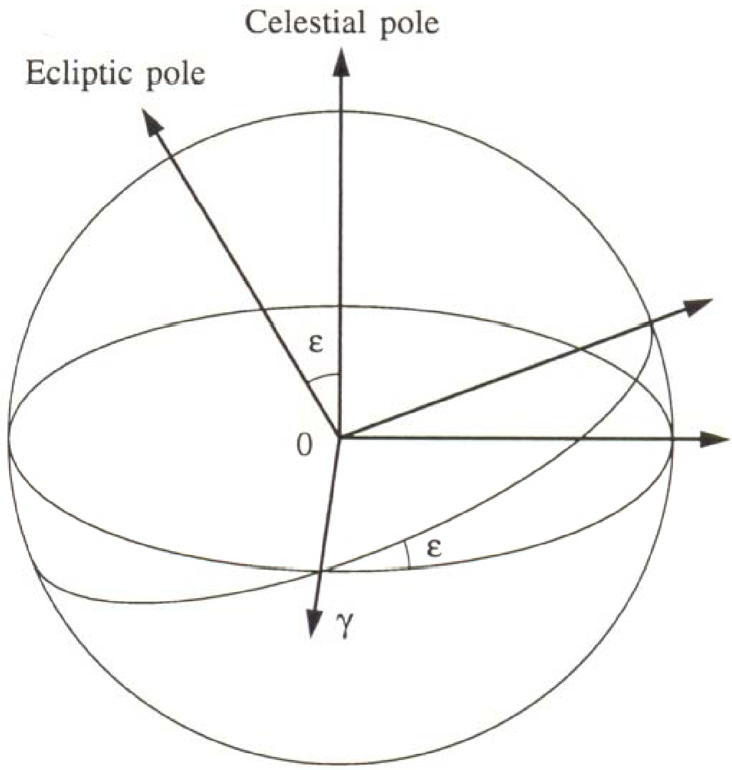
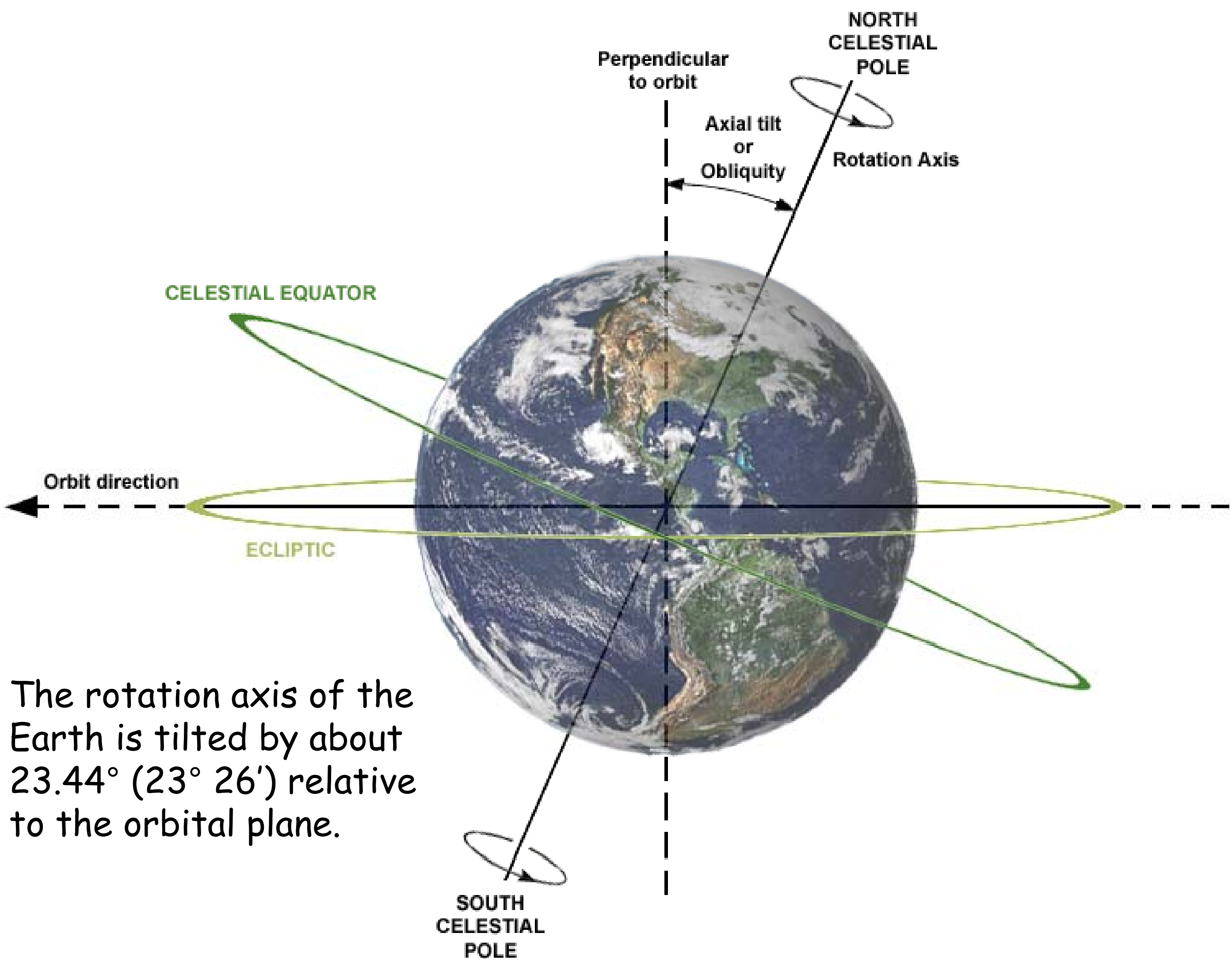
1. Coordinate Systems



Based on "Observational Astrophysics" (Springer) by P. Lena, F. Lebrun & F. Mignard, 2nd edition, 1998 – Chapter 8



source: Wikipedia

The rotation axis of the Earth is tilted by about 23.44° (23° 26') relative to the orbital plane.

