# Astronomische Waarneemtechnieken (Astronomical Observing Techniques)

## 6th Lecture: 8 October 2012
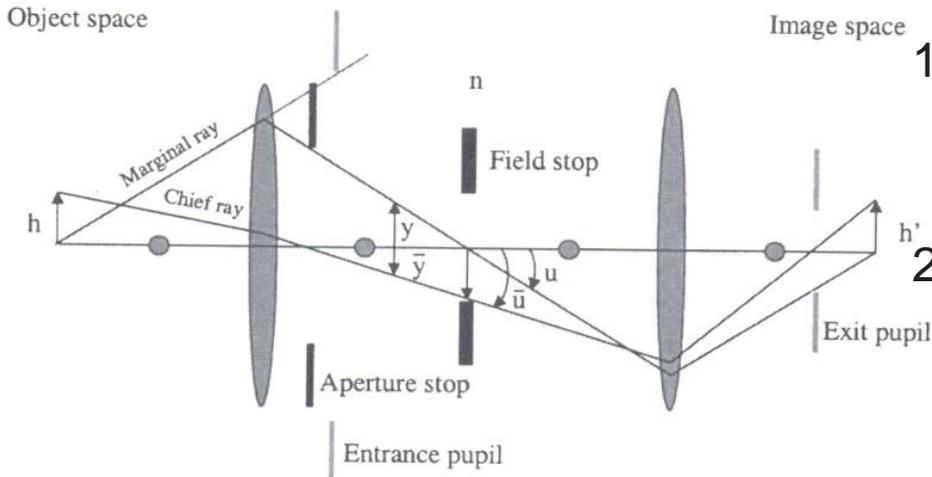


1. Geometrical Optics
   - Definitions
   - Aberrations
2. Diffraction Optics
   - Fraunhofer Diff.
   - PSF, MTF
   - SR & EE
   - high contrast im.

Based on "Astronomical Optics" by Daniel J. Schroeder, "Principles of Optics" by Max Born & Emil Wolf, the "Optical Engineer's Desk Reference" by William L. Wolfe, Lena book, and Wikipedia

# Part I Geometrical Optics

# Part II Diffraction Optics

# IMPORTANT DEFINITIONS

## Aperture and Field Stops
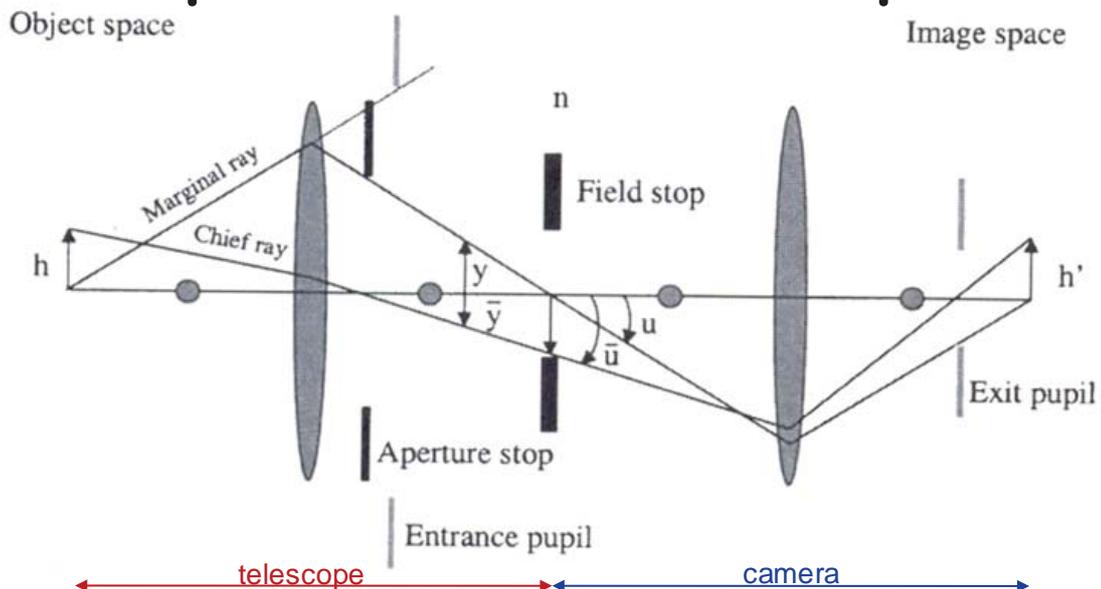


| | |
|---|---|
| Aperture stop: | determines the diameter of the light cone from an axial point on the object. |
| Field stop: | determines the field of view of the system. |
| Entrance pupil: | image of the aperture stop in the object space |
| Exit pupil: | image of the aperture stop in the image space |
| Marginal ray: | ray from an object point on the optical axis that passes at the edge of the entrance pupil |
| Chief ray: | ray from an object point at the edge of the field, passing through the center of the aperture stop. |

# The Speed of the System
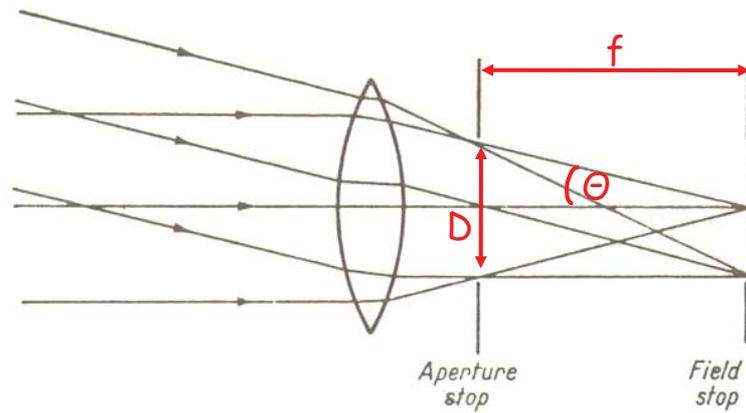


The speed of an optical system is described by the numerical aperture NA and the F number, where:

$$NA = n \cdot \sin\theta \qquad \text{and} \qquad F \equiv \frac{f}{D} = \frac{1}{2(NA)}$$

*Generally, fast optics (large NA) has a high light power, is compact, but has tight tolerances and is difficult to manufacture. Slow optics (small NA) is just the opposite.*

# Étendue and AxΩ

The geometrical étendue (frz. `extent') is the product of area A of the source times the solid angle Ω (of the system's entrance pupil as seen from the source).

The étendue is the maximum beam size the instrument can accept.

Hence, the étendue is also called acceptance, throughput, or A×Ω product.

> The étendue never increases in any optical system. A perfect optical system produces an image with the same étendue as the source.

