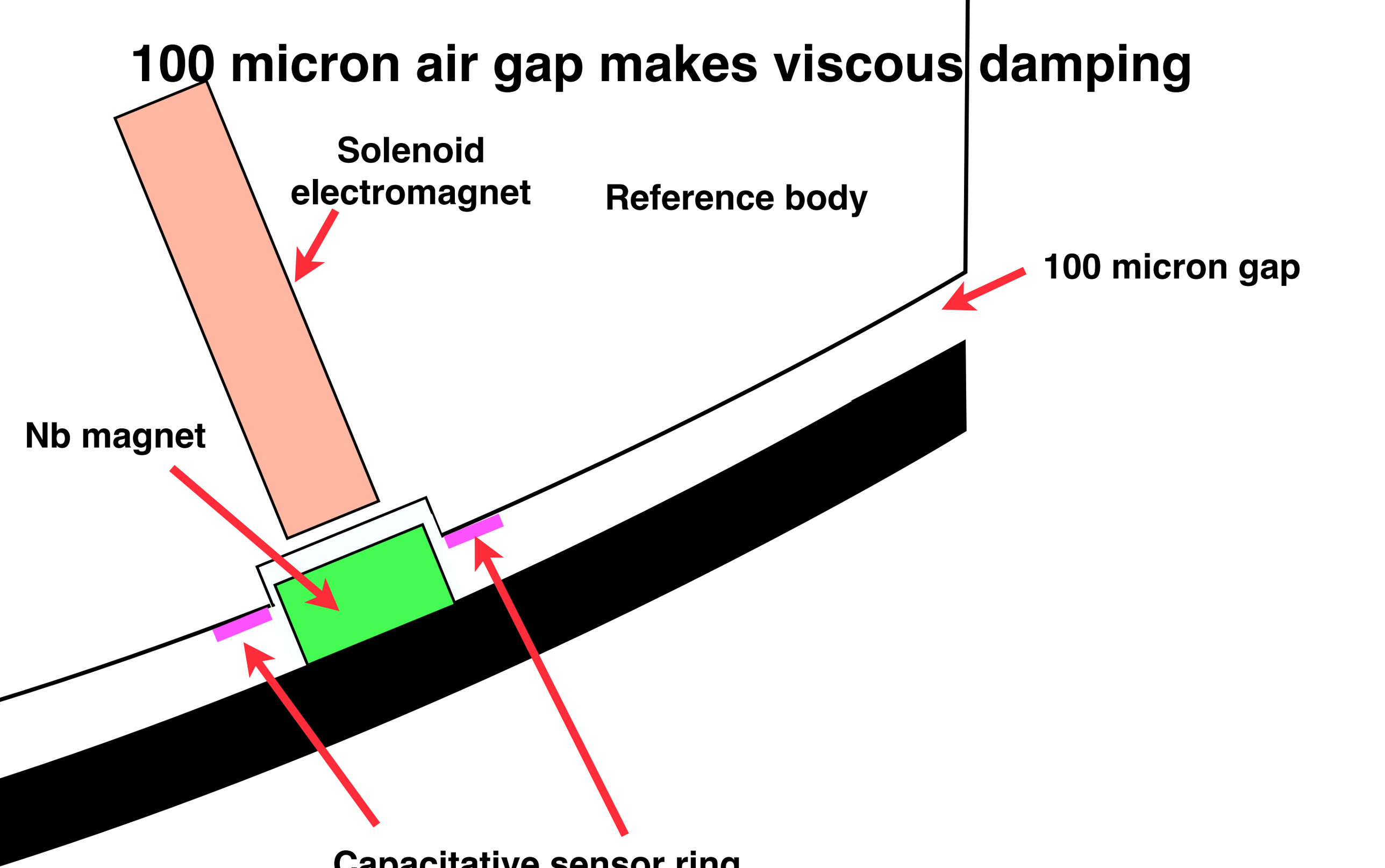
336 voice coil actuators

Undersized pupil for IR observations (effective D=6.35m)

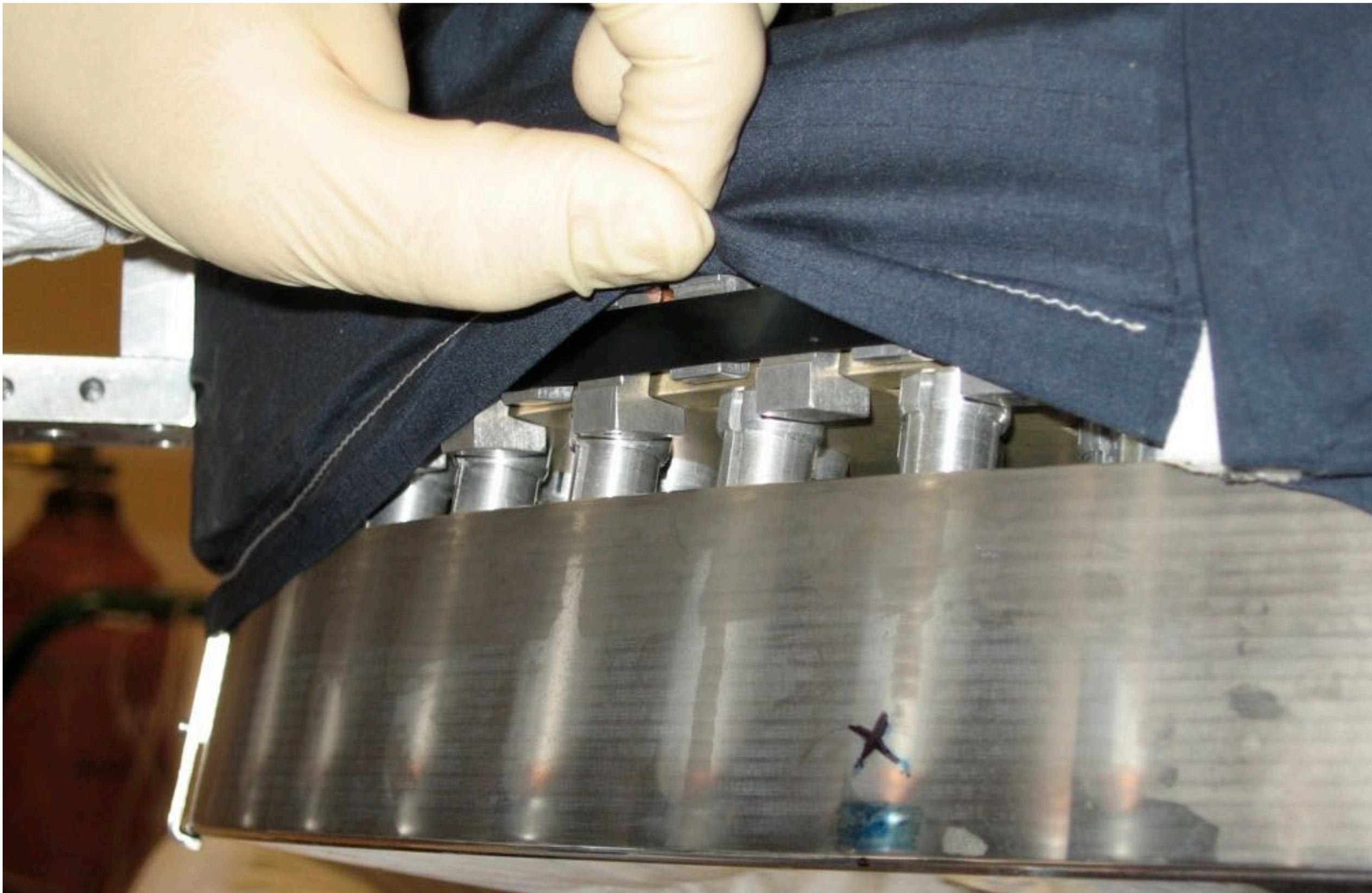
Deformable Secondary Mirror

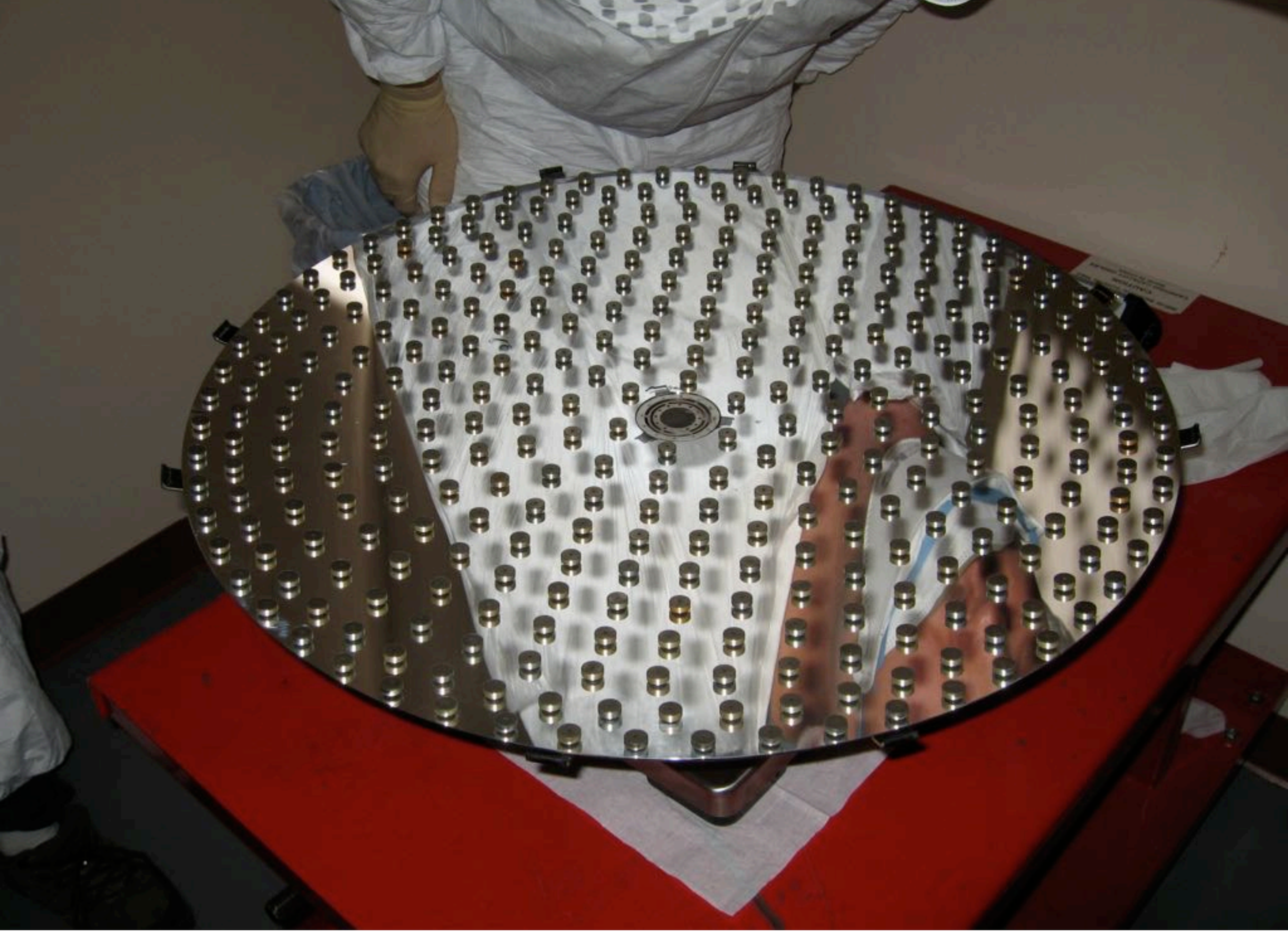


100 micron air gap makes viscous damping

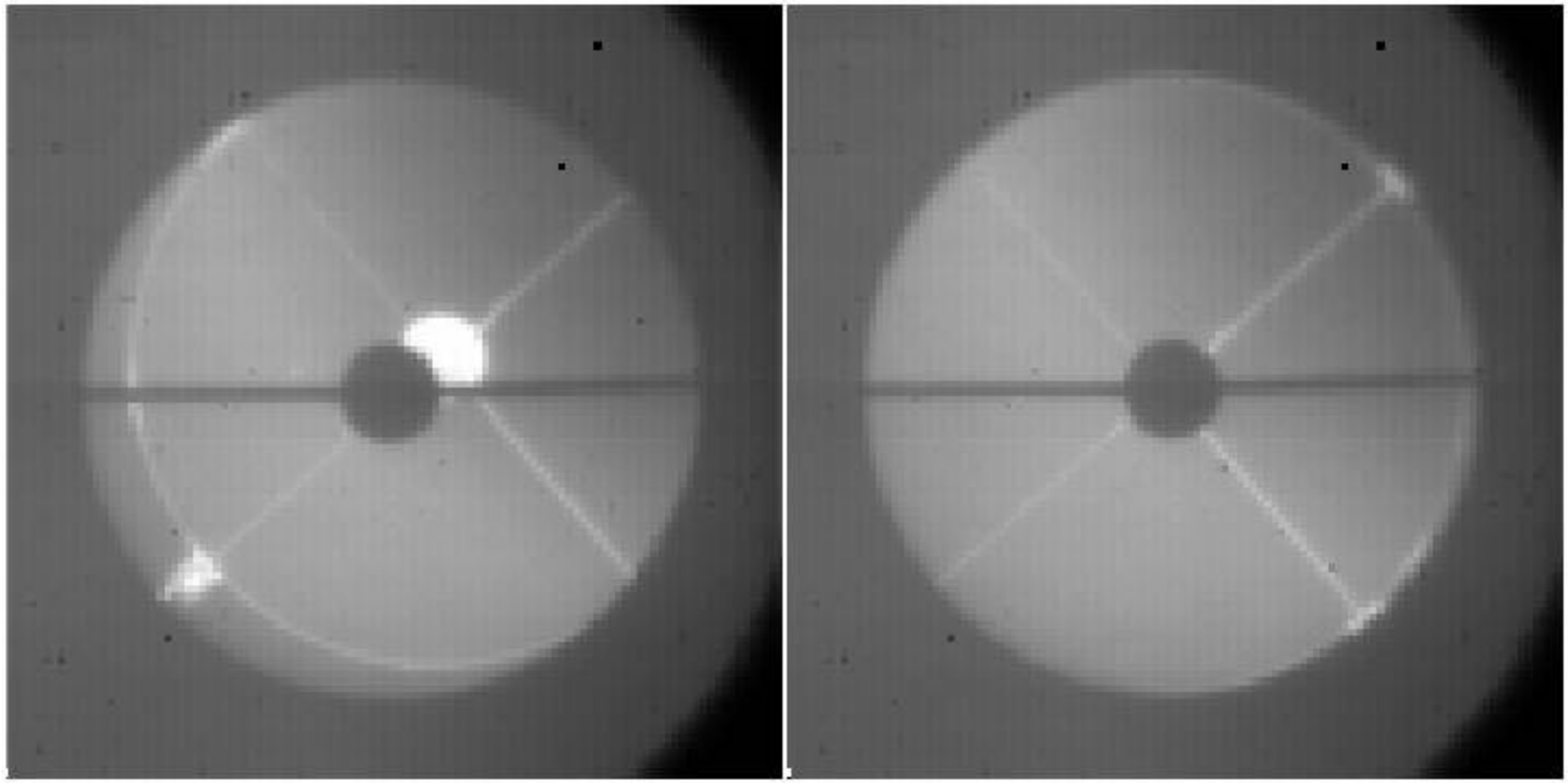


Deformable Secondary Mirror





Thermal IR Performance



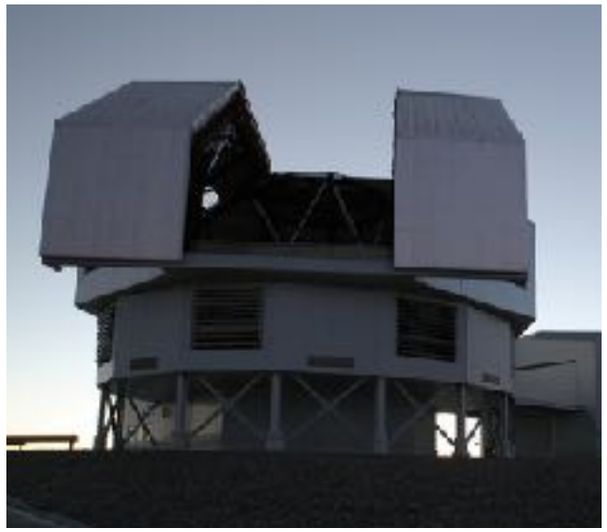
- 7% emissivity compared to Keck's 25-50%
- Very clean pupil, ideal for 5 micron and longer wavelengths

