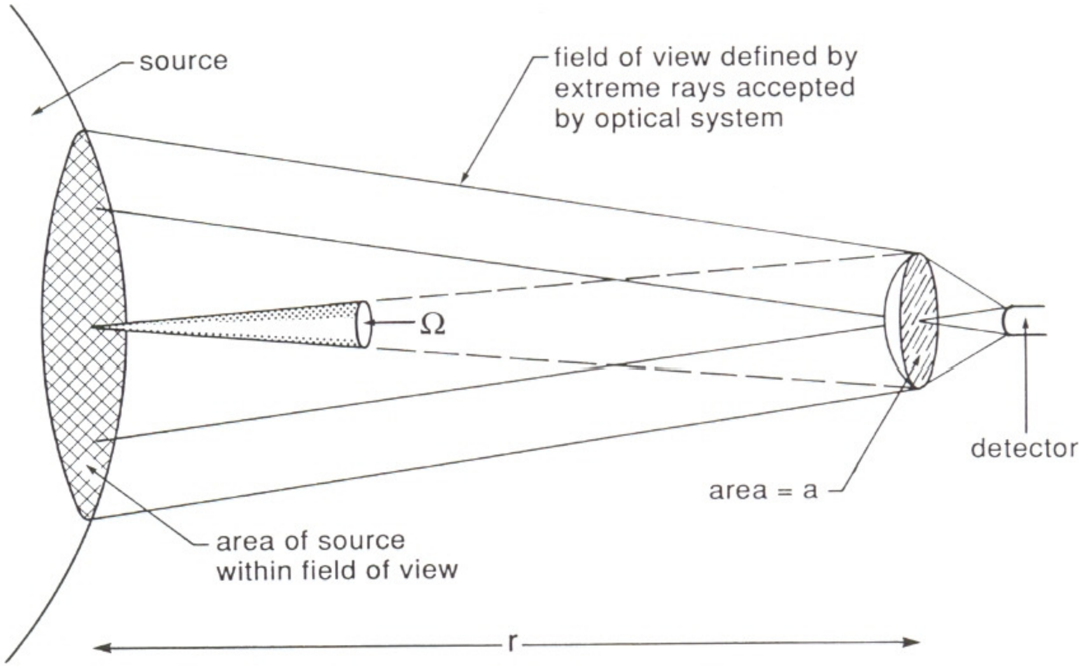
Astronomische Waarneemtechnieken (Astronomical Observing Techniques)

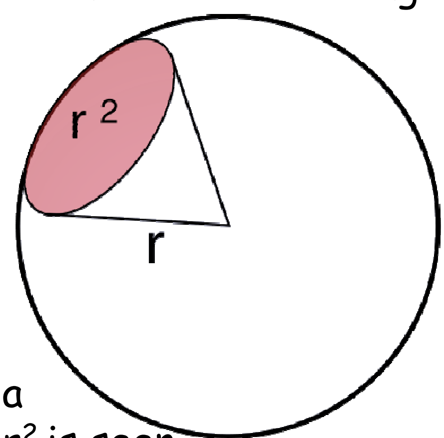
2^{nd} Lecture: 15 September 2010



Preface: Definition of the Angle

Babylonians: one degree = 1/360th of a full circle
Better measure: θ = (arc length s) / (radius of the circle r) in radian

Wikipedia: The solid angle Ω is the 2D angle in 3D space that an object subtends at a point (the 2D analogon to a linear angle) [stereos = Greek for solid].
It is a measure of how large that object *appears* to an observer looking from that point.



In three dimensions, the solid angle in steradians is the area it cuts out:
Ω = (surface area S) / (radius of the sphere r)

One steradian is the solid angle at the center of a sphere of radius r under which a surface of area r^2 is seen.

A complete sphere = 4π sr.

1 sr = $(180\text{deg}/\pi)^2$ = 3282.80635 deg^2.

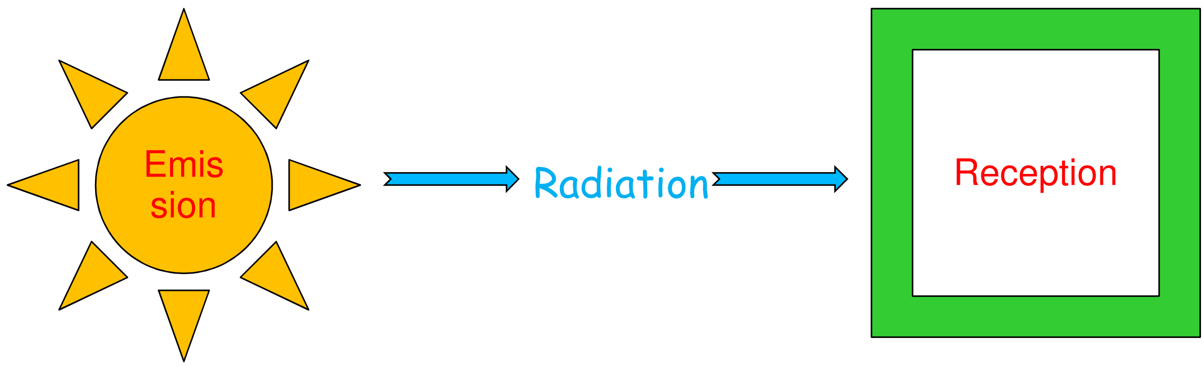
1. Radiometry

Radiometry = the physical quantities associated with the energy transported by electromagnetic radiation.