*Shrinking the field size A makes the beam faster (Ω bigger).*

Area $A = h^2\pi$;
Solid angle Ω given
by the marginal ray.
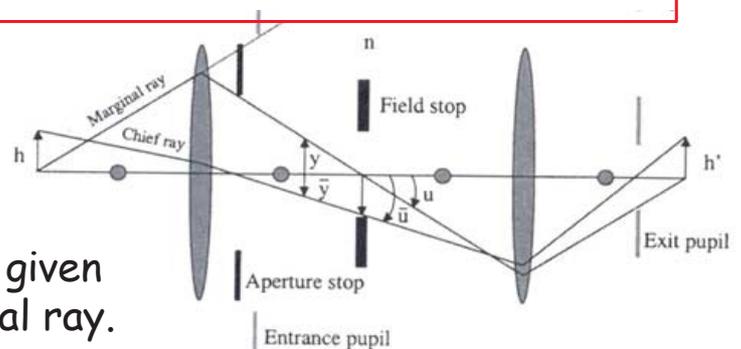
# ABERRATIONS

## Aberrations

Generally, aberrations are departures of the performance of an optical system from the predictions of paraxial optics.

Until photographic plates became available, only *on-axis* telescope performance was relevant.

There are two categories of aberrations:

1. On-axis aberrations (defocus, spherical aberration)

2. Off-axis aberrations:

   a) Aberrations that degrade the image: coma, astigmatism

   b) Aberrations that alter the image position: distortion, field curvature

# Relation between Wave and Ray Aberrations

Consider a reference sphere S of curvature R for a off-axis point P' and an aberrated wavefront W.

An "aberrated" ray from the object intersects the image plane at P".

The ray aberration is P'P".

The wave aberration is $n \cdot \overline{QQ}$
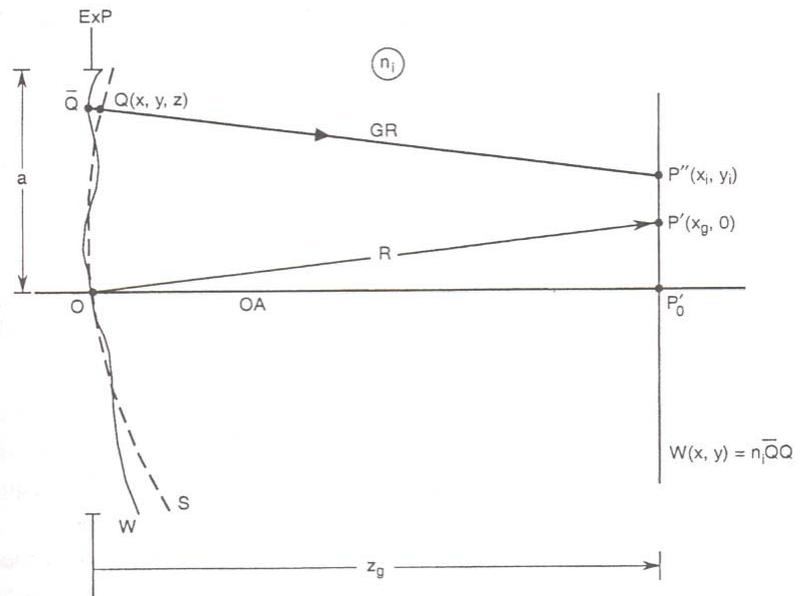


For small FOVs and a radially symmetric aberrated wavefront W(r) we can approximate the intersection with the image plane: $\quad r_i = \dfrac{R}{n_i} \dfrac{\partial W(r)}{\partial r}$

---

# Defocus

Defocus means "out of focus".
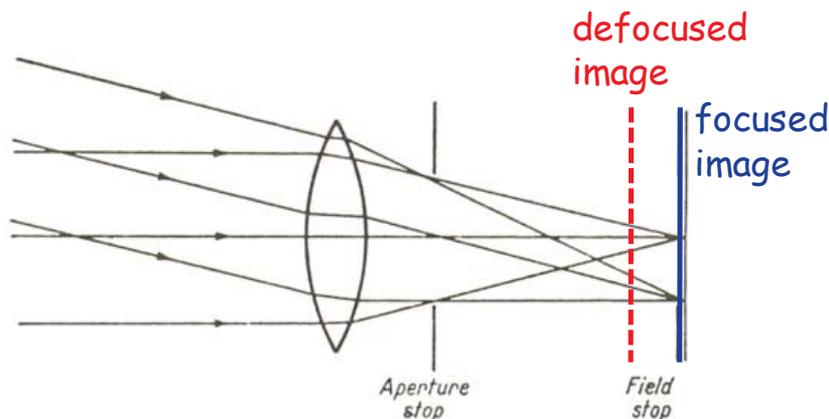


The amount of defocus can be characterized by the depth of focus:

$$\delta = 2\lambda F^2 = \frac{\lambda}{2}\left(\frac{1}{NA}\right)^2$$

The depth of focus usually refers to an optical path difference of λ/4.

# Spherical Aberration

Rays further from the optical axis have a different focal point than rays closer to the optical axis:
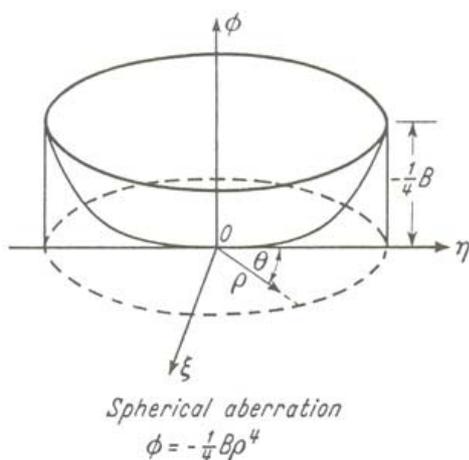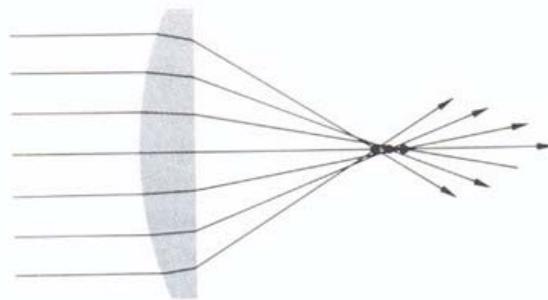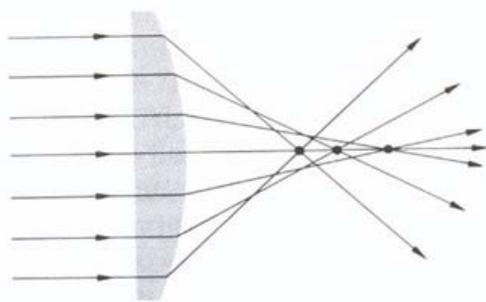




$\phi$ is the wave aberration
$\Theta$ is the angle in the pupil plane
$\rho$ is the radius in the pupil plane
$\xi = \rho \sin\Theta$; $\eta = \rho \cos\Theta$

Spherical aberration
$$\phi = -\tfrac{1}{4}B\rho^4$$

---

# Side note: the HST mirror

Optical problem: HST primary mirror suffers from spherical aberration.

