

Origins & Evolution of the Universe

an introduction to cosmology — Fall 2018

Lecture 9: Recombination, Cosmic Microwave Background

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Layout of the Course

Sep 24: Introduction and Friedmann Equations

Oct 1: Fluid and Acceleration Equations

Oct 8: Introductory GR, Space Time Metric, Proper Distance

Oct 15: Redshift, Horizons, Observable Distances

Oct 17: Problem Class #1

Oct 22: Observable Distances, Parameter Constraints

Oct 29: Thermal History, Early Universe

Nov 5: Early Universe, Inflation

Nov 12: Inflation, Lepton Era, Big Bang Nucleosynthesis

Nov 14: Problem Class #2

Nov 19: Recombination, Cosmic Microwave Background Radiation

Nov 26: Introduction to Structure Formation

Dec 3: Cosmic Microwave Background Radiation (I)

Dec 5: Problem Class #3

Dec 10: Cosmic Microwave Background Radiation (II)

Dec 21: Final Exam

Review Last Week

The lepton era

Important events during the lepton era are:

- annihilation of muons at $T < 10^{12} \text{K}$ (early-on)
- annihilation of electrons at $T < 5 \times 10^9 \text{K}$ (at the end)
- nucleosynthesis

Statistical equilibrium distributions

A relativistic particle species in thermal equilibrium follows a Fermi-Dirac(+) or Bose-Einstein(-) distribution:

$$n(T) = \int_0^{\infty} \frac{g}{2\pi^2 \hbar^3 c^3} \frac{E^2 dE}{e^{E/k_B T} \pm 1} \simeq 0.1216 \binom{3/4}{1} g \left(\frac{k_B T}{\hbar c} \right)^3$$

The energy density is given by:

$$\rho(T) c^2 = \int_0^{\infty} \frac{g}{2\pi^2 \hbar^3 c^3} \frac{E^3 dE}{e^{E/k_B T} \pm 1} = \binom{7/8}{1} \frac{g}{2} \sigma_r T^4$$

The total energy density $\rho(T) c^2 = (g^* \sigma_r / 2) T^4$.

Important Concept in Temperature Evolution of Evolving Universe

We have told you that the temperature of the radiation component of the universe scales as follows:

$$T(t) = T_p a(t_p)/a(t)$$

More precisely, it is the entropy which is conserved during the expansion of the universe (since the reactions are in equilibrium):

$$S = (\rho c^2 + P)V / T = 4/3 \rho c^2 V / T = 4/3 g^*(T) (1/2) \sigma T^3 V$$

where g^* gives the statistical weight (\sim degrees of freedom)

$$\Rightarrow S \propto g^* T^3 V \text{ is conserved}$$

$$\Rightarrow T \propto (g^*)^{-1/3}/a$$

Lepton Era

At the start of the Lepton era, $g^*(T < T_\pi) = 4 \times 2 \times 7/8 + N_\nu \times 2 \times 7/8 + 2 = 14.25$

$$e^+ e^- \mu^+ \mu^-: g = 2 \quad \text{neutrinos: } g = 1 \quad \text{if } N_\nu = 3$$

$$\text{photons: } g = 2$$

How does the conservation of g^*T^3 impact the evolution of the temperature as various species annihilate?

The first instance in the lepton era is when muons annihilate at $T \sim 10^{12}$ K

$$g^* \text{ falls from } 6 \times (7/8) + 4 \times 7/8 \times 2 + 2 = 14.25 \text{ to } 6 \times (7/8) + 2 \times 7/8 \times 2 + 2 = 10.75$$

T of the constituent radiative components of the universe of rises by $(14.25/10.75)^{1/3} - 1 \sim 9.8\%$

Annihilation of Electrons-Positrons / Ratio of T_γ / T_ν

At the time of decoupling, the temperature of the neutrinos coincides with the temperature of the other constituents. After decoupling, the temperature T of neutrinos evolve as $1/a$, such that ρ evolves as $1/a^4$.

The e^+ , e^- , γ component follows the same evolution, but at e^+ , e^- annihilation (5×10^9 K: 4 seconds after Big Bang) the temperature is increased again; this ratio persists until $T_\gamma / T_\nu = (11/4)^{1/3} > 1$

Once e^+ , e^- annihilation is over in the universe, the universe is dominated by a radiation background with thermal black-body spectrum.