Photon energy: $E_{ph} = h\nu = \frac{hc}{\lambda}$

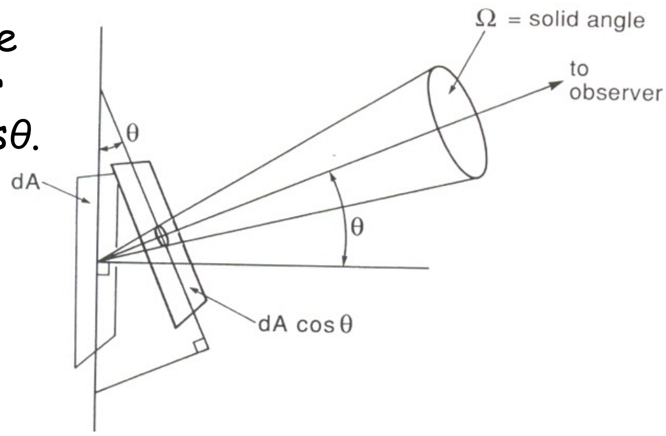
with h = Planck's constant [$6.626 \cdot 10^{-34}$ Js]

See also
http://web.archive.org/web/20080527160217/www.optics.arizona.edu/Palmer/rpfaq/rpfaq.htm



Emission (1): Radiance L or Intensity I

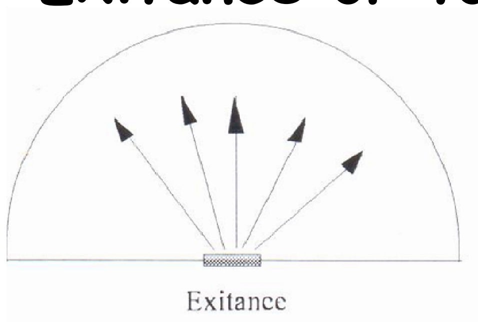
Consider a projected area of a surface element dA onto a plane perpendicular to the direction of observation dA cosθ.
θ is the angle between both planes.



The spectral radiance L_ν or specific intensity I_ν is the power *leaving* a unit projected area [m^2] into a unit solid angle [sr] and unit frequency interval [Hz].
It is measured in units of [W m^{-2} sr^{-1} Hz^{-1}] in frequency space L_ν, or [W m^{-3} sr^{-1}] in wavelength space L_λ.

The radiance L or intensity I is the spectral radiance integrated over all frequencies or wavelengths. Units are [W m^{-2} sr^{-1}].

Emission (2): Exitance or total Power M

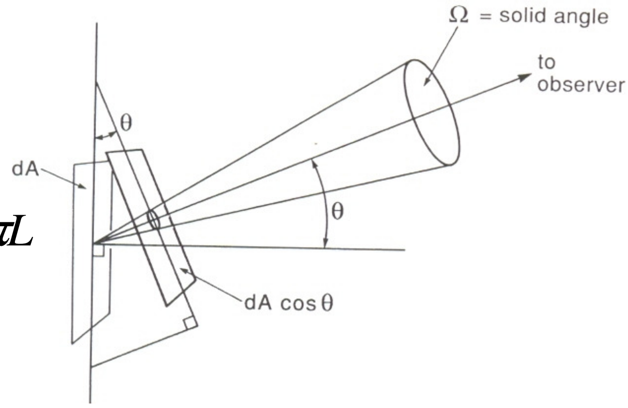


The radiant exitance M is the integral of the radiance over the solid angle Ω.

It measures the total power emitted per unit surface area.
Units are [W m^{-2}].

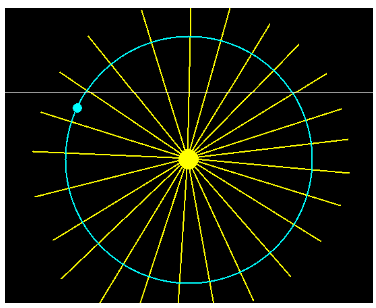
For Lambertian sources (see below) we get:

$$M = \int L\cos\theta d\Omega = 2\pi L \int_0^{\pi/2} \sin\theta \cos\theta d\theta = \pi L$$



Emission (3): Flux Φ and Luminosity L

The flux Φ or luminosity L emitted by the source is the product of radiant exitance and total surface area of the source.
It is the power emitted by the *entire* source.



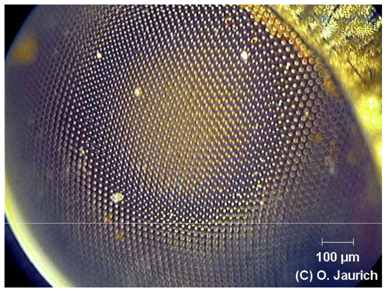
Units are [W] or [erg s^{-1}]

Example, a source of radius R (e.g., a star) has:

$$\Phi = 4\pi R^2 M \underset{\uparrow \atop M = \pi L}{=} 4\pi^2 R^2 L$$

The Field of View (FOV)

A detector system usually accepts radiation only from a limited range of directions, determined by the geometry of the optical system, the field of view (FOV).



The relevant area of the source, which produces a signal that can be observed depends on *(i)* FOV and *(ii)* distance r.

The *received* power [W] is then:

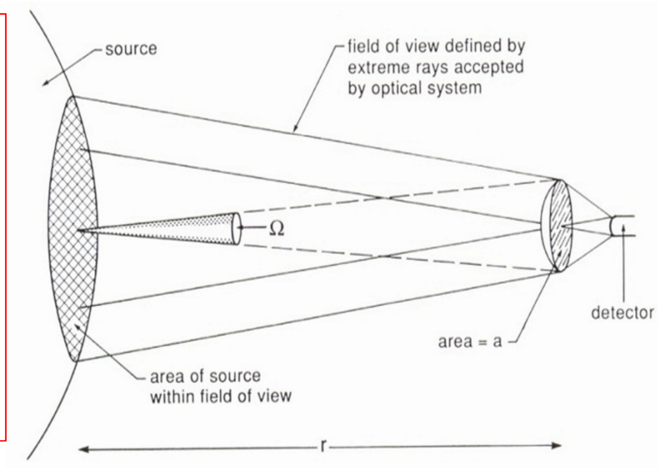
Radiance *L* [W m^{-2} sr^{-1}]