Reason: the *null corrector* used to measure the mirror shape had been incorrectly assembled (one lens was misplaced by 1.3 mm).

Management problem: The mirror manufacturer had analyzed its surface with other null correctors, which indicated the problem, but the test results were ignored because they were believed to be less accurate.

*A null corrector cancels the non-spherical portion of an aspheric mirror figure. When the correct mirror is viewed from point A the combination looks precisely spherical.*
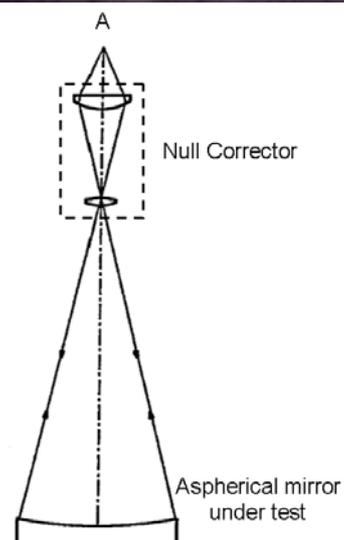
# Coma

Coma appears as a variation in magnification across the entrance pupil. Point sources will show a cometary tail. Coma is an inherent property of telescopes using parabolic mirrors.
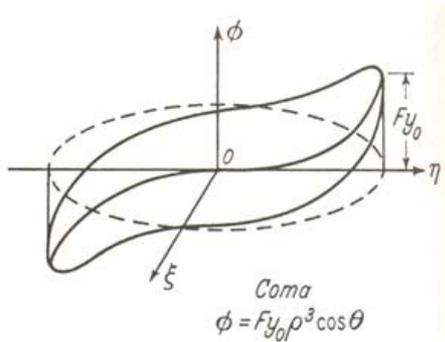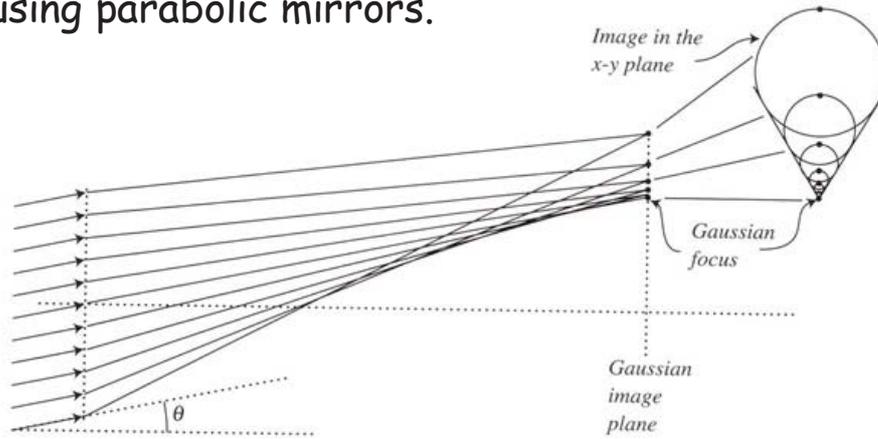


$\phi$ is the wave aberration
$\Theta$ is the angle in the pupil plane
$\rho$ is the radius in the pupil plane
$\xi = \rho \sin\Theta$; $\eta = \rho \cos\Theta$
$y_0$ = position of the object in the field

Coma
$$\phi = F y_0 \rho^3 \cos\theta$$

# Astigmatism

Consider an off-axis point A. The lens does not appear symmetrical from A but shortened in the plane of incidence, the tangential plane. The emergent wave will have a smaller radius of curvature for the tangential plane than for the plane normal to it (sagittal plane) and form an image closer to the lens.
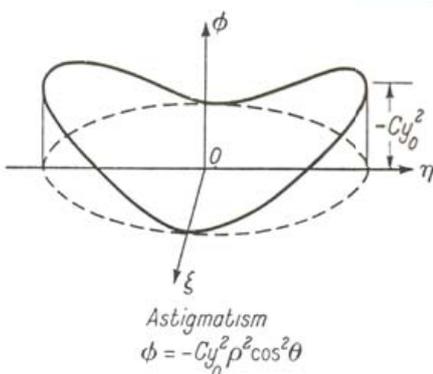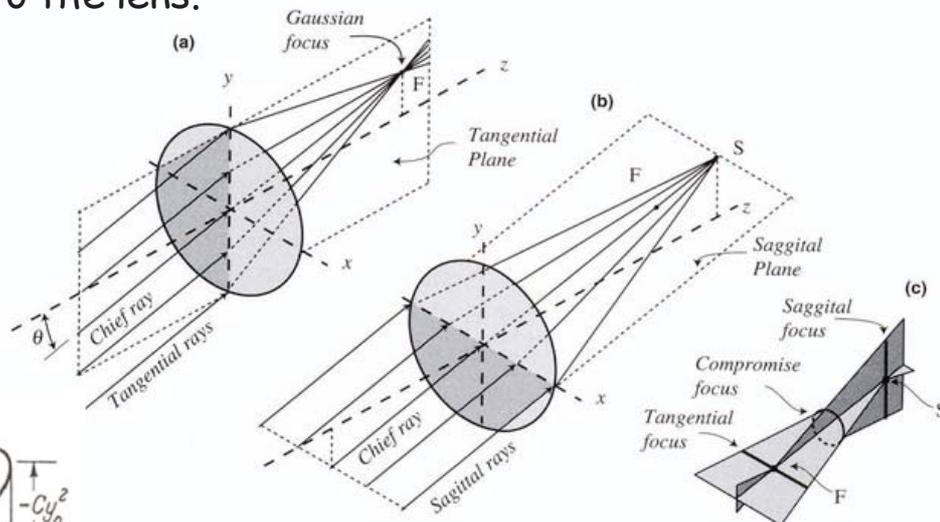


$\phi$ is the wave aberration
$\Theta$ is the angle in the pupil plane
$\rho$ is the radius in the pupil plane
$\xi = \rho \sin\Theta$; $\eta = \rho \cos\Theta$
$y_0$ = position of the object in the field

Astigmatism
$$\phi = -C y_0^2 \rho^2 \cos^2\theta$$

# Field Curvature

Only objects close to the optical axis will be in focus on a flat image plane.   Close-to-axis and far off-axis objects will have different focal points due to the OPL difference.