Big Bang Nucleosynthesis

The following is the elemental abundance in the universe:

$${}^4\text{He}: Y = 0.25$$

$${}^3\text{He}: \sim 10^{-3}Y$$

$${}^2\text{H or D}: \sim 2 \times 10^{-2}Y$$

It is the ratio of the mass of a particular element to the total mass in baryons.

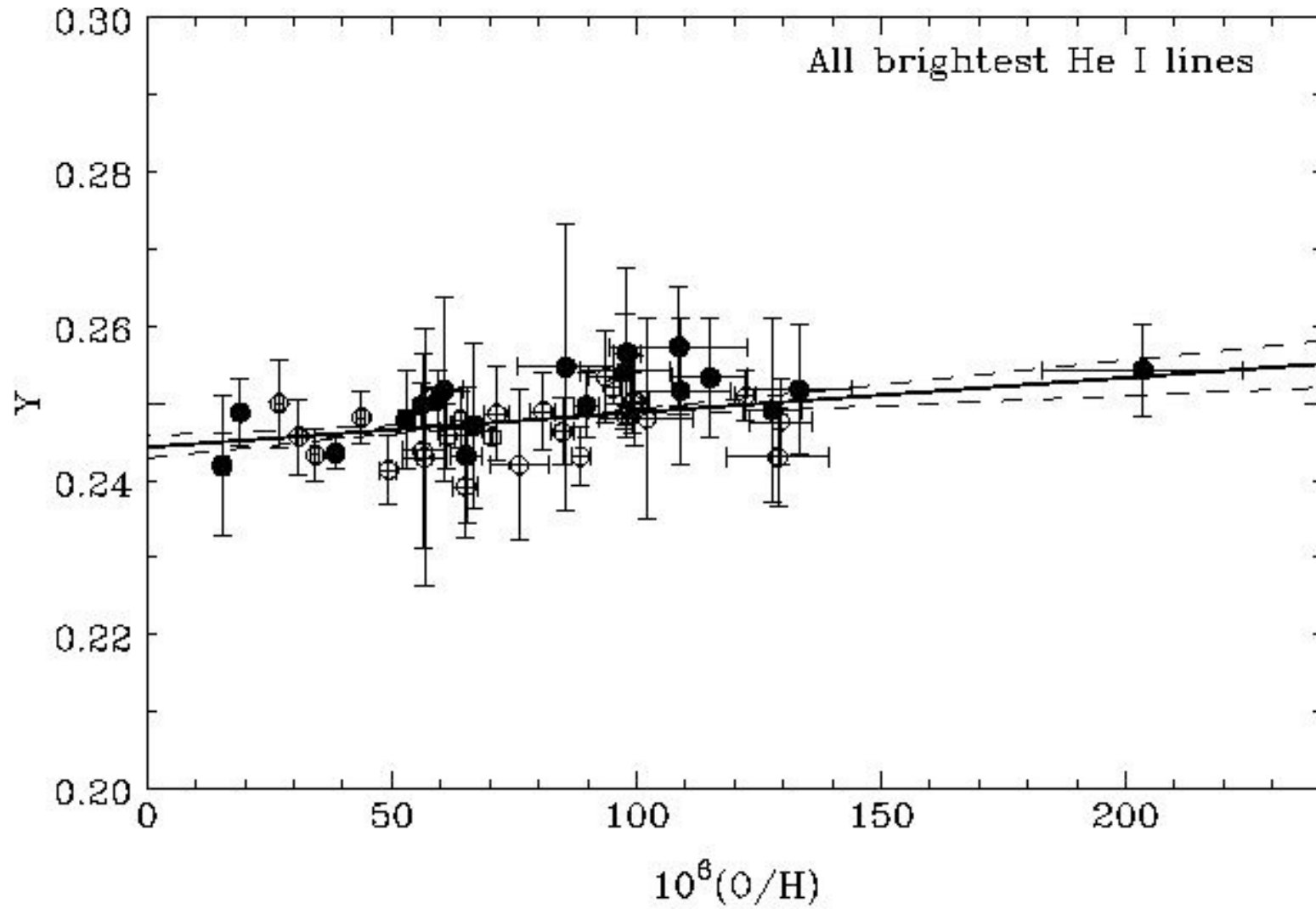
Observational challenge — dealing with late-time effects that can impact abundance of different elements (production in stars, cosmic-ray bombardment)

Could the helium be produced in the stars? No! Just 1% of the nucleons would undergo fusion if galaxies remains at fixed luminosity for 10^{10} years.

