Astronomische Waarneemtechnieken (Astronomical Observing Techniques)

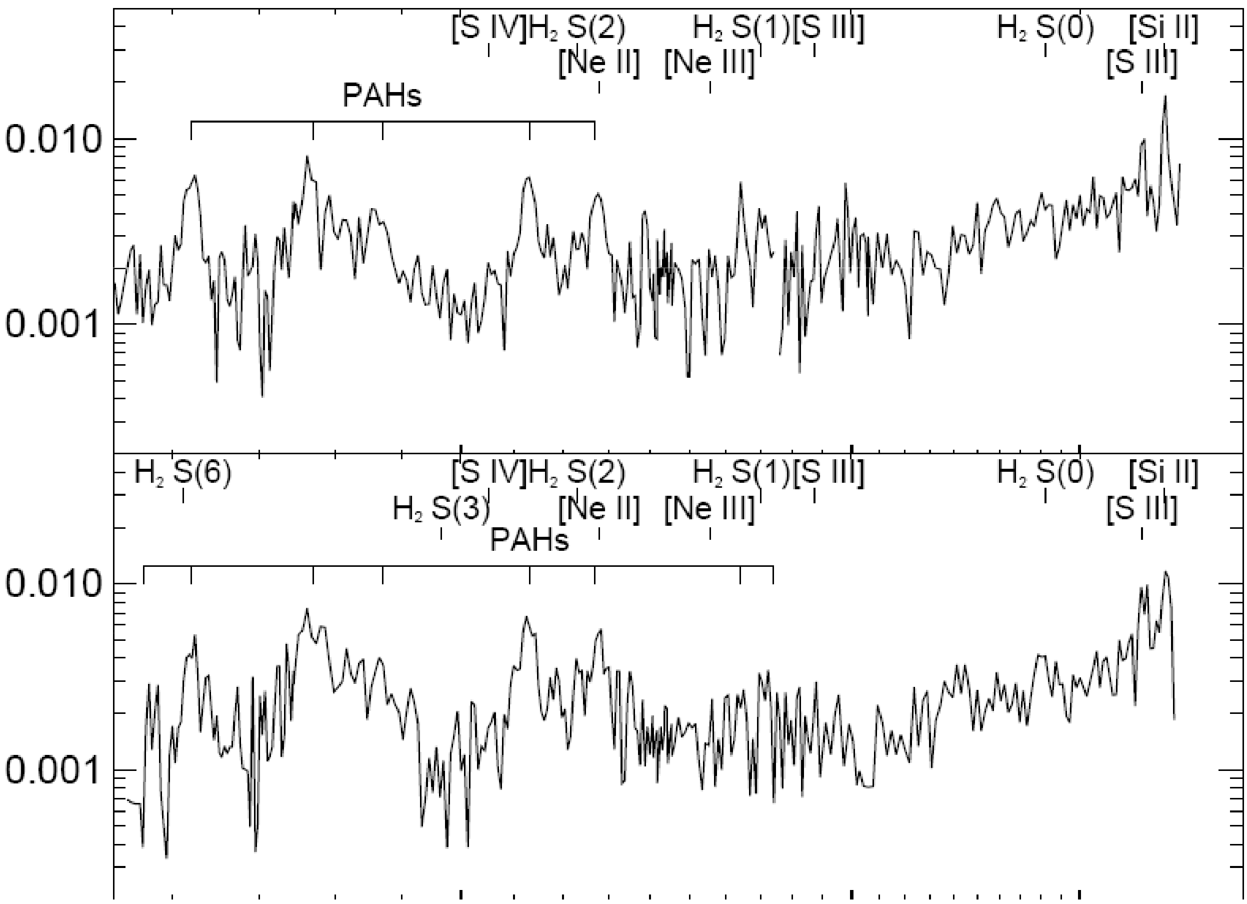
8th Lecture: 12 November 2008

$$S_{cont} = \frac{\sigma h \lambda \sqrt{n_{pix}}\, 10^{30}}{SR\Delta\lambda A_{tel}\eta_D G \eta_{atm}\eta_{tot} t_{\text{int}}} \sqrt{\frac{2hc^2}{\lambda^5}\left(\frac{\varepsilon_T}{\exp\left[\frac{hc}{kT_T\lambda}\right]-1}+\frac{\varepsilon_A}{\exp\left[\frac{hc}{kT_A\lambda}\right]-1}\right)\eta_{tot}} \cdot$$

$$\cdot \sqrt{2\pi\left(1-\cos\left(\arctan\left(\frac{1}{2F\#}\right)\right)\right)D^2{}_{pix}\cdot\frac{\eta_D G\lambda}{hc}\cdot\Delta\lambda\cdot t_{\text{int}} + I_d t_{\text{int}} + N^2_{read}\, n}$$

Based on "Observational Astrophysics" (Springer) by P. Lena, F. Lebrun & F. Mignard, 2nd edition – Chapter 6; and other sources

Noise ...?



