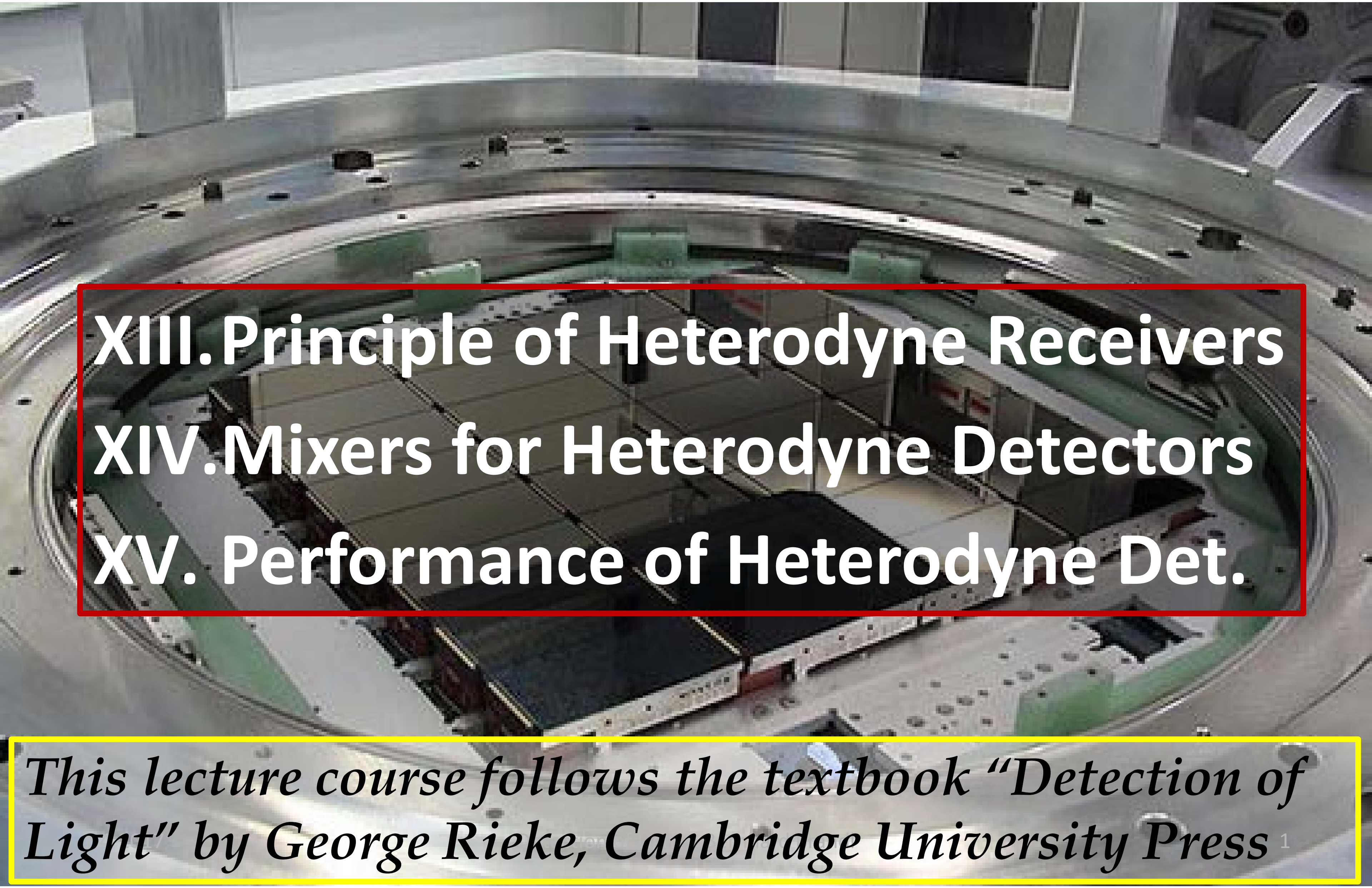
Detection of Light



XIII. Principle of Heterodyne Receivers

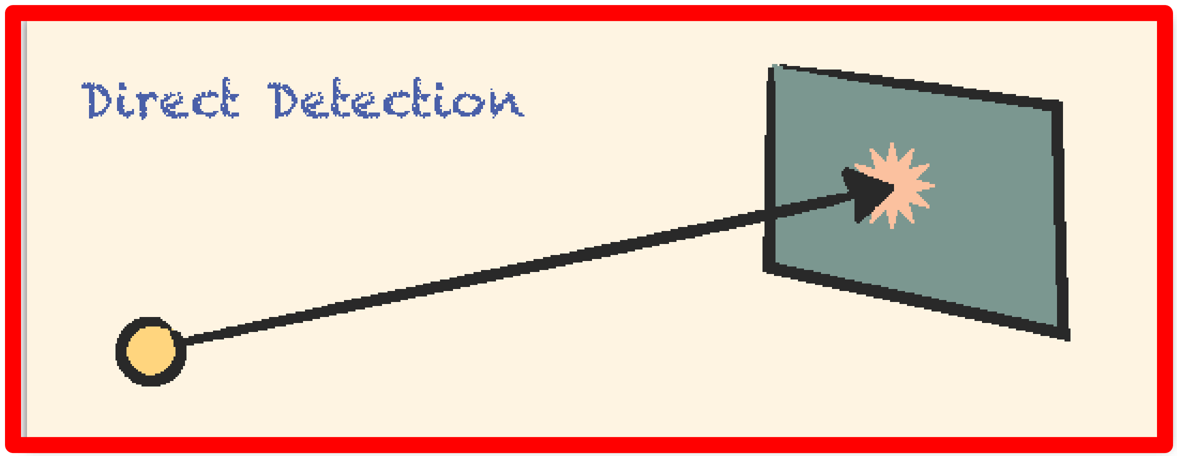
XIV. Mixers for Heterodyne Detectors

XV. Performance of Heterodyne Det.

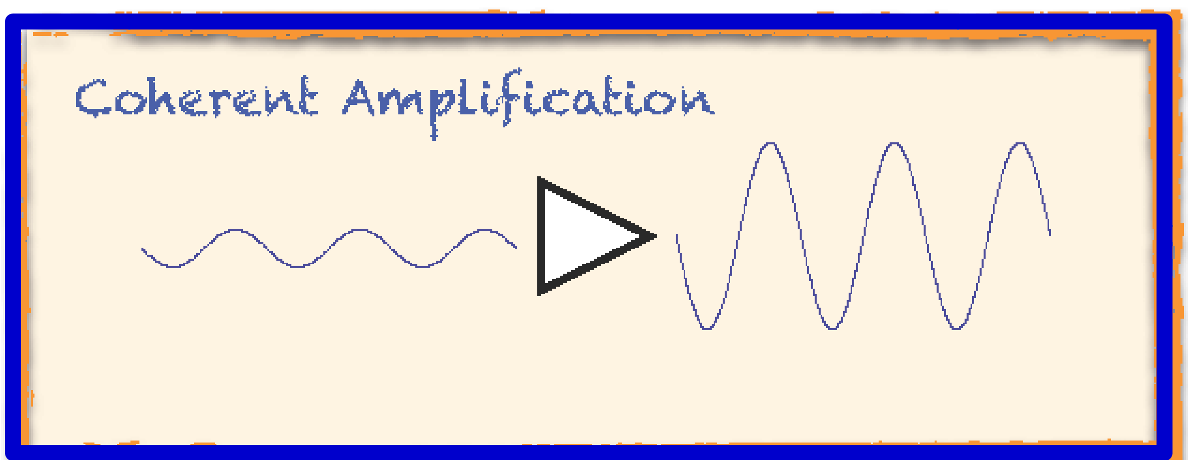
This lecture course follows the textbook "Detection of Light" by George Rieke, Cambridge University Press

Two Fundamental Principles of Detection

Respond to individual photon energy



Photons



Waves

Respond to electrical field strength and preserve phase

Principle of Heterodyne Detectors

Problem with "conventional" Detection of Waves rather than Particles or heat

Consider a detector that responds linearly within a response time τ to measure the resulting field:

If $\tau >> f$ the output will be zero on average

There may not be any detectors which have short enough response times τ

Wavelength	Frequency (GHz)	Frequency (THz)	Response time (s)
10 metres	0.03	0.00003	3.33E-08
1 metre	0.3	0.0003	3.33E-09
10cm	3	0.003	3.33E-10
1cm	30	0.03	3.33E-11
1mm	300	0.3	3.33E-12
0.1mm	3000	3	3.33E-13
10 microns	30000	30	3.33E-14
1 micron	300000	300	3.33E-15
0.1 microns	3000000	3000	3.33E-16

Heterodyne Receivers

Heterodyning is a radio signal processing technique invented in 1901 by Canadian inventor-engineer Reginald Fessenden

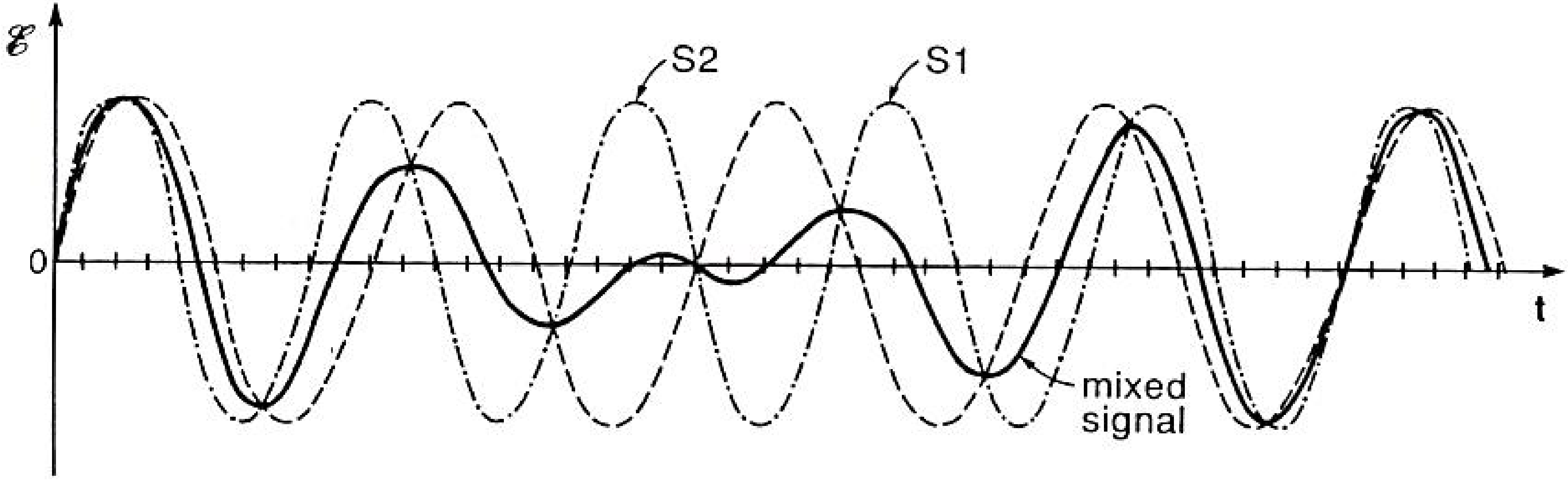
Reginald Aubrey Fessenden
(1866 – 1932)