×

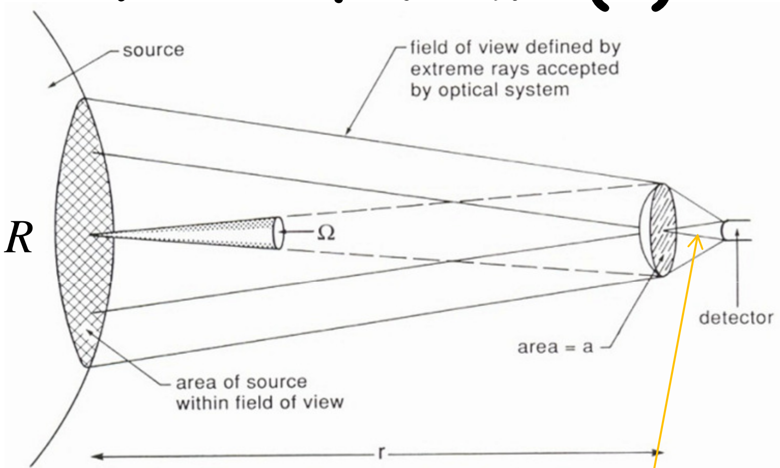
Source area *A* [m^2] within FOV

×

Solid angle Ω [sr] subtended by the optical system (as viewed from the source)



Field of View (2)



We assume that the entire source of radius R (or area πR^2) lies within the FOV.

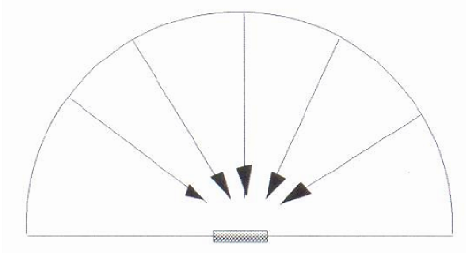
The solid angle subtended by the detector system is: $\Omega = \frac{a}{r^2}$,

where *a* is the area of the entrance aperture and *r* is the distance to the source.

For a circular aperture: $\Omega = 4\pi \sin^2\left(\frac{\theta}{2}\right)$

where θ is the half angle of the right cone.

Reception (1): the Irradiance E



The irradiance E is the power received at a unit surface element from the source.
Units are [W m^{-2}].

To compute E:

1. multiply M (=π·L) by surface area A of the source to get flux Φ.
2. divide flux Φ by the area of a sphere of radius r.

That yields: $E = \frac{AL}{4r^2}$

Reception (2): the Flux Density F_ν

The spectral irradiance E_ν or flux density F_ν is the irradiance per unit frequency or wavelength interval:

$$F_\nu = \frac{AL_\nu}{4r^2}$$

Units are [W m^{-2} Hz^{-1}] in frequency space or
[W m^{-3}] in wavelength space.

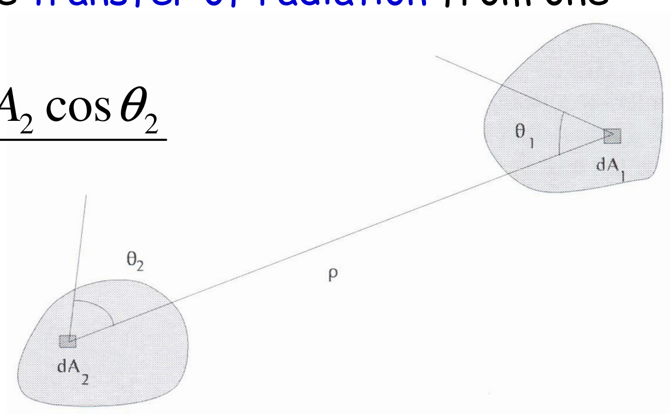
Note: 10^{-26} W $m^{-2}Hz^{-1}$ = 10^{-23} erg $s^{-1}cm^{-2}Hz^{-1}$ is also called 1 Jansky, named after US radio astronomer Karl Guthe Jansky.

Summary of Radiometric Quantities

Symbol	Name	Definition	Units	Equation	Alternate name	Alternate symbol
L_ν	Spectral radiance (frequency units)	Power leaving unit projected surface area into unit solid angle and unit frequency interval	W m^{-2} Hz^{-1} ster^{-1}		Specific intensity (frequency units)	I_ν
L_λ	Spectral radiance (wavelength units)	Power leaving unit projected surface area into unit solid angle and unit wavelength interval	W m^{-3} ster^{-1}		Specific intensity (wavelength units)	I_λ
L	Radiance	Spectral radiance integrated over frequency or wavelength	W m^{-2} ster^{-1}	$L = \int L_\nu d\nu$	Intensity or specific intensity	I
M	Radiant exitance	Power emitted per unit surface area	W m^{-2}	$M = \int L(\theta) d\Omega$		
Φ	Flux	Total power emitted by source of area A	W	$\Phi = \int M dA$	Luminosity	L
E	Irradiance	Power received at unit surface element; equation applies well removed from the source at distance r	W m^{-2}	$E = \frac{\int M dA}{(4\pi r^2)}$		
E_ν, E_λ	Spectral irradiance	Power received at unit surface element per unit frequency or wavelength interval	W m^{-2} Hz^{-1}, W m^{-3}		Flux density	S_ν, S_λ F_ν, F_λ

2. Radiative Transfer

Fundamental equation to describe the transfer of radiation from one surface to another in vacuum:



$$d\Phi = L\frac{dA_1 \cos\theta_1 dA_2 \cos\theta_2}{\rho^2}$$

where:
L – net radiance (1 ⇔ 2)
$A_{1,2}$ – areas
ρ – line of sight distance
$\theta_{1,2}$ – angles between surface normal and line of sight

Using the definition of the solid angle $d\Omega_{12} = \frac{dA_1 \cos\theta_1}{\rho^2}$ one can show that

$$d\Phi = LdZ$$

where *dZ* is the differential throughput.