Atmospheres of main sequence stars consist of ~25% helium with only weak trends with age or metallicity. This He is therefore not made in the stars.

Towards the end of the lepton era, nuclear physics begins to take place, ultimately resulting in H, He, and traces of D and Li. This phase is not important for the thermal history of the universe, but clearly important for our existence.

Big Bang Nucleosynthesis

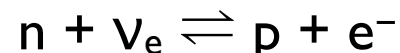


How many neutrons are bound into nuclei?

1. When does the neutron / proton equilibration interaction freeze out, i.e., fixing the ratio (except for neutron decay)?

The ratio of neutron to proton number densities n_n / n_p is $e^{-Q/kT} = e^{-(1.5e10/T)}$ as long as the neutrons and protons are in thermal equilibrium.

The equilibrium is maintained by weak nuclear reactions:



Because of the mass difference, there are more protons than neutrons.

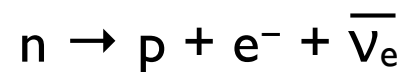
The equilibrium is maintained for $> 10^{10}$ K when the neutrinos decouple.

When this happens:

$$x_n = n/(n+p) = n / n_{\text{tot}} = (1 + e^{-1.5})^{-1} = 0.17 = x_n(0)$$

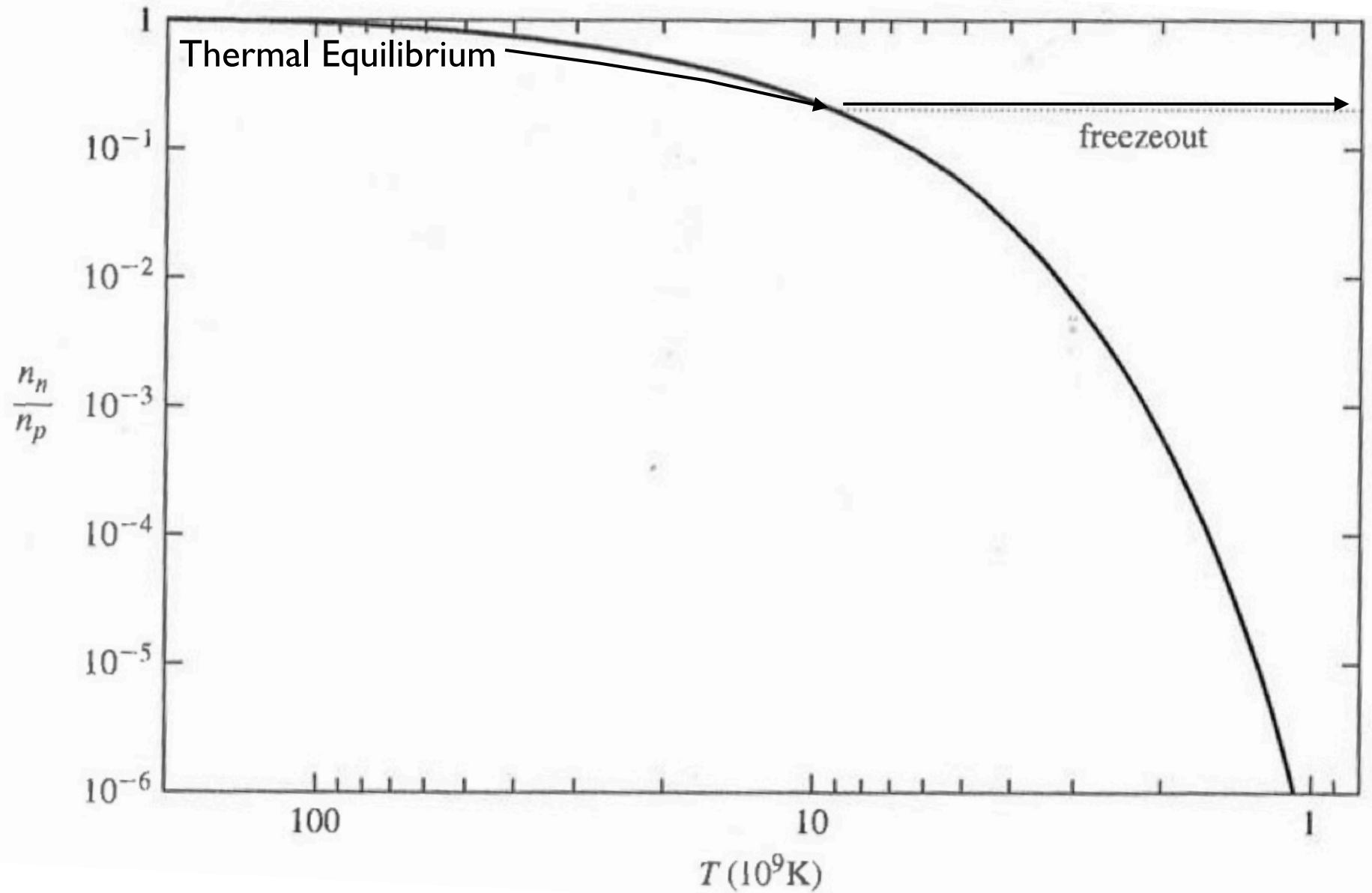
2. Then neutrons decay, but how many will have decayed before they are bound into nuclei?