Heterodyning = new frequencies are created by combining or mixing two frequencies

Mixing

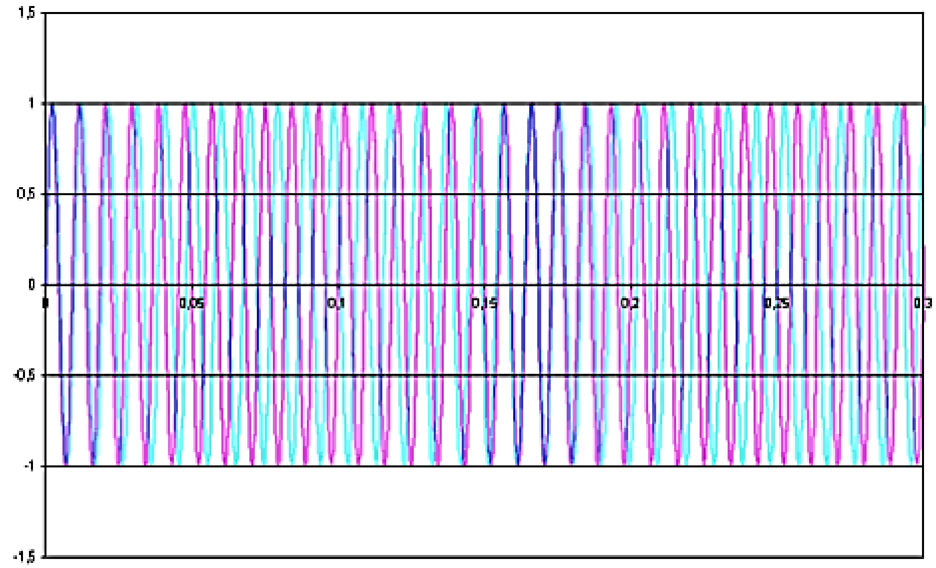
Directly mix incoming light with coherent light of *nearly* the same frequency:

Signal S1 is mixed with a local oscillating field S2

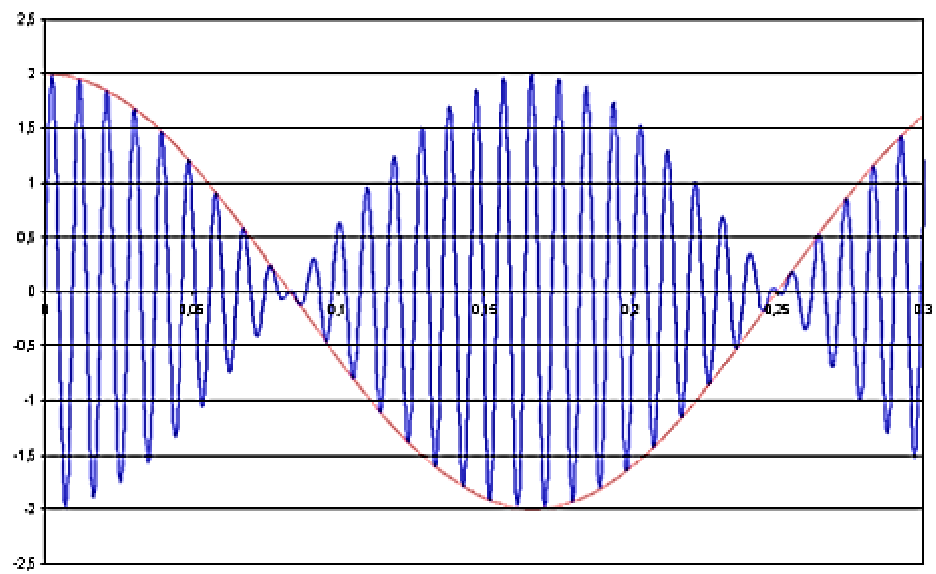


The Intermediate Frequency (IF)

The IF is the "beat frequency" and is the difference between the local oscillator and the signal frequency



The resultant mix produces a lower frequency envelope that we can measure.



The Math of Mixing (1)

Heterodyne receivers mix the signals of different frequencies and measure the amplitude of the (much lower frequency) modulated signal

(1) Take two waves: $V_{LO}\sin(\omega_{LO}t)$ and $V_S\sin(\omega_S t)$

(2) Add them together and square for Power:

$$V(t)^2 = [V_{LO}\sin(\omega_{LO}t) + V_S\sin(\omega_S t)]^2$$

$$V(t)^2 = V_{LO}^2\sin^2(\omega_{LO}t) + V_S^2\sin^2(\omega_S t) + 2V_{LO}V_S\sin(\omega_{LO}t)\sin(\omega_S t)$$

(3) Use the trigonometric identity:

$$\sin A\sin B \equiv \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

...and we get:

$$V(t)^2 = V_{LO}^2\sin^2(\omega_{LO}t) + V_S^2\sin^2(\omega_S t) + V_{LO}V_S[\cos(\omega_S - \omega_{LO})t - \cos(\omega_S + \omega_{LO})t]$$

The Math of Mixing (2)

Through a low pass filter, these terms average to zero:

$$V(t)^2 = V_{LO}^2 \sin^2(\omega_{LO} t) + V_S^2 \sin^2(\omega_S t) + V_{LO} V_S [\cos(\omega_S - \omega_{LO})t - \cos(\omega_S + \omega_{LO})t]$$

DIFFERENCE SUM frequency

The remaining term contains the power:

$$Power(t) \propto V_{LO} V_S \cos(\omega_S - \omega_{LO})t$$

By mixing two frequencies – of which one is known in amplitude and frequency – we can measure an unknown frequency which is much higher than the response time of our detector!

Advantages and Applications

Advantages:

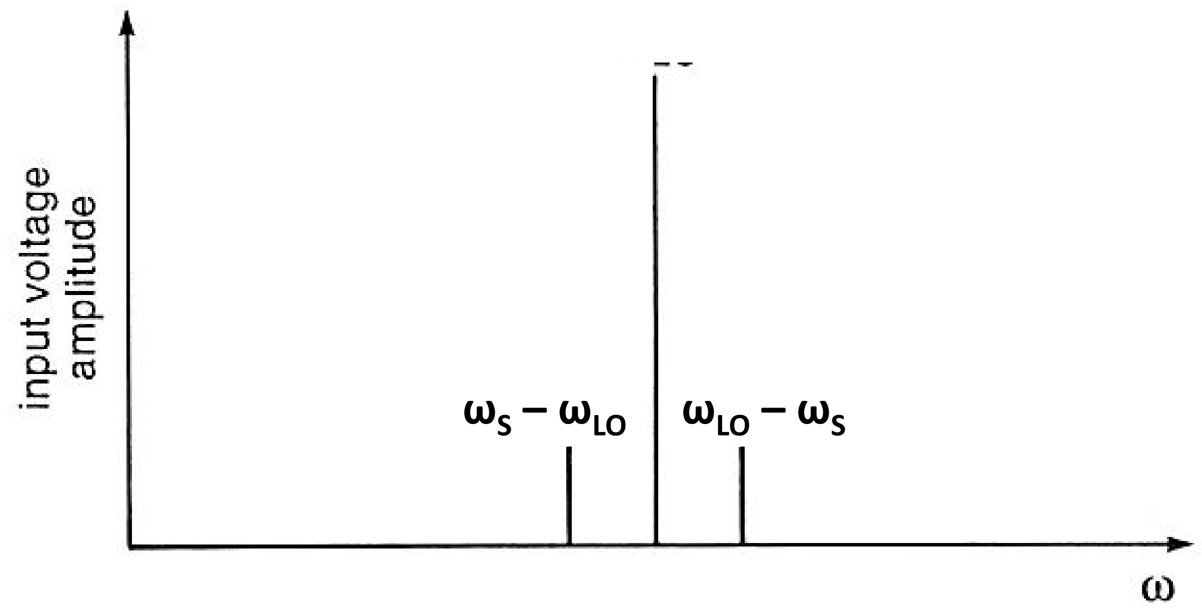
- Direct encoding of the spectrum of the incoming signal over a given wavelength range
- Recording of phase allows for interferometers - very long baseline interferometry (VLBI)
- Signals are down-converted to frequencies where low noise electronics can be used

Applications:

- Very common for sub-mm receivers, where bolometers provide photometry and heterodyne techniques provide efficient spectroscopy
- Possible in optical/IR but cannot make large imaging arrays

Sidebands

There are always *two* Sidebands (1)



The mixed signal is amplitude modulated at the intermediate frequency $\omega_{IF} = |\omega_S - \omega_{LO}|$.

The mixer produces the same result at ω_{IF} no matter if $\omega_S > \omega_{LO}$ or $\omega_S < \omega_{LO}$.

There are always *two* Sidebands (2)

Same result at ω_{IF} for both $\omega_S > \omega_{LO}$ and $\omega_S < \omega_{LO}$ – is this a problem?