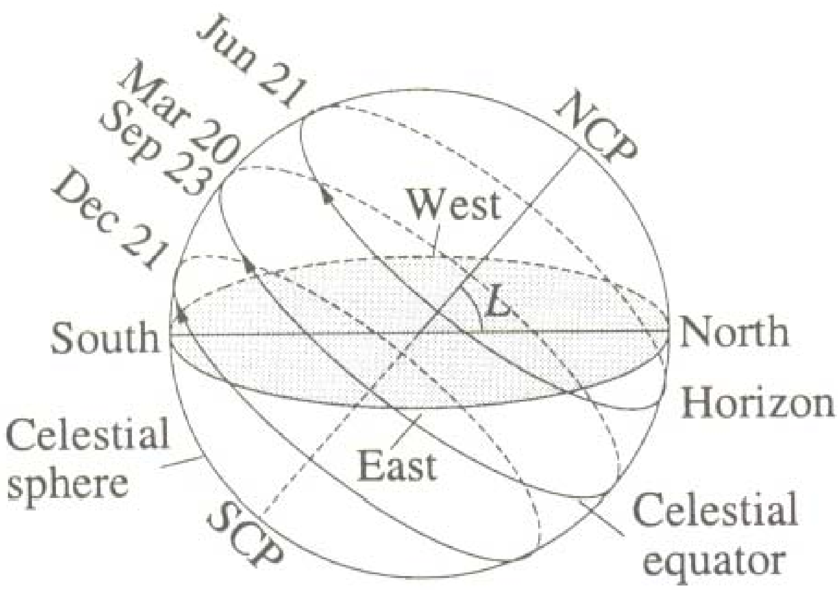
Everything is changing ... [axis tilt]

source: Wikipedia

Equinox = when the centre of the Sun can be observed to be directly above the Earth's equator.

Equinox derives from `aequus' (equal) and `nox' (night).

At an equinox, the Sun will spend equal amounts of time above and below the horizon at every location on Earth.



UTC date and time of solstices and equinoxes[1]

year	Equinox Mar		Solstice June		Equinox Sept		Solstice Dec	
	day	time	day	time	day	time	day	time
2002	20	19:16	21	13:24	23	04:55	22	01:14
2003	21	01:00	21	19:10	23	10:47	22	07:04
2004	20	06:49	21	00:57	22	16:30	21	12:42
2005	20	12:33	21	06:46	22	22:23	21	18:35
2006	20	18:26	21	12:26	23	04:03	22	00:22
2007	21	00:07	21	18:06	23	09:51	22	06:08
2008	20	05:48	20	23:59	22	15:44	21	12:04
2009	20	11:44	21	05:45	22	21:18	21	17:47
2010	20	17:32	21	11:28	23	03:09	21	23:38
2011	20	23:21	21	17:16	23	09:04	22	05:30
2012	20	05:14	20	23:09	22	14:49	21	11:11
2013	20	11:02	21	05:04	22	20:44	21	17:11
2014	20	16:57	21	10:51	23	02:29	21	23:03

Everything is changing ... [orbital motion]

Parallax = semi-angle of inclination between two sightlines to a star. Definition:

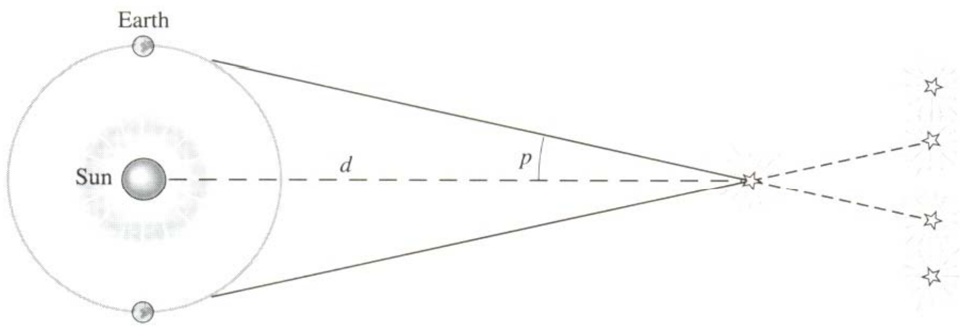
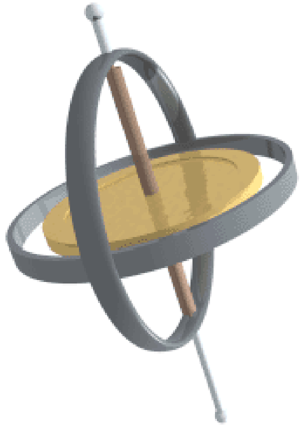


Figure 3.2 Stellar parallax: $d = 1/p''$ pc.