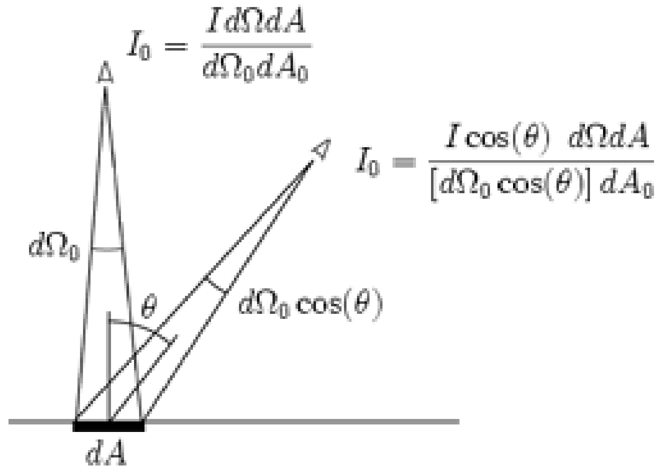
dZ is also called the étendue (extent, size) or the A-Ω product.

Note: L is property of the source, dZ a property of the geometry.

Lambertian Emitters

The radiance of Lambertian emitters is independent of the direction θ of observation (i.e., isotropic).

When a Lambertian surface is viewed from an angle θ, then dΩ is decreased by cos(θ) but the size of the observed area A is increased by the corresponding amount.



Example: the Sun is almost a perfect Lambertian radiator (except for the limb) with a uniform brightness across the disk.

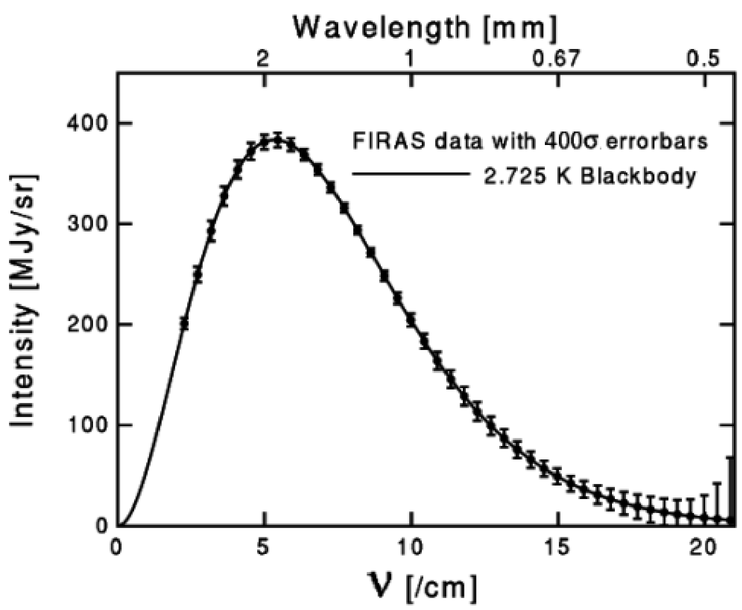
Perfect black bodies obey Lambert's law (1760)

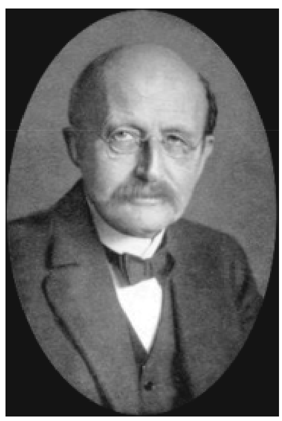
Johann Heinrich Lambert

3. Black Body Radiation

- A black body (BB) is an idealized object that absorbs all EM radiation
- BBs appear black when they are cold (no emitted or reflected light)
- At T > 0 K BBs absorb and re-emit a characteristic EM spectrum
- Many astronomical sources emit close to a black body.