Two noisy spectra... What part is noise? What is real information?

General Overview

Detected signal = Source Signal + Background

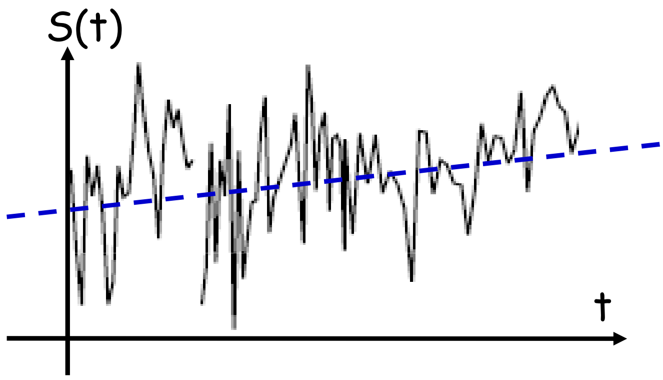
Background:

- background signal (sky background, thermal emission, cosmics, …)
- background noise (noise associated with the background signal)
- detectors noise (see next lecture)

Source signal:

- fundamental (physical limitations)
- observational/practical limitations
- transmission noise (scintillation)

Gaussian Noise

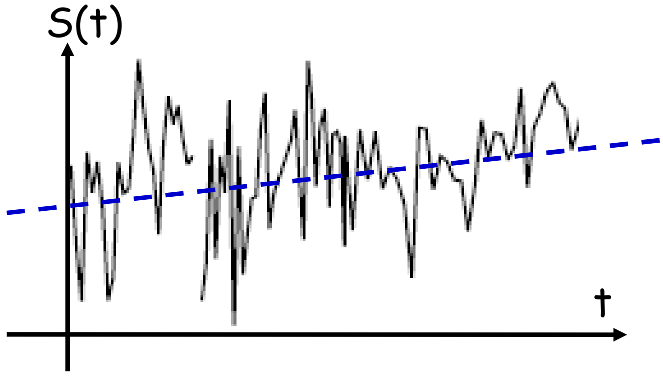


Gaussian noise – noise with a Gaussian amplitude distribution (normal distribution), i.e., the noise values are Gaussian-distributed*.

$$S = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

*Often incorrectly labeled white noise, which refers to the (un-)correlation of the noise.

Poisson Noise



Poisson noise – noise following a Poissonian distribution:

$$P \sim \frac{e^{-\overline{N}}\,\overline{N}^{k}}{k!}$$

For instance, fluctuations in the detected photon flux between finite time intervals Δt. Detected are *k* photons, while expected are on average $\overline{N}$ photons.

Note that the standard deviation of *P* is $\sqrt{\overline{N}}$.

Fundamental Fluctuations

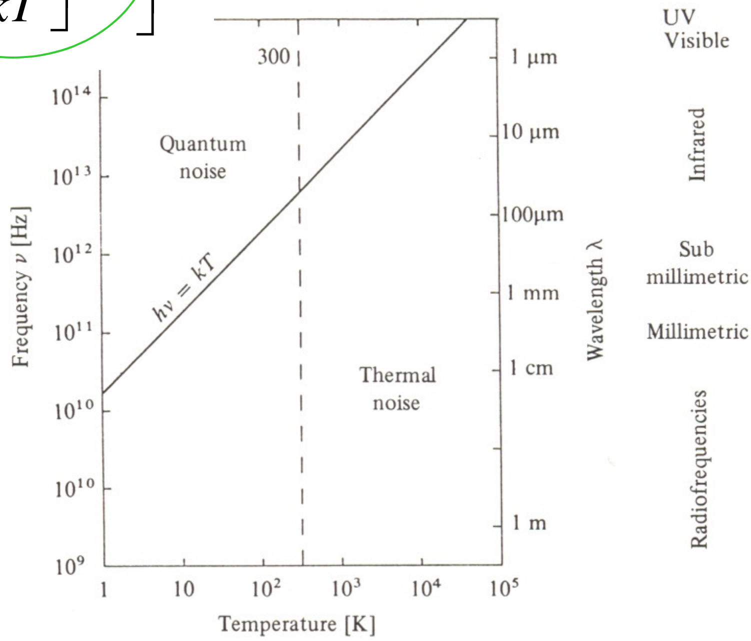
Consider a thermodynamic system of monochromatic power P(ν) and mean energy <P(ν)>. The actual energy of the system fluctuates around this mean with a variance of:

$$\left\langle [\Delta P(\nu)]^2 \right\rangle = P(\nu)\cdot h\nu \cdot \left[1 + \frac{1}{\exp\left[\frac{h\nu}{kT}\right] - 1} \right]$$

