XIII. Principle of Heterodyne Receivers
XIV. Mixers for Heterodyne Detectors
XV. Performance of Heterodyne Det.
Two Fundamental Principles of Detection

Respond to individual photon energy

Direct Detection

Photons

Respond to electrical field strength and preserve phase

Coherent Amplification

Waves
Principle of Heterodyne Detectors
Light as a Wave

\[ E(z) = E_0 \cdot \sin(kz - \omega t + \phi) \]

Phase

\[ \omega = \frac{2\pi}{\lambda} \]

Intensity \( \propto (E_0)^2 \)

Direction of propagation (\( z \))

<table>
<thead>
<tr>
<th>10 mm</th>
<th>3 mm</th>
<th>1 mm</th>
<th>300 ( \mu )m</th>
<th>100 ( \mu )m</th>
<th>30 ( \mu )m</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 GHz</td>
<td>100 GHz</td>
<td>300 GHz</td>
<td>1 THz</td>
<td>3 THz</td>
<td>10 THz</td>
</tr>
<tr>
<td>( 3 \times 10^{10} )</td>
<td>( 1 \times 10^{11} )</td>
<td>( 3 \times 10^{11} )</td>
<td>( 1 \times 10^{12} )</td>
<td>( 3 \times 10^{12} )</td>
<td>( 1 \times 10^{13} )</td>
</tr>
</tbody>
</table>

RADIO

MICROWAVE

SUB-MM

INFRARED

Detection of Light – Bernhard Brandl

1-4-2015
Problem with “conventional” Detection

Consider a detector that responds linearly within a response time $\tau$ to measure the resulting field:

If $\tau \gg f$ the output will be zero on average

There may not be any detectors which have short enough response times $\tau$

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Frequency (GHz)</th>
<th>Frequency (THz)</th>
<th>Response time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 metres</td>
<td>0.03</td>
<td>0.000003</td>
<td>3.33E-08</td>
</tr>
<tr>
<td>1 metre</td>
<td>0.3</td>
<td>0.0003</td>
<td>3.33E-09</td>
</tr>
<tr>
<td>10cm</td>
<td>3</td>
<td>0.003</td>
<td>3.33E-10</td>
</tr>
<tr>
<td>1cm</td>
<td>30</td>
<td>0.3</td>
<td>3.33E-11</td>
</tr>
<tr>
<td>1 mm</td>
<td>300</td>
<td>3</td>
<td>3.33E-12</td>
</tr>
<tr>
<td>0.1 mm</td>
<td>3000</td>
<td>30</td>
<td>3.33E-13</td>
</tr>
<tr>
<td>10 microns</td>
<td>30000</td>
<td>300</td>
<td>3.33E-14</td>
</tr>
<tr>
<td>1 micron</td>
<td>300000</td>
<td>3000</td>
<td>3.33E-15</td>
</tr>
<tr>
<td>0.1 micron</td>
<td>3000000</td>
<td>30000</td>
<td>3.33E-16</td>
</tr>
</tbody>
</table>
Heterodyne Receivers

Heterodyning is a radio signal processing technique invented in 1901 by Canadian inventor-engineer Reginald Fessenden.

Reginald Aubrey Fessenden (1866 – 1932)

Heterodyning = new frequencies are created by combining or mixing two frequencies
Mixing (1)

Directly mix incoming light with coherent light of nearly the same frequency:

**Signal S1** is mixed with a local oscillating field S2
Mixing (2)

The mix produces a lower frequency converted intermediate frequency or beat frequency at:

\[ \omega_{S1} - \omega_{S2} \text{ and } \omega_{S1} + \omega_{S2} \]

The resultant mix produces a lower frequency envelope that we can measure.
The Intermediate Frequency (IF)

The IF is the “beat frequency” and is the difference between the local oscillator and the signal frequency.