$\phi$ is the wave aberration
$\Theta$ is the angle in the pupil plane
$\rho$ is the radius in the pupil plane
$\xi = \rho \sin\Theta$;  $\eta = \rho \cos\Theta$
$y_0$ = position of the object in the field

Curvature of field

$$\phi = -\tfrac{1}{2} D y_0^2 \rho^2$$

---
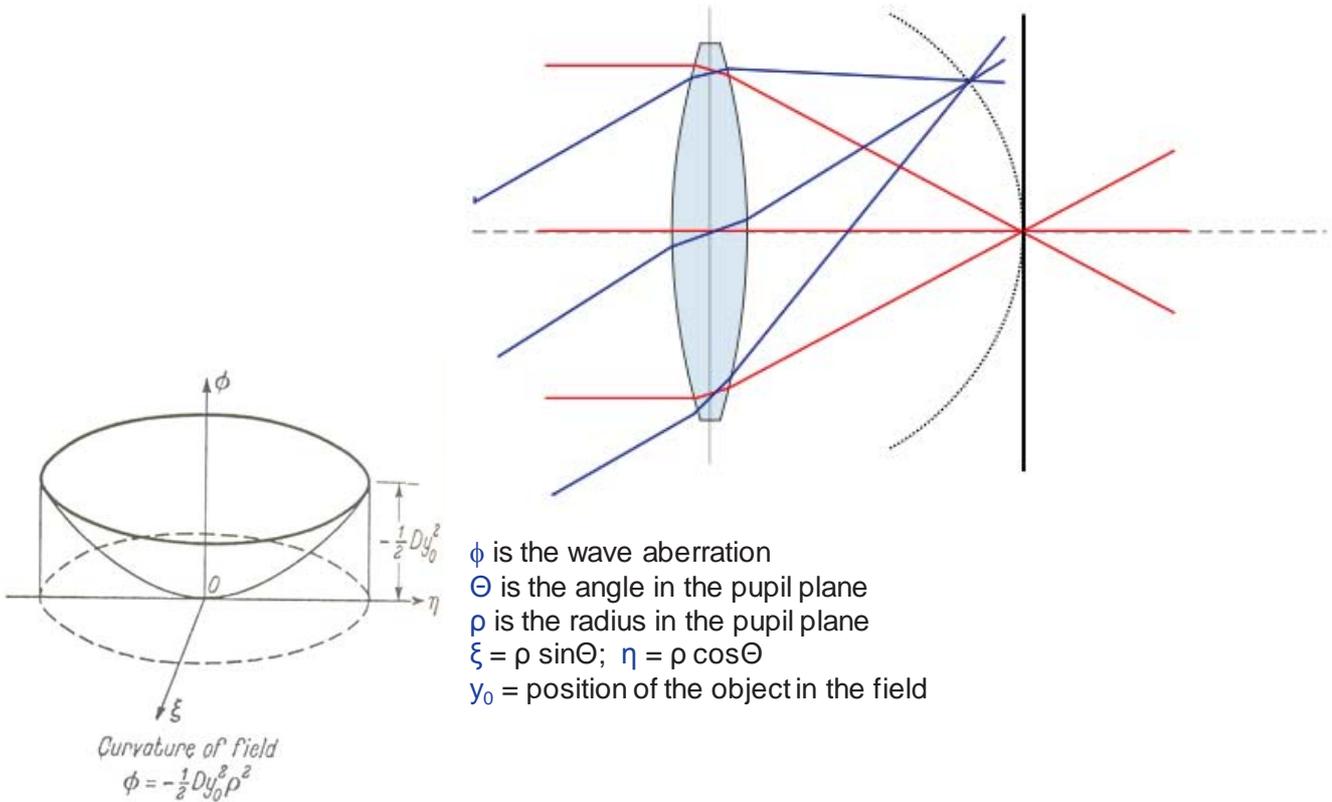
# Distortion  (1)

Straight lines on the sky become curved lines in the focal plane.  The transversal magnification depends on the distance from the optical axis.
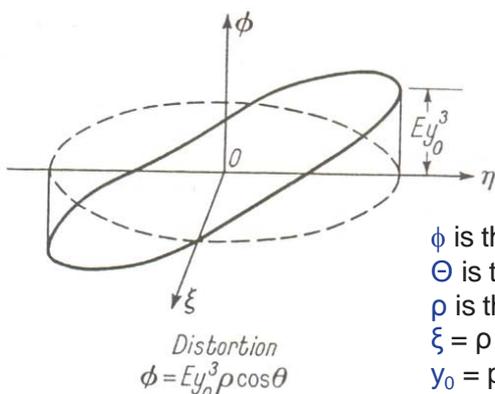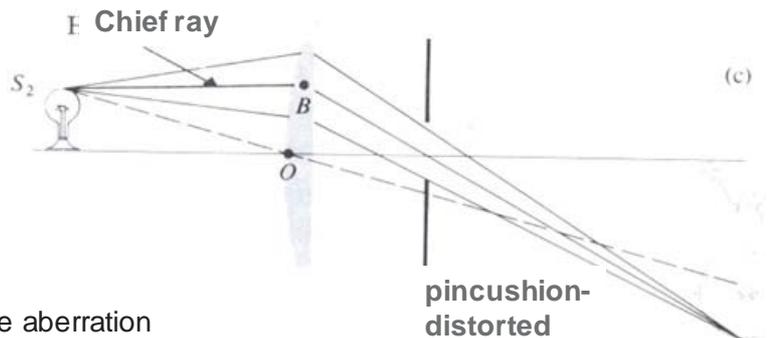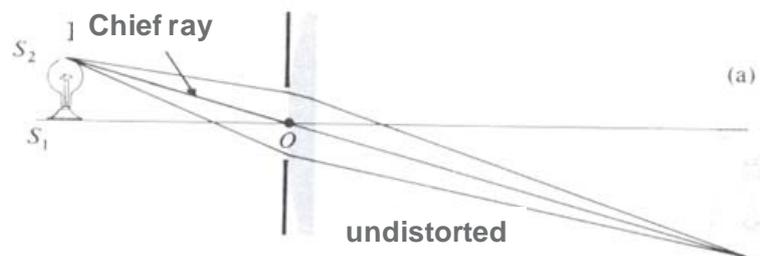


Chief ray

(a)

undistorted

Chief ray

(c)

pincushion-distorted

$\phi$ is the wave aberration
$\Theta$ is the angle in the pupil plane
$\rho$ is the radius in the pupil plane
$\xi = \rho \sin\Theta$;  $\eta = \rho \cos\Theta$
$y_0$ = position of the object in the field