http://instruct1.cit.cornell.edu/courses/astro101/java/parallax/parallax.html

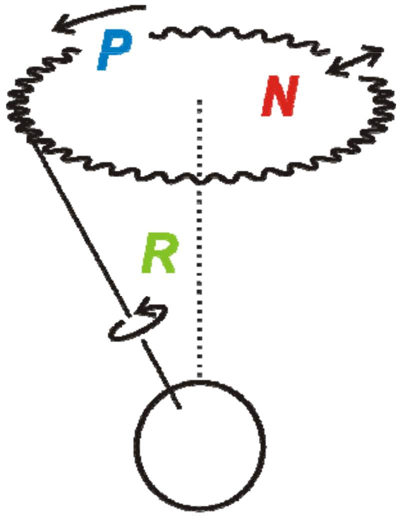
Everything is changing ... [tidal forces]

Precession = movement of the direction of the Earth's rotational axis, caused mainly by Sun and Moon.
period: 25,764 years
magnitude: ~50"

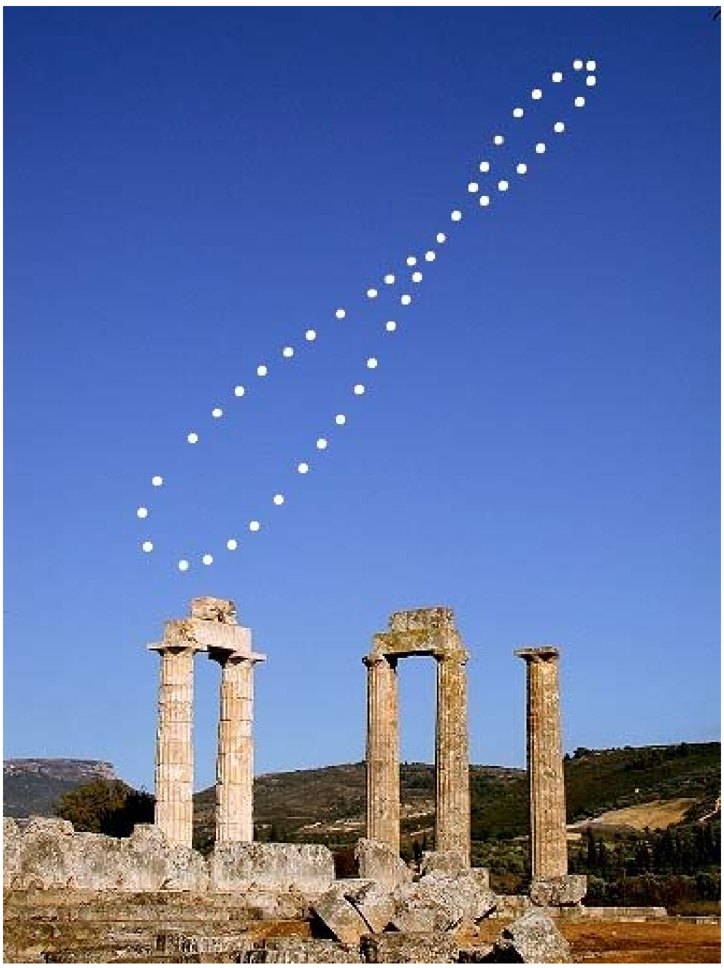


source: Wikipedia

Nutation = tidal forces of Sun and Moon are continuously changing with their relative positions → cause precession to vary over time.
period: 18.6 years (largest component)
magnitude: ~9.2″



Everything is changing ... [position of the Sun]



The Analemma

Positions of the Sun at 12:00hr

↔ elliptical orbit + axis tilt

↑↓ axis tilt

The Horizontal Frame

Origin: Position of the observer

Axes: local vertical (gravity vector)
horizon

Coordinates: azimuthal angle A
height h

Plane containing the local vertical and the direction to the celestial pole is the meridian plane.

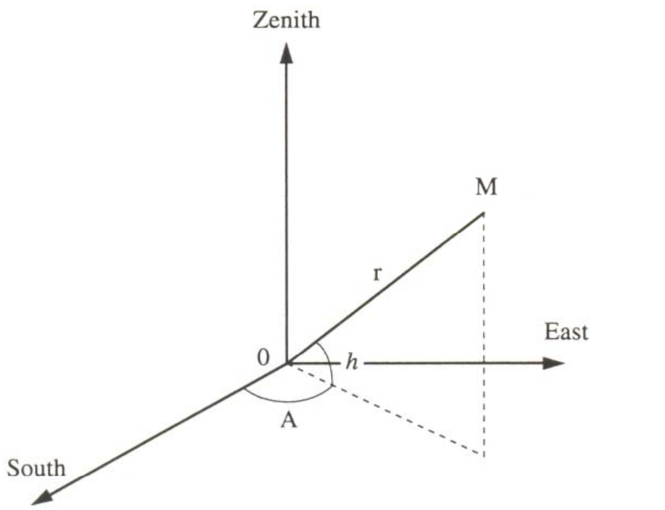


Fig. 8.2. Definition of the horizontal reference system

$$OM(A,h) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cos A \cos h \\ \sin A \cos h \\ \sin h \end{bmatrix}$$

Problem: apparent motion of the celestial sphere.