Current and planned DSM facilities

Large Binocular Telescope

Credit: David Steele, AP

Magellan 6.5m

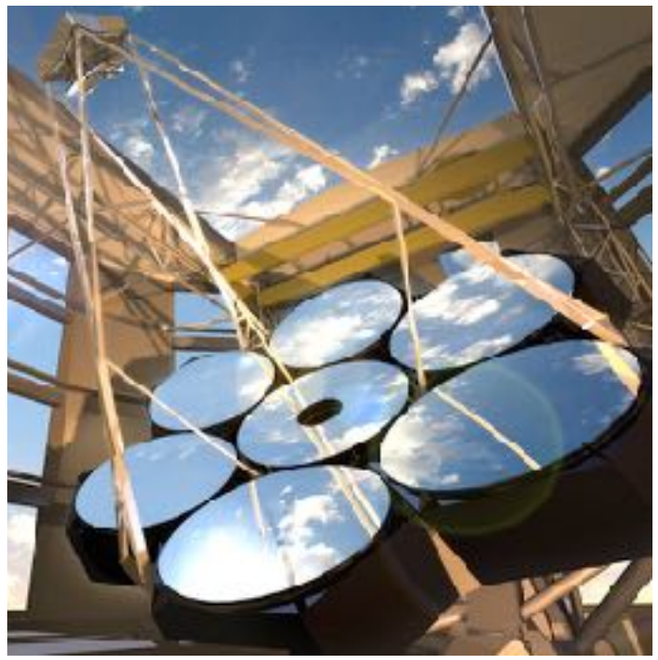


LCO Website

MMTO Telescope

H. Lester/MMTO

Giant Magellan Telescope



Very Large Telescope

G. Hüdepohl/ESO

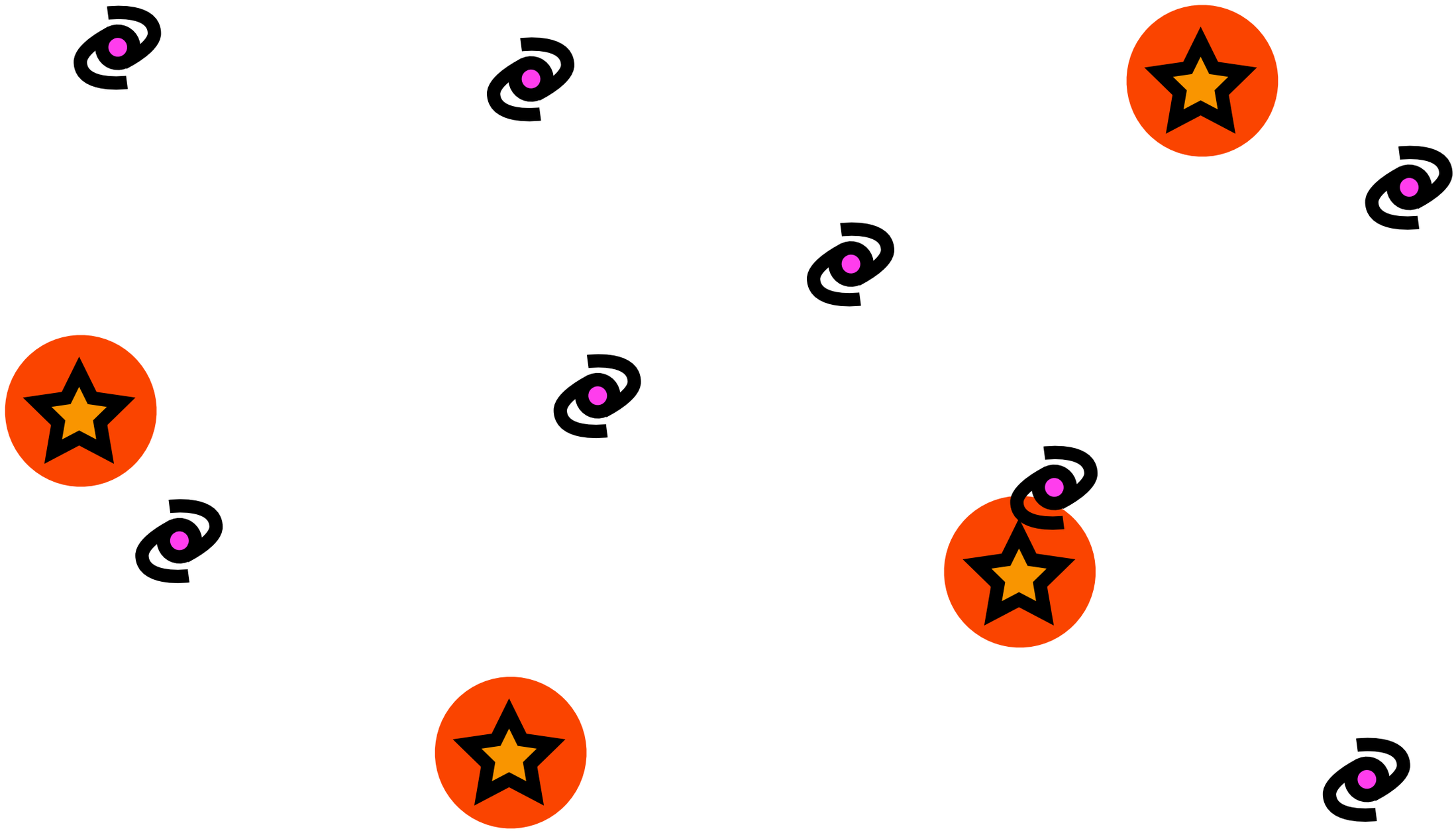
Isoplanatic Angle

$$\theta_{iso}$$

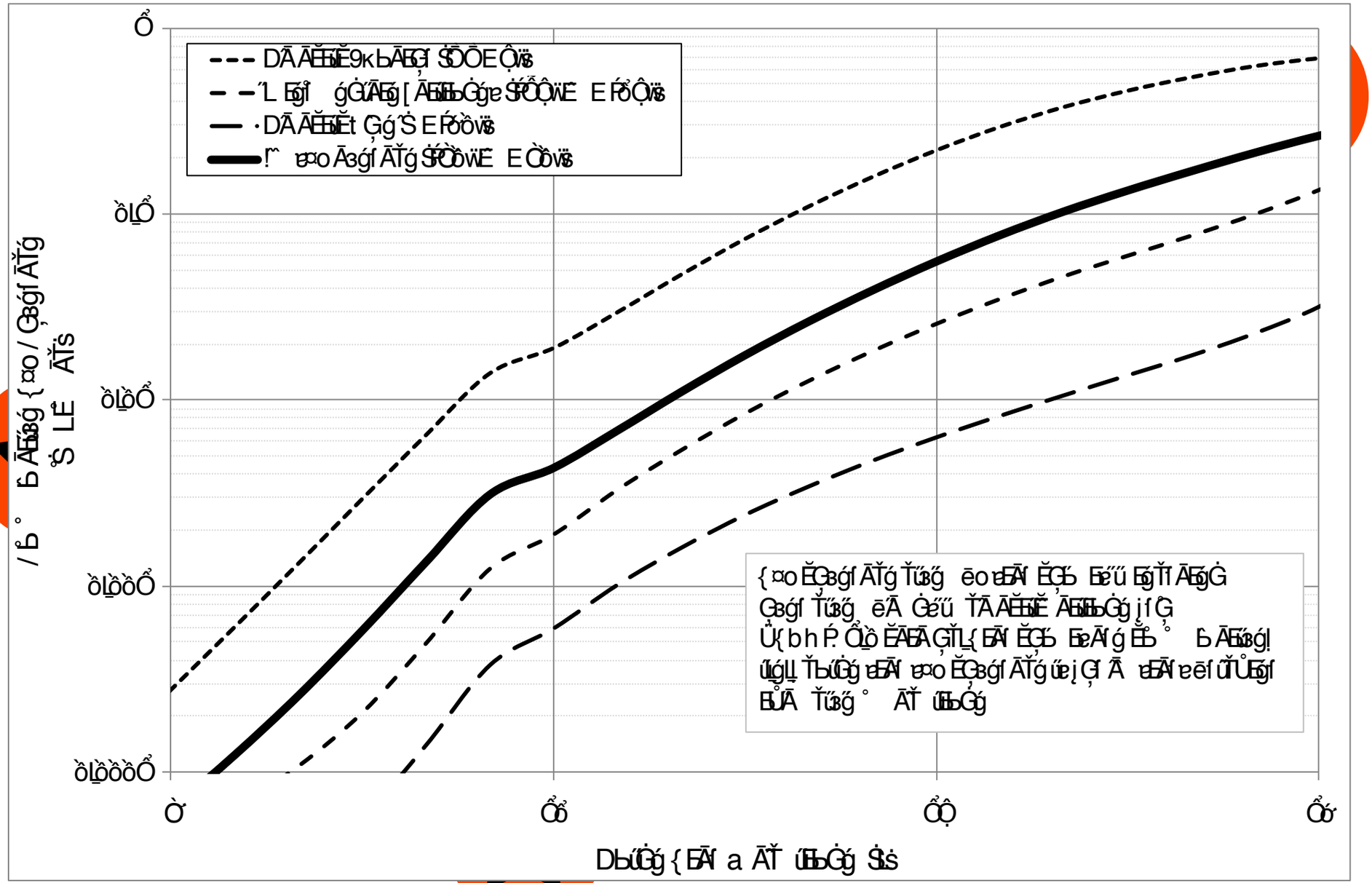
10-20 arcsec for IR

2-4 arcsec for visible

Not enough Natural Guide Stars for complete sky coverage

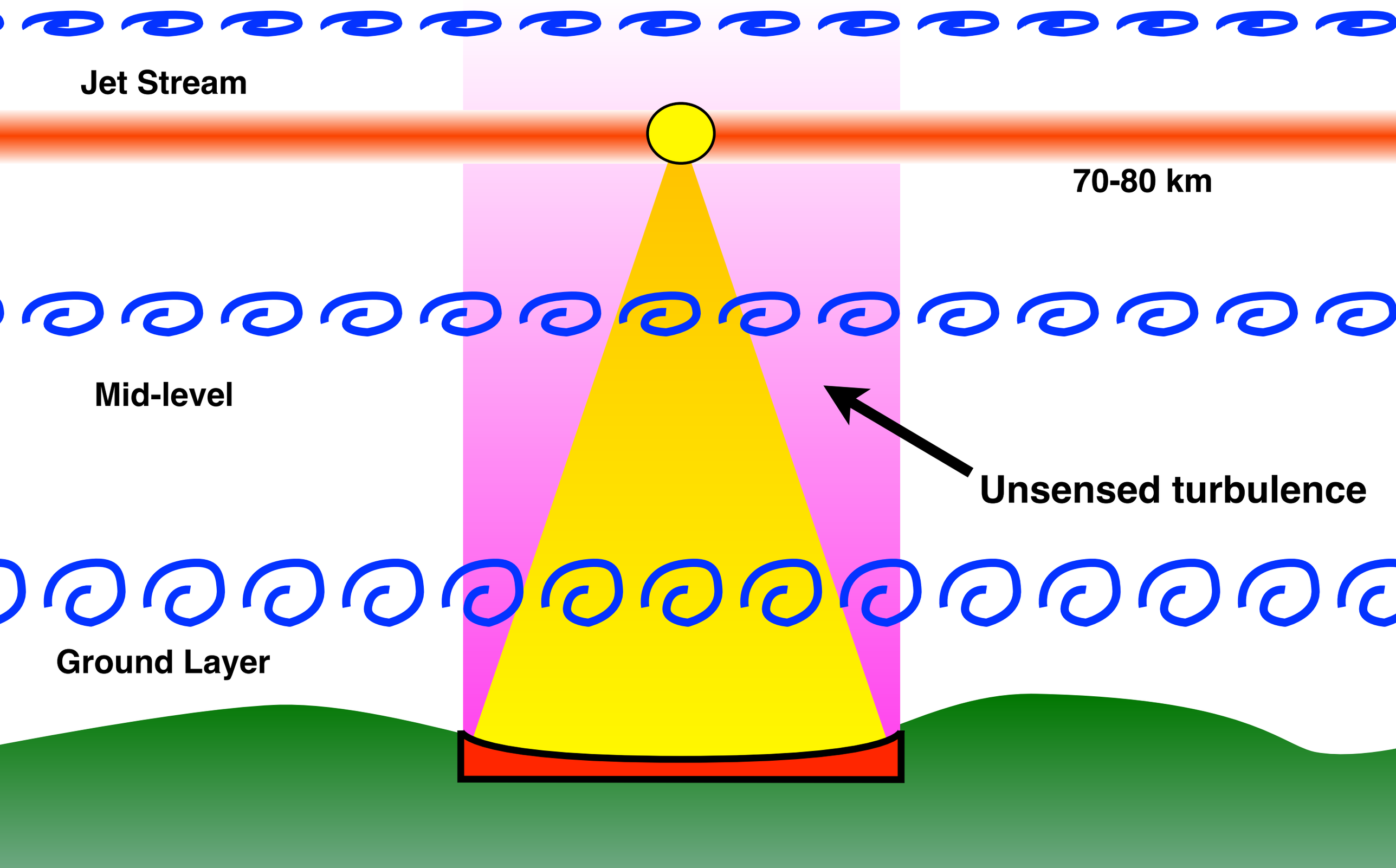