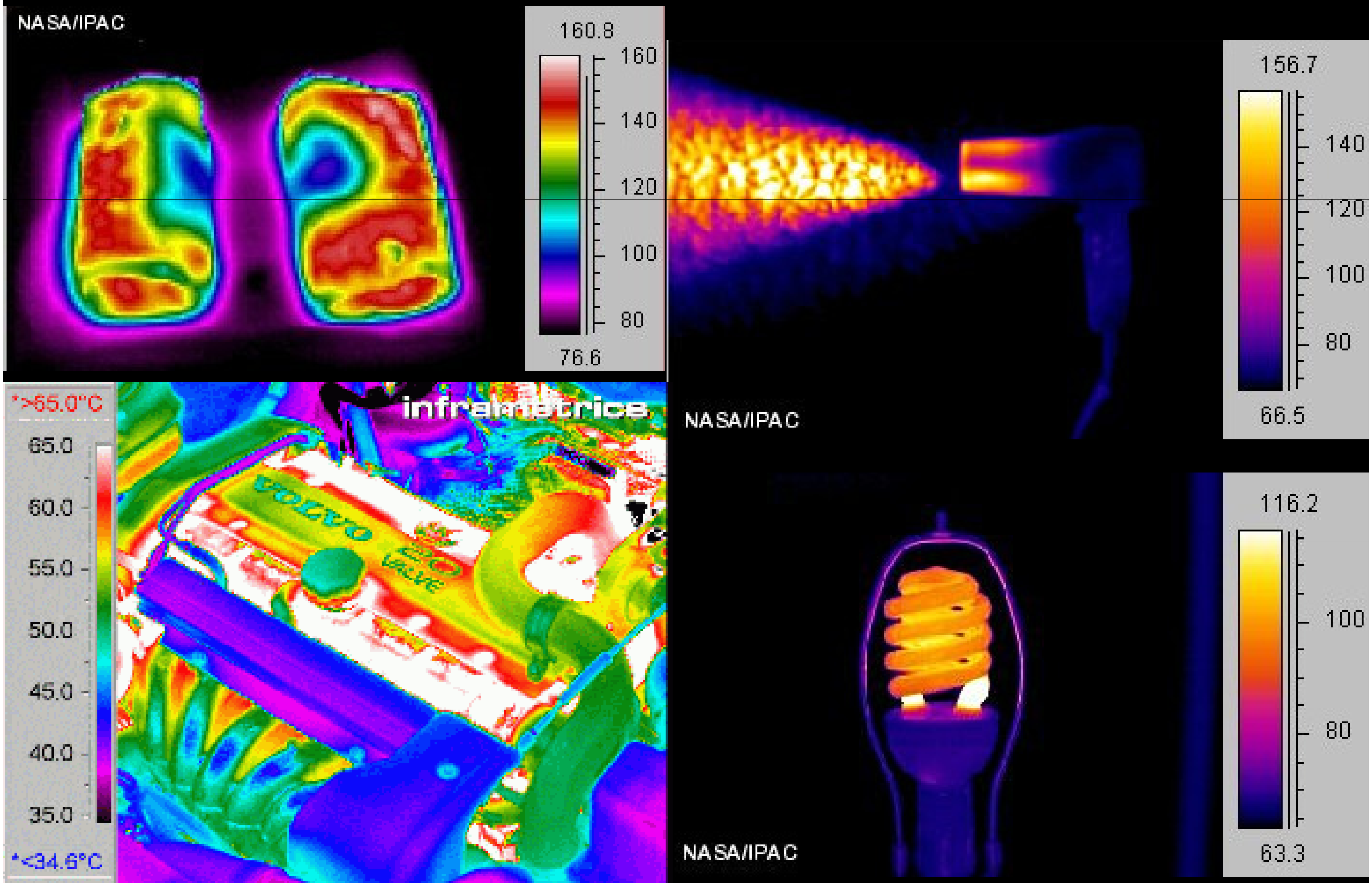
Example: COBE measurement of the cosmic background





Max Planck, Nobel Prize 1918

Temperature ⇔ Radiation



Black Body Emission

The specific intensity I_ν of a blackbody is given by Planck's law as:

$$I_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(\frac{h\nu}{kT}\right)-1}$$ in units of [W m^{-2} sr^{-1} Hz^{-1}]

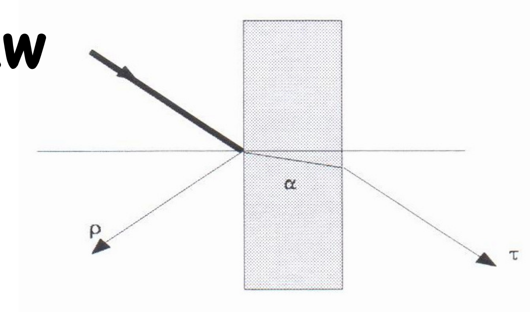
In terms of wavelength units this corresponds to:

$$I_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda kT}\right)-1}$$ in units of [W m-1 sr-1]

Note for the conversion of frequency ⇔ wavelength:

$$dv = \frac{c}{\lambda^2}d\lambda \quad \text{or} \quad d\lambda = \frac{c}{\nu^2}d\nu$$

Kirchhoff's Law

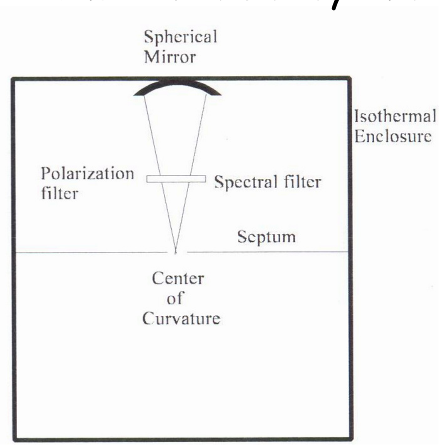


Conservation of power requires that:

$$\alpha + \rho + \tau = 1$$

With α = absorptivity, ρ = reflectivity, and τ = transmissivity

Consider a cavity in thermal equilibrium with completely opaque sides:



$$\left.\begin{array}{c} \varepsilon = 1 - \rho \\ \alpha + \rho + \tau = 1 \\ \tau = 0 \end{array}\right\} \quad \boxed{\alpha = \varepsilon}$$

This is Kirchhoff's law, which applies to a perfect black body

A radiator with ε = ε(λ) <~ 1 is often called a grey body

Useful Approximations

At high frequencies (hν >> kT) we get Wien's law:

$$I_\nu(T) = \frac{2h\nu^3}{c^2}\exp\left(-\frac{h\nu}{kT}\right)$$

At low frequencies (hν << kT) we get Rayleigh-Jeans' law:

$$I_\nu(T) \approx \frac{2\nu^2}{c^2}kT = \frac{2kT}{\lambda^2}$$

The total radiated power per unit surface is proportional to the fourth power of the temperature:

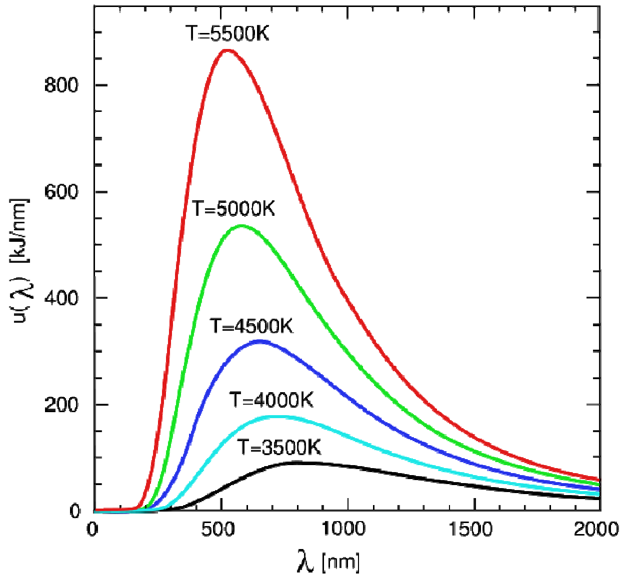
$$\int_\Omega \int_\nu I_\nu(T)\, d\nu\, d\Omega = \sigma T^4$$

$\sigma = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4} is the Stefan-Boltzmann constant.