Two terms:

the quantum noise (the photon number fluctuation), and the thermal noise (phase differences of the wave fields).

Characteristic domains:



Signal Processing: Sampling

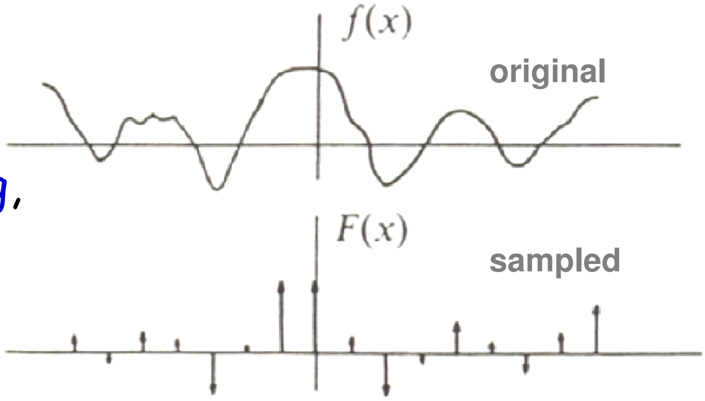
Sampling means multiple measurements of the signal – either in time (Δt) or in space (Δx).

The interval between two measurements is called sampling rate.

The Nyquist-Shannon sampling theorem (1949) states that

If a function x(t) contains no frequencies higher than f_s, it is completely determined by measuring its values at a series of points spaced $1/(2f_s)$ apart.

Sampling above or below the sampling rate $1/(2f_s)$ is called oversampling or undersampling, respectively.



Signal Processing: Digitization

Digitization = converting an analog signal into a digital signal using an Analog-to-Digital Converter (ADC).

The number of bits determines the dynamic range of the ADC. The resolution is 2^n, where n is the number of bits.

Typical ADCs have:

12 bit: 2^{12} = 4096 quantization levels

16 bit: 2^{16} = 65636 quantization levels

Compare this to the detector pixel capacity (number of electrons)!

Instrument Sensitivities – Overview

The signal detection depends on the two main components:

1. the strength of the detected signal S_{el}
2. the total noise N_{tot} nof the system,

and can be characterized by the statistical significance of the detection σ (= signal-to-noise S/N)

$$\sigma = \frac{S_{el}}{N_{tot}}$$

Notes:

(i) in this discussion we neglect quantum (shot) noise from the source.

(ii) we consider only point sources.

(iii) typically, the threshold for a "real" detection is taken as 3σ.

Instrument Sensitivities (1)

The detected signal S_{el} depends on:

- the source flux density S_{src} [photons s^{-1} cm^{-2} μm^{-1}]
- the integration time t_{int} [s]
- the telescope aperture A_{tel} [m^2]
- the transmission of the atmosphere η_{atm}
- the total throughput of the system η_{tot}, which includes the reflectivity of all telescope mirrors and the reflectivity (or transmission) of all instrument components, such as mirrors, lenses, filters, beam splitters, grating efficiencies, slit losses, etc.
- the Strehl ratio SR
- the detector responsivity $\eta_D G$, and
- the spectral bandwidth $\Delta\lambda$ [μm]

$$S_{el} = S_{src} SR \Delta\lambda A_{tel} \eta_D G \eta_{atm} \eta_{tot} t_{\mathrm{int}}$$

Instrument Sensitivities (2)

The total noise N_{tot} depends on:

- the number of pixels n_{pix} of one resolution element
- the background noise per pixel N_{back}

$$N_{tot} = N_{back}\sqrt{n_{pix}}$$

Where the total background noise N_{back} depends on:

- the background flux density S_{back}
- the integration time t_{int}
- the detector dark current I_d
- the number of reads (N) and detector frames (n)

$$N_{back} = \sqrt{S_{back} t_{\text{int}} + I_d t_{\text{int}} + N_{read}^2 n}$$