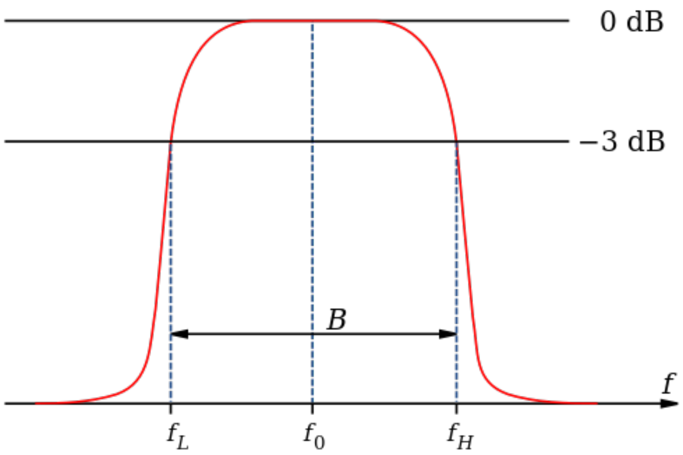
- For continuum sources: not really, if they vary slowly with wavelength.
- For spectral lines: yes, big problem! (if ω_{line} is not known)

SOLUTIONS:

1. tune the mixer to remove the degeneracy
2. "Image rejection" narrowband filter in front of the receiver

Bandwidth

Bandwidth Basics

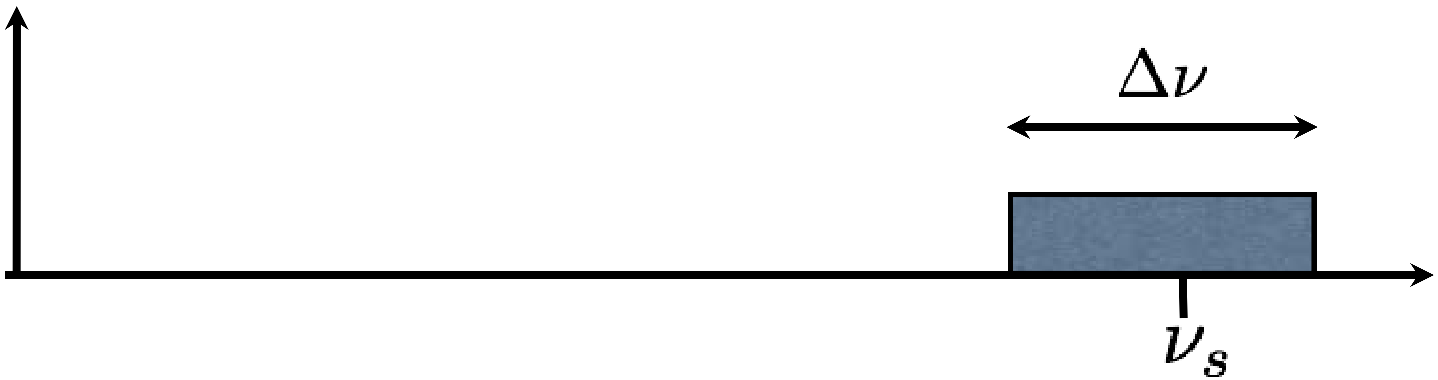


from Wikipedia:

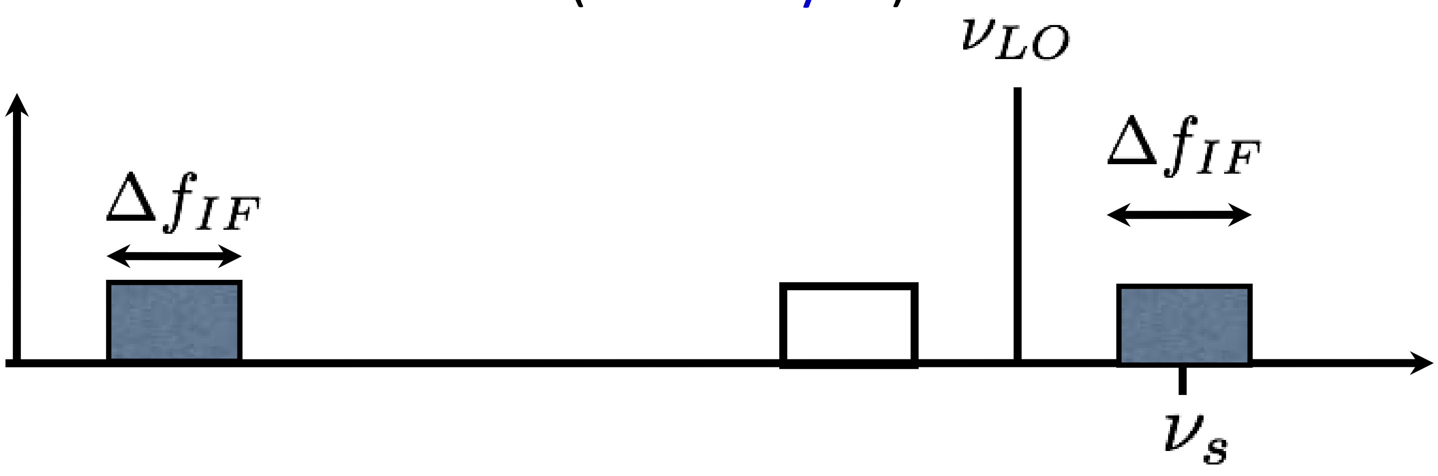
- Bandwidth is the difference between high and low frequencies f_H and f_L in a continuous set of frequencies.
- Bandwidth is typically measured in Hertz [s^{-1}]
- any band of a given width can carry the same amount of information, regardless of where that band is located in the frequency spectrum
- The equivalent to spectral resolution^{-1} $R^{-1} = \Delta\lambda / \lambda$ would be the percent bandwidth $(f_H - f_L) / f_C$, which can be 200% at max.

Comparison of Receiver Technologies

INCOHERENT receivers (bolometer, photoconductor)



COHERENT receivers (heterodyne)



The intermediate Frequency Bandwidth Δ_{IF}

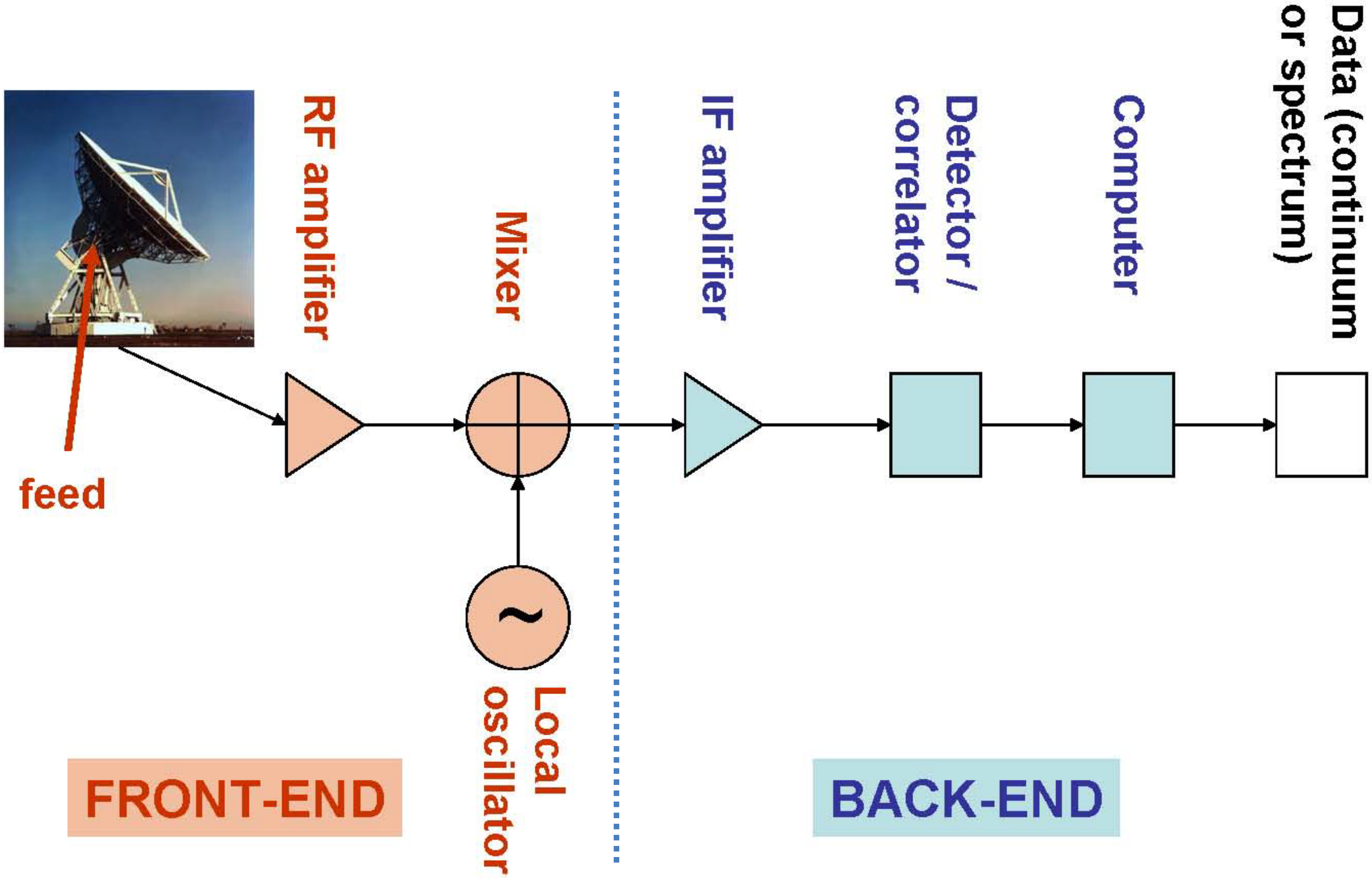
- The bandwidth of even the best photodiode mixers is usually small compared to the signal frequency, typically one part in 100 or 1000
- Heterodyne receivers operating at short wavelengths have poor S/N on continuum sources
- ...so their best use is spectral line measurement at extremely high resolutions
- If the bandwidth Δ_{IF} is sufficiently wide the *IF* output can be sent to a set of parallel narrowband filters
- The time response of a heterodyne receiver is $1/f_{IF}$ and can be as short as a few nanoseconds.