Note: due to tidal forces the gravity vector changes and hence the coordinates relative to surface structures by typically 0.015"

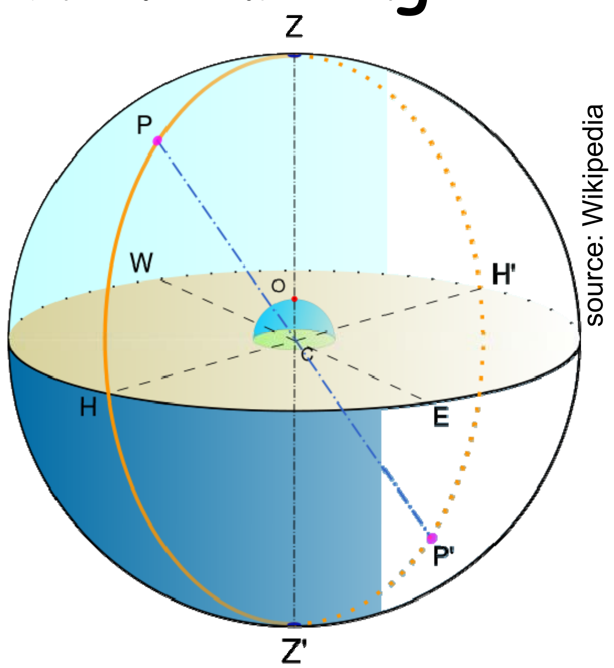
Side notes: Meridian and Hour Angle

A meridian is a great circle on the celestial sphere passing through:

- the north point on the horizon,
- the celestial pole
- the zenith
- the south point on the horizon

It is perpendicular to the local horizon.

Stars will appear to drift past the local meridian as the Earth spins.



An hour angle is the angle described by the earth rotation in one hour = 360°/24hr = 15°.

The hour angle is also the difference between local siderial time (LST) and right ascension α.

The Hour Frame

Improvement: choose celestial equator as fundamental plane.

Origin: Position of the observer

Axes: vector to celestial pole
vector to celestial equator (rot)

Coordinates: declination δ (latitude)
hour angle H

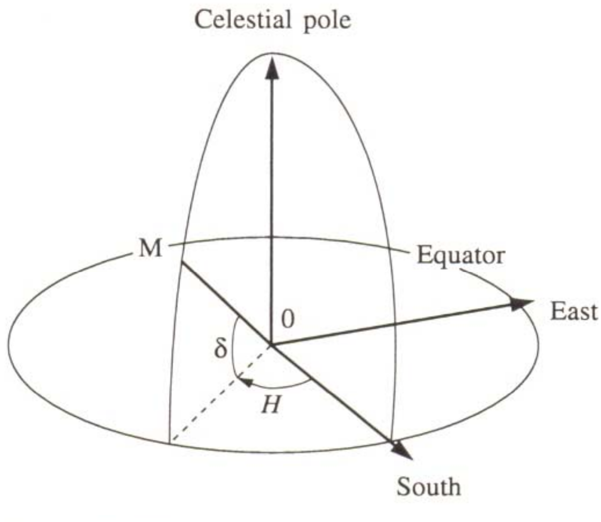


Fig. 8.3. The hour system of coordinates

The hour angle is the angle between the local meridian and the plane containing both the star and the celestial pole.

$$OM(H,\delta) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cos H \cos\delta \\ -\sin H \cos\delta \\ \sin\delta \end{bmatrix}$$

Problem: refers to the observer's local meridian.

The Equatorial Frame

Improvement: removes the largest component of Earth rotation .

Axes: vector to celestial pole
vector to celestial equator (fixed)

Coordinates: declination δ (lattitude)
right ascension α (RA)

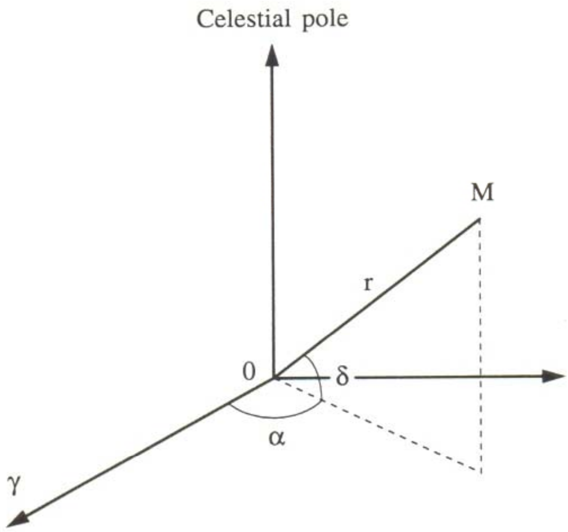


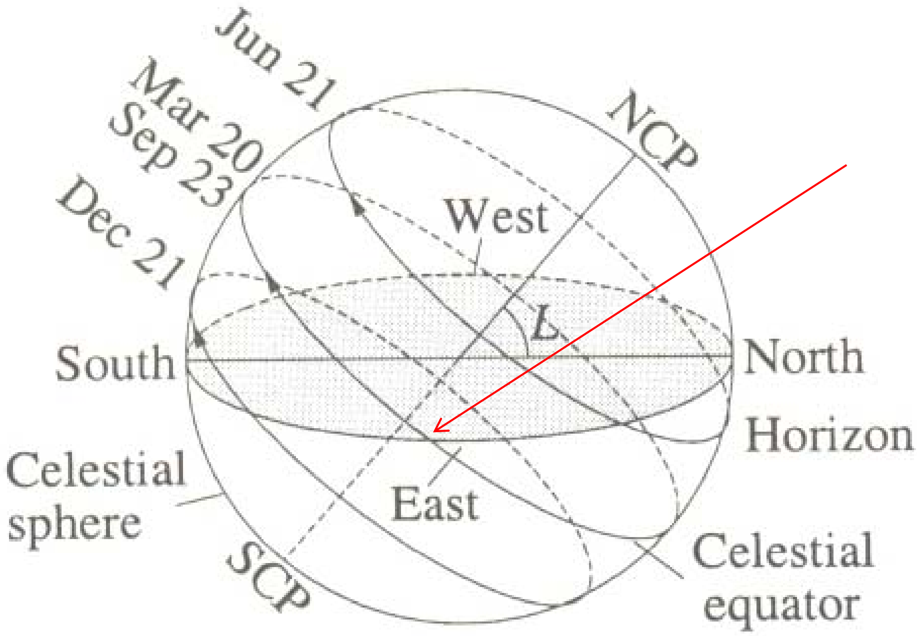
Fig. 8.4. The equatorial reference system