The goal is to measure the outer envelope’s frequency.
Heterodyne receivers mix the signals of different frequencies and measure the amplitude of the (much lower frequency) modulated signal

(1) Take two waves: \( V_{LO} \sin(\omega_{LO}t) \) and \( V_{S} \sin(\omega_{ST}t) \)

(2) Add them together and square for Power:

\[
V(t)^2 = [V_{LO} \sin(\omega_{LO}t) + V_{S} \sin(\omega_{ST}t)]^2
\]

\[
V(t)^2 = V_{LO}^2 \sin^2(\omega_{LO}t) + V_{S}^2 \sin^2(\omega_{ST}t) + 2V_{LO}V_{S} \sin(\omega_{LO}t) \sin(\omega_{ST}t)
\]

(3) Use the trigonometric identity:

\[
\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]
\]

...and we get:

\[
V(t)^2 = V_{LO}^2 \sin^2(\omega_{LO}t) + V_{S}^2 \sin^2(\omega_{ST}t) + V_{LO}V_{S} [\cos(\omega_{S} - \omega_{LO})t - \cos(\omega_{S} + \omega_{LO})t]
\]
The Math of Mixing (2)

Through a low pass filter, these terms average to zero:

\[ V(t)^2 = V_{LO}^2 \sin^2(\omega_{LO}t) + V_S^2 \sin^2(\omega_st) + V_{LO}V_S[\cos(\omega_s - \omega_{LO})t - \cos(\omega_s + \omega_{LO})t] \]

The remaining term contains the power:

\[ \text{Power}(t) \propto V_{LO}V_S \cos(\omega_s - \omega_{LO})t \]

By mixing two frequencies we can measure a much higher frequency than the response time for our detector!
Advantages and Applications

Advantages:

• **Direct encoding** of the spectrum of the incoming signal over a given wavelength range

• **Recording of phase** allows for interferometers - very long baseline interferometry (VLBI)

• Signals are down-converted to frequencies where **low noise electronics** can be used

Applications:

• Used in **radio** as they boost the angular resolution through interferometry

• Very common for **sub-mm** receivers where bolometers cannot provide efficient spectroscopy

• Possible in **optical/IR** but cannot make large imaging arrays
Sidebands
There are always *two* Sidebands (1)

The mixed signal is amplitude *modulated at the intermediate frequency* $\omega_{\text{IF}} = |\omega_S - \omega_{\text{LO}}|$.

The mixer produces the *same result* at $\omega_{\text{IF}}$ no matter if $\omega_S > \omega_{\text{LO}}$ or $\omega_S < \omega_{\text{LO}}$. 
There are always two Sidebands (2)

Same result at $\omega_{\text{IF}}$ for both $\omega_S > \omega_{\text{LO}}$ and $\omega_S < \omega_{\text{LO}}$ – is this a problem?

- For continuum sources: not really, if they vary slowly with wavelength.
- For spectral lines: yes, big problem!

SOLUTIONS:

1. tune the mixer to remove the degeneracy
2. “Image rejection” narrowband filter in front of the receiver
Bandwidth
from Wikipedia:

- Bandwidth is the difference between high and low frequencies $f_H$ and $f_L$ in a continuous set of frequencies.
- Bandwidth is typically measured in Hertz [s$^{-1}$]
- any band of a given width can carry the same amount of information, regardless of where that band is located in the frequency spectrum
- The equivalent to spectral resolution$^{-1}$ $R^{-1} = \Delta \lambda / \lambda$ would be the percent bandwidth $(f_H - f_L) / f_C$, which can be 200% at max.
Comparison of Receiver Technologies

**INCOHERENT** receivers (bolometer, photoconductor)

**COHERENT** receivers (heterodyne)
The intermediate Frequency Bandwidth $\Delta_{IF}$

- The bandwidth of even the best photodiode mixers is usually small compared to the signal frequency, typically one part in 100 or 1000.
- Heterodyne receivers operating at short wavelengths have poor S/N on continuum sources.
- ...so their best use is spectral line measurement at extremely high resolutions.
- If the bandwidth $\Delta_{IF}$ is sufficiently wide the IF output can be sent to a set of parallel narrowband filters.
- The time response of a heterodyne receiver is $1/f_{IF}$ and can be as short as a few nanoseconds.
The IF Bandwidth $\Delta f_{IF}$ (2)

- The IF bandwidth $\Delta f_{IF}$ depends on:
  1. frequency response of the mixer
  2. signal amplifier
  3. signal filter

- The $\Delta f_{IF}$ provided by photoconductor mixers in the radio/sub-mm are usually narrow $\Delta f_{IF} < \text{few} \times 10^9 \text{ Hz}$ – limited by the carrier recombination time:
  - for Ge: $\Delta f_{IF} < 10^8 \text{ Hz}$
  - for InSb (hot electron bolometers): $\Delta f_{IF} < 10^6 \text{ Hz}$