The IF Bandwidth Δ_{IF} (2)

- The IF bandwidth Δf_{IF} depends on:
  1. frequency response of the mixer
  2. signal amplifier
  3. signal filter
- The Δf_{IF} provided by photoconductor mixers in the radio/sub-mm are usually narrow $\Delta f_{IF} <$ few $\times$ 10^9 Hz – limited by the carrier recombination time:
  - for Ge: $\Delta f_{IF} < 10^8$ Hz
  - for InSb (hot electron bolometers): $\Delta f_{IF} < 10^6$ Hz
- Δf_{IF} is even narrower in the infrared. Example: mixer at 10μm → $\nu = 3 \cdot 10^{13}$ Hz, $\Delta f_{IF} \sim 10^9$ Hz → bandwidth is only 0.01% of λ

System Components
I. Overview

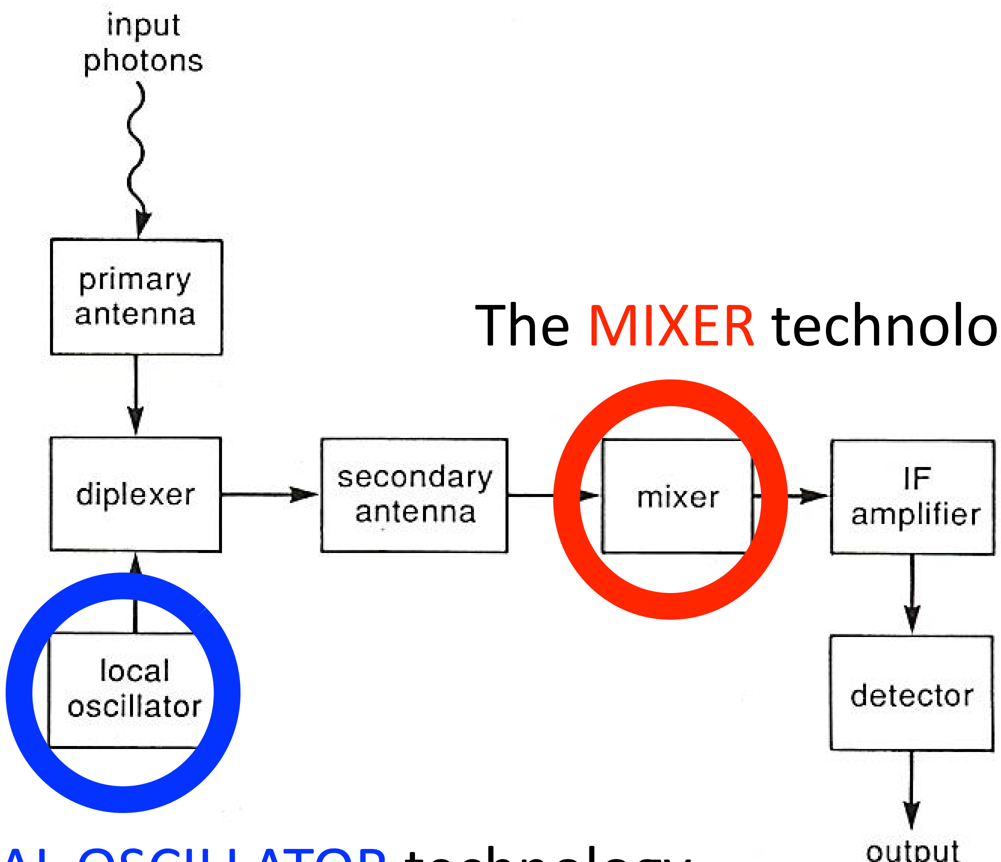
Setup of a Coherent Detection System
I. Radio/Sub-mm Receivers



taken from Sandor Frey's Summer School presentation at
http://www.vlti.org/events/assets/4/documents/RadioInterferometry-Frey.pdf

Setup of a Coherent Detection System
II. Visible/Infrared Heterodyne Detection

The same principles from IR heterodyne apply to sub-mm, apart from…:



The MIXER technology

The LOCAL OSCILLATOR technology

Differences in LO Technology

The LO power can be fed to the mixer via a second waveguide or from a diplexer:

$10mm$	$3mm$	$1mm$	$300\mu m$	$100\mu m$	$30\mu m$
30 GHz	100 GHz	300 GHz	1 THz	3 THz	10 THz
3×10^{10}	1×10^{11}	3×10^{11}	1×10^{12}	3×10^{12}	1×10^{13}

Low frequencies, use an electronic LO

+ easily tunable in frequency

- low output power at high frequencies

High frequencies, use a continuous wave laser

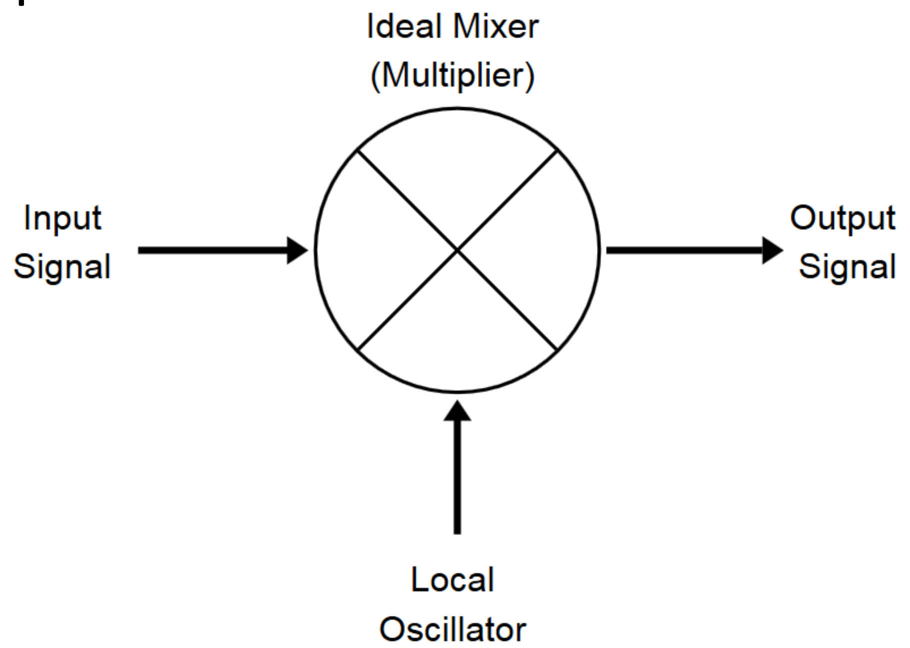
+ high output power

- discrete frequencies

System Components
II. Mixers

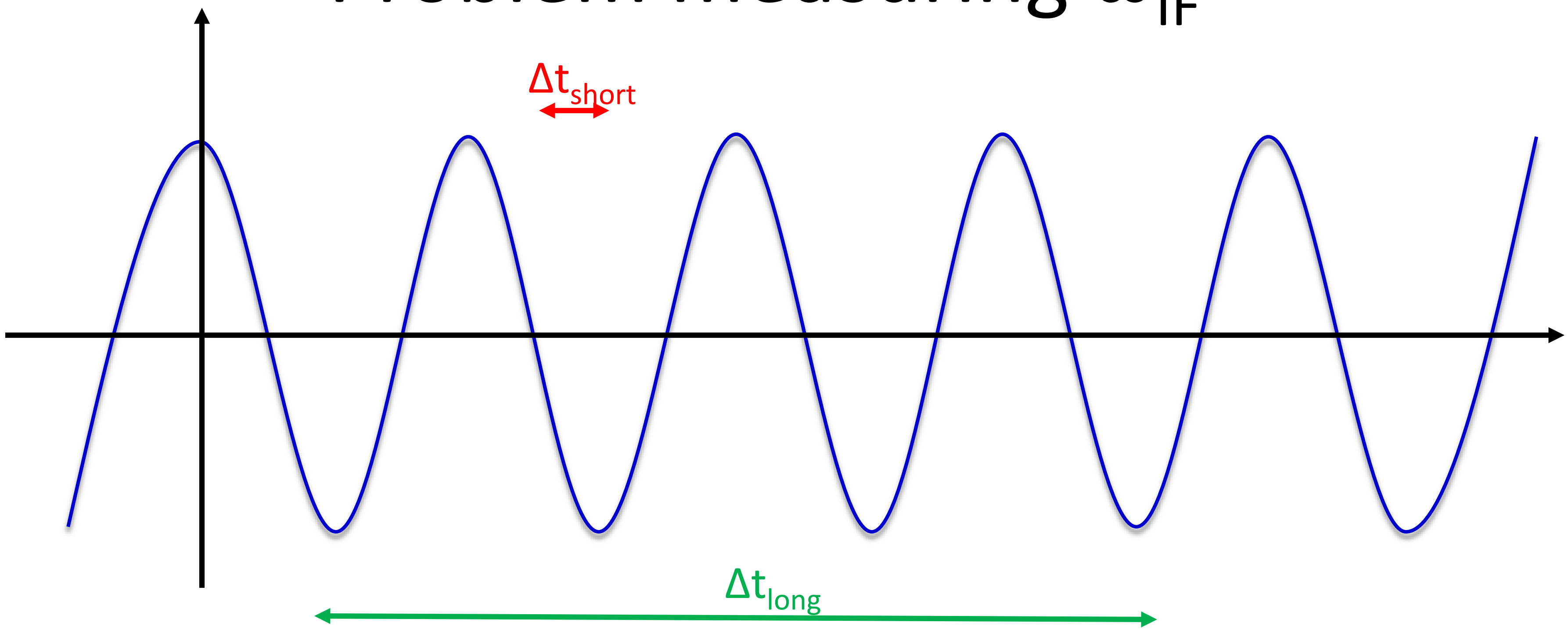
Mixer Basics

Wikipedia again:
a mixer is a nonlinear electrical circuit that creates new frequencies from two signals applied to it.



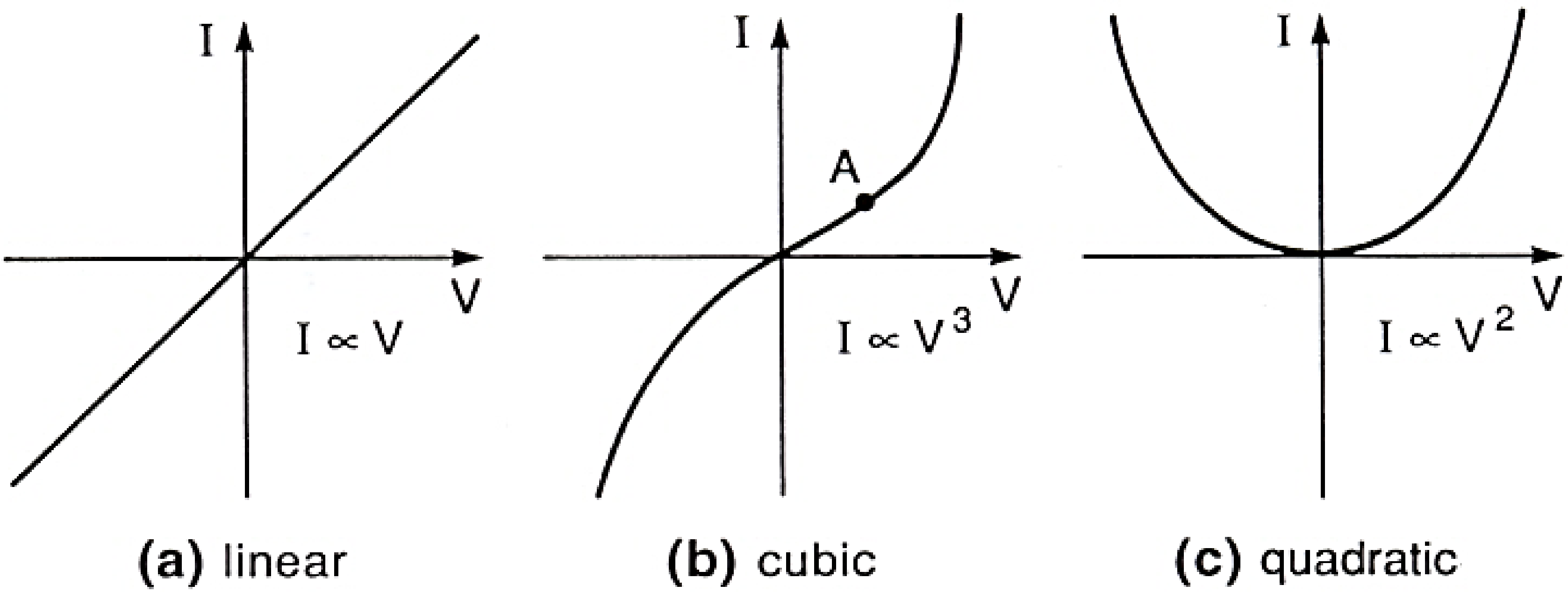
Why a *nonlinear* electrical circuit ?