Distortion

$$\phi = E y_0^3 \rho \cos\theta$$

# Distortion (2)

Generally there are two cases:

1. Outer parts have smaller magnification  → barrel distortion
2. Outer parts have larger magnification  → pincushion distortion
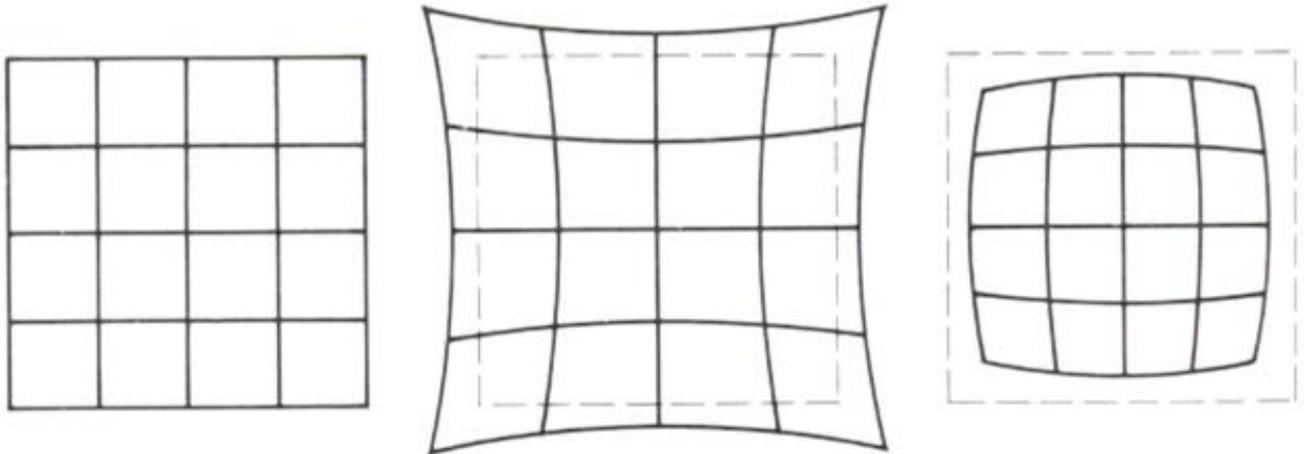


# Summary: Primary Wave Aberrations

| | On-axis focus | On-axis defocus | Off-axis | Off-axis defocus | Dependence on pupil size | Dependence on image size |
|---|---|---|---|---|---|---|
| Spherical aberration | | | | | $\sim\rho^4$ | const. |
| Coma | | | | | $\sim\rho^3$ | $\sim y$ |
| Astigmatism | | | | | $\sim\rho^2$ | $\sim y^2$ |
| Field curvature | | | | | $\sim\rho^2$ | $\sim y^2$ |
| Distortion | | | | | $\sim\rho$ | $\sim y^3$ |
| Defocus | | | | | $\sim\rho^2$ | const. |

# Chromatic Aberration

Since the refractive index $n = f(\lambda)$, the focal length of a lens $= f(\lambda)$ and different wavelengths have different foci. (Mirrors are usually achromatic).

*What's wrong with this figure (taken from Wikipedia)?*
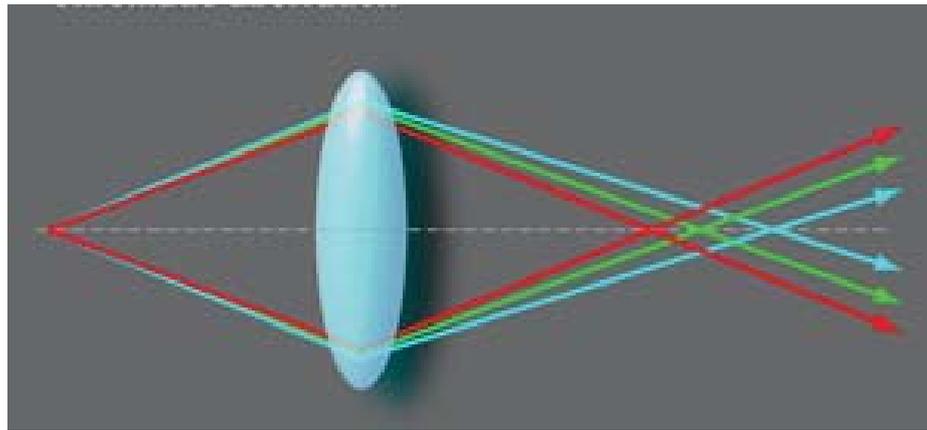
Mitigation: use two lenses of different material with different dispersion → achromatic doublet

Crown — Flint

*Achromatic doublet*
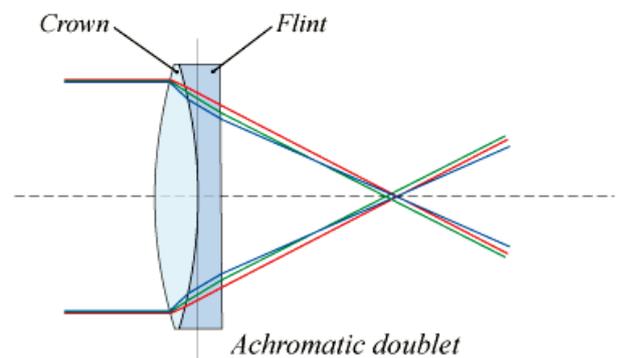
# Part I
# Geometrical Optics

# Part II
# Diffraction Optics

# FRAUNHOFER DIFFRACTION

---

# Fresnel and Fraunhofer Diffraction

Fresnel diffraction = near-field diffraction

When a wave passes through an aperture and diffracts in the near field it causes the observed diffraction pattern to differ in size and shape for different distances.

For Fraunhofer diffraction at infinity (far-field) the wave becomes planar.
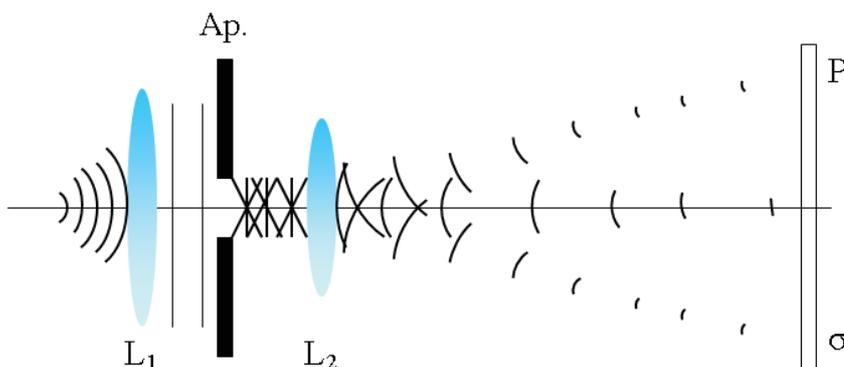


Fresnel: $\quad F = \dfrac{r^2}{d \cdot \lambda} \geq 1$

Fraunhofer: $\quad F = \dfrac{r^2}{d \cdot \lambda} \ll 1$

(where F = Fresnel number, r = aperture size and d = distance to screen).