- $\Delta f_{IF}$ is even narrower in the infrared. Example: mixer at 10$\mu$m $\rightarrow \nu = 3 \cdot 10^{13} \text{ Hz}$, $\Delta f_{IF} \sim 10^9 \text{ Hz} \rightarrow$ bandwidth is only 0.01% of $\lambda$
System Components

I. Overview
Setup of a Coherent Detection System

I. Radio/Sub-mm Receivers

- Feed
- RF amplifier
- Mixer
- IF amplifier
- Detector/correlator
- Computer
- Data (continuum or spectrum)

FRONT-END

Local oscillator

BACK-END

Setup of a Coherent Detection System
II. Visible/Infrared Heterodyne Detection

The same principles from IR heterodyne apply to sub-mm, apart from…:

The MIXER technology

The LOCAL OSCILLATOR technology
Differences in LO Technology

The LO power can be fed to the mixer via a second waveguide or from a diplexer:

<table>
<thead>
<tr>
<th>Waveguide Width</th>
<th>LO Frequency Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>10mm</td>
<td>Low frequencies, use an electronic LO + easily tunable in frequency - low output power at high frequencies</td>
</tr>
<tr>
<td>3mm</td>
<td>High frequencies, use a continuous wave laser + high output power - discrete frequencies</td>
</tr>
<tr>
<td>1mm</td>
<td></td>
</tr>
<tr>
<td>300μm</td>
<td></td>
</tr>
<tr>
<td>100μm</td>
<td></td>
</tr>
<tr>
<td>30μm</td>
<td></td>
</tr>
<tr>
<td>30 GHz</td>
<td>3 × 10^{10}</td>
</tr>
<tr>
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</tr>
<tr>
<td>10 THz</td>
<td>1 × 10^{13}</td>
</tr>
</tbody>
</table>
System Components

II. Mixers
Wikipedia again: a mixer is a **nonlinear electrical circuit** that creates new frequencies from two signals applied to it.

A non-linear device will convert power from the original frequencies to the beat frequency.
Problem measuring the outer Envelope

• With short time constant, you trace the **high frequency component** which you’re not sensitive to - you see ZERO.

• With a long time constant, the outer envelope is **symmetric** about zero and so this *also* averages out to ZERO.
2\textsuperscript{nd} Component must be a non-linear Device

- If the mixer is linear then the conversion efficiency is ZERO
- Even if it’s an odd function of voltage about the origin the conversion efficiency is zero (but biased above zero at A can work)
- The quadratic case is called a square law device
Generally, a useful mixer has an I-V curve that can be approximated by a Taylor series around the operating voltage $V_0$:

\[
I(V) = I(V_0) + \left( \frac{dI}{dV} \right)_{V=V_0} dV + \frac{1}{2!} \left( \frac{d^2 I}{dV^2} \right)_{V=V_0} dV^2 + \frac{1}{3!} \left( \frac{d^3 I}{dV^3} \right)_{V=V_0} dV^3 + \frac{1}{4!} \left( \frac{d^4 I}{dV^4} \right)_{V=V_0} dV^4 + \ldots
\]

- **DC Voltage**: $I(V_0)$
- **Square law mixer**: $\left( \frac{dI}{dV} \right)_{V=V_0}$
- **Zero response**: $\frac{1}{2!} \left( \frac{d^2 I}{dV^2} \right)_{V=V_0}$
- **Negligible if $dV = V - V_0$ is small**: $\frac{1}{3!} \left( \frac{d^3 I}{dV^3} \right)_{V=V_0}$, $\frac{1}{4!} \left( \frac{d^4 I}{dV^4} \right)_{V=V_0}$
Let’s try a diode as a mixer $I = I_0(e^{qV_B/kT} - 1)$, which can be expanded as:

$e^{qV_B/kT} - 1 \approx \frac{qV_B}{kT} + \frac{1}{2} \left(\frac{qV_B}{kT}\right)^2 + \ldots$

The current is proportional to $(voltage)^2$ – that’s what we want:

$I \propto V^2 \propto E^2 \propto P$
Example: 230 GHz Balanced Mixer

Mixer block hardware of the 180-280 GHz Balanced Mixer
http://www.submm.caltech.edu/cso/receivers/
More Example: Mixers

SIS front-end receiver for balloon heterodyne receiver TELIS

183 GHz fixed-tuned sub-harmonic mixer

2.5THz Schottky diode mixer

560 GHz micro-machined sub-harmonic mixer

http://www.sstd.rl.ac.uk/mmt/components_mixers.php
Because of the low power in the IF signal, the amplifier following the mixer is critical.