Problem measuring ω_{IF}



- With a long time constant, the outer envelope is symmetric about zero and averages out to zero.
- With short time constant, you could trace the signal at high frequency but a device with linear response will also give zero response.

Linear and non-linear Devices



- As shown on the previous slide, if the mixer is linear (a) then the conversion efficiency is ZERO
- Even if it's an odd function of voltage about the origin (b) the conversion efficiency is zero (but biased above zero at A can work)
- The quadratic case (c) is called a square law device.

Mixer Math

Generally, a useful mixer has an I-V curve that can be approximated by a Taylor series around the operating voltage V_0:

$$I(V) = I(V_0) + \left(\frac{dI}{dV}\right)_{V=V_0} dV + \frac{1}{2!}\left(\frac{d^2I}{dV^2}\right)_{V=V_0} dV^2 + \frac{1}{3!}\left(\frac{d^3I}{dV^3}\right)_{V=V_0} dV^3 + \frac{1}{4!}\left(\frac{d^4I}{dV^4}\right)_{V=V_0} dV^4 + \ldots$$

DC Voltage

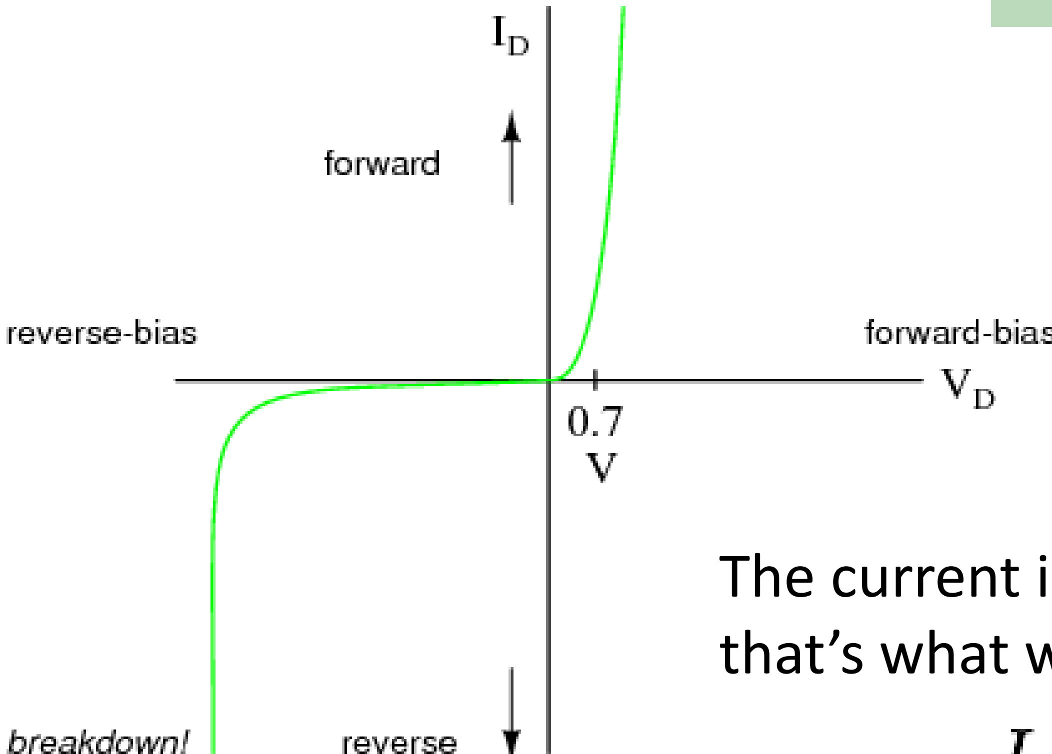
Zero response

Square law mixer

Negligible if $dV = V - V_0$ is small

A Diode as a Mixer

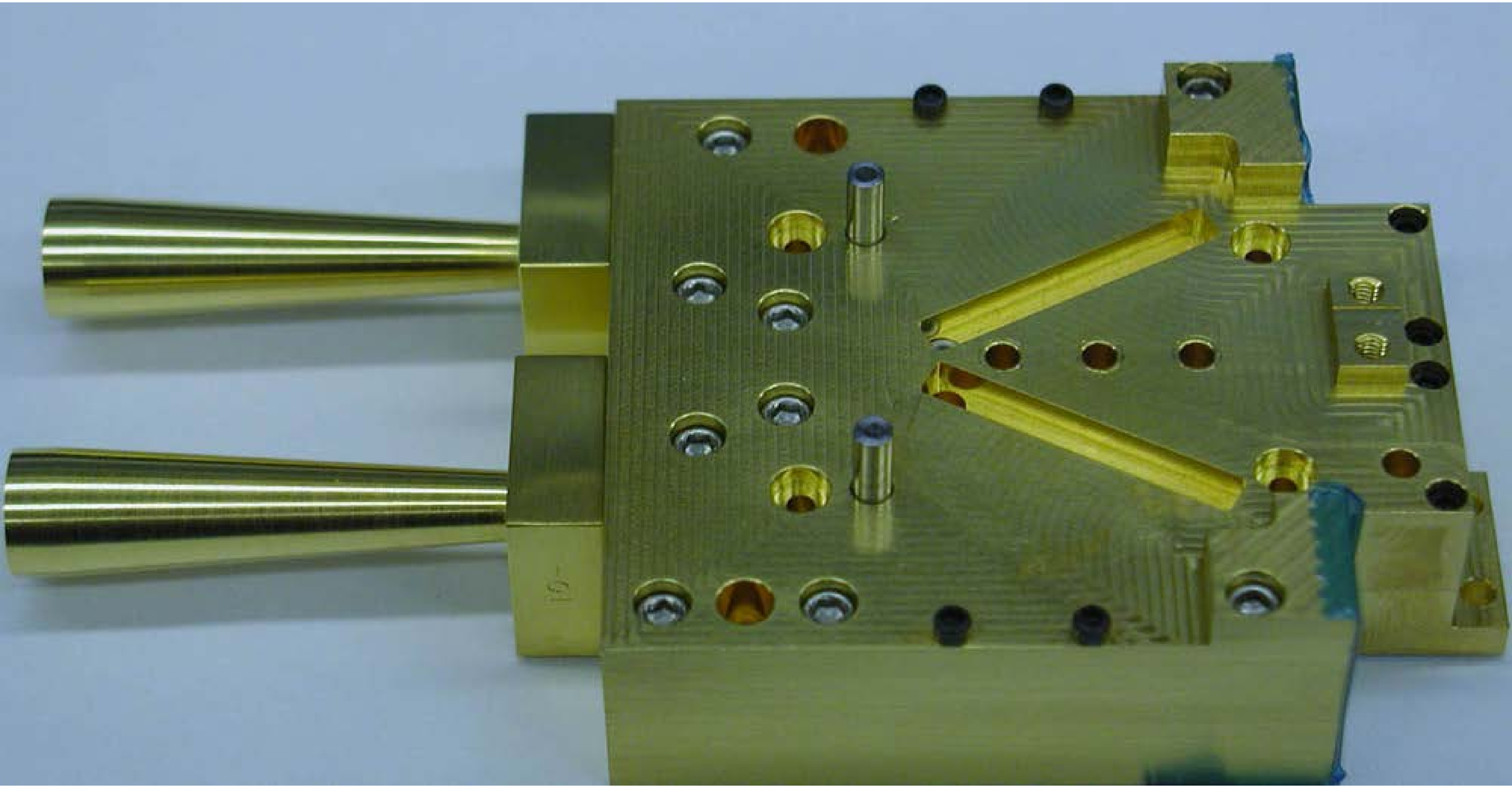
Let's try a diode as a mixer $I = I_0(e^{qV_B/kT} - 1)$, which can be expanded as: $e^{qV_B/kT} - 1 \approx \frac{qV_B}{kT} + \frac{1}{2}\left(\frac{qV_B}{kT}\right)^2 + \dots$



The current is proportional to (voltage)^2 – that's what we want:

$$I \propto V^2 \propto E^2 \propto P$$

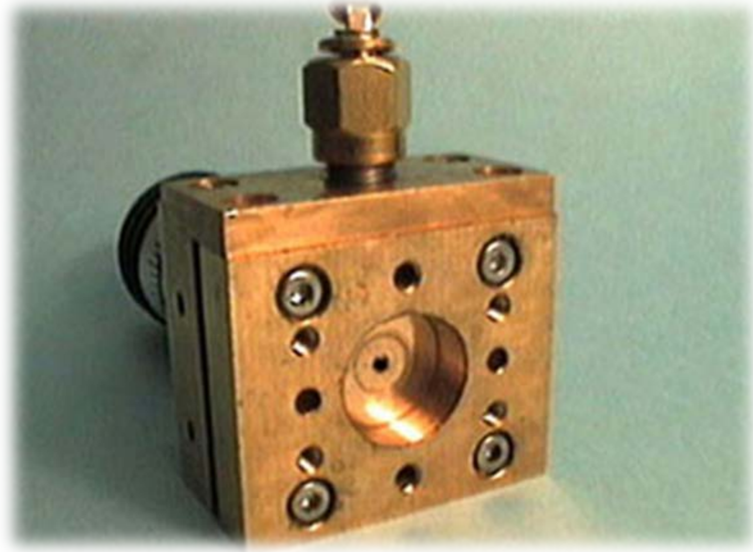
Example: 230 GHz Balanced Mixer



Mixer block hardware of the 180-280 GHz Balanced Mixer
http://www.submm.caltech.edu/cso/receivers/

