Detection of Light. Problem Set 1

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1 Fermi energy

Consider the Fermi distribution for the probability of finding an electron (or any other fermion) with an energy E in a given physical system:

$$f(E) = \frac{1}{1 + e^{(E-E_{\mathrm{F}})/kT}}$$

a What can you say about the symmetry properties of $f(E)$ with respect to the *Fermi energy*, E_{F}? Is $f(E)$ symmetric or antisymmetric?

b In an isolated system, what is the probability of finding an electron with an energy equal to the Fermi energy? Does this depend on the temperature T of the system?

c Sketch the shape of $f(E)$ in the case of $T = 0$ (absolute zero). What does this mean in terms of the occupation of states in the system?

d Using the symmetry properties of $f(E)$, show that, in an intrinsic semiconductor, the Fermi energy lays in the middle of the band gap E_{g}[1]

e In an intrinsic semicondutor, is it possible to find electrons with energy E_{F}?

[1] This is actually not entirely true, since the position of the Fermi level depends on the ratio of the equivalent masses of electrons and holes (see Problem 2). The difference is, however, very small.

2 Carrier Concentration in Intrinsic and Extrinsic Semiconductors

The Fermi distribution gives us only the probability of finding an electron with certain energy E. But in a given physical system, each energy has several states associated with it. Hence, to account for the actual number of electrons within an infinitesimal energy range dE, we need to know the *density of states* (cm^{-3}) of the system, $N(E)$. We can then calculate the concentration of electrons n_0 in an energy range (E_1, E_2) as follows:

$$n_0 = \int_{E_1}^{E_2} f(E)N(E)\,\mathrm{dE}$$

For a semiconductor material, $N(E)$ can be calculated by solving the Schrödinger equation for the electron in the potential energy of the the lattice. For the conduction band, it can be shon that the density of states adopts the form:

$$N(E) = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}(E - E_c)^{1/2}\,\mathrm{dE}$$

where m^* is an effective mass of the carrier, associated with the potential energy in the lattice, and $\hbar$ is the Planck constant divided by 2π.

a Consider the concentration of carriers (electrons) in the conduction band of a semiconductor. Show that in the limit where $E - E_\mathrm{F} >> kT$, the concentration of electrons is given by:

$$n_0 = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} e^{(E_\mathrm{F}-E_\mathrm{c})/kT}$$

where E_c is the bottom energy edge of the conduction band.

Tip:

$$\int_0^\infty x^{1/2}e^{-ax}\,\mathrm{dx} = \frac{\sqrt{\pi}}{2\mathrm{a}\sqrt{\mathrm{a}}}$$

b Discuss how n_0 depends on the energy difference between the Fermi level E_F and the edge of the conduction band E_c. In a n-type semiconductor, an intermediate level E_d is created very close to the conduction band. Intuitively, what do you think is the effect of this on the position of the Fermi level? Does the concentration of carriers in the conduction band increases or decreases with respect to the intrinsic case?

c In a similar way, it can be shown that the concentration of holes in the valence band can be written as:

$$p_0 = 2\left(\frac{2\pi m_\mathrm{p}^* kT}{h^2}\right)^{3/2} e^{-(E_\mathrm{F}-E_\mathrm{v})/kT}$$

where E_v is the top energy edge of the valence band.

Show that for a given material at a given temperature, the product of n_0 and p_0 is a constant that depends only on the energy

d Show that in the case of an intrinsic semiconductor, the concentration of electrons in the conduction band (and holes in the valence band) is given by:

$$n_\mathrm{i} = \sqrt{N_\mathrm{c} N_\mathrm{v}} e^{-E_\mathrm{g}/2kT}$$

where $N_\mathrm{c} = 2\left(\frac{2\pi m_\mathrm{n}^* kT}{h^2}\right)^{3/2}$ and $N_\mathrm{v} = 2\left(\frac{2\pi m_\mathrm{p}^* kT}{h^2}\right)^{3/2}$.

Where is the Fermi level in comparison with E_v and E_c if $N_\mathrm{c} = N_\mathrm{v}$?

e Now let's plug in some numbers. Calculate the intrinsic concentration for Si ($E_\mathrm{g} = 1.11\mathrm{eV}$) at room temperature (300 K) using the previous expression. Assume $m_\mathrm{n}^* = 1.1\, m_0$ and $m_\mathrm{p}^* = 0.56\, m_0$. You also need to know:

$m_0 = 9.11 \times 10^{-28}$ g (rest mass of the electron)
$k = 1.381 \times 10^{-16}\,\mathrm{g\,cm^2\,s^{-2}\,K^{-1}} = 8.617 \times 10^{-5}\,\mathrm{eV\,K^{-1}}$ (Boltzmann constant)
$h = 6.626 \times 10^{-27}\,\mathrm{g\,cm^2\,s^{-1}}$ (Planck constant)

f Finally, suppose that the Si sample is dopped with 10^{17} As atoms/cm^3 (at room temperature, the atomic density of Si is $4.96 \times 10^{22}\,\mathrm{cm^{-3}}$; hence, this doping is of about 1 As atom in 10^5 Si atoms). What is the equilibrium hole concentration p_0 at 300 K? Where is E_F relative to E_i?