Best performance: high electron mobility transistors (HEMTs) on GaAs or InP up to \( \sim 10^{11} \) Hz.

**HEMT setup:**

- MESFET (metal semiconductor field effect transistor) grown on heavily doped GaAlAs over GaAs.
- \( E_F \) of GaAlAs can be above \( E_c \) of GaAs
- electrons gather in thin GaAs layer where the mobility is high
Differences in Mixer Technology (1)

Good *and fast* photon detectors do not exist for wavelengths $\lambda > 40\mu m$.

**PHOTODIODE mixers have a frequency response limited to less than 1 GHz due to the recombination time of the charge carriers that have crossed the junction!**

<table>
<thead>
<tr>
<th>Material</th>
<th>$\tau_{\text{recombination}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>100 $\mu$s</td>
</tr>
<tr>
<td>Ge</td>
<td>10000 $\mu$s</td>
</tr>
<tr>
<td>PbS</td>
<td>20 $\mu$s</td>
</tr>
<tr>
<td>InSb</td>
<td>0.1 $\mu$s</td>
</tr>
<tr>
<td>GaAs</td>
<td>1 $\mu$s</td>
</tr>
<tr>
<td>InP</td>
<td>$\sim$1 $\mu$s</td>
</tr>
</tbody>
</table>

→ Common mixer devices:
- SIS junctions
- Schottky diodes
- Hot electron bolometers (HEB)
Differences in Mixer Technology (2)

<table>
<thead>
<tr>
<th></th>
<th>10mm</th>
<th>3mm</th>
<th>1mm</th>
<th>300μm</th>
<th>100μm</th>
<th>30μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>30 GHz</td>
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<td>1 THz</td>
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<td>$1 \times 10^{12}$</td>
<td>$3 \times 10^{12}$</td>
<td>$1 \times 10^{13}$</td>
</tr>
</tbody>
</table>

- **SIS Pb**
  - 150GHz bandwidth

- **SIS NbTiN**
  - SIS, Schottky diodes and HEB all become less effective above 1THz

- **Schottky**

- **HEB**
  - 150GHz bandwidth

- **Superconducting HEB**
System Components

III. Detectors
Detector Stage (1)

It’s the detectors job to rectify the input signal and send it through the low pass filter so that the output is a ‘slowly’ varying output.

Blocks DC components
Detector Stage (2)

- If the IF signal contains important frequency components it should not be smoothed directly.
- Instead, the signal can be sent to a bank of narrow-band electronic filters, operating in parallel – with a smoothing detector for each filter output.
- Hence, the filter bank can provide a spectrum of the source. (A back-end spectrometer could consist of several filters tuned to different frequencies with detectors on their outputs.)

This spectral multiplexing is one of the most useful features of heterodyne receivers.
Example: Herschel/HIFI
Herschel / HIFI

- Seven spectral bands:
  - Two polarization components each
  - Resolving power up to $10^7$
  - Down-converted $\omega_{IF}$ is centered at 6 GHz
  - Bandwidth $\Delta \omega_{IF} = 4$ GHz

\[
d\lambda = \frac{c}{\nu^2} = \frac{3 \times 10^8 \text{ m.s}^{-1}}{(1920 \times 10^9 \text{ Hz})^2} = 4 \times 10^9 \text{ Hz} = 0.33 \mu\text{m}
\]
Throughput
Two factors limit the throughput of a heterodyne system:

1. Only components of the signal electric field vector parallel to the laser field can interfere (incl. polarization!)

The signal beam will strike the mixer in a range of angles relative to the LO (or laser) beam. The conditions for interference limit the maximum angular displacement. Full cancellation occurs only when the offset $\sim \lambda$

Since the LO / laser field is polarized only one polarization component of the source can interfere and produce a signal so heterodyne receivers = single-mode detectors.

$$l \sin \theta_{\text{max}} = \lambda \approx l \theta_{\text{max}}$$
Throughput \((2) - A\Omega\)

Two factors limit the throughput of a heterodyne system:

2. A coherent receiver should operate at the **diffraction limit** of the telescope

\[
l \cdot \sin(\theta_{\text{max}}) = \lambda \approx l \cdot \theta_{\text{max}}
\]

\[
\Omega = 4\pi \sin^2(\theta/2) \rightarrow \theta^2 \approx \Omega
\]

\[A\Omega \approx \lambda^2\]

The throughput \(A\Omega\) (collecting area \(\times\) field of view) is also called the **Etendue** and is invariant in any aberration-free optical system.