Neutrons decay via beta decay with mean lifetime of 900 s



$$x_n(t) = x_n(0)e^{-t/(900s)}$$

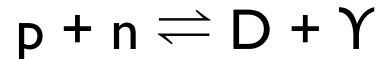
Neutron - Proton Ratio



When Thermodynamics Favors Nucleosynthesis

3. At which point does the temperature favor baryons being bound in nuclei than remaining unbound (at high temperatures, nucleosynthesis is not favored)?

At $t \sim 2s$, the proton-neutron freeze-out occurs, the neutrinos are decoupled, but the photons are still strongly coupled. To build heavier nuclei, we need a series of 2-particle interactions:



the binding energy $B_D = (m_p + m_n - m_D)c^2 = 2.22 \text{ MeV}$

γ with $E > B_D$ can destroy D

During D formation, the relative number densities are given by the equation:

$$n_D / (n_p n_n) = g_D / (g_p + g_n) (m_D / (m_p m_n))^{3/2} (kT / 2\pi\hbar^2)^{-3/2} e^{B_D/kT}$$

Note that $n_D = g_D (2\pi\hbar^2 m_D / kT)^{3/2} e^{-m_D/kT}$

As $m_p = m_n = m_D/2$ and $g_D = 3, g_p = 2, g_n = 2$

$$n_D / (n_p n_n) = 6 (m_p kT / \pi\hbar^2)^{-3/2} e^{B_D/kT}$$

Deuterium is expected in the limit $T \rightarrow 0$, where p^+, n are favored when $T \rightarrow \infty$

When Thermodynamics Favors Nucleosynthesis

So when is $n_D / n_n = 1$ (half neutrons fused)?

$$n_D / n_n = 6 n_p (m_n kT / \pi \hbar^2)^{-3/2} e^{B_D/kT}$$

Approximately 80% of the baryons are in the form of H:

$$n_p = 0.8 n_{\text{baryon}} = 0.8 n_\gamma \eta = 0.8 \eta (0.243 (kT/\hbar c)^3) \quad \text{where } \eta = n_{\text{baryon}} / n_\gamma$$

$$n_D/n_n = 6.5 \eta (kT/m_n c^2)^{3/2} e^{B_D/kT}$$

The ratio is unity for $T_{\text{nuc}} \sim 8 \times 10^8 \text{ K} \Rightarrow 200 \text{ s}$ after Big Bang

The time delay is non-negligible compared to the neutron decay time (900 s).

$$\text{So } n_n / n_p = e^{-(200/900)} / (5 + (1 - e^{-200/900})) = 0.8/5.2 \sim 0.154$$