Not enough Natural Guide Stars for complete sky coverage



Laser Guide Stars

Sodium Laser Guide Star

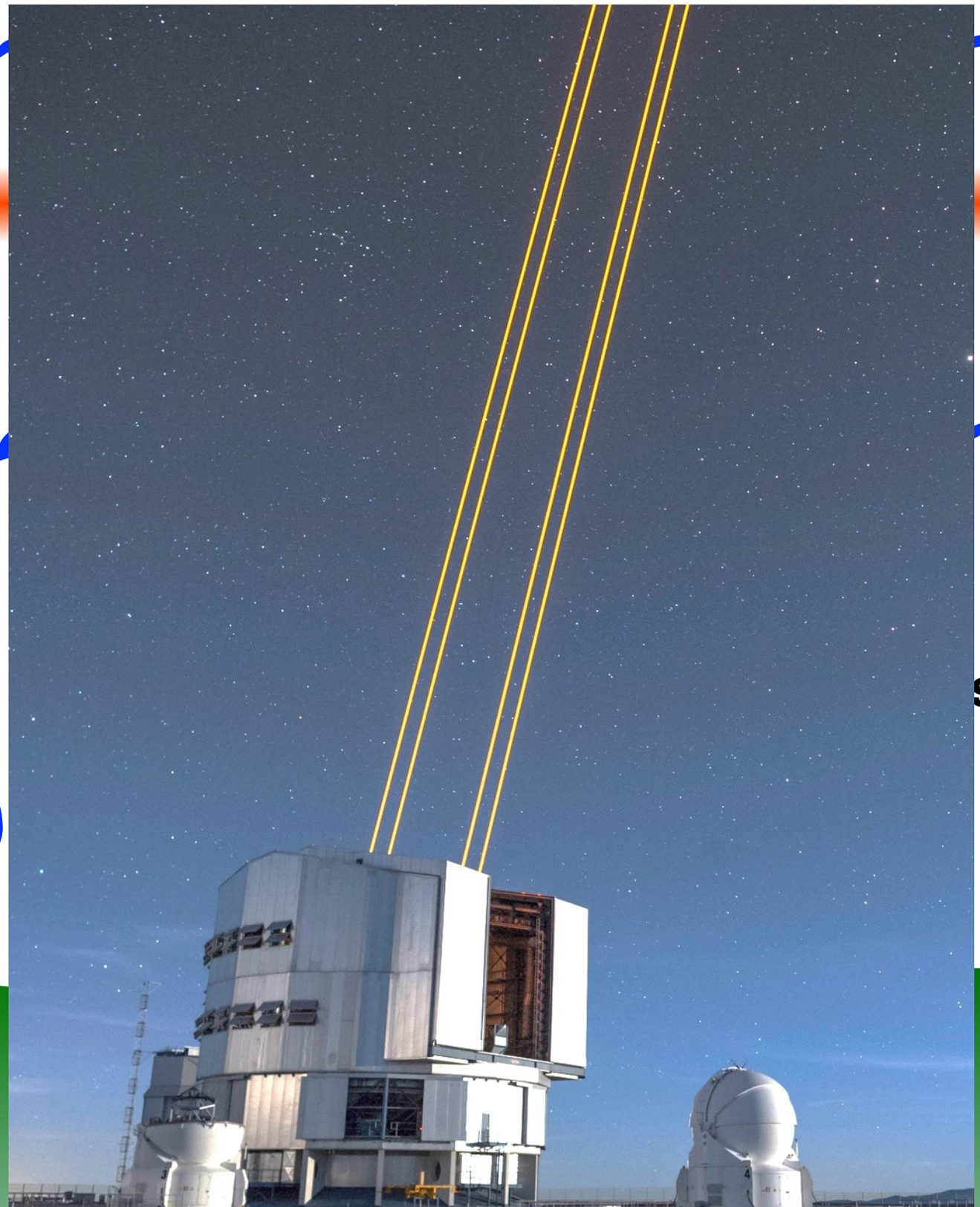


Sodium Laser Guide Star

Jet Stream

Mid-level

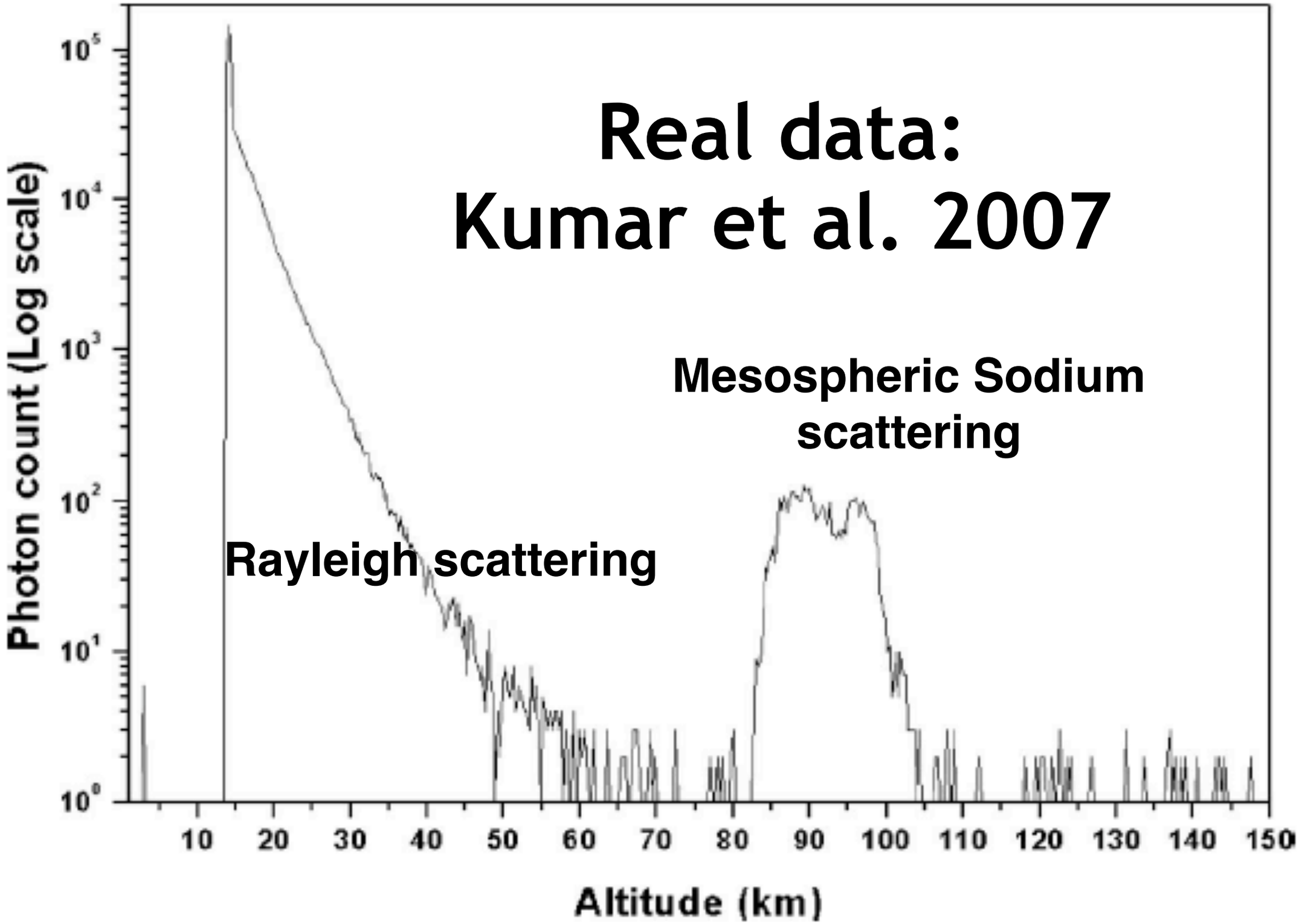
Ground Layer



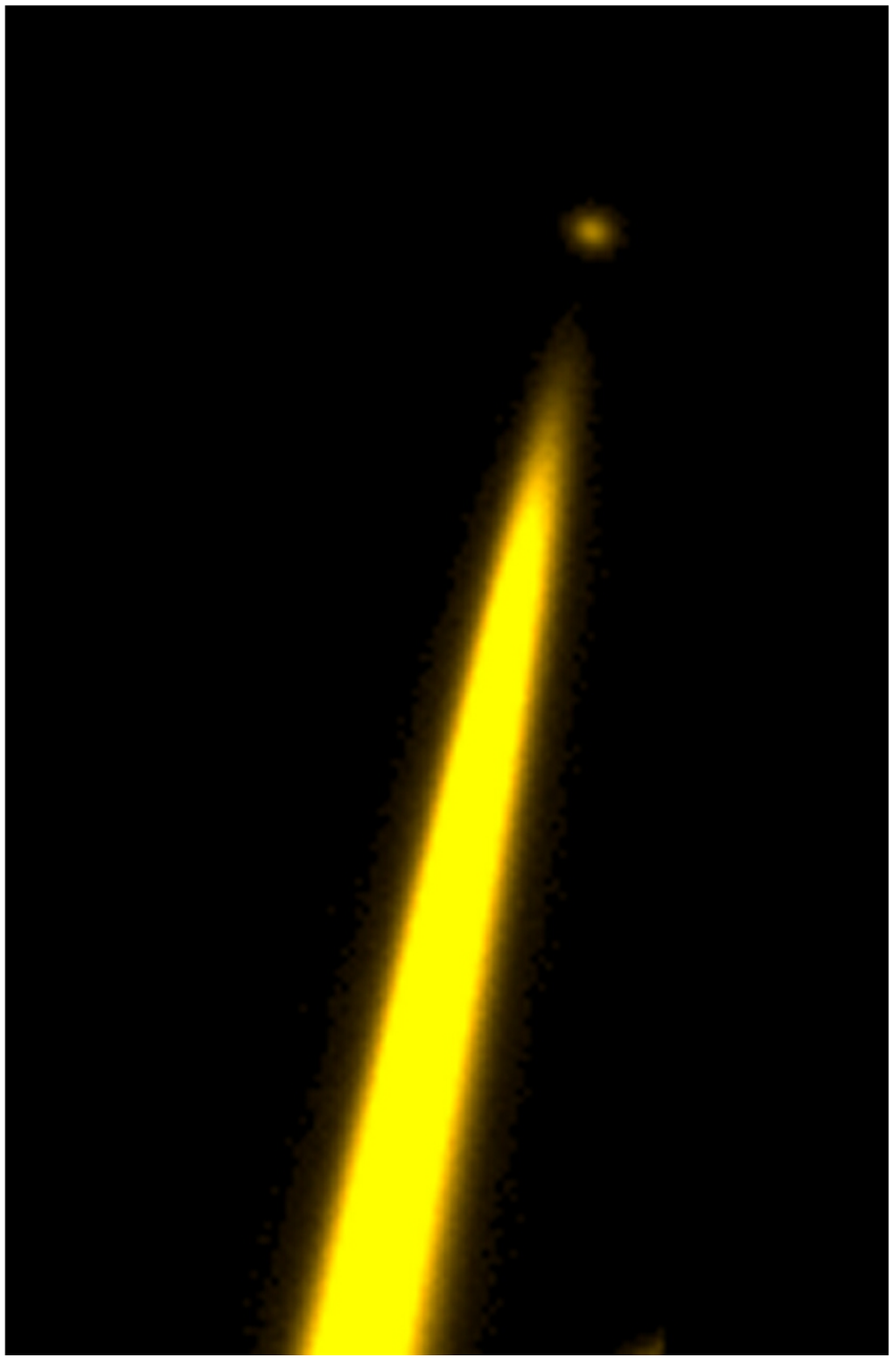
70-80 km

sensed turbulence

Rayleigh goes as exponential ATM pressure



Looking up at Na LGS at Keck

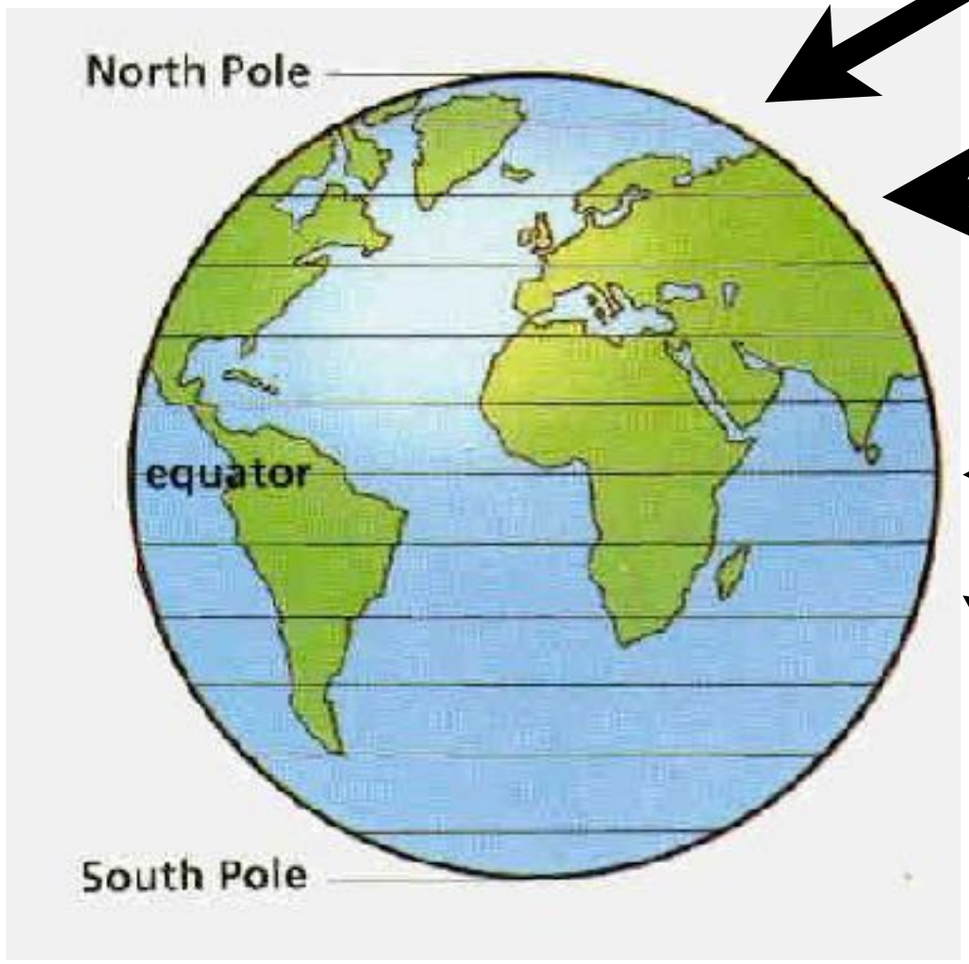


Mesospheric Sodium scattering

Rayleigh scattering

Credit: Claire Max at CfAO

Na varies with season and location....