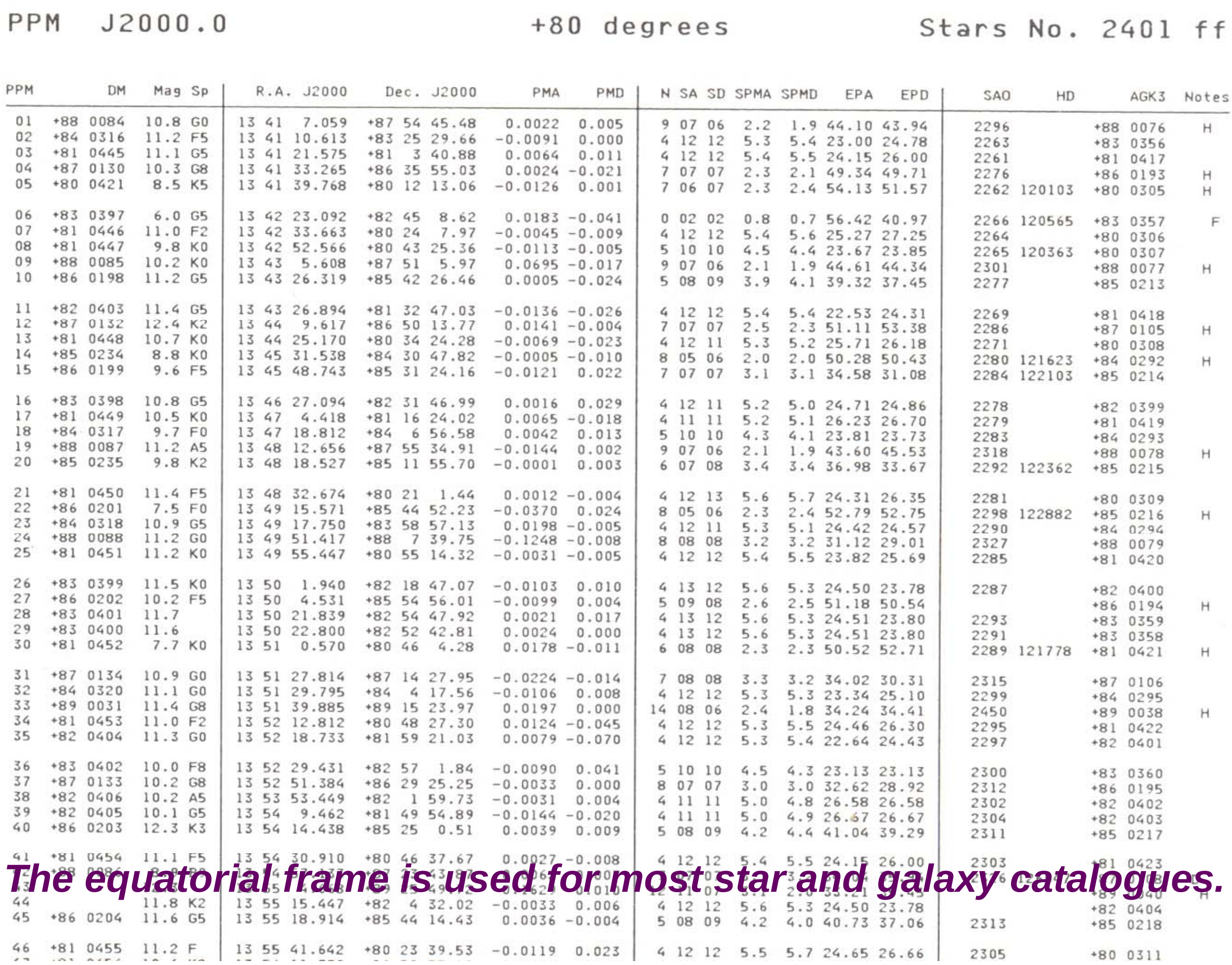
The right ascension is the angle to the intersection between the orbital plane of the Earth and the instantaneous celestial equator (= vernal equinox γ).

$$OM(\alpha,\delta) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cos\alpha \cos\delta \\ \sin\alpha \cos\delta \\ \sin\delta \end{bmatrix}$$

The right ascension is the angle to the intersection between the orbital plane of the Earth and the instantaneous celestial equator (= vernal equinox γ).

At the (vernal) equinox the Sun is at one of two opposite points on celestial sphere where the celestial equator (i.e., declination δ=0) and ecliptic intersect.