*An example of an optical setup that displays Fresnel diffraction occurring in the **near-field**. On this diagram, a wave is diffracted and observed at point σ. As this point is moved further back, beyond the Fresnel threshold or in the **far-field**, Fraunhofer diffraction occurs.*

# Fraunhofer Diffraction at a Pupil

Consider a circular pupil function G(r) of unity within A and zero outside.
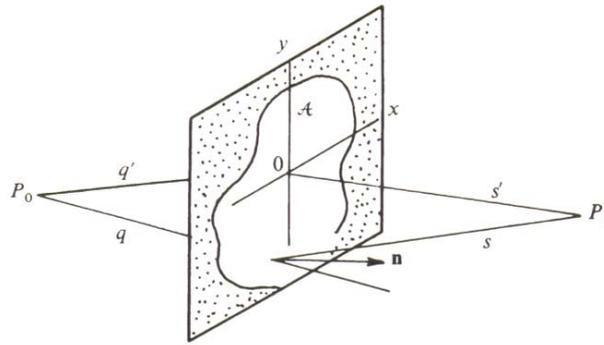


Theorem: *When a screen is illuminated by a source at infinity, the amplitude of the field diffracted in any direction is the Fourier transform of the pupil function characterizing the screen A.*

Mathematically, the amplitude of the diffracted field can be expressed as (see Lena book pp. 120ff for details):

$$V_1(\theta_1, t) = \lambda \sqrt{\frac{E}{A}} \iint\limits_{screenA} G\left(\frac{r}{\lambda}\right) e^{-i2\pi(\theta_1-\theta_0)\cdot\frac{r}{\lambda}} \frac{dr}{\lambda^2}$$

# Imaging and Filtering

$V(\Theta_0)$, $V(\Theta_1)$: complex field amplitudes of points in object and image plane
$K(\Theta_0;\Theta_1)$:      "transmission" of the system



Object          Pupil          Image
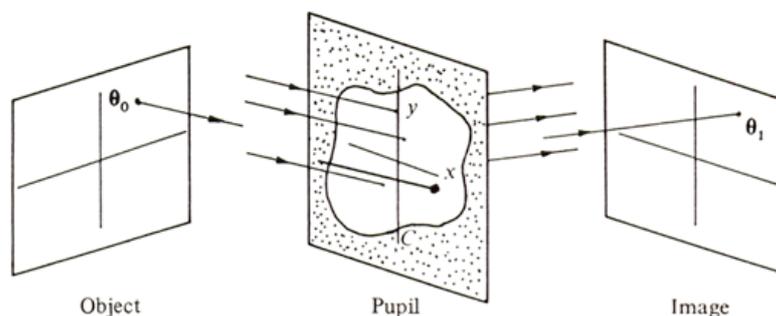
Then the image of an *extended* object can be described by:

$$V(\theta_1) = \underbrace{\iint\limits_{object} V_0(\theta_0) K(\theta_1-\theta_0) d\theta_0}_{\text{convolution}} \quad \text{where} \quad K(\theta) = \iint G(r) e^{-i2\pi\theta\frac{r}{\lambda}} \frac{dr}{\lambda^2}$$

In Fourier space: $FT\{V(\theta_1)\} = FT\{V_0(\theta_0)\} \cdot FT\{K(\theta_0)\} = FT\{V_0(\theta_0)\} \cdot G(r)$

*The Fourier transform of the image equals the product of Fourier transform of the object and the pupil function G, which acts as a linear spatial filter.*

# POINT SPREAD FUNCTION (PSF)

## Point Spread Function (1)

When the circular pupil is illuminated by a point source $I_0(\theta) = \delta(\theta)$ then the resulting PSF is described by a $1^{st}$ order Bessel function:

$$I_1(\theta) = \left( \frac{2J_1(2\pi\, r_0\theta/\lambda)}{2\pi\, r_0\theta/\lambda} \right)^2$$

This is also called the Airy function.
The radius of the first dark ring (minimum) is at:

$$r_1 = 1.22\lambda F \qquad \text{or} \qquad \alpha_1 = \frac{r_1}{f} = 1.22\frac{\lambda}{D}$$
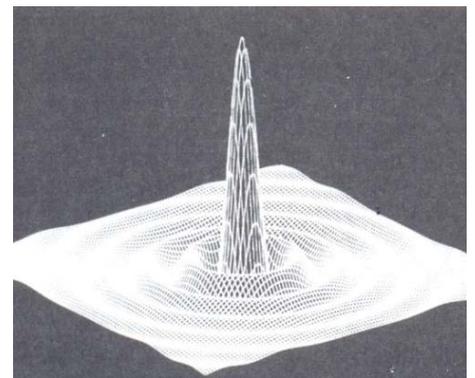


The PSF is often simply characterized by the half power beam width (HPBW) or full width half maximum (FWHM) in angular units.

According to the Nyquist-Shannon sampling theorem $I(\Theta)$ (or its FWHM) shall be sampled with a rate of at least:

$$\Delta\theta = \frac{1}{2\omega_c}$$

# Point Spread Function  (2)

Most "real" telescopes have a central obscuration, which modifies our simplistic pupil function $\qquad G(r) = \Pi(r/2r_0)$

The resulting PSF can be described by a modified Airy function:

$$I_1(\theta) = \frac{1}{\left(1-\varepsilon^2\right)^2}\left(\frac{2J_1\left(2\pi\, r_0\theta/\lambda\right)}{2\pi\, r_0\theta/\lambda} - \varepsilon^2\,\frac{2J_1\left(2\pi\, r_0\varepsilon\theta/\lambda\right)}{2\pi\, r_0\varepsilon\theta/\lambda}\right)^2$$

where $\varepsilon$ is the fraction of central obscuration to total pupil area.