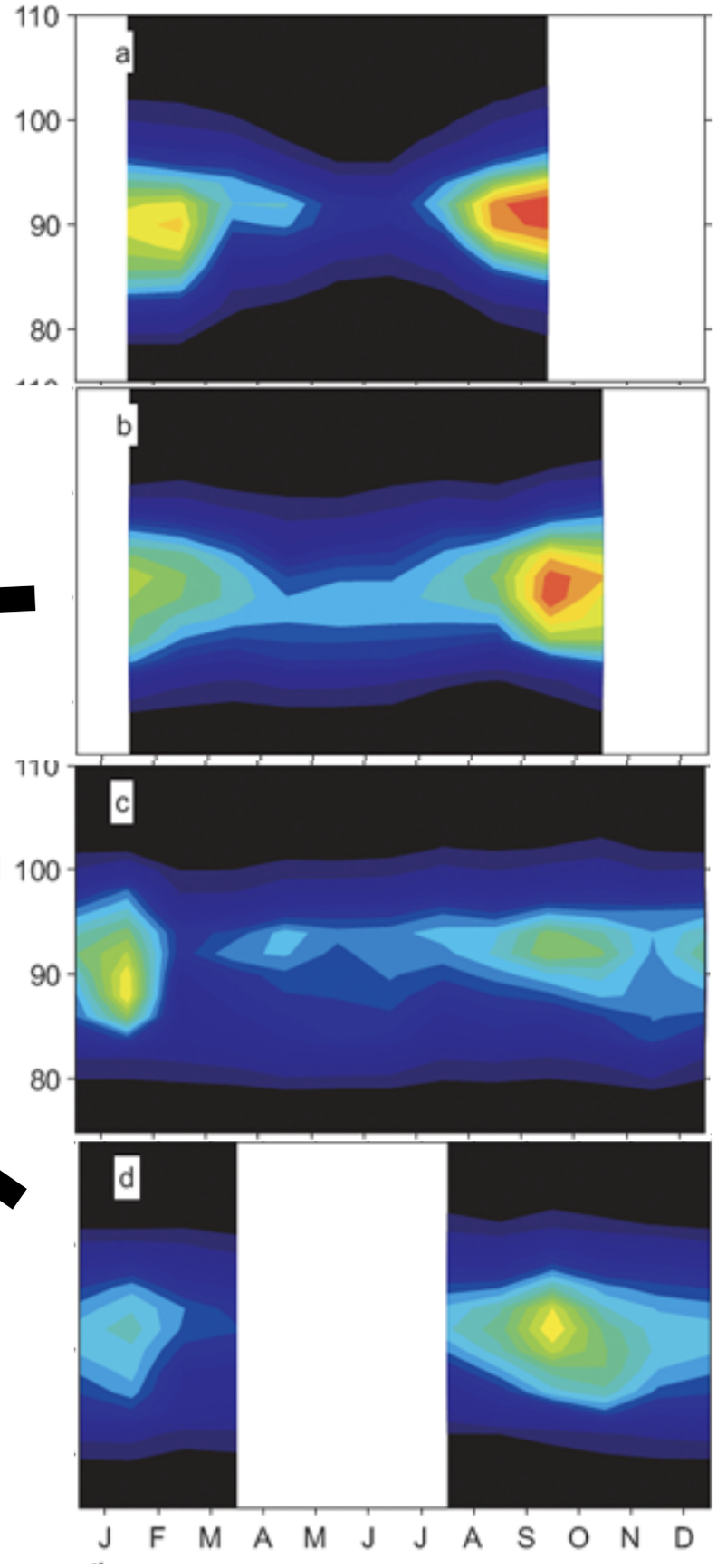


Fig. 3. Seasonal variation of the zonally- averaged Na density profile (units: atom cm^{-3}) at four latitude bands centred at **(a)** 70° N, **(b)** 40° N, **(c)** the equator, and **(d)** 20° S.

Satellite measurements of the global mesospheric sodium layer

Z. Y. Fan[1], J. M. C. Plane[2], J. Gumbel[3], J. Stegman[3], and E. J. Llewellyn[4]

Credit: Claire Max at CfAO

...and on smaller timescales

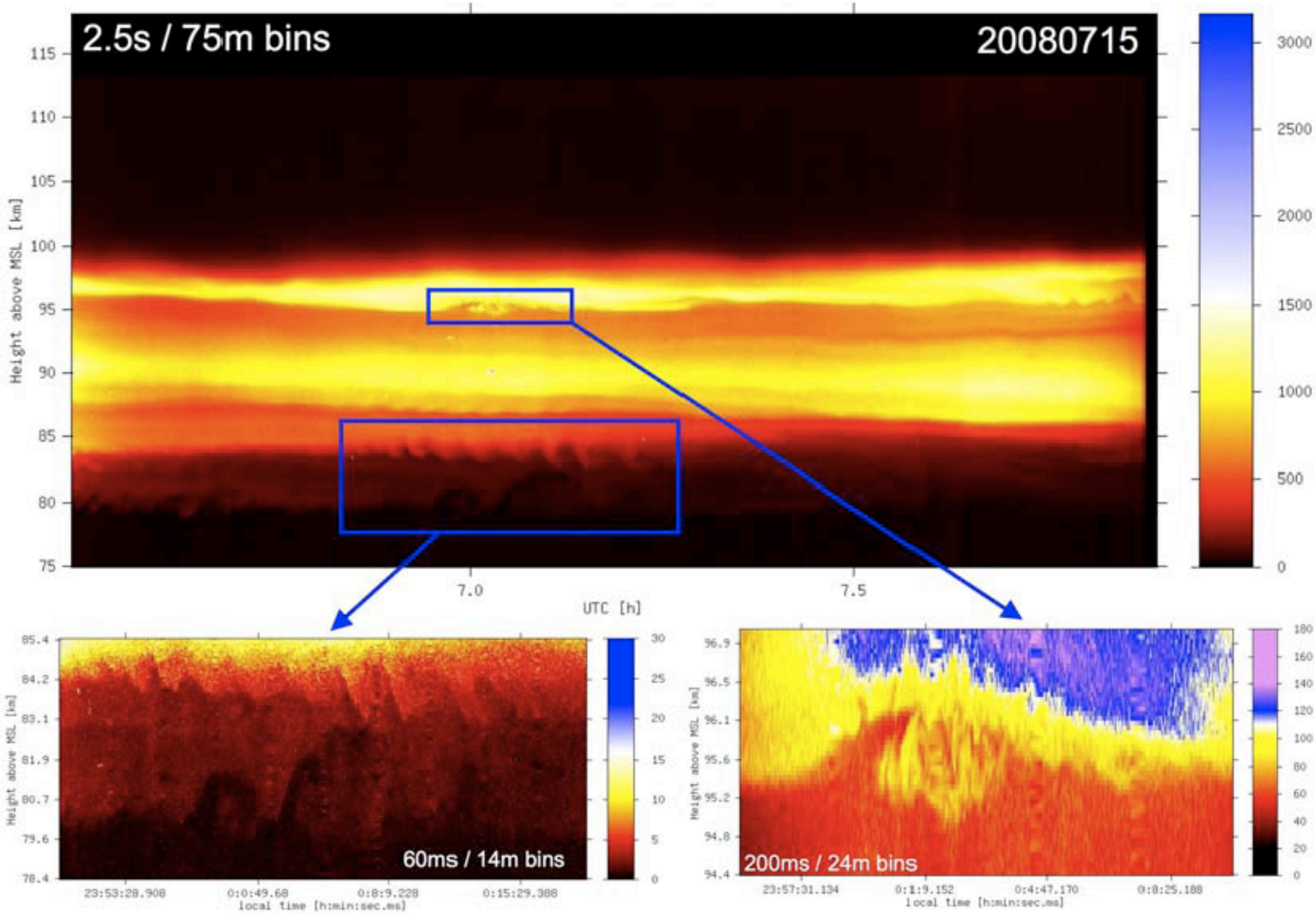
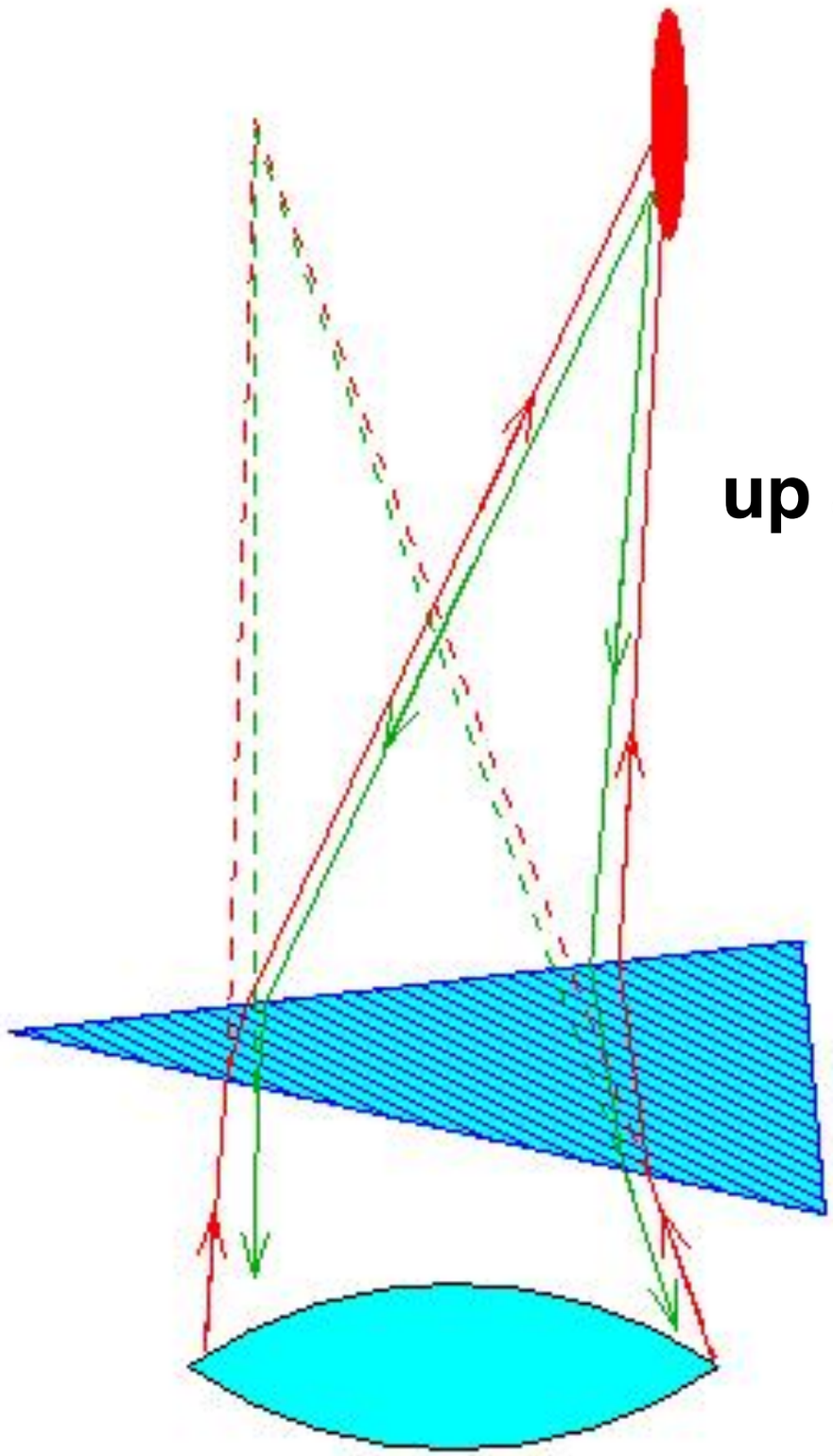


Fig. 1. Sodium layer density map with data from July 15th 2008. Color coded is the number of returned photons per bin (2.5 s / 75 m).