More Example: Mixers

SIS front-end receiver for balloon heterodyne receiver TELIS



2.5THz Schottky diode mixer

183 GHz fixed-tuned sub-harmonic mixer

560 GHz micro-machined sub-harmonic mixer

http://www.sstd.rl.ac.uk/mmt/components_mixers.php

Mixer Choices for Wavelengths λ > 40μm

Fast photon detectors do *not* exist for wavelengths λ > 40μm

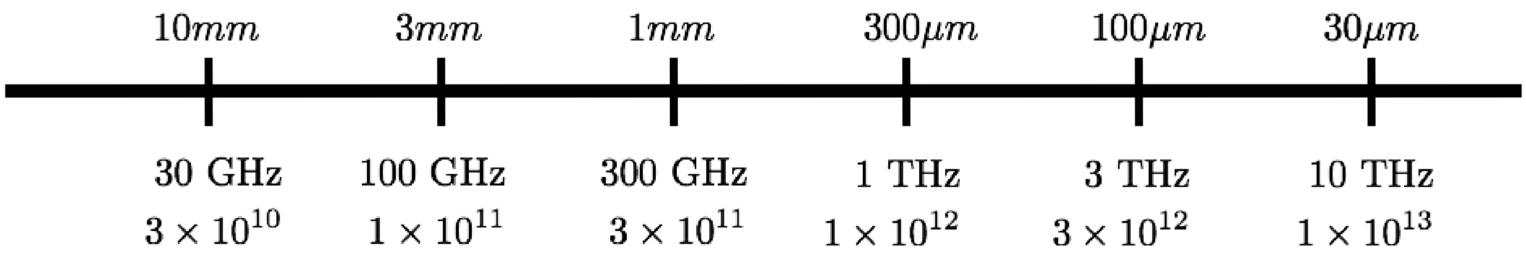
PHOTODIODE mixers have a frequency response limited to less than 1 GHz due to the recombination time of the charge carriers that have crossed the junction!

Material	$\tau_{recombination}$
Si	100 μs
Ge	10000 μs
PbS	20 μs
InSb	0.1 μs
GaAs	1 μs
InP	~1 μs

→ Common mixer devices:

- SIS junctions
- Schottky diodes
- Hot electron bolometers (HEB)

Mixer Technologies



SIS Pb

SIS NbTiN

150GHz bandwidth

SIS, Schottky diodes and HEB all become less effective above 1THz

Schottky

HEB

150GHz bandwidth

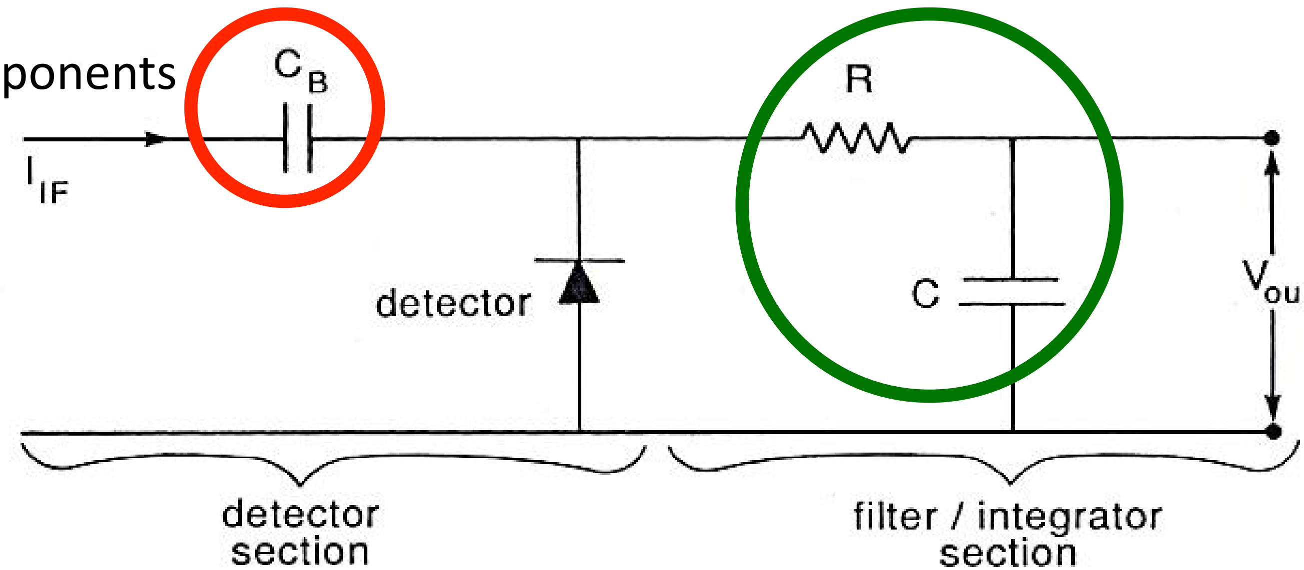
Superconducting HEB

System Components
III. Detectors

Tasks of a Detector Stage

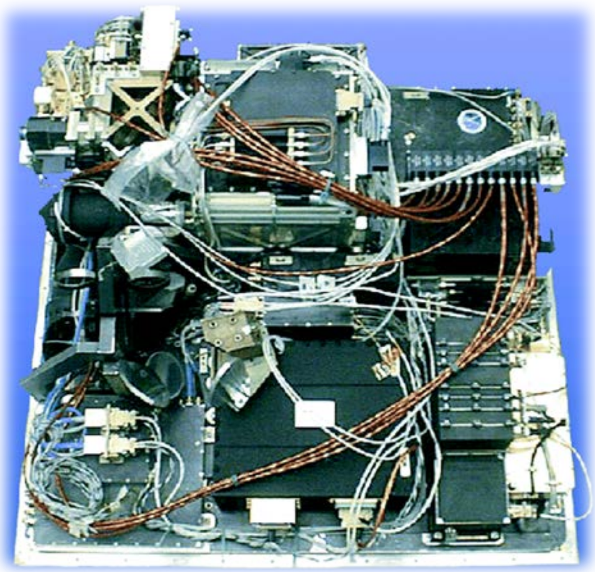
The detector sends the signal through a low pass filter so that the output is a smooth, ‘slowly’ varying output.

Blocks DC components



Filter Banks

- If the IF signal contains important frequency ("color") components it should not be smoothed directly.
- Instead, the signal can be sent to a bank of narrow-band electronic filters, operating in parallel – with a smoothing detector for each filter output.
- Hence, the filter bank can provide a spectrum of the source. (A back-end spectrometer could consist of several filters tuned to different frequencies with detectors on their outputs.)



This spectral multiplexing is one of the most useful features of heterodyne receivers.

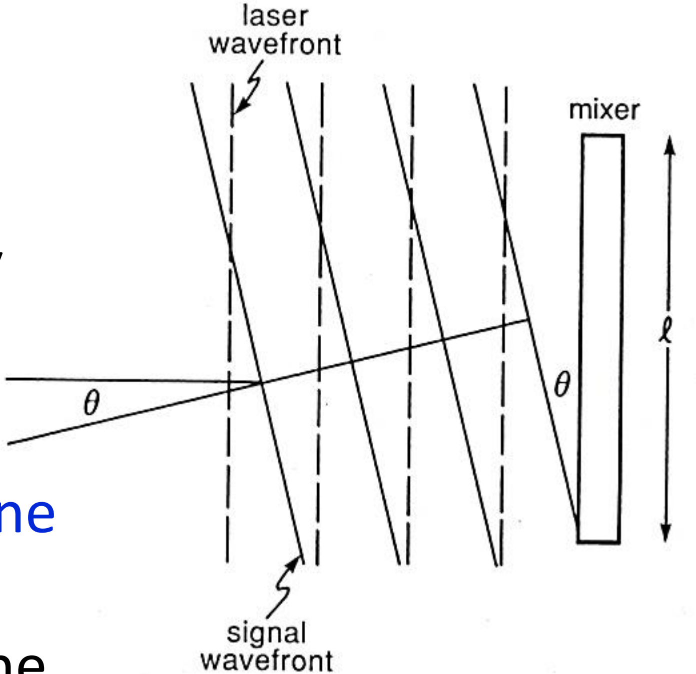
Throughput

Throughput (1)

Factors that limit the throughput of a heterodyne system:

1. Only components of the signal electric field vector parallel to the laser field can interfere (incl. polarization!)

The signal beam will strike the mixer in a range of angles relative to the LO (or laser) beam. The conditions for interference limit the maximum angular displacement. Full cancellation occurs only when the offset ~ λ



$$l \sin\theta_{max} = \lambda \approx l\theta_{max}$$

Since the LO / laser field is polarized only one polarization component of the source can interfere and produce a signal so heterodyne receivers = single-mode detectors.

Throughput (2) – Antenna Theorem

The angular diameter of the FOV on the sky is given by:

$$\Phi \approx \frac{\lambda}{D} \approx \text{Rayleigh criterion}$$

A coherent receiver should operate at the diffraction limit of the telescope. ← This is the "Antenna theorem" (applies to all heterodyne detectors).

- If the receiver only accepts a smaller FOV there is significant loss.
- If the receiver accepts much more it leads to a higher background and limited throughput (factor 1 above).