*Astronomical instruments sometimes use a phase mask to reduce the secondary lobes of the PSF (from diffraction at "hard edges"). Phase masks introduce a position dependent phase change.  This is called apodisation.*

Radii of Dark Rings in Airy Pattern[a,b]

| $\varepsilon$ | $w_1$ | $w_2$ | $w_3$ |
|------|------|------|------|
| 0.00 | 1.220 | 2.233 | 3.238 |
| 0.10 | 1.205 | 2.269 | 3.182 |
| 0.20 | 1.167 | 2.357 | 3.087 |
| 0.33 | 1.098 | 2.424 | 3.137 |
| 0.40 | 1.058 | 2.388 | 3.300 |
| 0.50 | 1.000 | 2.286 | 3.491 |
| 0.60 | 0.947 | 2.170 | 3.389 |

[a] Subscript on $w$ is the number of the dark ring starting at the innermost ring.
[b] $w = v/\pi$.

# Optical/Modulation Transfer Function

Remember: so far we have used the Rayleigh criterion to describe resolution:  two sources can be resolved if the peak of the second source is no closer than the 1st dark Airy ring of the first source.

$$\sin\Theta = 1.22\,\frac{\lambda}{D}$$
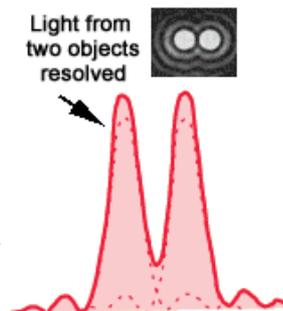
A better measure of the resolution that the system is capable of is the optical transfer function (OTF):
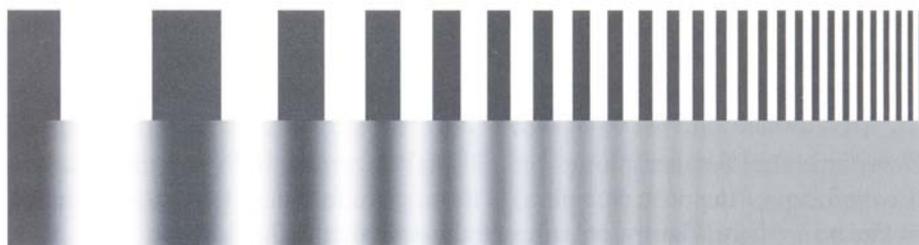


Figure 2.10. Bar chart test of resolution. The upper half shows the object imaged, while the lower half is the image and shows the blurring due to the optical system. Based on material from Norman Koren (n.d.), with permission.

(a) Input × (b) output

$$MTF(f) = \frac{C(f)}{C_0}, \qquad \text{where } C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$
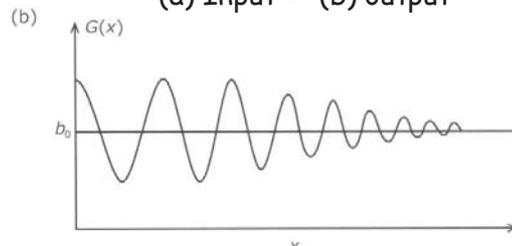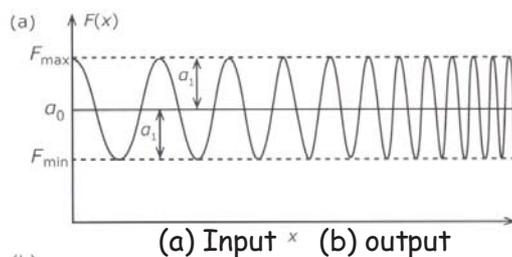
# Optical/Modulation Transfer Function  (2)

The Optical Transfer Function (OTF) describes the spatial signal variation as a function of spatial frequency.  With the spatial frequencies $(\xi, \eta)$, the OTF can be written as…

$$OTF(\xi, \eta) = MTF(\xi, \eta) \cdot PTF(\xi, \eta)$$

$$MTF(\xi, \eta) = |OTF(\xi, \eta)|$$

$$PTF(\xi, \eta) = e^{-i2\pi\lambda(\xi, \eta)}$$

…where the Modulation Transfer Function (MTF) describes its magnitude, and the Phase Transfer Function (PTF) the phase.

Example:
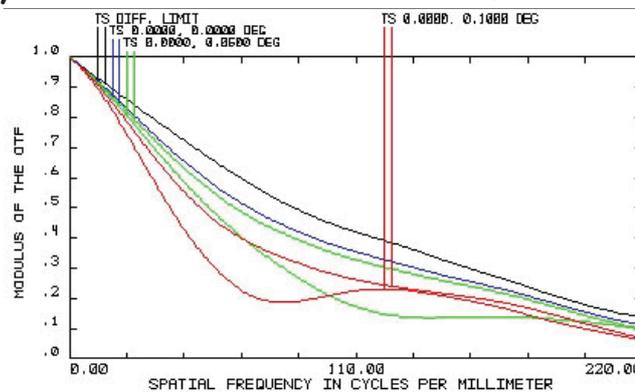


**SR & EE**

# Strehl Ratio

A convenient measure to assess the quality of an optical system is the Strehl ratio.

The Strehl ratio (SR) is the ratio of the observed *peak intensity* of the PSF compared to the theoretical maximum peak intensity of a point source seen with a perfect imaging system working at the diffraction limit.

Using the wave number $k=2\pi/\lambda$ and the RMS wavefront error $\omega$ one can calculate that:

$$SR = e^{-k^2\omega^2} \approx 1 - k^2\omega^2$$

Examples:

- A SR > 80% is considered diffraction-limited → average WFE ~ $\lambda/14$

- A typical adaptive optics system delivers SR ~ 10-50% (depends on $\lambda$)

- A seeing-limited PSF on an 8m telescope has a SR ~ 0.1-0.01%.

# Encircled Energy

Q: What is the maximum concentration of light within a small area? The fraction of the total PSF intensity within a certain radius is given by the encircled energy (EE):

$$EE(r) = 1 - J_0^2\left(\frac{\pi r}{\lambda F}\right) - J_1^2\left(\frac{\pi r}{\lambda F}\right) \qquad F \text{ is the f/\# number}$$

Note that the EE depends strongly on the central obscuration $\varepsilon$ of the telescope:
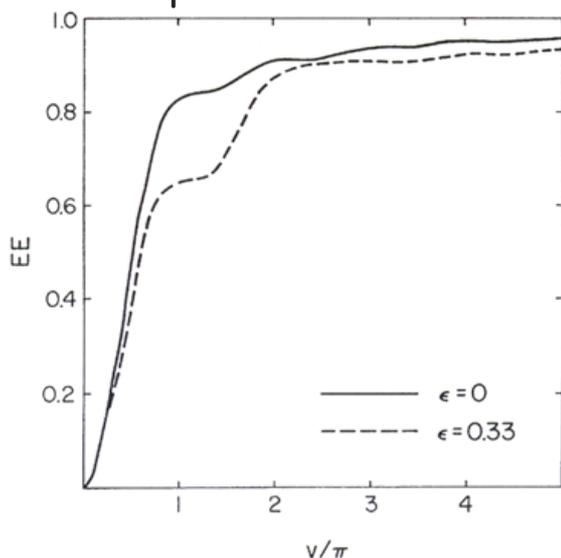