From Pfrommer et al. Credit: Claire Max at CfAO

LGS still needs a Tip Tilt NGS

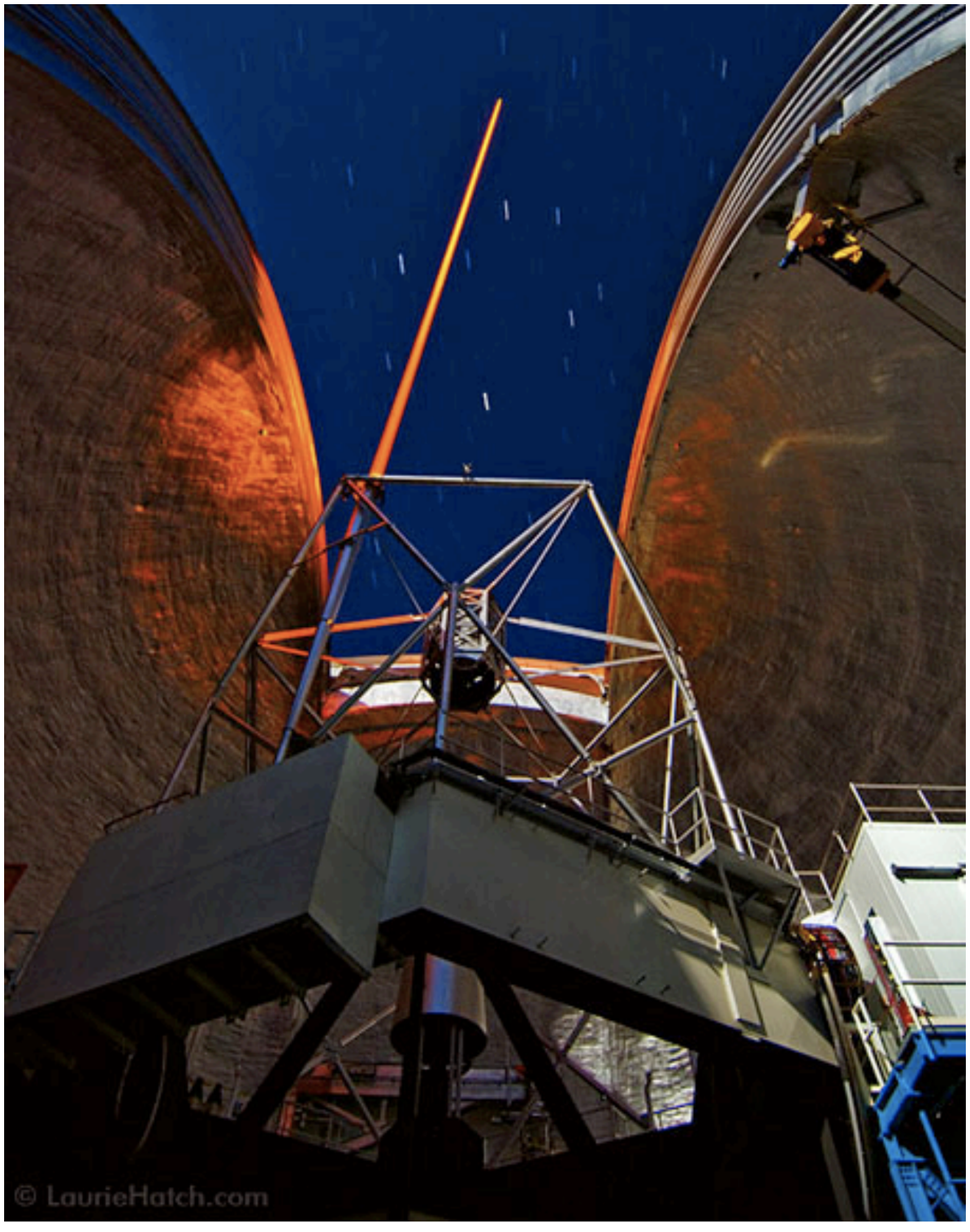


TT is the same going up and going down for LGS

Only TT sensing needed, so guide star can be fainter

Credit: Tokovinin / Claire Max at CfAO

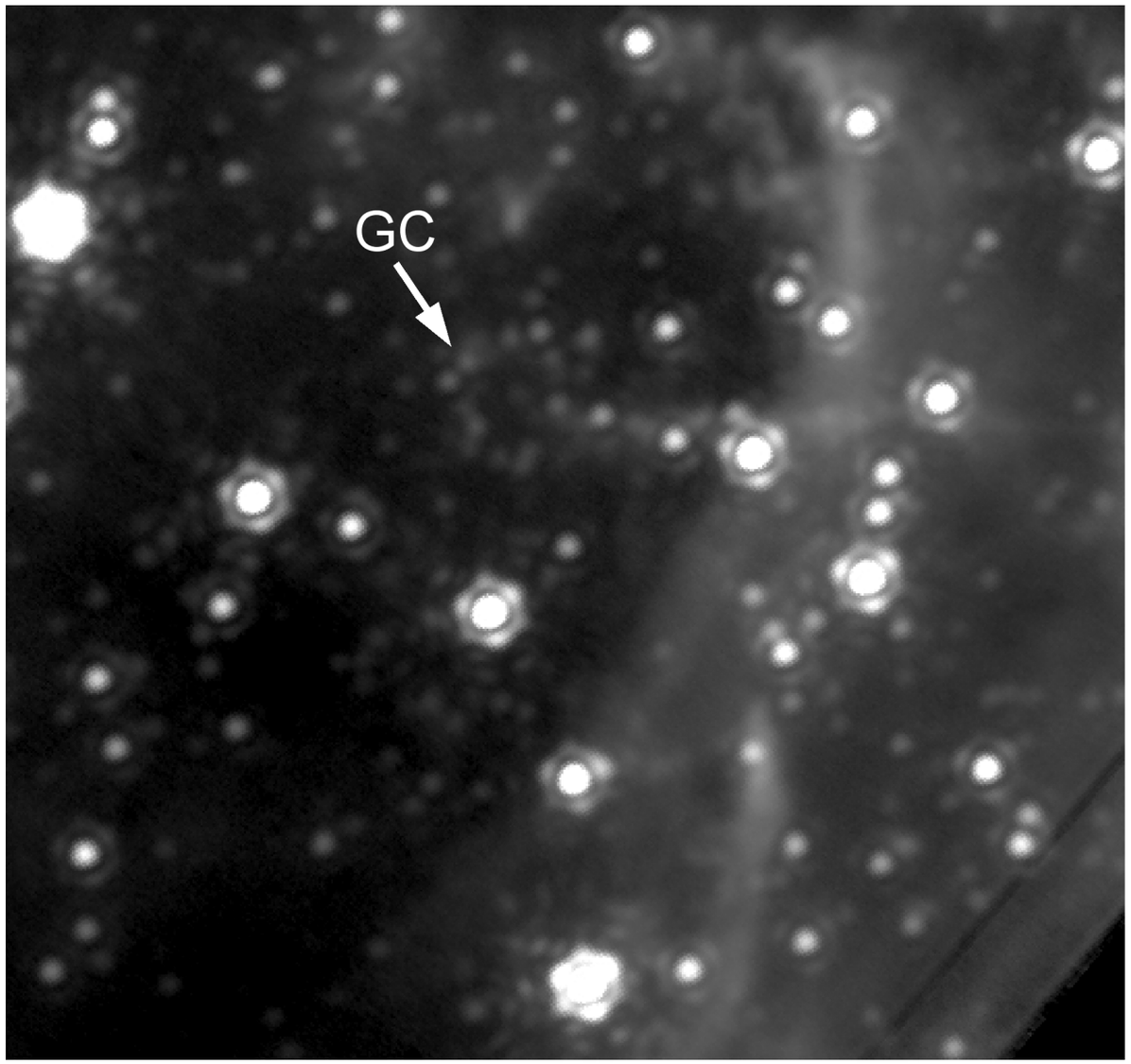
Keck Observatory LGS



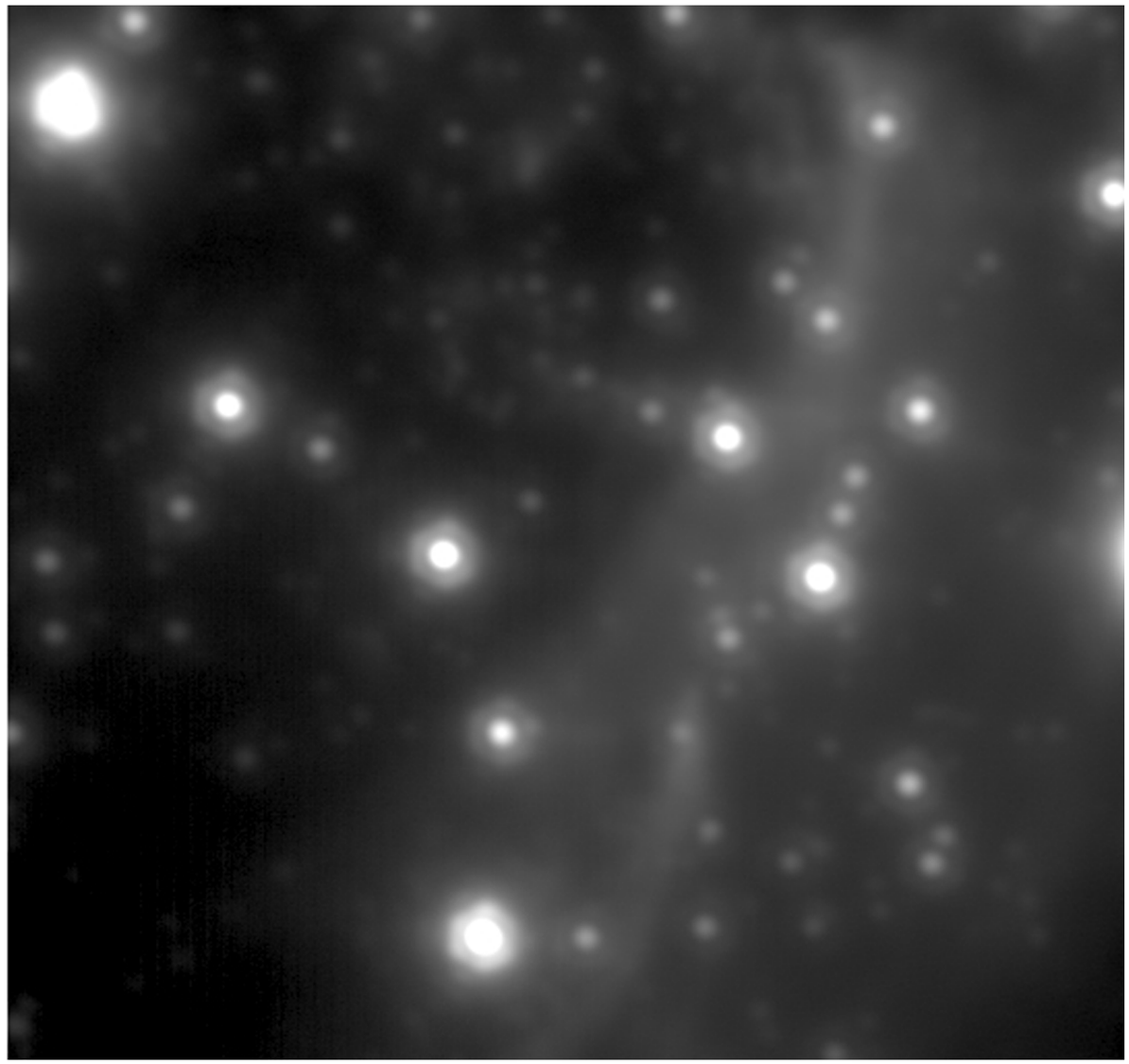
Credit: Claire Max at CfAO

Keck LGS Science of the Galactic Centre

LGS



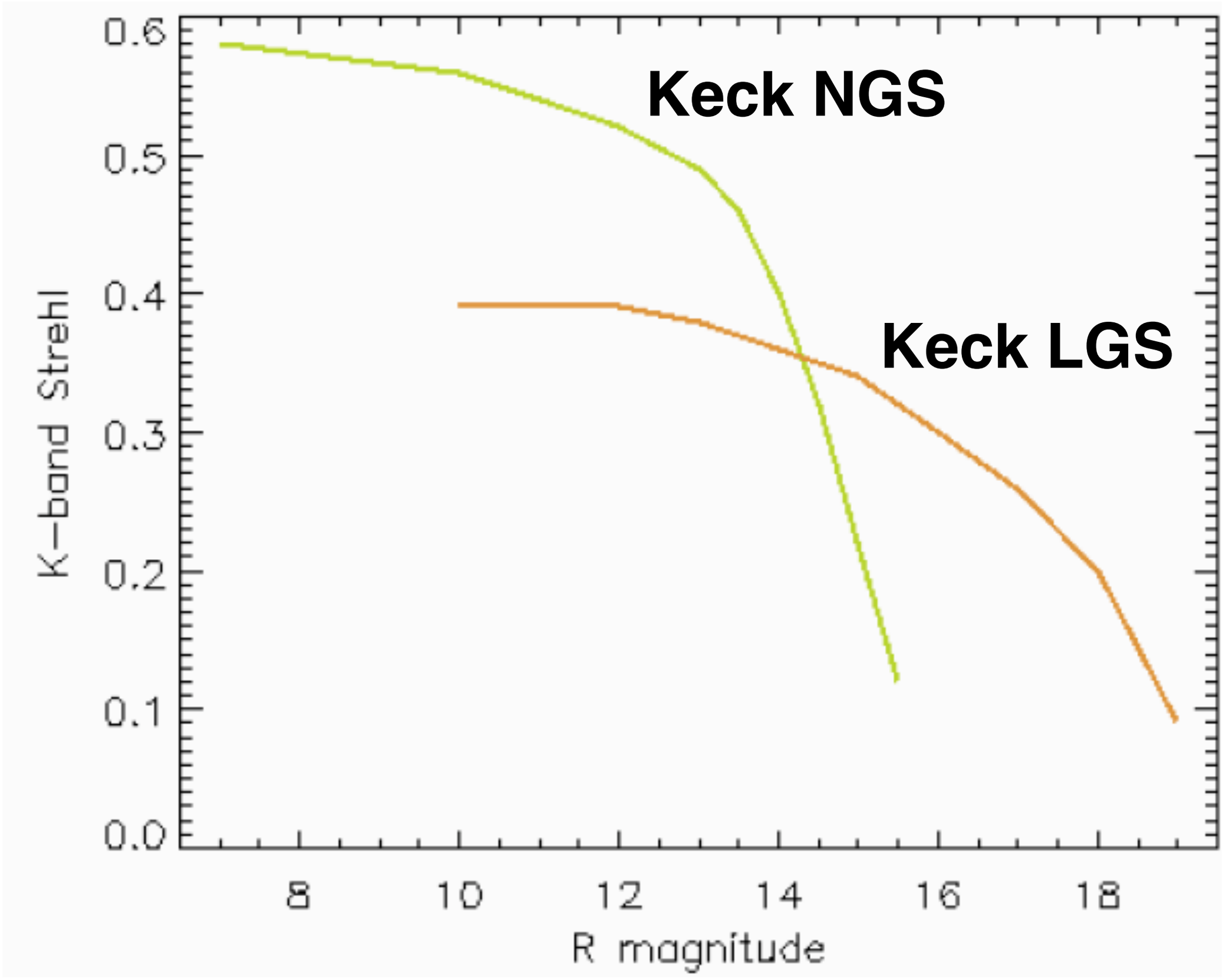
Best NGS



Andrea Ghez Group at UCLA