Example: Herschel/HIFI

Herschel / HIFI

- Seven spectral bands:

SIS Technology | HEB Technology

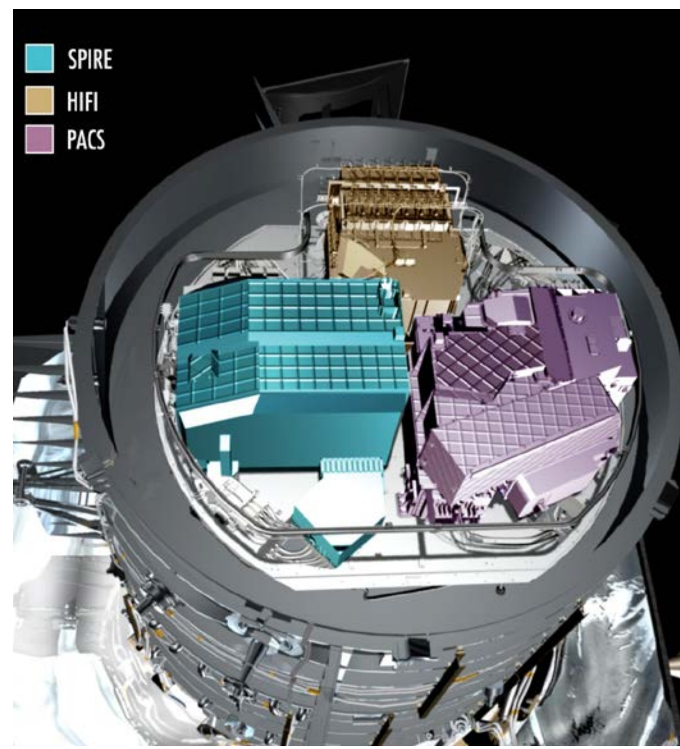
GHz: 480 → 626 → 799 → 949 → 1108 → 1280 | 1421 → 1691 → 1911

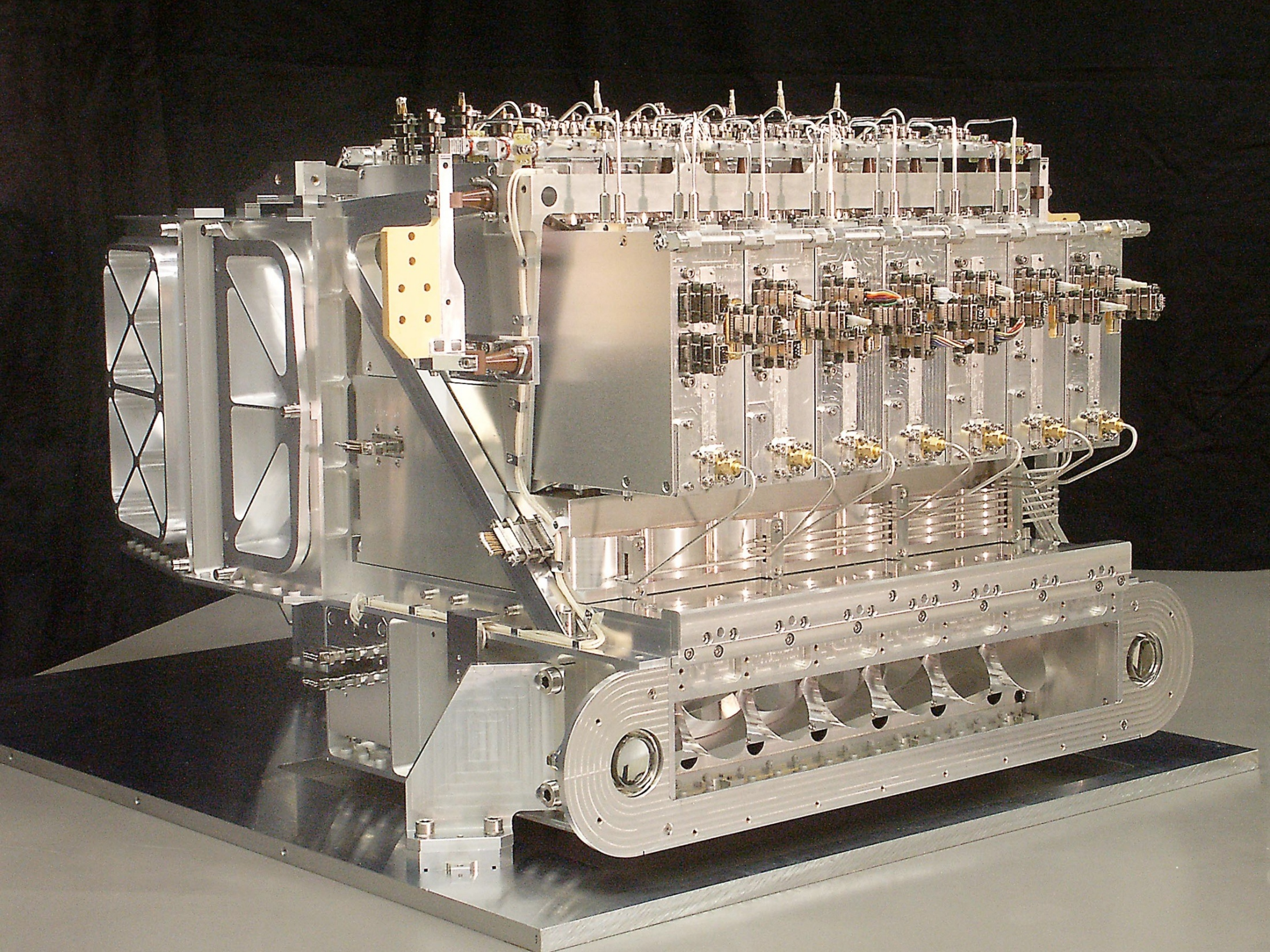
HIFI Bands	1	2	3	4	5	6	7

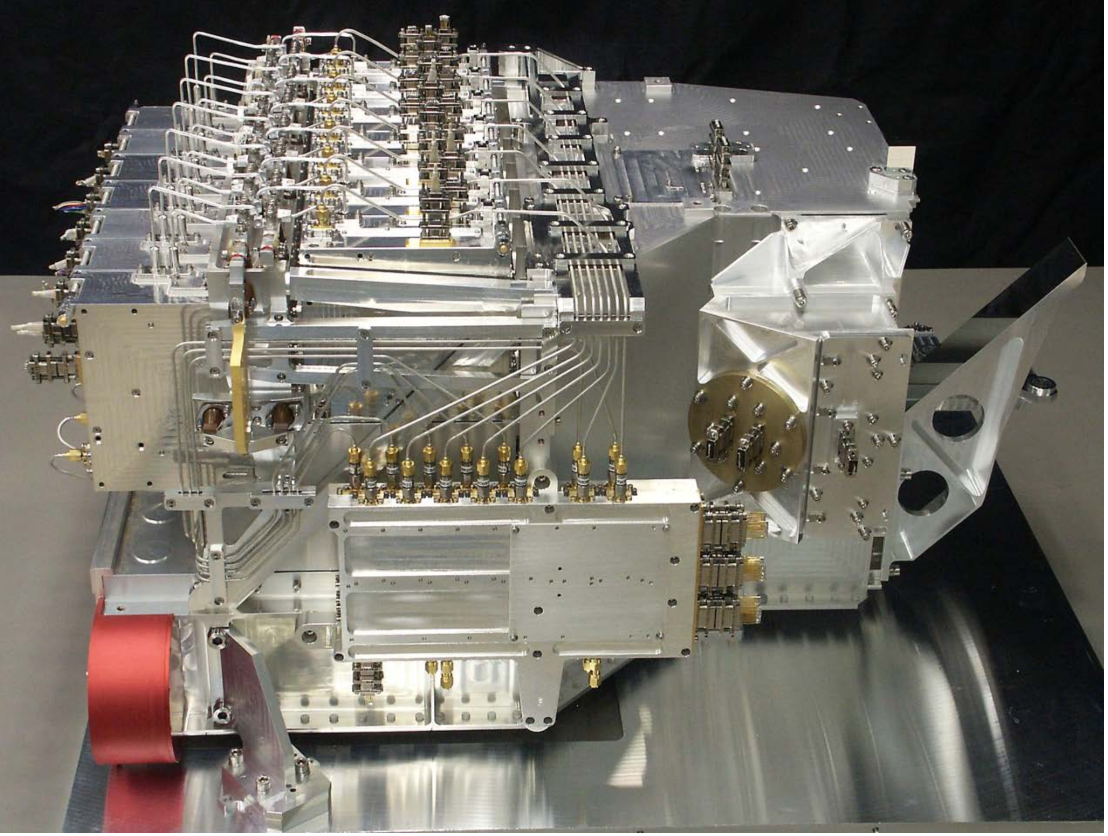
μm: 612 → 473 → 375 → 313 → 271 → 240 | 208 → 178 → 157

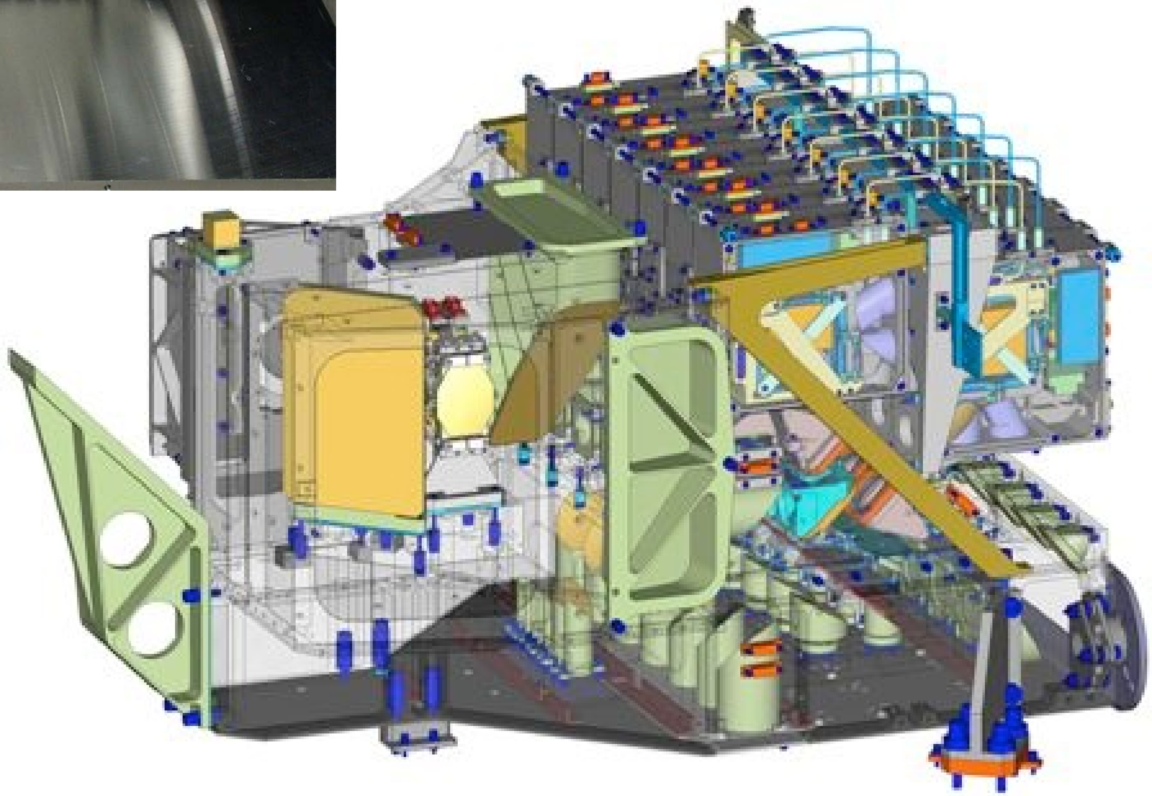
- two polarization components each
- resolving power up to 10^7
- down-converted ω_{IF} is centered at 6 GHz
- bandwidth $\Delta\omega_{IF}$ = 4 GHz

$$d\lambda = \frac{c}{\nu^2} = \frac{3 \times 10^8 \text{ m.s}^{-1}}{(1920 \times 10^9 \text{ Hz})^2} = 4 \times 10^9 \text{ Hz} = 0.33\mu m$$









