Exercises Astronomical Observing Techniques, Set 6

Exercise 1

The 1D Fourier pairs $f(x)$ and $F(s)$ are defined as follows:
$\int_{-\infty}^{+\infty} f(x)e^{-2\pi ixs}\,dx = F(s)$: $\mathcal{F}\{f(x)\} = F(s)$, the Fourier transform of $f(x)$ and
$\int_{-\infty}^{+\infty} F(s)e^{2\pi ixs}\,ds = f(x)$: $\hat{\mathcal{F}}\{F(s)\} = f(x)$, the inverse Fourier transform of $F(s)$
(g(x) and G(s) are also a Fourier pair)

The 2D Fourier pairs $f(x,y)$ and $F(u,v)$ are defined as follows:
$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)e^{-2\pi i(xu+vy)}\,dxdy = F(u,v)$: $\mathcal{F}\{f(x,y)\} = F(u,v)$, the Fourier transform of $f(x,y)$ and
$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(u,v)e^{2\pi i(xu+vy)}\,dudv = f(x,y)$: $\hat{\mathcal{F}}\{F(u,v)\} = f(x,y)$, the inverse Fourier transform of $F(u,v)$

Compute the 2D Fourier transforms of:

a) $\delta(x,y)$

b) $\delta(x-a, y-b)$

Exercise 2

The convolution of $f(x)$ with $g(x)$ is defined as $h(x) = f(x) * g(x) = \int_{-\infty}^{+\infty} f(u)g(x-u)\,du$

a) Show that $\mathcal{F}\{f(x) * g(x)\} = F(s)G(s)$

b) Give a proof of Parsevals' theorem (Rayleighs' energy theorem): $\int_{-\infty}^{+\infty} |f(x)|^2\,dx = \int_{-\infty}^{+\infty} |F(s)|^2\,ds$

Exercise 3

We have a diffraction limited 0.1 arcsec image of a star from the Hubble Space Telescope, describe what happens if we convolve this image with a Gaussian having a width of about 2 arcsec?

Exercise 4

Calculate the depth of focus for an 8.2m telescope with focal length of 14.4 m operating at 0.5μm.

Exercise 5

A spherical galaxy has a magnitude of $m_V = 18$ and spans 30 arcsec on the sky. Calculate the surface brightness of this galaxy in mag arcsec^{-2} (you can assume the light is homogeneously distributed over the galaxy).