PPM J2000.0 +80 degrees Stars No. 2401 ff

PPM	DM	Mag	Sp	R.A. J2000	Dec. J2000	PMA	PMD	N	SA	SD	SPMA	SPMD	EPA	EPD	SAO	HD	AGK3	Notes
01	+88 0084	10.8	G0	13 41 7.059	+87 54 45.48	0.0022	0.005	9	07	06	2.2	1.9	44.10	43.94	2296		+88 0076	H
02	+84 0316	11.2	F5	13 41 10.613	+83 25 29.66	-0.0091	0.000	4	12	12	5.3	5.4	23.00	24.78	2263		+83 0356	
03	+81 0445	11.1	G5	13 41 21.575	+81 3 40.88	0.0064	0.011	4	12	12	5.4	5.5	24.15	26.00	2261		+81 0417	
04	+87 0130	10.3	G8	13 41 33.265	+86 35 55.03	0.0024	-0.021	7	07	07	2.3	2.1	49.34	49.71	2276		+86 0193	H
05	+80 0421	8.5	K5	13 41 39.768	+80 12 13.06	-0.0126	0.001	7	06	07	2.3	2.4	54.13	51.57	2262	120103	+80 0305	H
06	+83 0397	6.0	G5	13 42 23.092	+82 45 8.62	0.0183	-0.041	0	02	02	0.8	0.7	56.42	40.97	2266	120565	+83 0357	F
07	+81 0446	11.0	F2	13 42 33.663	+80 24 7.97	-0.0045	-0.009	4	12	12	5.4	5.6	25.27	27.25	2264		+80 0306	
08	+81 0447	9.8	K0	13 42 52.566	+80 43 25.36	-0.0113	-0.005	5	10	10	4.5	4.4	23.67	23.85	2265	120363	+80 0307	
09	+88 0085	10.2	K0	13 43 5.608	+87 51 5.97	0.0695	-0.017	9	07	06	2.1	1.9	44.61	44.34	2301		+88 0077	H
10	+86 0198	11.2	G5	13 43 26.319	+85 42 26.46	0.0005	-0.024	5	08	09	3.9	4.1	39.32	37.45	2277		+85 0213	
11	+82 0403	11.4	G5	13 43 26.894	+81 32 47.03	-0.0136	-0.026	4	12	12	5.4	5.4	22.53	24.31	2269		+81 0418	
12	+87 0132	12.4	K2	13 44 9.617	+86 50 13.77	0.0141	-0.004	7	07	07	2.5	2.3	51.11	53.38	2286		+87 0105	H
13	+81 0448	10.7	K0	13 44 25.170	+80 34 24.28	-0.0069	-0.023	4	12	11	5.3	5.2	25.71	26.18	2271		+80 0308	
14	+85 0234	8.8	K0	13 45 31.538	+84 30 47.82	-0.0005	-0.010	8	05	06	2.0	2.0	50.28	50.43	2280	121623	+84 0292	H
15	+86 0199	9.6	F5	13 45 48.743	+85 31 24.16	-0.0121	0.022	7	07	07	3.1	3.1	34.58	31.08	2284	122103	+85 0214	
16	+83 0398	10.8	G5	13 46 27.094	+82 31 46.99	0.0016	0.029	4	12	11	5.2	5.0	24.71	24.86	2278		+82 0399	
17	+81 0449	10.5	K0	13 47 4.418	+81 16 24.02	0.0065	-0.018	4	11	11	5.2	5.1	26.23	26.70	2279		+81 0419	
18	+84 0317	9.7	F0	13 47 18.812	+84 6 56.58	0.0042	0.013	5	10	10	4.3	4.1	23.81	23.73	2283		+84 0293	
19	+88 0087	11.2	A5	13 48 12.656	+87 55 34.91	-0.0144	0.002	9	07	06	2.1	1.9	43.60	45.53	2318		+88 0078	H
20	+85 0235	9.8	K2	13 48 18.527	+85 11 55.70	-0.0001	0.003	6	07	08	3.4	3.4	36.98	33.67	2292	122362	+85 0215	
21	+81 0450	11.4	F5	13 48 32.674	+80 21 1.44	0.0012	-0.004	4	12	13	5.6	5.7	24.31	26.35	2281		+80 0309	
22	+86 0201	7.5	F0	13 49 15.571	+85 44 52.23	-0.0370	0.024	8	05	06	2.3	2.4	52.79	52.75	2298	122882	+85 0216	H
23	+84 0318	10.9	G5	13 49 17.750	+83 58 57.13	0.0198	-0.005	4	12	11	5.3	5.1	24.42	24.57	2290		+84 0294	
24	+88 0088	11.2	G0	13 49 51.417	+88 7 39.75	-0.1248	-0.008	8	08	08	3.2	3.2	31.12	29.01	2327		+88 0079	
25	+81 0451	11.2	K0	13 49 55.447	+80 55 14.32	-0.0031	-0.005	4	12	12	5.4	5.5	23.82	25.69	2285		+81 0420	
26	+83 0399	11.5	K0	13 50 1.940	+82 18 47.07	-0.0103	0.010	4	13	12	5.6	5.3	24.50	23.78	2287		+82 0400	
27	+86 0202	10.2	F5	13 50 4.531	+85 54 56.01	-0.0099	0.004	5	09	08	2.6	2.5	51.18	50.54			+86 0194	H
28	+83 0401	11.7		13 50 21.839	+82 54 47.92	0.0021	0.017	4	13	12	5.6	5.3	24.51	23.80	2293		+83 0359	
29	+83 0400	11.6		13 50 22.800	+82 52 42.81	0.0024	0.000	4	13	12	5.6	5.3	24.51	23.80	2291		+83 0358	
30	+81 0452	7.7	K0	13 51 0.570	+80 46 4.28	0.0178	-0.011	6	08	08	2.3	2.3	50.52	52.71	2289	121778	+81 0421	H
31	+87 0134	10.9	G0	13 51 27.814	+87 14 27.95	-0.0224	-0.014	7	08	08	3.3	3.2	34.02	30.31	2315		+87 0106	
32	+84 0320	11.1	G0	13 51 29.795	+84 4 17.56	-0.0106	0.008	4	12	12	5.3	5.3	23.34	25.10	2299		+84 0295	
33	+89 0031	11.4	G8	13 51 39.885	+89 15 23.97	0.0197	0.000	14	08	06	2.4	1.8	34.24	34.41	2450		+89 0038	H
34	+81 0453	11.0	F2	13 52 12.812	+80 48 27.30	0.0124	-0.045	4	12	12	5.3	5.5	24.46	26.30	2295		+81 0422	
35	+82 0404	11.3	G0	13 52 18.733	+81 59 21.03	0.0079	-0.070	4	12	12	5.3	5.4	22.64	24.43	2297		+82 0401	
36	+83 0402	10.0	F8	13 52 29.431	+82 57 1.84	-0.0090	0.041	5	10	10	4.5	4.3	23.13	23.13	2300		+83 0360	
37	+87 0133	10.2	G8	13 52 51.384	+86 29 25.25	-0.0033	0.000	8	07	07	3.0	3.0	32.62	28.92	2312		+86 0195	
38	+82 0406	10.2	A5	13 53 53.449	+82 1 59.73	-0.0031	0.004	4	11	11	5.0	4.8	26.58	26.58	2302		+82 0402	
39	+82 0405	10.1	G5	13 54 9.462	+81 49 54.89	-0.0144	-0.020	4	11	11	5.0	4.9	26.67	26.67	2304		+82 0403	
40	+86 0203	12.3	K3	13 54 14.438	+85 25 0.51	0.0039	0.009	5	08	09	4.2	4.4	41.04	39.29	2311		+85 0217	
41	+81 0454	11.1	F5	13 54 30.910	+80 46 37.67	0.0027	-0.008	4	12	12	5.4	5.5	24.15	26.00	2303		+81 0423	
42	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
43	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
44		11.8	K2	13 55 15.447	+82 4 32.02	-0.0033	0.006	4	12	12	5.6	5.3	24.50	23.78			+82 0404	
45	+86 0204	11.6	G5	13 55 18.914	+85 44 14.43	0.0036	-0.004	5	08	09	4.2	4.0	40.73	37.06	2313		+85 0218	
46	+81 0455	11.2	F	13 55 41.642	+80 23 39.53	-0.0119	0.023	4	12	12	5.5	5.7	24.65	26.66	2305		+80 0311	