Signal-to-Noise

Noise Sources

There are *two types* of noise in heterodyne receivers:

1. Noise independent of the LO generated current I_{LO}

   Fundamental noise from the thermal background detected by the system

2. Noise dependent on the LO generated current I_{LO}, which are fundamental noise limits for heterodyne receivers

   Noise in the mixer from the generation of charge carriers by the LO power

S/N, Quantum and Thermal Limit

The S/N is given by (Rieke, p. 291):

$$\left(\frac{S}{N}\right)_{IF} = \frac{\eta P_S}{h\nu\Delta_{IF}\left[\frac{a}{G} + \frac{2\eta\varepsilon}{e^{h\nu/kT_B}-1}\right]}$$

a ≈ 1 for a photodiode mixer and a ≈ 2 for a photoconductor; G = gain, ε = BB emissivity

We can distinguish two cases:

1. QUANTUM LIMIT ($h\nu >> kT_B$): G-R noise in the mixer dominates.
2. THERMAL LIMIT ($h\nu << kT_B$): noise from thermal background dominates.

The dividing line between the two cases is roughly at:

$$\frac{a}{G} \approx \frac{2\eta\varepsilon}{e^{h\nu/kT_B} - 1}$$

NEP

Remember: The noise equivalent power (NEP) is the signal that can be detected at a S/N of unity within unity frequency bandwidth Δf:

The NEP in the quantum limit is:

$$\mathrm{NEP}_{ql} = \frac{P}{\left(\frac{S}{N}\right)_{out} (\Delta f)^{1/2}} = \frac{h\nu a}{\eta G} (2\Delta f_{IF})^{1/2}$$

The NEP in the thermal limit is:

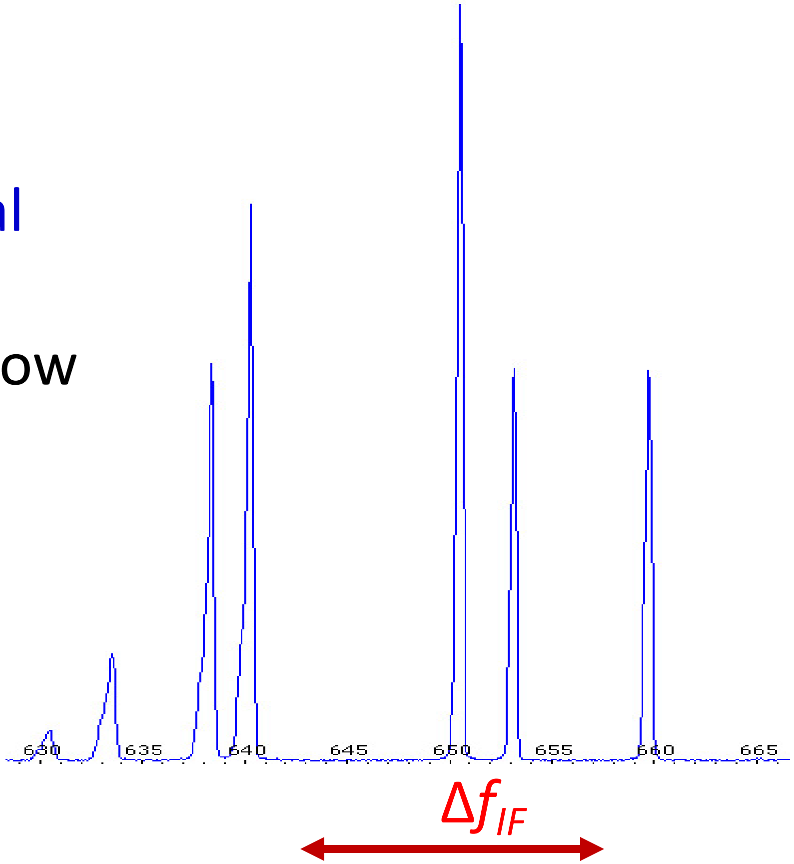
$$\mathrm{NEP}_{th} = \frac{P}{\left(\frac{S}{N}\right)_{out} (\Delta f)^{1/2}} = \frac{2h\nu\varepsilon}{e^{h\nu/kT_B} - 1} (\Delta f)^{1/2}$$

Noise Temperature

NEP and Bandwidth

So far: $NEP_H \propto \sqrt{2\Delta f_{IF}}$ implies that the NEP decreases (i.e., the S/N increases) when narrowing down the bandwidth!

However, this is only correct if all signal power falls within an interval, which is smaller than Δf_{IF}. This is given for narrow emission lines but not for continuum sources.



The Noise Temperature

Let's define a noise temperature T_N such, that a matched blackbody at the receiver input at a temperature T_N produces a S/N = 1.

The concept of noise temperatures offers a convenient means to quantify the LO-independent components, such as amplifier noise.

This amplifier noise is usually Johnson noise: $\left\langle I_A^2 \right\rangle = \frac{4kT_N \Delta f_{IF}}{R_A}$,

where R_A and T_N are the amplifier input resistance and noise temperature, respectively.

The lower limit for the noise temperature is given by $T_N \approx \frac{h\nu}{k}$

- For an amplifier operating at 32 GHz T_N ~ 1.5K.
- For a good HEMT amplifier T_A ~ 10K.

Noise Temperatures at the Limits

First we estimate the noise temperature T_N in the *thermal* limit.

- If the BB emissivity $\varepsilon = 1$ then: $T_N = T_B$
- If the BB emissivity $\varepsilon < 1$ then: $T_N = \frac{h\nu}{k} \frac{1}{\ln(\varepsilon - 1 + e^{h\nu/kT_B}) - \ln\varepsilon}$

Similarly, the noise temperature in the quantum limit (double sideband) is: $T_N = \frac{h\nu}{k\ln\left(1 + \frac{2G\eta}{a}\right)} \approx \frac{h\nu}{k}$

Ideally: $G = \eta = a = 1$

How to measure Noise Temperatures

Take two blackbody emitters with well spaced different temperatures T_{hot} and T_{cold}.

If V is the output voltage of the receiver we can define a "Y" factor:

$$Y = \frac{V_{hot}}{V_{cold}}$$

...which can be measured by alternately placing the blackbodies over the receiver input:

$$Y = \frac{T_{hot} + T_N}{T_{cold} + T_N}$$

...and solve it for the receiver noise temperature T_N:

$$T_N = \frac{T_{hot} - YT_{cold}}{Y - 1}$$

Antenna or Source Temperature

Just like the noise temperature T_N describes the strength of the noise background, we can assign the source flux an antenna temperature T_S :

We get for a blackbody-type source in the Rayleigh-Jeans approximation ($h\nu << kT$):

$$A\Omega = \lambda^2 = \frac{c^2}{\nu^2}$$

$$P_S = L_\nu T_S A\Omega\Delta f_{IF} = \frac{2kT_S\nu^2}{c^2} A\Omega\Delta f_{IF} = 2\Delta f_{IF} kT_s$$

where $2\Delta f_{IF}$ is the frequency bandpass for a double sideband receiver.

→ The antenna temperature is linearly related to the input flux density: $P_s \sim T_S$

Coherent ⇔ Incoherent Receivers

Performance Ratio of In/coherent Receivers

The achievable S/N for a coherent receiver in terms of antenna and system noise temperatures is given by the Dicke radiometer equation:

$$\left(\frac{S}{N}\right)_{coh} \approx \frac{T_S}{T_N^{sys}} (\Delta f_{IF} \Delta t)^{1/2}$$

....so the signal to noise for an incoherent receiver operating at the diffraction limit is:

$$\left(\frac{S}{N}\right)_{inc} = \frac{2kT_S \Delta\nu_{inc} \Delta t^{1/2}}{\mathrm{NEP}_{inc}}$$

Hence, the performance ratio between these two types of receivers is:

$$\frac{(S/N)_{coh}}{(S/N)_{inc}} = \frac{\mathrm{NEP}_{inc} (\Delta f_{IF})^{1/2}}{2kT_N^{sys} \Delta\nu_{inc}}$$

Operation at the thermal (Background) Limit

Consider a bolometer operating at the background limit (BLIP) and a heterodyne receiver operating in the thermal limit:

$$\frac{(S/N)_{coh}}{(S/N)_{inc}} = \left[\left(\frac{1}{\eta}\right)\left(\frac{\Delta f_{IF}}{\Delta\nu_{inc}}\right)\left(\frac{h\nu}{kT_B}\right)\right]^{1/2}$$

→ the bolometer will perform better unless $\Delta f_{IF} \gg \eta\Delta\nu$

The *latter case* ($\Delta f_{IF} \gg \eta\Delta\nu$) will be given for measurements at high spectral resolution, much higher than the *IF* bandwidth.

Operation at the Quantum Limit

Consider a detector noise-limited bolometer and a heterodyne receiver operating at the quantum limit:

$$\frac{(S/N)_{coh}}{(S/N)_{inc}} = \frac{\mathrm{NEP}_{inc}(\Delta f_{IF})^{1/2}}{2h\nu\Delta\nu_{inc}}$$

In the case of narrow bandwidth and high spectral resolution, the heterodyne receiver will outperform the bolometer

→ *heterodyne receivers are best for high spectral resolution applications in the sub-mm!*

If you keep the spectral resolution $\nu/\Delta\nu$ constant (typically given) in the above equation, then the relative figure of merit goes as $1 / \nu^2$ ➔ transition from case favoring incoherent over coherent detectors is relatively abrupt.

Outlook: DTL Part-II

Date	Speaker	Affiliation	Topic
20-04-17	Akira Endo	TU Delft	
04-05-17	Edoardo Charbon	TU Delft	
11-05-17	Marco Beijersbergen	cosine, U Leiden	
18-05-17	Alessandra Menicucci	TU Delft	
01-06-17	Derek Ives	ESO	
08-06-17	Jian-Rong Gao	SRON	
15-06-17	Jochem Baselmans	SRON/TU Delft	