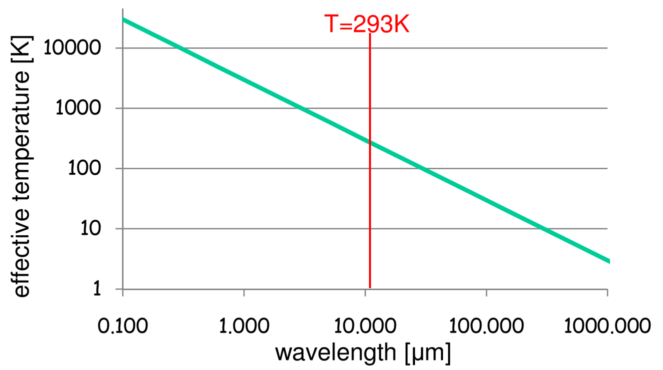
Black Body "Peak" Temperatures

The temperature corresponding to the maximum specific intensity is given by:

$$\frac{c}{\nu_{max}} T = 5.096 \cdot 10^{-3}\ \text{mK} \quad \text{or} \quad \lambda_{max} T = 2.98 \cdot 10^{-3}\ \text{mK}$$



Hence, cooler BBs have their peak emission (effective temperatures) at longer wavelengths and at lower intensities:



Assuming BB radiation, astronomers often describe the emission from objects via their effective temperature.

4. Magnitudes

This system has its origins in the Greek classification of stars according to their visual brightness. The brightest stars were m = 1, the faintest detected with the bare eye were m = 6.

Later formalized by Pogson (1856):
a 1st mag star is 100 times brighter than a 6th mag star.

(Apparent) magnitude = *relative* measure of the monochromatic flux density F(λ) of a source:

$$m_{\lambda_0} = -2.5 \log \frac{F(\lambda_0)}{F_0} = -2.5 \log F(\lambda_0) + q_{\lambda_0}$$

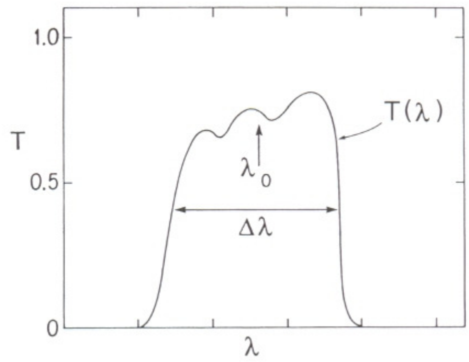
The constant q_0 defines magnitude zero.

Note: Magnitudes are units to describe unresolved (pointlike) objects. When referring to surface brightness one uses mag/sr or mag/arcsec^2.

Photometric Systems

In practice, measurements are done through a transmission filter $t_0(\lambda)$ that defines a finite bandwidth:

$$m_{\lambda_0} = -2.5\log\int_0^\infty t_0(\lambda)F(\lambda)d\lambda + 2.5\log\int_0^\infty t_0(\lambda)d\lambda + q_{\lambda_0}$$



As filters differ there are many different photometric systems:

- Johnson UBV system
- Gunn griz
- USNO
- SDSS
- 2MASS JHK
- HST filter system (STMAG)
- ...
- AB magnitude system

m(AB) = -2.5 log(F_ν[W/cm²/Hz]) - 48.60

Photometric System	Reference
AAO	Allen & Cragg (1983) Elias et al. (1983)
ARNICA	Hunt et al. (1988)
Bessell & Brett	Bessell & Brett (1988)
CIT	Elias et al. (1982) Elias et al. (1983)
ESO	van der Bliek et al. (1996)
Koornneef	Koornneef (1983)
LCO	Persson et al. (1998)
MKO	UKIRT web site (2002)
MSSSO	McGregor (1994)
SAAO	Carter (1990) Carter & Meadows (1995)
UKIRT	Hawarden et al. (2001)

Standard Photometry

Name	λ_0 [μm]	$\Delta\lambda_0$ [μm]	F_λ [W m^{-2} μm^{-1}]	F_ν [Jy]	
U	0.36	0.068	4.35×10^{-8}	1 880	Ultraviolet
B	0.44	0.098	7.20×10^{-8}	4 650	Blue
V	0.55	0.089	3.92×10^{-8}	3 950	Visible
R	0.70	0.22	1.76×10^{-8}	2 870	Red
I	0.90	0.24	8.3×10^{-9}	2 240	Infrared
J	1.25	0.30	3.4×10^{-9}	1 770	Infrared
H	1.65	0.35	7×10^{-10}	636	Infrared
K	2.20	0.40	3.9×10^{-10}	629	Infrared
L	3.40	0.55	8.1×10^{-11}	312	Infrared
M	5.0	0.3	2.2×10^{-11}	183	Infrared
N	10.2	5	1.23×10^{-12}	43	Infrared
Q	21.0	8	6.8×10^{-14}	10	Infrared

1 Jy = 10^{-26} W m^{-2} Hz^{-1}.

Bolometric Magnitude

Bolometric magnitude = integral of the monochromatic flux over all wavelengths:

$$m_{bol} = -2.5 \log \frac{\int_0^\infty F(\lambda) d\lambda}{F_{bol}}$$

with $F_{bol} = 2.52 \cdot 10^{-8}$ W/m^2

If the source radiates isotropically one gets:

$$m_{bol} = -0.25 + 5 \log D - 2.5 \log \frac{L}{L_\Theta}$$

where $L_o = 3.827 \cdot 10^{26}$ W is the luminosity of the Sun.