Credit: Claire Max at CfAO

Keck AO Performance



MMT Rayleigh LGS

Slides: Michael Hart, Steward Observatory

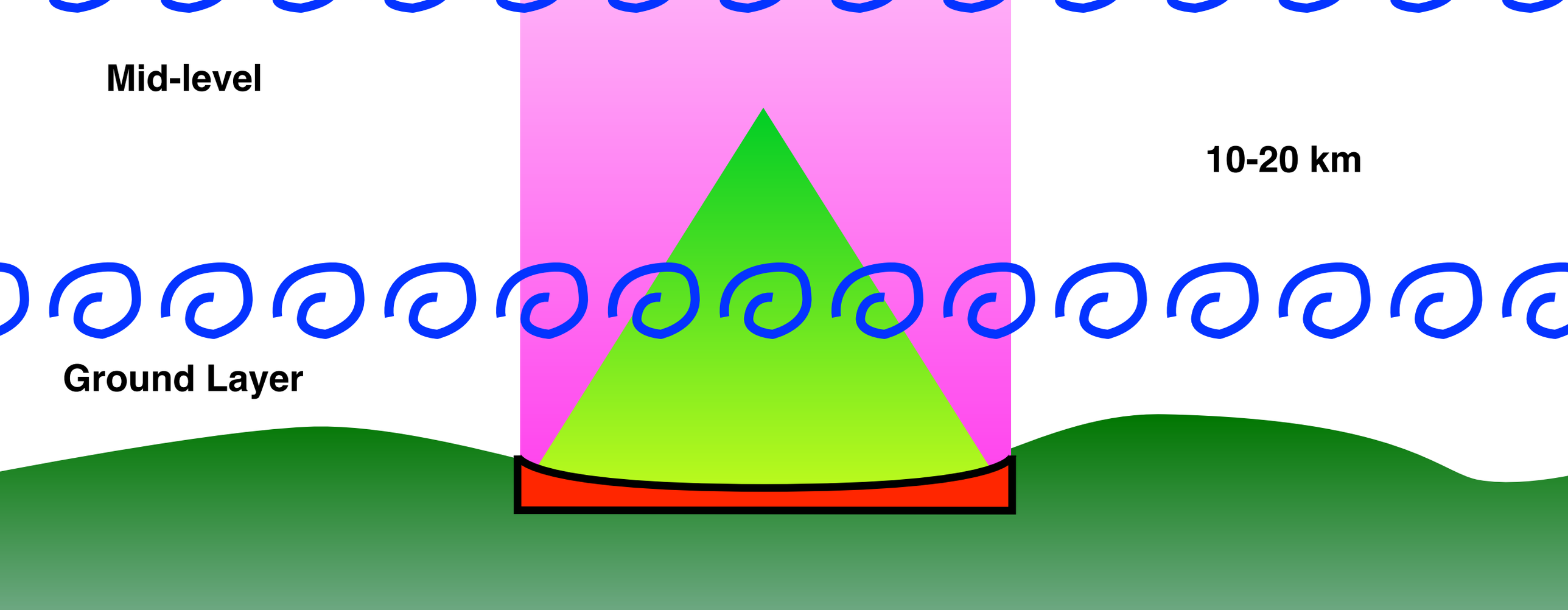
Rayleigh Laser Guide Star

Jet Stream

Mid-level

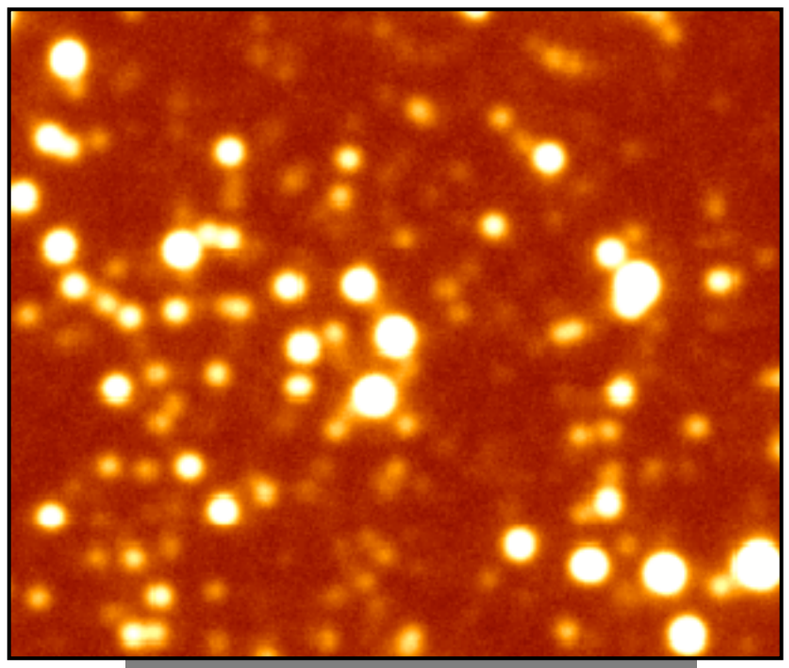
10-20 km

Ground Layer

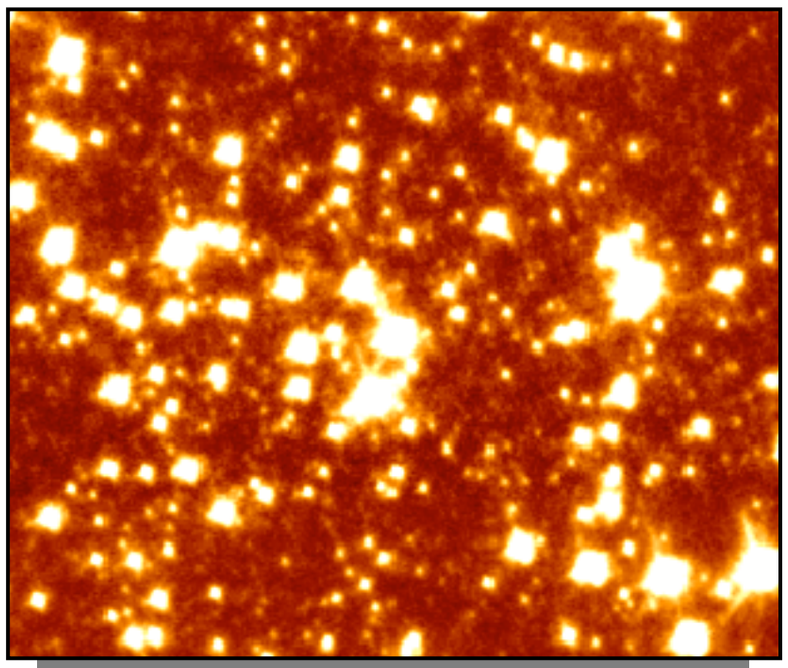


Multiple Rayleigh Laser Guide Stars

MMTO 6.5m GLAO System

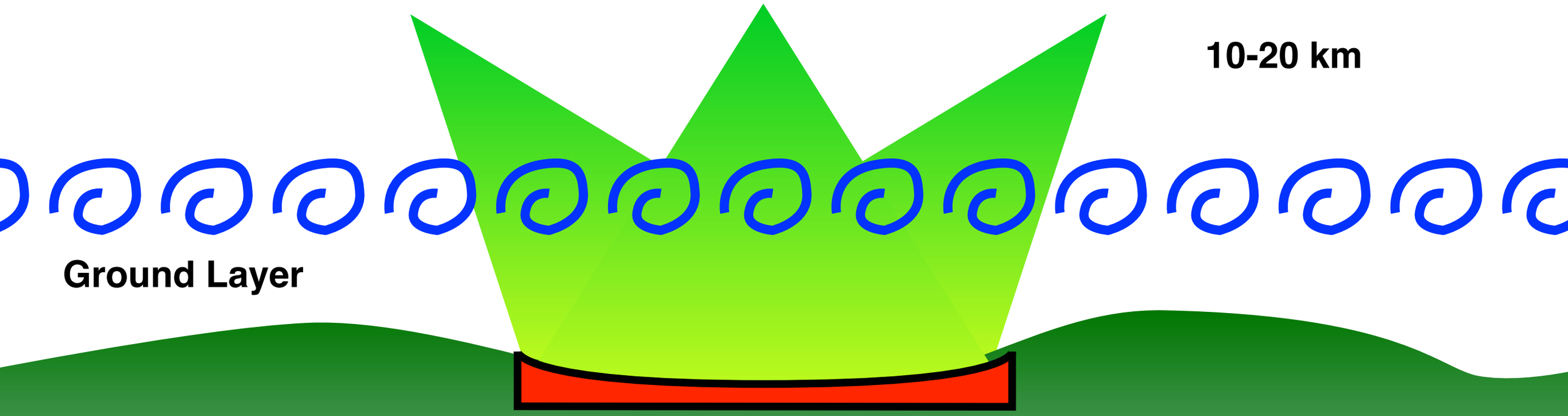


Native seeing: 0.70”

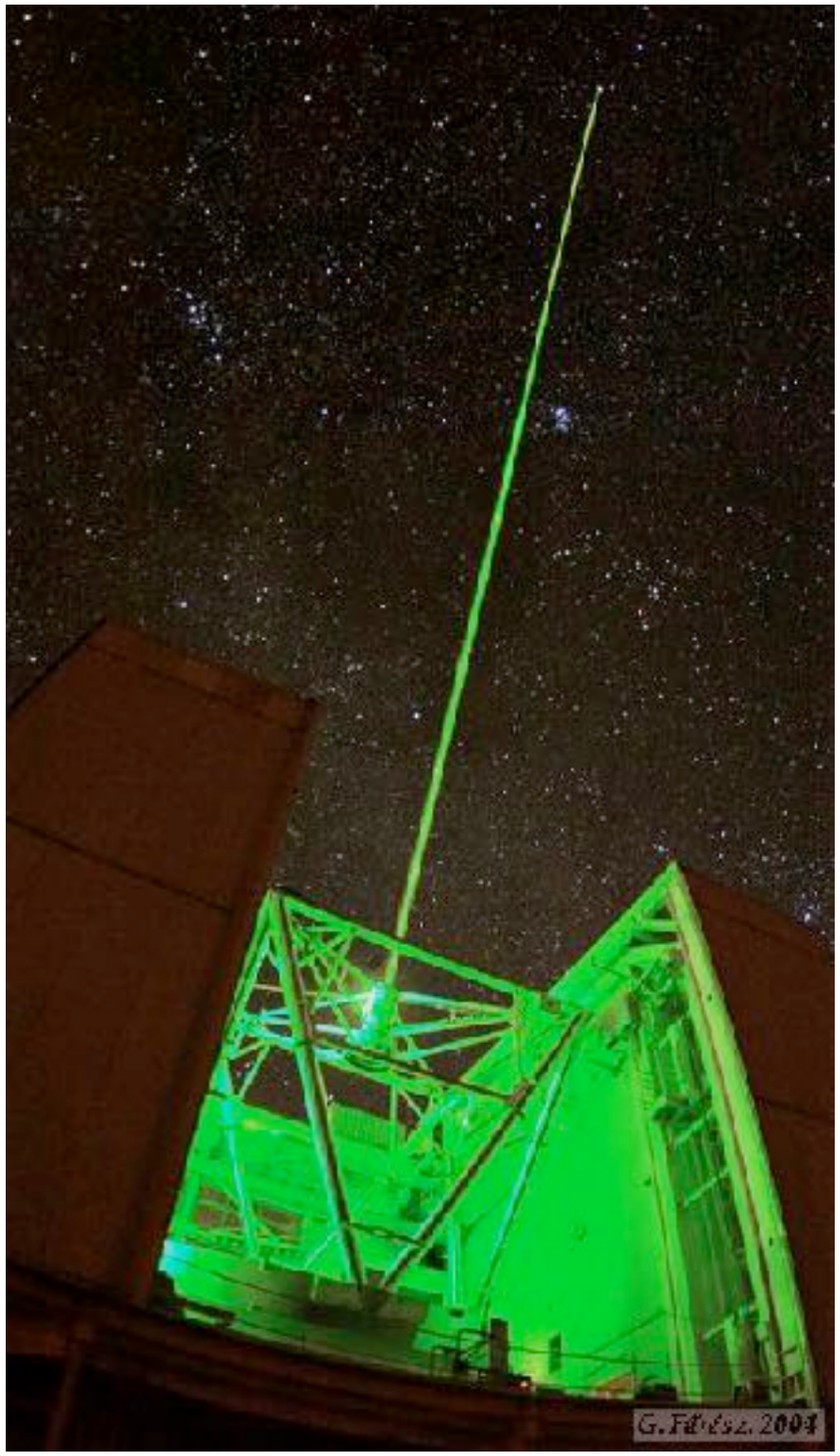


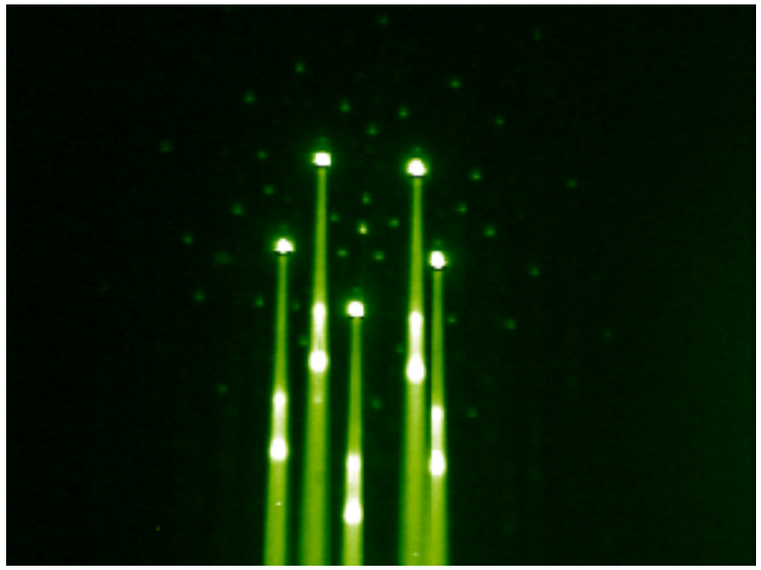
GLAO image width: 0.30”