Therefore, \(A\Omega\) sets a **constraint on the beam** that can be accepted by a telescope of diameter \(D\) (or any other optical system).
Throughput (2) – Antenna Theorem

The angular diameter of the FOV on the sky is given by:

$$\Phi \approx \frac{\lambda}{D} \approx \text{Rayleigh criterion}$$

A coherent receiver should operate at the diffraction limit of the telescope. ⇐ This is the “Antenna theorem” (applies to all heterodyne detectors).

- If the receiver only accepts a smaller FOV there is significant loss.
- If the receiver accepts much more it leads to a higher background and limited throughput (factor 1 above).
Signal-to-Noise
There are two types of noise in heterodyne receivers:

1. Noise independent of the LO generated current $I_{LO}$
2. Noise dependent on the LO generated current $I_{LO}$, which are fundamental noise limits for heterodyne receivers

Fundamental noise sources are from:

- Noise in the mixer from the generation of charge carriers by the LO power, and
- Noise from the thermal background detected by the system
The S/N is given by (Rieke, p. 291):

\[
\left( \frac{S}{N} \right)_{IF} = \frac{\eta P_S}{h \nu \Delta_{IF} \left[ \frac{a}{G} + \frac{2\eta \varepsilon}{e^{h \nu / kT_B} - 1} \right]}
\]

There are two cases:

1. **QUANTUM LIMIT** \((h \nu >> kT_B)\): G-R noise from the LO power dominates.

2. **THERMAL LIMIT** \((h \nu << kT_B)\): noise from thermal background dominates.

The **dividing line between the two cases** is roughly at:

\[
\frac{a}{G} \approx \frac{2\eta \varepsilon}{e^{h \nu / kT_B} - 1}
\]

*Note: \(a \approx 1\) for a photodiode mixer*
NEP

Remember: The noise equivalent power (NEP) is the signal that can be detected at a S/N of unity within unity frequency bandwidth $\Delta f$:

The **NEP in the quantum limit** is:

$$\text{NEP}_{ql} = \frac{P}{\left(\frac{S}{N}\right)_{out} (\Delta f)^{1/2}} \quad \text{or:} \quad \text{NEP}_{ql} = \frac{h \nu a}{\eta G} (2\Delta f_{IF})^{1/2}$$

The **NEP in the thermal limit** is:

$$\text{NEP}_{th} = \frac{P}{\left(\frac{S}{N}\right)_{out} (\Delta f)^{1/2}} \quad \text{or:} \quad \text{NEP}_{th} = \frac{2h \nu \epsilon}{e h \nu / k T_B - 1} (\Delta f)^{1/2}$$
Noise Temperature
Our goal is to compare the theoretical performance of heterodyne detectors with that of incoherent receivers (e.g. bolometers).

So far: $NEP_H \propto \sqrt{2\Delta f_{IF}}$ implies that the NEP decreases (i.e., the S/N increases) when narrowing down the bandwidth!

However, this is only correct if all power falls within an interval, which is smaller than $\Delta f_{IF}$. This is given for narrow emission lines but not for continuum sources.
Noise Temperature (1)

Let’s define a noise temperature $T_N$ such, that a matched blackbody at the receiver input at a temperature $T_N$ produces a S/N = 1.

First we estimate the noise temperature $T_N$ in the thermal limit.

- If the BB emissivity $\varepsilon = 1$ then: $T_N = T_B$
- If the BB emissivity $\varepsilon < 1$ then: $T_N = \frac{h\nu}{k} \frac{1}{\ln(\varepsilon - 1 + e^{h\nu/kT_B}) - \ln \varepsilon}$

Similarly, the noise temperature in the quantum limit (double sideband) is: $T_N = \frac{h\nu}{k \ln \left(1 + \frac{2G\eta}{a}\right)} \approx \frac{h\nu}{k}$

Ideally: $G = \eta = a = 1$
Why $T_N$? The concept of noise temperatures offers a convenient means to quantify the LO-independent components, such as amplifier noise.