Theoretical Maximum Production of $^4\text{Helium}$ Allowed

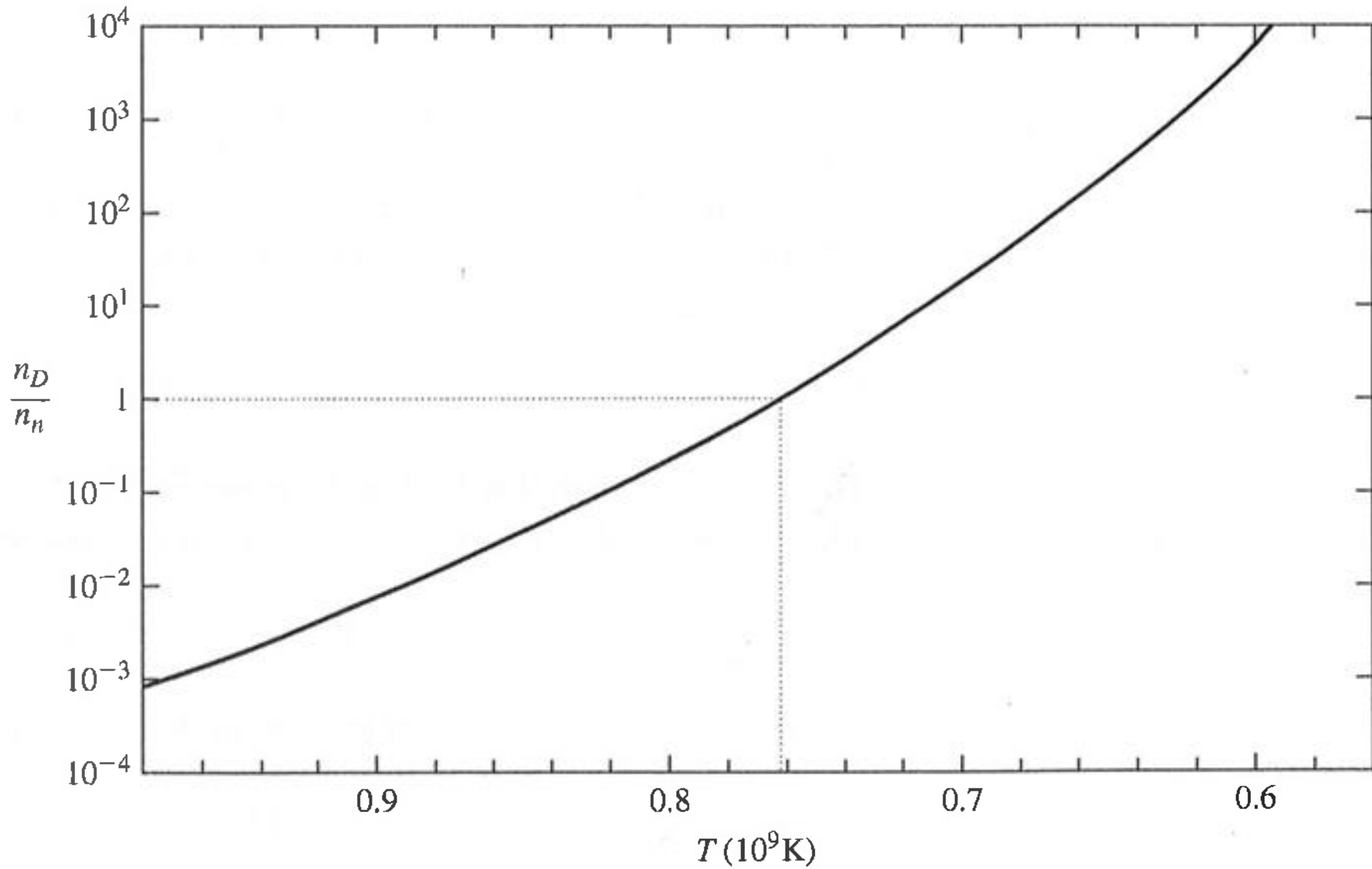
As $p + n \Rightarrow D + \gamma$ is so much more efficient than reactions involving two protons, BBN is limited by the # of neutrons.

This allows us to compute the maximum value for Y based on the neutron to proton ratio

$$\text{generally if } f = n_n/n_p \Rightarrow Y_{\text{max}} = 2f/(1+f) \quad Y_{\text{max}} = 4/12 = 1/3 \text{ for } f = 0.2$$