The equatorial frame is used for most star and galaxy catalogues.

The Ecliptic Frame

Best for charting solar system objects

Axes: normal to the orbital plane
vector to vernal equinox

Coordinates: ecliptic lattitude β
ecliptic longitude λ

Problem: orbital plane (Earth-Sun vector) is sensitive to perturbations by the other planets → oscillations of ~1″ about mean plane.

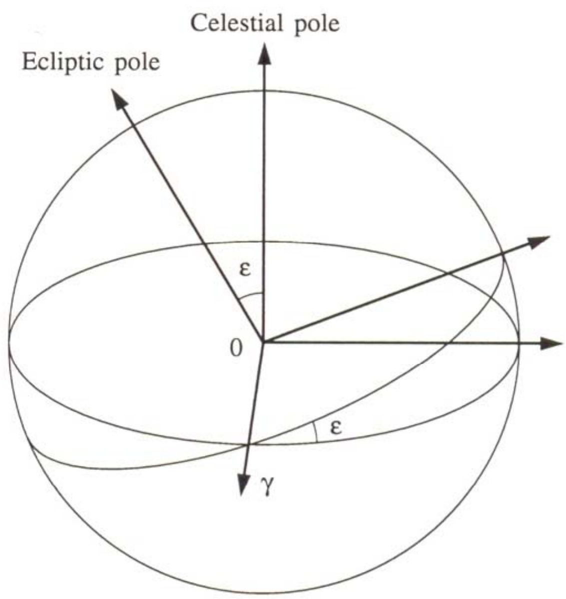


Fig. 8.5. Position of the ecliptic in relation

The Galactic Frame

Useful to chart e.g. stellar populations in the MilkyWay.

Origin: Sun

Axes: rotation vector of the Galaxy
vector to the Galactic Center

Coordinates: galactic lattitude b
galactic longitude l

Definition:
North Galactic pole at $12^h\ 51^m\ 26.282^s$ +27° 07' 42.01" (J2000)
zero longitude at position angle 122.932°
zero point is at $17^h\ 45^m\ 37.224^s$ –28° 56' 10.23" (J2000), close to Sgr A*

The Supergalactic Frame

Equator aligned with the supergalactic plane formed by the nearby galaxy clusters Virgo, the Great Attractor and the Pisces-Perseus supercluster.