Globular Cluster M3 at K band



Five lasers on the sky

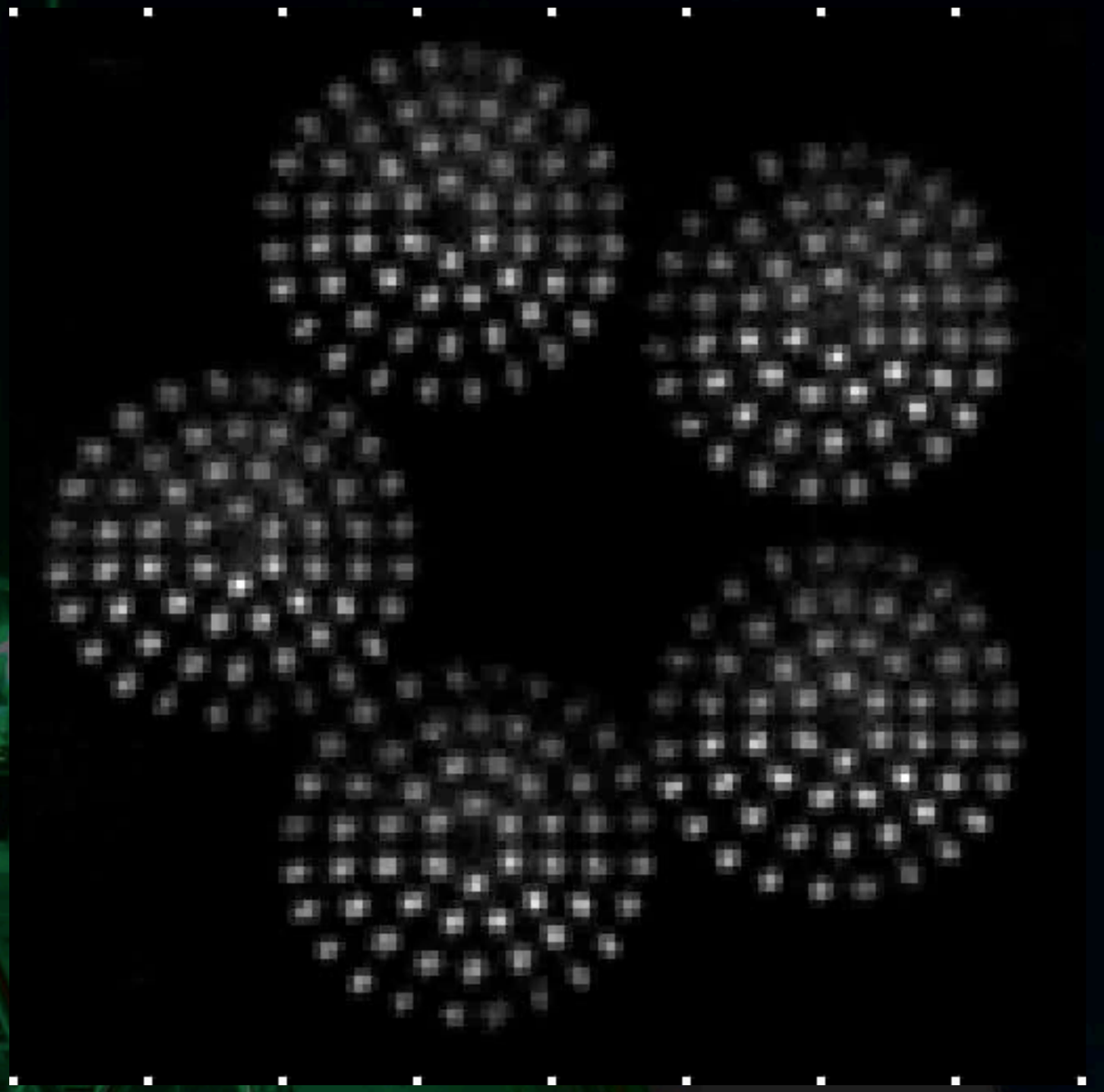




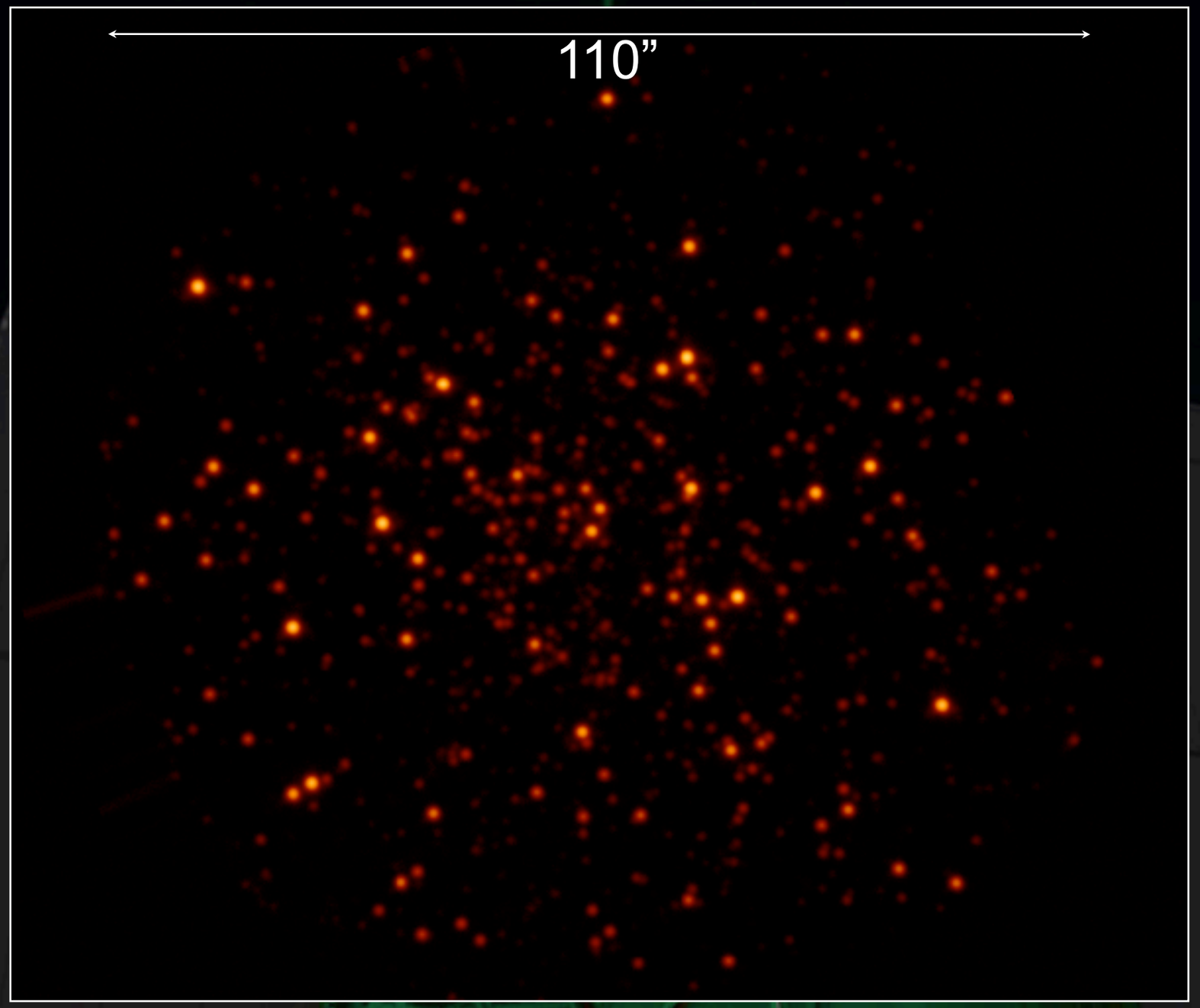
Laser type	2 x doubled YAG (15 W each)
Wavelength	532 nm
Pulse rep rate	5.2 kHz
Average power	30 W
Launch telescope location	Behind secondary mirror
Number of beacons	5, arranged as a regular pentagon
Enclosed field of view	2 arcminutes
Beacon type	Rayleigh scattering
Range gate	20-29 km with dynamic refocusing

Dynamic refocus in operation

- The lasers are pulsed at 5 kHz
- Each laser pulse is tracked as it rises through the atmosphere by refocusing the telescope *very* fast
- If we didn't do that, the pulses would appear on the wavefront sensor as streaks, and all useful information would be lost



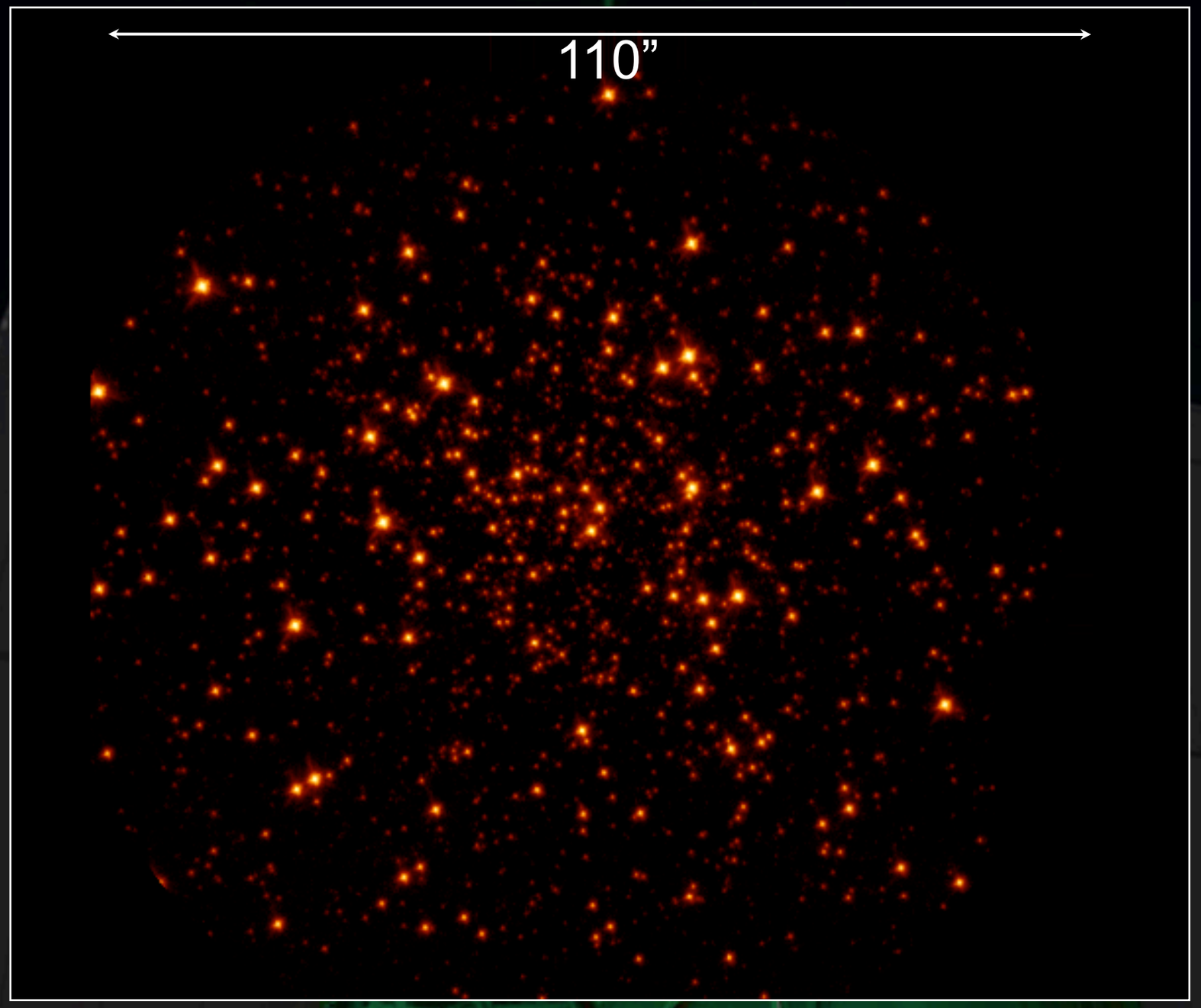
MMT results: M3



Open loop, 2.2 μm filter, seeing 0.70”

Logarithmic scale

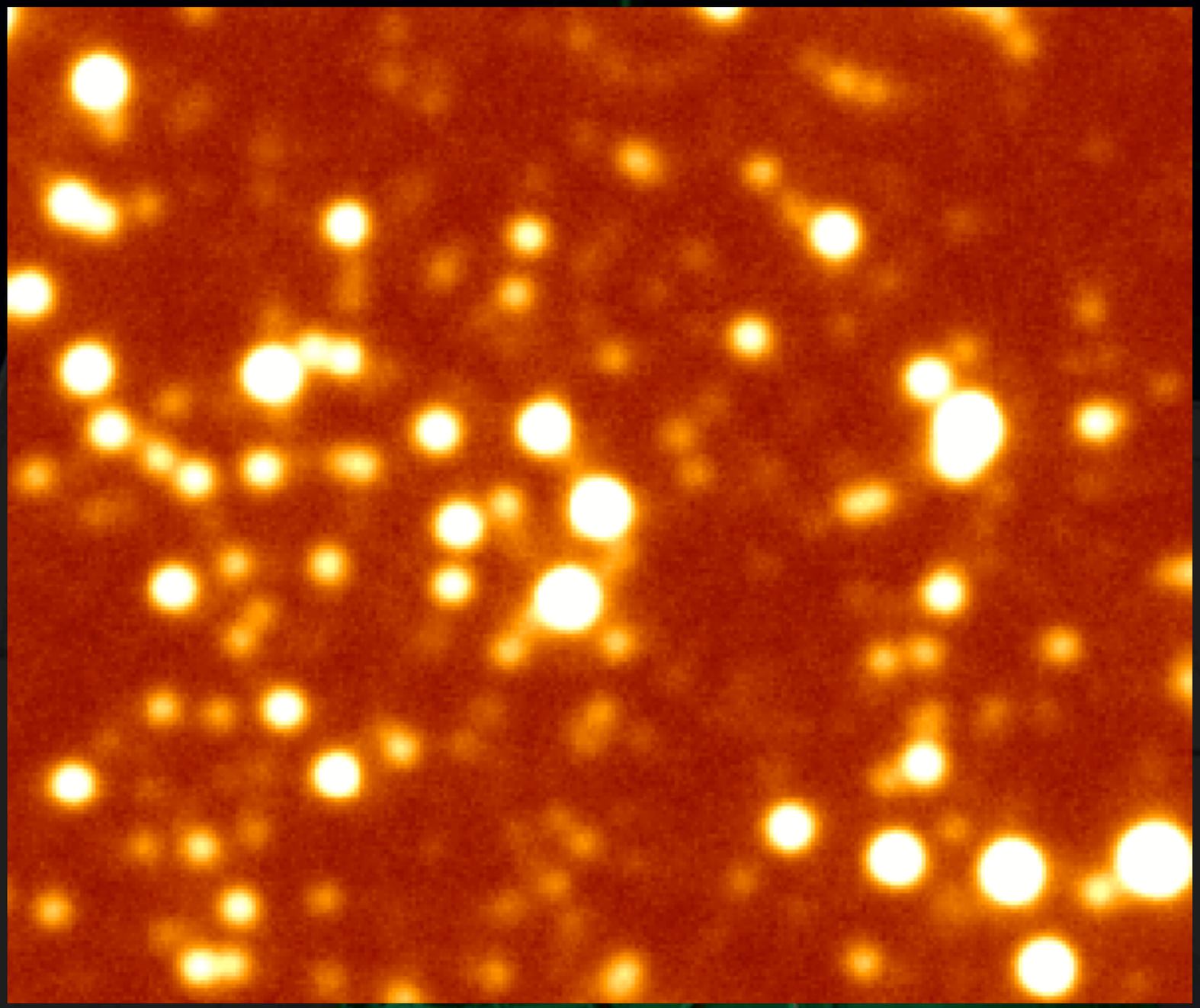
MMT results: M3



Closed loop, 2.2 μm filter, seeing 0.30”