Absolute Magnitude and Color Indices

Absolute magnitude = apparent magnitude of the source if it were at a distance of D = 10 parsecs.

Including a term *A* for interstellar absorption we get:

$$M = m + 5 - 5 \log D - A$$

Color indices = difference of magnitudes at different wavebands = ratio of fluxes at different wavelengths.

Important:

- The color indices of an A0 dwarf star are about zero longward of V.
- The color indices of a blackbody in the Rayleigh-Jeans tail are:

  B-V = -0.46, U-B = -1.33, V-R = V-I = ... = V-N = 0.0

5. Coherence of Light

Coherence (from Latin *cohaerere* = to be connected) of EM waves enables temporally and spatially constant interference.

Best case of an uni-directional monochromatic wave (perfect laser): it is possible to define the relative phase at two arbitrary points along **k**.

Worst case (in terms of coherence): black-body radiation.

Two types of coherence:

1. spatial coherence → image formation
2. temporal coherence → spectral analysis

First we consider the wave aspect of light...

Degree of Coherence

Consider a complex field V(t) as a stationary random process with power spectrum S(ν) and time average <V(t)> = 0.

Measure the fields at any two points in space $V_1(t)$ and $V_2(t)$. The cross correlation between these measurements is given by

$$\Gamma_{12}(\tau) = \left\langle V_1(t) V_2^*(t+\tau) \right\rangle$$

whereas the *mean* intensity at point 1 can be described by

$$\Gamma_{11}(0) = \left\langle V_1(t) V_1^*(t) \right\rangle$$

The (mutual) degree of coherence can then be defined as:

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{[\Gamma_{11}(0)\Gamma_{22}(0)]^{1/2}}$$

Note that γ_{12} includes both spatial (points 1,2) and temporal (τ) coherence.

Quasi-Monochromatic Radiation

It can be shown that the relation between spectral width $\Delta\nu$ and temporal width τ_c is:

$$\tau_c \Delta\nu \cong 1$$

The coherence length l_c is the length over which the field retains the memory of its phase – the distance beyond which the waves λ and $\lambda+\Delta\lambda$ are out of step by λ: $l_c = c\tau_c = \frac{\lambda_0^2}{\Delta\lambda}$

For $l \ll c\tau_c$ it follows that: $\gamma_{12}(\tau) \sim \gamma_{12}(0)e^{-2i\pi\nu_0\tau}$

and the coherence is determined by $\gamma_{12}(0)$.

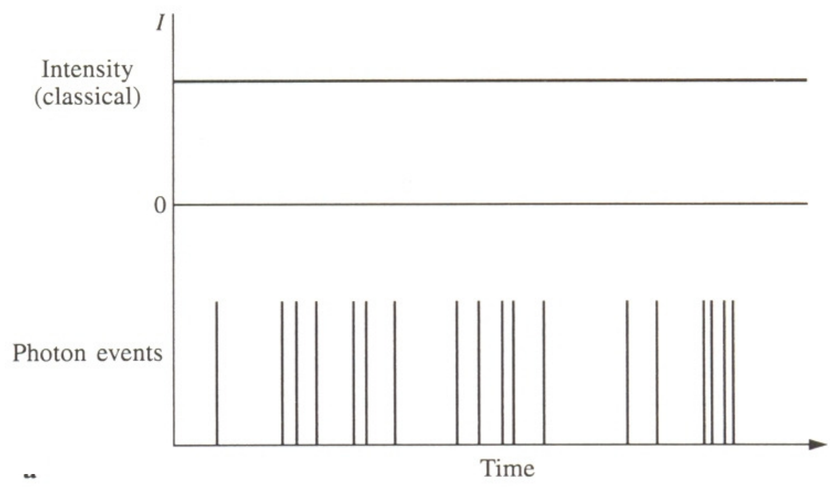
For purely monochromatic radiation, τ is infinite.

Photon Statistics (1): Poissonian

Now we consider the particle aspect of light. There are several cases:

1. For any time τ the number of photons n obeys a Poissonian distribution with variance and standard deviation σ:

$$\left\langle \Delta n^2 \right\rangle = \bar{n}\tau \quad \text{or} \quad \sigma = \sqrt{\bar{n}\tau}$$



Constant classical intensity and photon events following a Poissonian distribution

Photon Statistics (2): Bose-Einstein

2. Quasi-monochromatic radiation (e.g., a spectral line) with finite coherence time $\tau_c \sim 1/\Delta\nu$ (where $\Delta\nu$ is the line width). If $\tau \gg \tau_c$ the photon fluctuation is affected by the Bose-Einstein

   distribution: $$f(E) = \frac{1}{Ae^{E/kT} - 1}$$

   and the photon statistics is given as:

$$\langle \Delta n^2 \rangle = \bar{n}\,\tau \left(1 + \frac{1}{e^{h\nu/kT} - 1} \right)$$

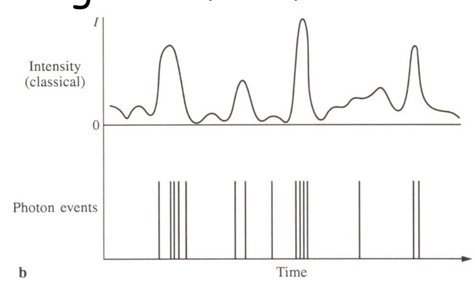
Photon Statistics (3): Bunching

- Statistical tendency for multiple photons to arrive simultaneously
- Classical view: non-interacting particles should arrive independently of one another
- Quantum mechanics (wave effect): a property of all bosons (due to the Pauli exclusion principle, fermions show the opposite effect)
- Experimentally known as Hanbury-Brown and Twiss effect (→intensity interferometer)
- → R.J. Glauber, Nobel Prize 2005

3. For thermal radiation, if $\tau < \tau_c$, the photons will no longer obey Poissonian statistics but group together (bunching). This becomes significant when:

$$\frac{1}{e^{h\nu/kT} - 1} \sim 1$$

4. For non-thermal radiation, if $\tau \gg \tau_c$, the bunching becomes more significant as degeneracy $\frac{1}{e^{h\nu/kT} - 1}$ increases.