This amplifier noise is usually Johnson noise: 

$$\langle I_A^2 \rangle = \frac{4kT_N\Delta f_{IF}}{R_A},$$

where $R_A$ and $T_N$ are the amplifier input resistance and noise temperature, respectively.

The lower limit for the noise temperature is given by $T_N \approx \frac{h\nu}{k}$

- For an amplifier operating at 32 GHz $T_N \sim 1.5$K.
- For a good HEMT amplifier $T_A \sim 10$K.
How to Measure Noise Temperature

Take two blackbody emitters with well spaced different temperatures $T_{\text{hot}}$ and $T_{\text{cold}}$.

If $V$ is the output voltage of the receiver we can define a “$Y$” factor:

$$Y = \frac{V_{\text{hot}}}{V_{\text{cold}}}$$

...which can be measured by alternately placing the blackbodies over the receiver input:

$$Y = \frac{T_{\text{hot}} + T_N}{T_{\text{cold}} + T_N}$$

...and solve it for the receiver noise temperature $T_N$:

$$T_N = \frac{T_{\text{hot}} - YT_{\text{cold}}}{Y - 1}$$
Antenna or Source Temperature

Just like the noise temperature $T_N$ describes the strength of the noise background, we can assign the **source flux** an **antenna temperature** $T_S$:

We get for a blackbody-type source in the Rayleigh-Jeans approximation ($h\nu << kT$):

$$P_S = L_\nu T_SA\Omega\Delta f_{IF} = \frac{2kT_S\nu^2}{c^2}A\Omega\Delta f_{IF} = 2\Delta f_{IF}kT_S$$

where $2\Delta f_{IF}$ is the frequency bandpass for a double sideband receiver.

→ The antenna temperature is linearly related to the input flux density: $P_s \sim T_S$
Coherent ↔ Incoherent Receivers
Performance Ratio of In/coherent Receivers

The achievable S/N for a **coherent receiver** in terms of antenna and system noise temperatures is given by the **Dicke radiometer equation**:

\[
\left( \frac{S}{N} \right)_{coh} \approx \frac{T_S}{T_N} \left( \Delta f_{IF} \Delta t \right)^{1/2}
\]

....so the signal to noise for an **incoherent receiver** operating at the diffraction limit is:

\[
\left( \frac{S}{N} \right)_{inc} = \frac{2kT_S \Delta \nu_{inc} \Delta t^{1/2}}{\text{NEP}_{inc}}
\]

Hence, the **performance ratio** between these two types of receivers is:

\[
\frac{(S/N)_{coh}}{(S/N)_{inc}} = \frac{\text{NEP}_{inc} \left( \Delta f_{IF} \right)^{1/2}}{2kT_N^{sys} \Delta \nu_{inc}}
\]
Consider a bolometer operating at the **background limit (BLIP)** and a heterodyne receiver operating in **the thermal limit**:

\[
\frac{(S/N)_{coh}}{(S/N)_{inc}} = \left[ \left( \frac{1}{\eta} \right) \left( \frac{\Delta f_{IF}}{\Delta \nu_{inc}} \right) \left( \frac{h \nu}{kT_B} \right) \right]^{1/2}
\]

→ the bolometer will perform better unless \( \Delta f_{IF} \gg \eta \Delta \nu \)

The *latter case* \( \Delta f_{IF} \gg \eta \Delta \nu \) will be given for **measurements at high spectral resolution**, much higher than the *IF bandwidth*. 
Consider a **detector noise-limited** bolometer and a heterodyne receiver operating at the quantum limit:

\[
\frac{(S/N)_{coh}}{(S/N)_{inc}} = \frac{\text{NEP}_{inc}(\Delta f_{IF})^{1/2}}{2h\nu\Delta\nu_{inc}}
\]

In the case of **narrow bandwidth and high spectral resolution**, the heterodyne receiver will outperform the bolometer

→ **heterodyne receivers are best for high spectral resolution applications in the sub-mm!**

If you keep the spectral resolution \(\nu/\Delta\nu\) constant (typically given) in the above equation, then the relative figure of merit goes as \(1/\nu^2\) → transition from case favoring incoherent over coherent detectors is relatively abrupt.