The purpose of this document is to answer two questions: (i) is the recent increase in atmospheric CO_2 likely to disturb the planetary heat flow? (ii) How much of the increase is likely due to human activity?

1 Expected temperature rise from CO_2 increase

Caveats: this is purely an approximate, 'back of the envelope' calculation. Besides the fact that it is a simplistic treatment of a complicated problem in radiative transfer, among the factors I have ignored are the effect of clouds, and the nonlinear relation between the amount of CO_2 and its fractional absorption. There are many other details glossed over. An estimate of this nature cannot be expected to return accurate numbers, but it is nevertheless a valuable way to estimate the approximate size of the effect. A detailed and correct treatment might change the numbers by a factor of 2 but would not be expected to change them by a factor of 10 or more.

The starting point is the fact that, at equilibrium, the energy coming into the planet must equal the energy flowing out. This is simple energy bookkeeping. The energy coming in consists, to excellent approximation, almost entirely of the electromagnetic radiation from the sun. This is distributed across the frequency range in a nearly black-body spectrum with a maximum at about 500 nm, i.e. in the range visible to our eyes. The amount coming in is reasonably constant, varying cyclically over the year by a few percent, and somewhat less regularly over the 11-year solar cycle, also by a few percent. The amount reaching the earth's surface depends on the amount of cloud cover, something which is difficult to estimate a priori. I will assume a constant value ϕ for the energy flux reaching the earth's surface.

The heat escaping the earth is again radiative in nature and its distribution again approximates that of a black body but with a maximum in the infrared (IR), at about 10 μm. It is important to realize that the atmosphere is not transparent at IR wavelengths: many of its constituent molecules have absorption bands in the IR, most importantly water vapour, followed by CO_2. There is a simple relationship between the heat flux and the temperature of the earth which is known as the Stefan-Boltzmann law:

$$\phi = \epsilon \sigma T^4. \tag{1}$$

Here ϕ is the energy flux in W m^{-2}, which must equal the constant amount coming in from the sun for the books to balance; ϵ is effectively the transparency of the intervening atmosphere; σ is a constant; and T is the temperature in units of kelvin, which is degrees centigrade with 273.15 added.

An increase in the amount of CO_2 reduces ϵ. We first have to estimate by how much, and then we have to calculate what increase in T there has to be to offset the ϵ decrease so as to maintain ϕ constant.

The fractional increase in CO_2 since pre-industrial times is about 0.4. The fractional contribution of CO_2 to atmospheric absorption varies because the concentration of the major contributor, water vapour, varies strongly with location and condition, but may be taken to be roughly 0.1. Multiplying these two numbers gives 0.04 for the fractional decrease in the transparency ϵ, assuming the relation between transparency and CO_2 concentration to be linear; however since some of the CO_2 lines are saturated, the relationship is actually far from linear, and we would therefore expect the effect on ϵ to be reduced. Adopting an arbitrary fudge-factor of 2, we arrive at $\Delta\epsilon/\epsilon = -0.02$ (the negative value indicates a decrease).

We don't actually have to evaluate ϕ etc, we can use some calculus to avoid that. Differentiating equation 1 for ϵ with respect to T, setting $d\epsilon \sim \Delta\epsilon$ etc and finally dividing the result by equation 1 gives us

$$\frac{\Delta T}{T} \sim -0.25 \frac{\Delta \epsilon}{\epsilon}.$$

The approximation is accurate for small fractional values. This gives, for a 40% increase in CO_2, a temperature increase of 1.5 degrees C; which, coincidentally, is almost the exact amount observed.

Once again, this is not meant to be accurate, merely to provide a wet-finger answer to the question, can we reasonably expect 40% more CO_2 (and rising steadily) to cause climatic changes? Answer: YES.

2 Comparison of human-produced CO_2

Here I am going to compare three figures: the amount of extra CO_2 in the atmosphere, the amount of world coal production, and an estimate for the amount of CO_2 produced by the world's population. I'll calculate each number in moles of CO_2 (a mole is about 6×10^{23}), and each as a total per year, using 2016 figures for everything.

Useful numbers to aid in converting between moles and kilograms are:

- The mass of CO_2 molecules is 44 g/mol. Let's call this m_{CO2} for short.
- The mass per mole of carbon atoms (m_C) is 12 g/mol.
- The average mass per mol m_{atm} of the atmosphere as a whole is about 29 g/mol.

2.1 Measured CO_2 increase

The current amount of CO_2 in the atmosphere is given as about 400 parts per million (ppm), as a fraction of the total number of molecules. The yearly increase in this fraction is currently about 1.7 ppm or 1.7×10^{-6}. To find the absolute number of molecules of CO_2 entering the atmosphere each year we need to multiply this fraction by the total number of moles of all atmospheric constituents. The mass of atmosphere over 1 square metre is about 10^4 kg; surface area of the earth is about 5×10^{14} m^2; this gives us a total atmospheric mass of 5×10^{18} kg. Dividing this by m_{atm} (converted to kg) and multiplying by 1.7×10^{-6} gives a yearly increase of about 3×10^{14} moles of CO_2. That's equivalent to 13 billion tonnes per year.

2.2 CO_2 from coal

Ok I looked up the world production of coal on Wikipedia. The number for 2016 is given as about 10 billion short tons, say 9×10^{12} kg. Dividing this by $m_C = 1.2 \times 10^{-2}$ kg/mol gives 7.5×10^{14} moles. Since each carbon atom, if burnt, will generate 1 molecule of CO_2, the number of moles of CO_2 produced per year is the same as the number of moles of carbon atoms burnt. *Note that it is more than double the atmospheric increase per year.*

2.3 Estimate of human CO_2 production

Here I am just going to do a very rough, independent calculation. I am going to take the population of the whole earth and allocate a 1 kilowatt power usage to each person. 1 kilowatt is not very much power - a typical Western household would consume more than that per person, a household in the developing world possibly less. It is just meant as a plausible order-of-magnitude number. 1 kW per person evaluates to about 2.2×10^{20} joules energy consumption by the world's population per year.

Say that 1 kW is produced by burning coal. Heat of formation of CO_2 is about 4×10^5 J/mol. Let's assume 50% generation efficiency, which will provide us with 2×10^5 J/mol. If we divide the total energy consumption by this number we get 1.0×10^{15} moles of CO_2 per year.

2.4 What does this tell us?

All three figures were in the range $1 - 10 \times 10^{14}$ moles of CO_2 per year. Is the entire atmospheric CO_2 increase due to human causes? This is impossible to say - there are many sources and sinks for CO_2 in the planetary system. The fact however that the amount of CO_2 we undoubtedly do produce is roughly equivalent to the increase we measure says in no uncertain terms that we must certainly be making a strong contribution to the total. We can add to this the circumstantial but compelling evidence that the giant leap in human production of CO_2 and the 40% increase we see in atmospheric CO_2 above the almost million-year stable norm *both happened in the last 300 years.*

3 Conclusion

Is it reasonable to be concerned about the effect human-produced CO_2 may be be having on the planetary heat balance? MOST CERTAINLY.