

Exercises Astronomical Observing Techniques, Set 4

3 October 2012

Exercise 1

The Fourier pairs $f(x)$ and $F(s)$ are defined as follows:

$\int_{-\infty}^{+\infty} f(x)e^{-2\pi ixs} dx = F(s)$: $\mathcal{F}\{f(x)\} = F(s)$, the Fourier transform of $f(x)$ and
 $\int_{-\infty}^{+\infty} F(s)e^{2\pi ixs} ds = f(x)$: $\hat{\mathcal{F}}\{F(s)\} = f(x)$, the inverse Fourier transform of $F(s)$

a) show that: $\mathcal{F}\{a(f(x)) + b(g(x))\} = a\mathcal{F}\{f(x)\} + b\mathcal{F}\{g(x)\}$

b) show that: $\mathcal{F}\{f(x - a)\} = e^{-2\pi ias}F(s)$

c) show that: $\mathcal{F}\{f(ax)\} = \frac{1}{|a|}F(s/a)$

Exercise 2

Compute the Fourier transforms (definition in Exercise 1) of:

a) $\delta(x)$

b) $\delta(x + a)$

c) $e^{-x^2\pi}$

d) $\frac{1}{2}\{\delta(x + \frac{1}{2}) + \delta(x - \frac{1}{2})\}$

e) $\Pi(x)$, 1 for $|x| < \frac{1}{2}a$, else 0

Exercise 3

Show that $\mathcal{F}\{\frac{df(x)}{dx}\} = 2\pi isF(s)$

Exercise 4

The 2D Fourier pairs $f(x, y)$ and $F(u, v)$ are defined as follows:

$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y)e^{-2\pi i(xu+vy)} dx dy = F(u, v)$: $\mathcal{F}\{f(x, y)\} = F(u, v)$, the Fourier transform of $f(x, y)$ and

$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(u, v)e^{2\pi i(xu+vy)} du dv = f(x, y)$: $\hat{\mathcal{F}}\{F(u, v)\} = f(x, y)$, the inverse Fourier transform of $F(u, v)$

Compute the 2D Fourier transforms of:

a) $\delta(x, y)$

b) $\delta(x - a, y - b)$