Logarithmic scale

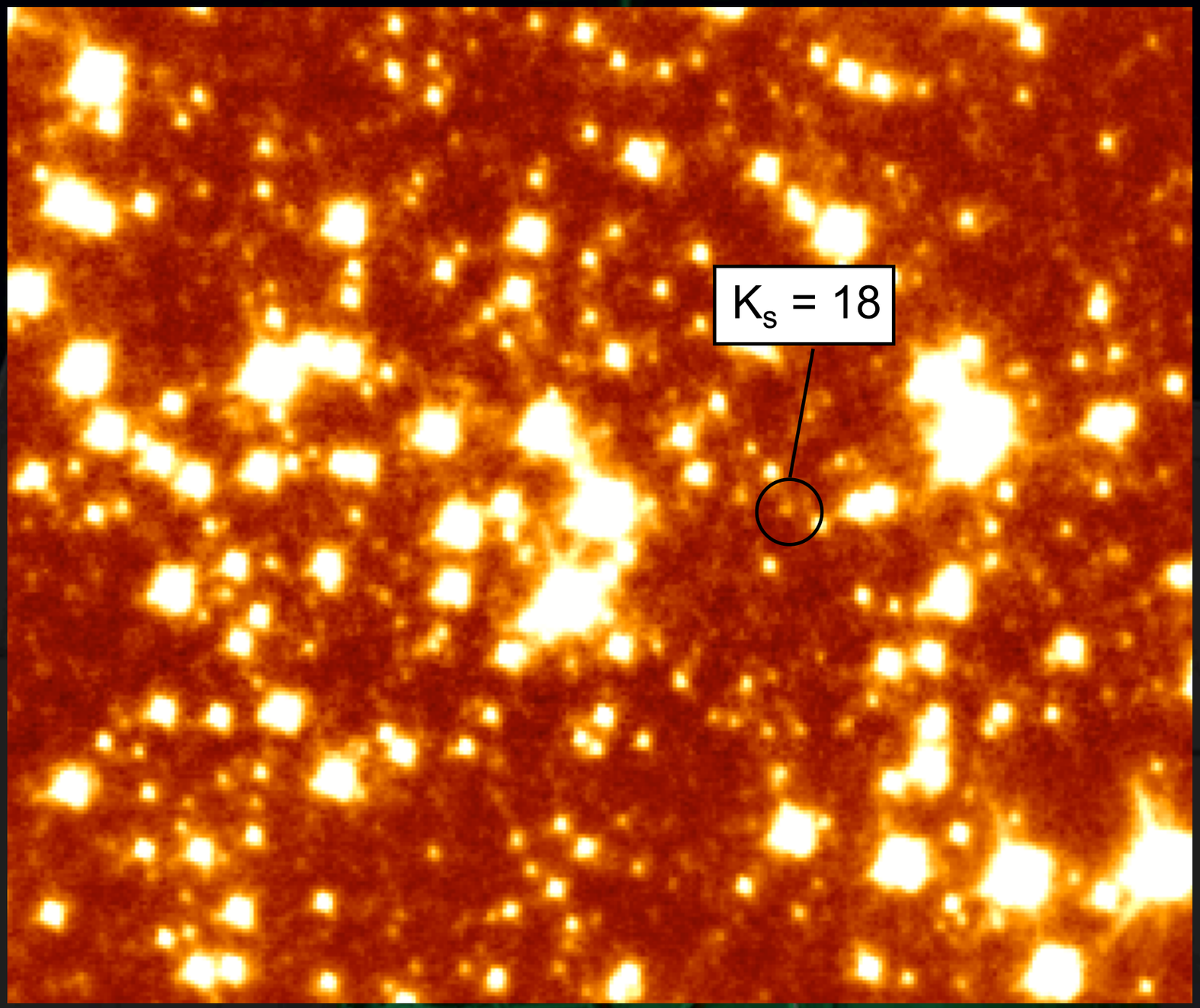
MMT results: M3 zoomed in



Open loop, 2.2 μm filter, seeing 0.70”

Linear scale

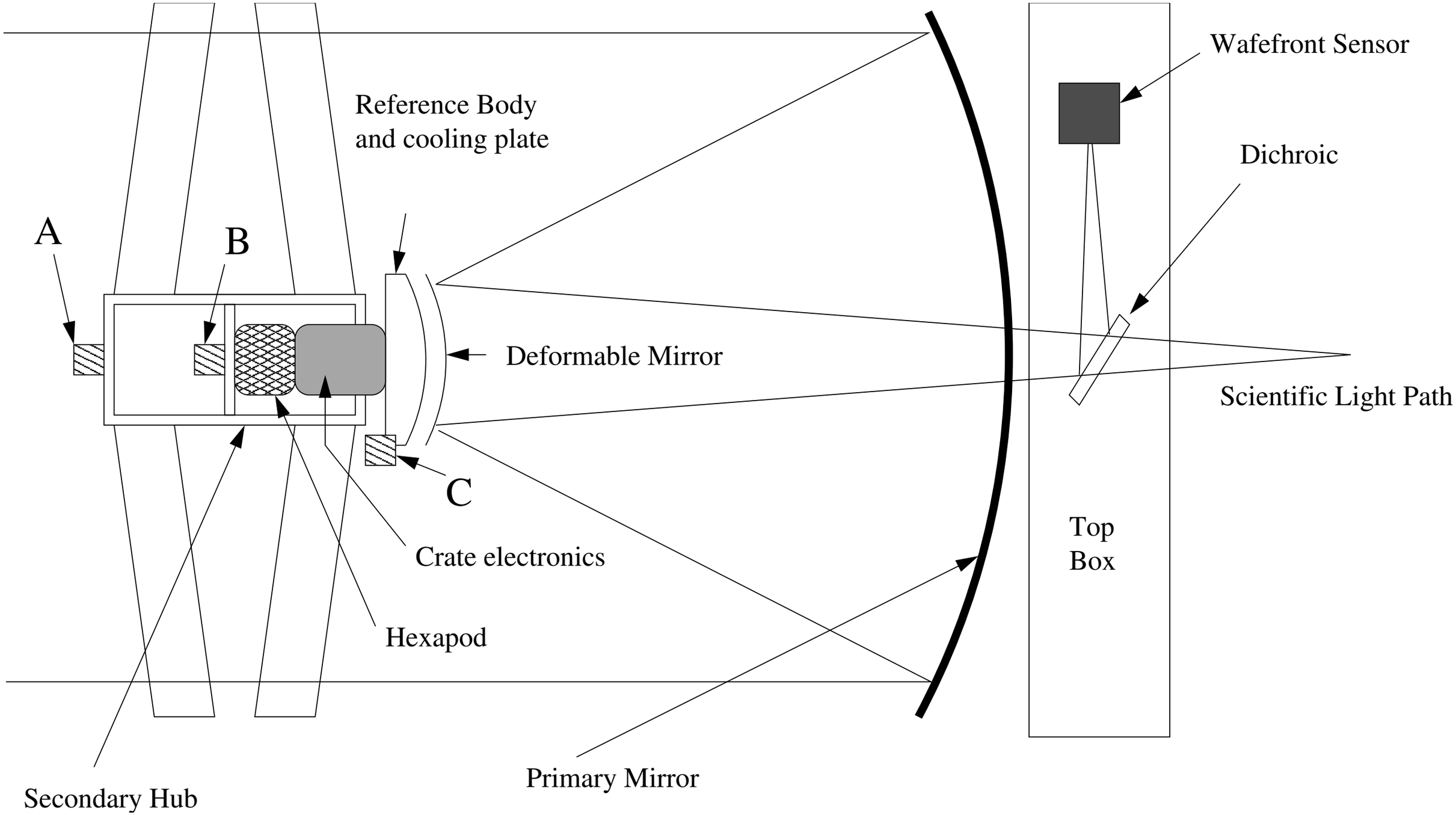
MMT results: M3 zoomed in



Closed loop, 2.2 μm filter, seeing 0.30”

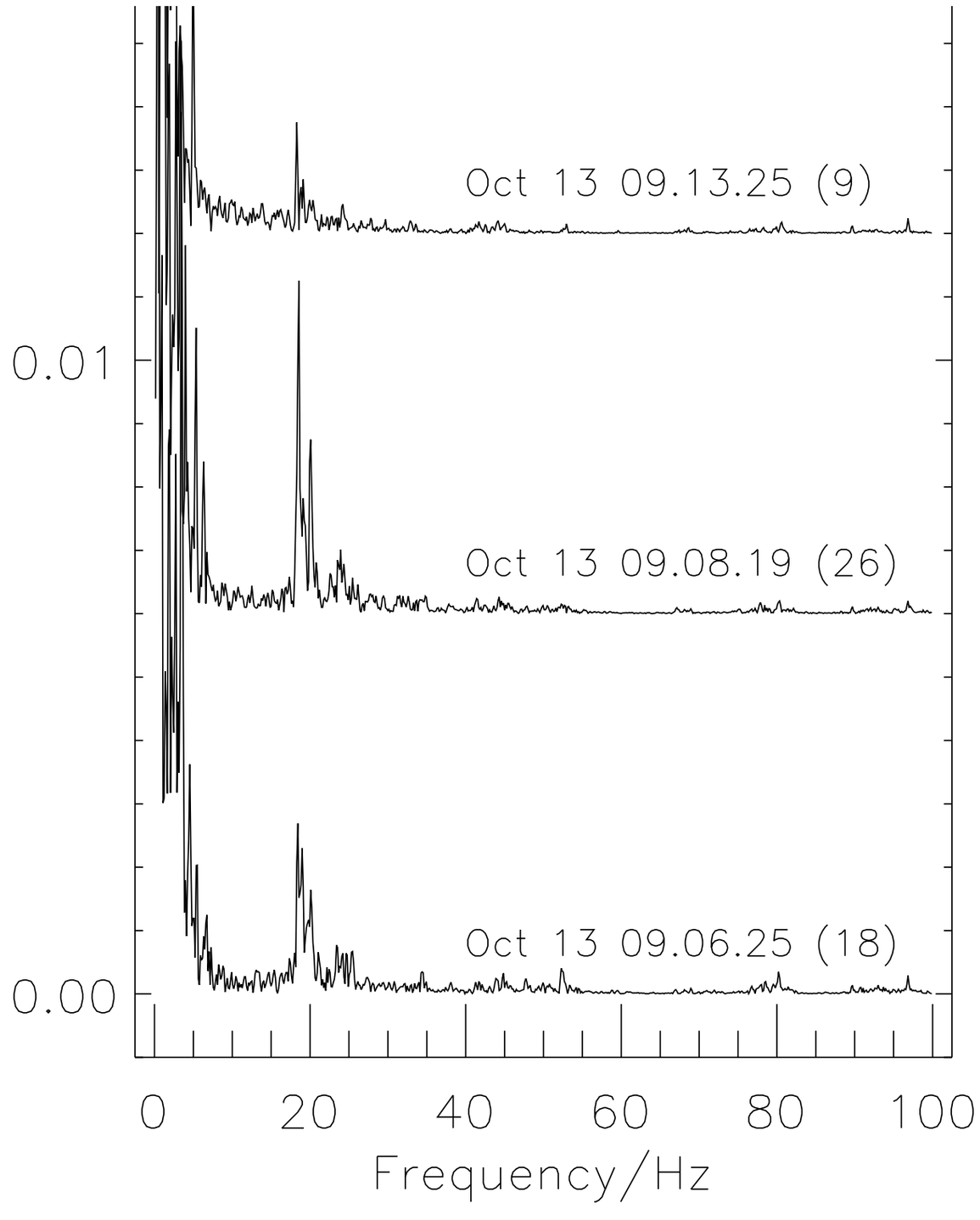
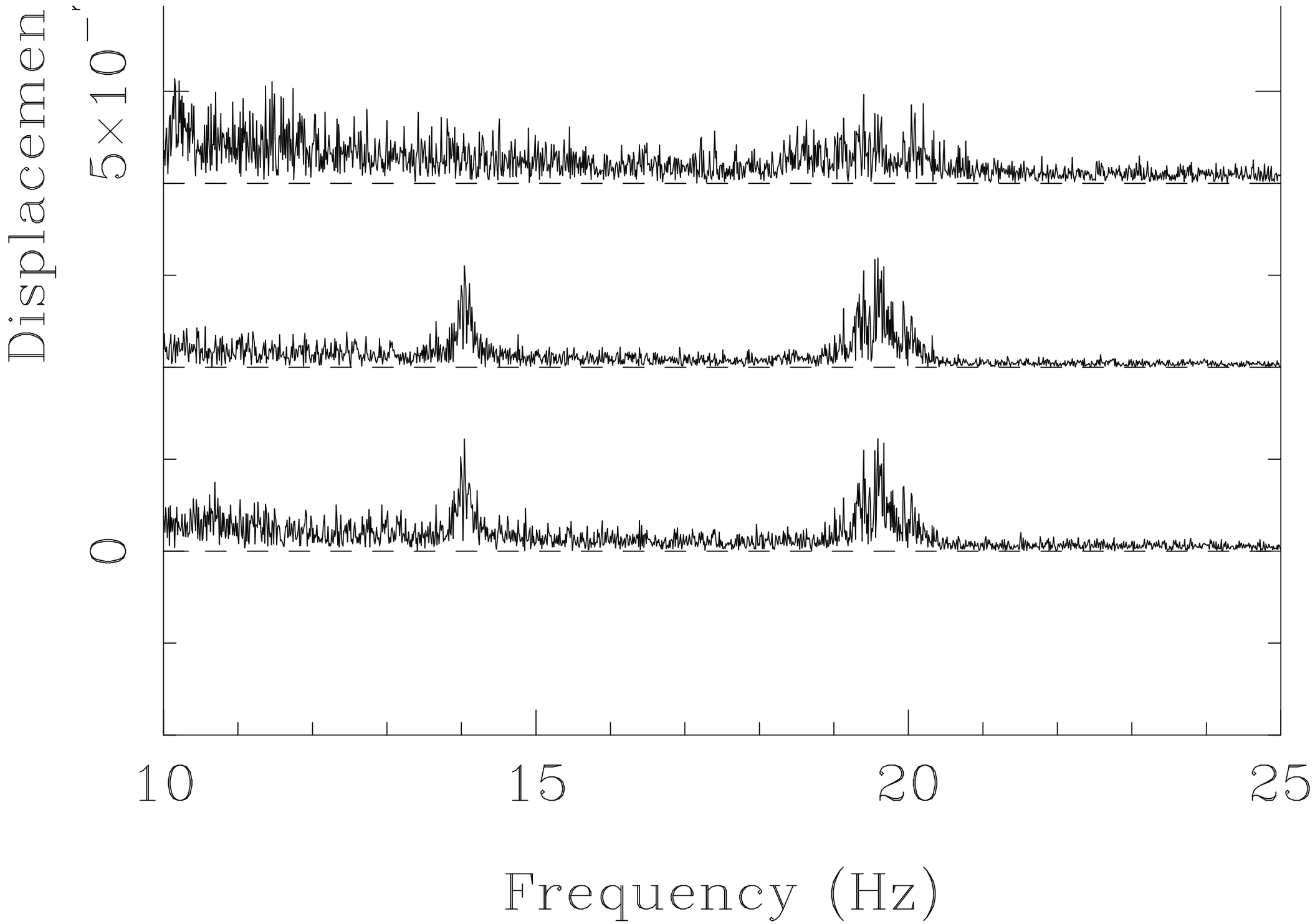
Linear scale

Telescope Vibrations



Telescope Vibrations

Arcseconds of vibration amplitude



Telescope Vibrations