Instrument Sensitivities (2b)

The background flux density S_{back} depends on:

- the total background intensity B_{tot}
  where B_T and B_A are the thermal emissions from telescope and atmosphere, approximated by black body emission

$$B_{tot} = (B_T + B_A)\eta_{tot}$$

$$B_{T,A} = \frac{2hc^2}{\lambda^5}\left[\frac{\varepsilon}{\exp\left[\frac{hc}{kT\lambda}\right] - 1}\right]$$

- the spectral bandwidth $\Delta\lambda$
- the pixel field of view $A\times\Omega$ $\quad A\times\Omega = 2\pi\left(1-\cos\left(\arctan\left(\frac{1}{2F\#}\right)\right)\right)D^2_{pix}$
- the detector responsivity $\eta_D G$, and
- the photon energy hc/λ

$$S_{back} = B_{tot}\cdot A\times\Omega\cdot\frac{\eta_D G\lambda}{hc}\cdot\Delta\lambda$$

Instrument Sensitivities (3)

Putting it all together, the minimum detectable source signal is:

$$S_{src} = \frac{\sigma_{S/N} N_{back} \sqrt{n_{pix}}}{SR\Delta\lambda A_{tel}\eta_D G \eta_{atm}\eta_{tot} t_{\text{int}}}$$

Now we can calculate the unresolved line sensitivity S_{line} [W/m²] from the source flux S_{src} [photons/s/cm²/μm]:

$$S_{line} = \frac{hc}{\lambda} S_{src} \Delta\lambda \cdot 10^4$$

Be aware of unit conversion issues

and with $S_\lambda\left[\frac{W}{m^2 \mu m}\right] = S_\nu[Jy] \cdot 10^{-26} \frac{c}{\lambda^2}$

we can calculate the continuum sensitivity S_{cont}:

$$S_{cont} = \frac{hc}{\lambda} S_{src} \cdot 10^4 \cdot \frac{\lambda^2}{c} \cdot 10^{26} = 10^{30} h\lambda S_{src}$$

SUMMARY

Summary: I. S/N Basics

Note: Signal = S; Background = B; Noise = N; Telescope diameter = D

Obviously: $\sigma = \frac{\text{Signal}}{\text{Noise}}$ ← measured as (S+B)-mean{B}

← total noise = $\sqrt{\sum (N_i)^2}$ if statist. independent

Noise: Poisson noise in B, read noise, dark current noise, ...

Both S and N should be in units of events (photons, electrons, data numbers) per unit area (pixel, PSF size, arcsec^2).

Standard case: N = Poisson shot noise in B = $\sqrt{B}$

Side note: noise between pixels is equivalent to successive measurements with one pixel – analogous to throwing 5 dices versus one dice 5 times.

Dependence on integration time t_{int}:

Consider integrating $n \times t_{int}$: $\sigma = \frac{n \cdot S}{\sqrt{n \cdot B}} = \sqrt{n}\frac{S}{N}$

➔ $\frac{S}{N} \propto \sqrt{t_{\mathrm{int}}}$ You need to integrate 4 times as long to get twice the S/N

Summary: II. S/N and Telescope Size

Note: Signal = S; Background = B; Noise = N; Telescope diameter = D

Case 1: seeing limited system & "point source" of θ_{seeing}

θ_{seeing} ~ const → if Nyquist sampled: S ~ D^2; B ~ D^2 → N ~ D

S/N ~ D → t_{int} ~ D^{-2}.

Case 2: diffraction limited system & extended source

"PSFØ" ~ const → if Nyquist sampled: S ~ D^2, pixel area ~ D^{-2}

D^2 and D^{-2} cancel each other; same for B

S/N ~ const → t_{int} ~ const → *no gain from larger telescopes!*

Case 2B: offline re-sampling by a factor x (= telescope x-times smaller)

S/N ~ $\sqrt{n_{pix}}$ → S/N ~ $\sqrt{x^2}$ = x → t_{int} ~ x^{-2}.

Case 3: diffraction limited system & point source

"S/N = $(S/N)_{\text{light bucket}} \cdot (S/N)_{\text{pixel scale}}$"

S ~ D^2; B ~ D^{-2} → N ~ D^{-1}; Nyquist: S ~ const; B/pix ~ D^{-2} → N/pix ~D^{-1}

combined S/N ~ D^2 → t_{int} ~ D^{-4} → *huge gain: 1hr ELT = 3 months VLT*