Coordinates: supergalactic lattitude SGB
supergalactic longitude SGL

Zero point for SGL is the intersection of the supergalactic plane with the Galactic plane

Definition:
North supergalactic pole at l =47.37°, b=+6.32° or RA=18.9^h, δ=+15.7° (J2000).
Zero point (SGB=0°, SGL=0°) at l=137.37°, b=0° or RA=2.82^h, δ=+59.5° (J2000).

Coordinate Frame Transformations

The easy way:
http://heasarc.gsfc.nasa.gov/cgi-bin/Tools/convcoord/convcoord.pl

The hard way: $\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = [R] \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, where:

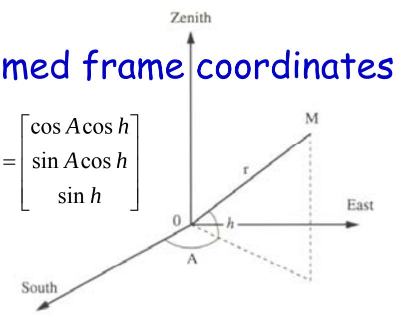
$$R_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_2(\beta) = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_3(\gamma) = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

➔see homework for how to apply these
1. Calculate Cartesian coordinates [x_1,x_2,x_3]
2. Apply rotation
3. Calculate transformed frame coordinates

$$OM(A,h) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cos A \cos h \\ \sin A \cos h \\ \sin h \end{bmatrix}$$



Example: from Equatorial to Galactic Frame

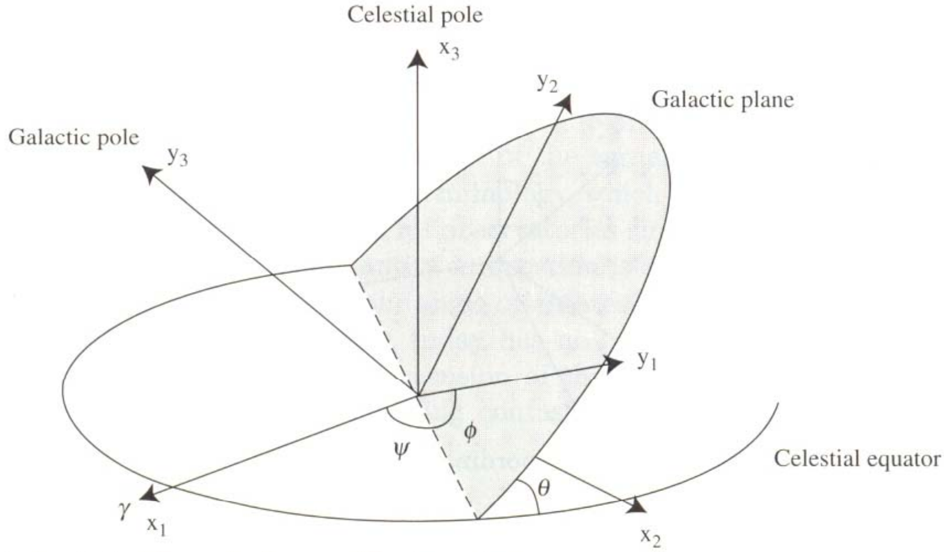


Fig. 8.6. The galactic coordinate system

$$OM(\lambda, \beta) = R_3(\phi) \cdot R_1(\theta) \cdot R_3(\psi) OM(\alpha, \delta)$$

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} -0.05611 & -0.87374 & -0.48315 \\ 0.49333 & -0.44498 & 0.74741 \\ -0.86803 & -0.19642 & 0.45601 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Ephemeris Time (ET)

Uniform motion ⇔ absence of force → ephemeris or position vector r(t)

t is found by solving the equation *OM* = r(t) → only need to define the units "day" or "year"

The independent variable in Simon Newcomb (1898) gravitational theory of the Sun and planets is the ephemeris time (ET).
Assumption: uniform motion of the Sun (mean of Jan 1900)
(not true: irregular and slowing down)

➔ SI definition : one second = 1/31,556,925.9747 of the tropical year for 1900 January 0 at 12 hours ephemeris time.

One Julian century has 36,525 days
One Julian day has 86,400 seconds

→ one tropical year has 365.24219879 days

Universal Time (UT)

Timescale based on the rotation of the Earth.

Reference location Greenwich meridian: Ideally, noon Greenwich Mean Time is when the Sun crosses the Greenwich meridian.

But: variations in the Earth's orbit (nutation, ellipticity, axial tilt) cause differences of up to 16 minutes.

→Use Newcomb's fictitious *mean* Sun – hence also called Greenwich mean time G.M.T.)

UT = 12 hours + Greenwich hour angle

Nowadays, use atomic standards (won't slow down) ...
...and provide connection to Earth orientation.
(undergoes one second discontinuities whenever necessary to keep the Earth rotation in phase with atomic clocks).

Sidereal Time (ST)

Sidereal time ≡ hour angle of the true vernal equinox

The sidereal time is proportional to the UTC.

Note: when ST = RA the object will be at culmination.

When corrected for nutation → mean sidereal time.

Local sidereal time (LST)

Leiden: 52°09'N 04°30'E

http://tycho.usno.navy.mil/sidereal.html

Sidereal Day

Every 24.00 hr the Sun reaches the highest position in the South (= synodic day)

In the meantime, the Earth has rotated a bit more than 360 degrees as it orbits the Sun, (namely 360°/365.25 = 0.986°).

Thus, a sidereal day is shorter: 24hr · 360°/360.986° = 23hr 56min