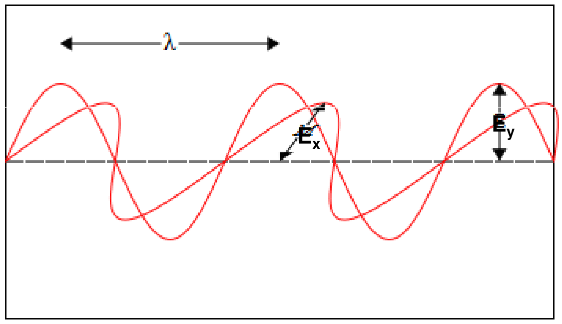


Classical intensity of a thermal source with a photon distribution that combines a Poisson process, Bose-Einstein distribution, and bunching.

6. Polarization of Light

The wave vectors of the electric field are given by:

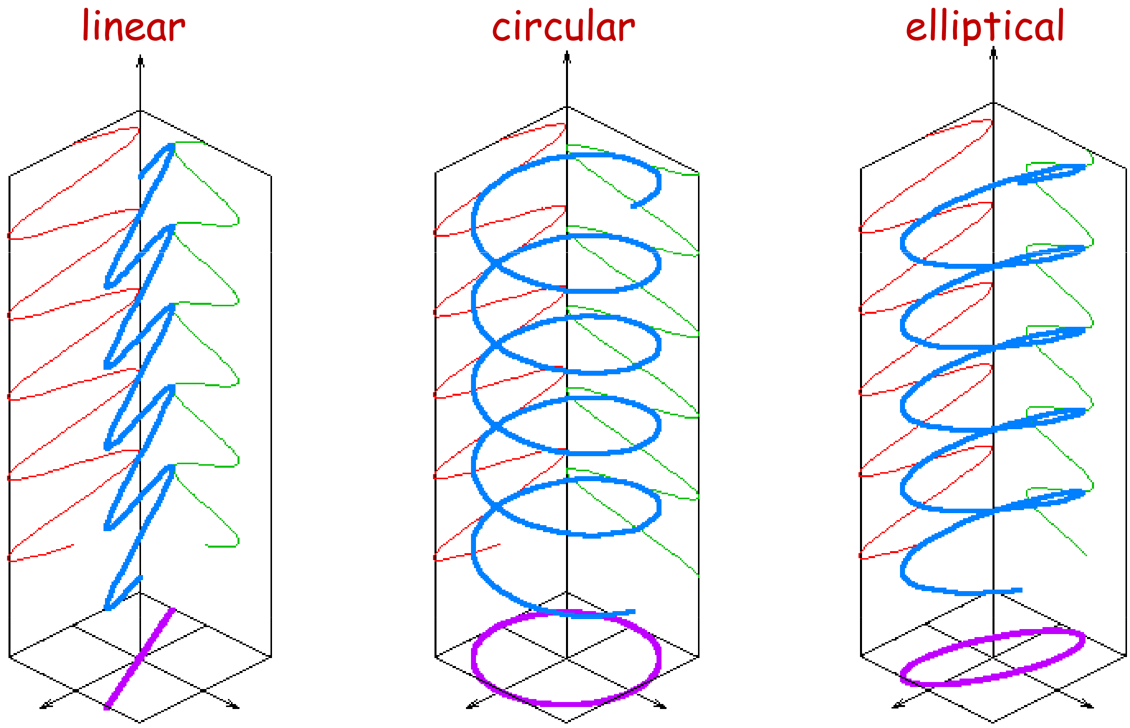
$$E_x = a_1 \cos(2\pi\nu t - k \cdot r + \phi_1)$$
$$E_y = a_2 \cos(2\pi\nu t - k \cdot r + \phi_2)$$



where a_i are the amplitudes, ν is the frequency, $k=2\pi/\lambda$ the wavevector, and Φ_i are the phases.

We also define $\Phi = \Phi_2 - \Phi_1$

Three Types of Polarized Waves



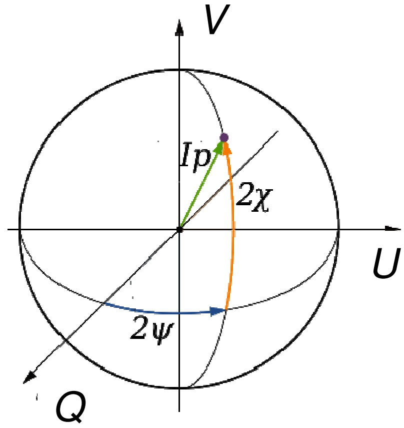
Type and degree of polarization is important as it carries information on the properties of the source (magnetic fields, dust grain alignment, etc.).

But telescope, instrument optics and detector may alter the polarization!

The Stokes Parameter

Polarization can be defined by the four Stokes parameters I, Q, U, V (1852) as follows:

$$I = a_1^2 + a_2^2$$
$$Q = a_1^2 - a_2^2 = I\cos 2\chi \cos 2\psi$$
$$U = 2a_1a_2\cos\phi = I\cos 2\chi \sin 2\psi$$
$$V = 2a_1a_2\sin\phi = I\sin 2\chi$$



Generally, the degree of polarization of a wave is:

$$\Pi = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

A plane wave has Π = 1 and the Stokes parameters are related as:

$$I^2 = Q^2 + U^2 + V^2$$

Examples

Polarizers can be used to filter out e.g., reflected light