Encircled Energy Fraction within Airy Dark Rings[a]

| $\varepsilon$ | $EE_1$ | $EE_2$ | $EE_3$ |
|---|---|---|---|
| 0.00 | 0.838 | 0.910 | 0.938 |
| 0.10 | 0.818 | 0.906 | 0.925 |
| 0.20 | 0.764 | 0.900 | 0.908 |
| 0.33 | 0.654 | 0.898 | 0.904 |
| 0.40 | 0.584 | 0.885 | 0.903 |
| 0.50 | 0.479 | 0.829 | 0.901 |
| 0.60 | 0.372 | 0.717 | 0.873 |

[a] Subscript on EE is number of dark ring starting at innermost ring.

# High Contrast Imaging

---

## One Motivation: Exoplanet Detections



HR 8799 - first directly imaged multi-exoplanet system

b
c
e
d

20 AU
0.5″

1 November 2009, L′ band          Marois et al (Nature, 2010)
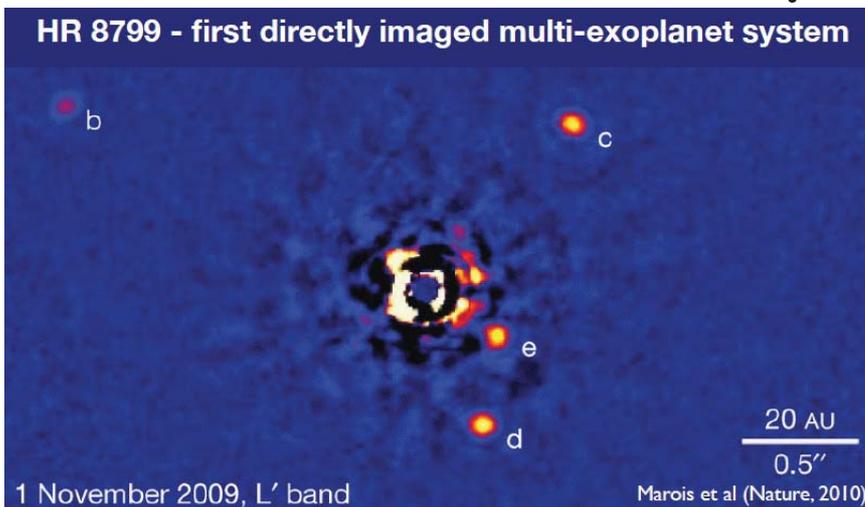
Goal: direct imaging of exoplanets…

But only few have been directly imaged so far:

| | **777 planets** |
|---|---|
| **Candidates detected by radial velocity or astrometry** *update : 05 July 2012* | 570 planetary systems 715 planets 96 multiple planet systems |
| **Transiting planets** *update : 05 July 2012* | 205 planetary systems 239 planets 30 multiple planet systems |
| **Candidates detected by microlensing** *update : 02 June 2012* | 15 planetary systems 16 planets 1 multiple planet systems |
| **Candidates detected by imaging** *update : 05 April 2012* | 27 planetary systems 31 planets 2 multiple planet systems |
| **Candidates detected by timing** *update : 15 June 2012* | 12 planetary systems 15 planets 2 multiple planet systems |

# The Challenge: Contrast
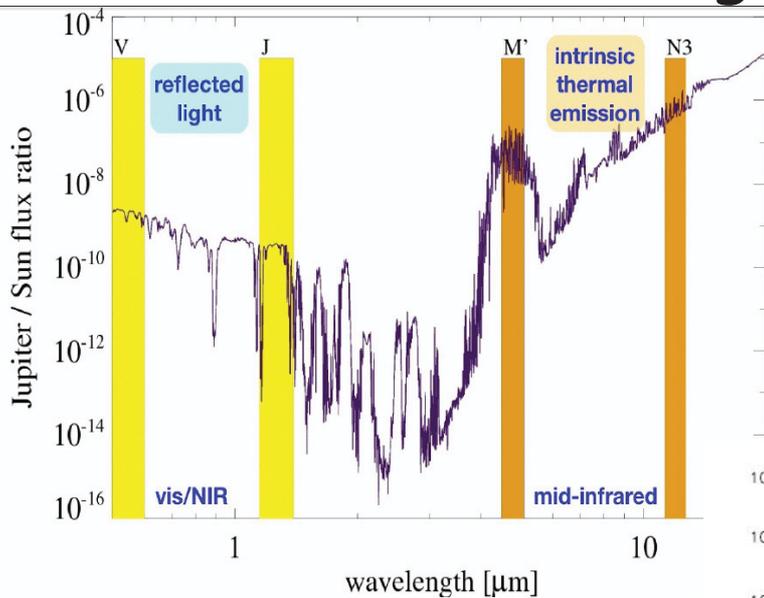


The brightness contrast between Jupiter and Sun

Achievable contrast with and without coronagraph (example)
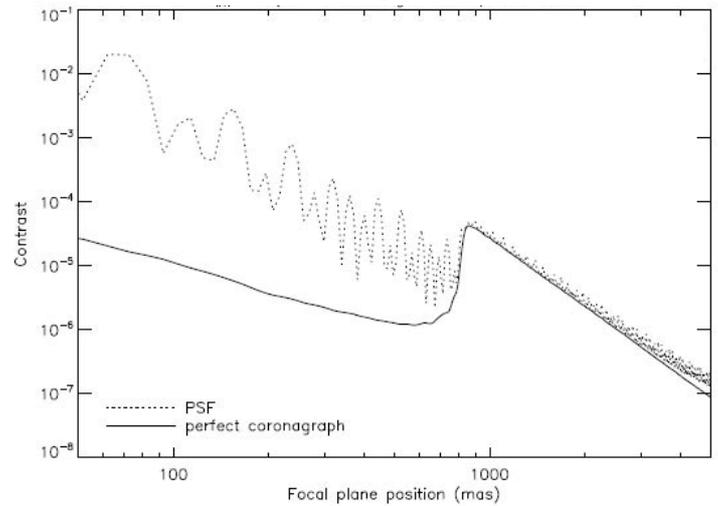
**Fusco et al. (2006)**

# Lyot Coronagraphy



Illuminated telescope

Diffraction limited image

Illumination input pattern to pupil

inputs →

**Output pattern after masking outer zones**

outputs →

(a)     (b)     (c)     (d)

aperture stop     occulting spot     Lyot stop     detector array