$$\text{For } f = 0.154, Y_{\text{max}} \text{ is } 0.27$$

When Thermodynamics Favors Nucleosynthesis



New Material for This Week

Neutron - Proton Ratio

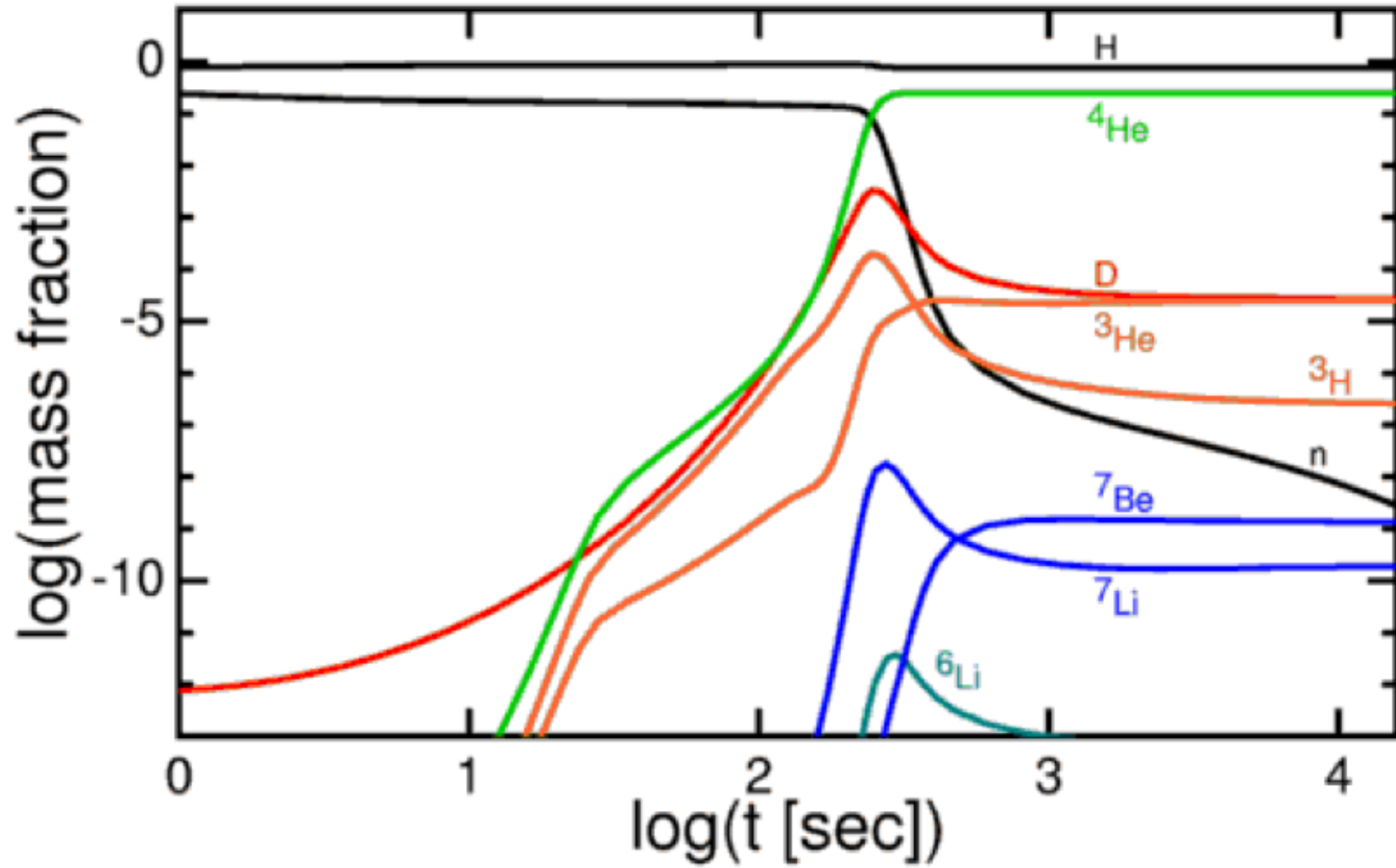
How does nucleosynthesis proceed after we form Deuterium?

The ratio n_D / n_n does not remain at the equilibrium values, once there is enough D we get

- proton capture $D + p \leftrightarrow {}^3\text{He} + \gamma$
- neutron capture $D + n \leftrightarrow {}^3\text{H} + \gamma$
- formation of α particle (rare) $D + D \leftrightarrow {}^4\text{He} + \gamma$
- Tritium formation $D + D \leftrightarrow {}^3\text{H} + p$
- ${}^3\text{He}$ formation $D + D \leftrightarrow {}^3\text{He} + n$

${}^3\text{H}$ decays into ${}^3\text{He} + e^- + \nu_e$, but has a decay time of 18 years, i.e. stable during BBN

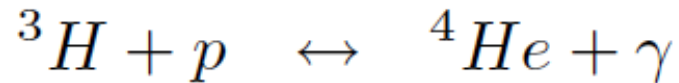
BBN calculations



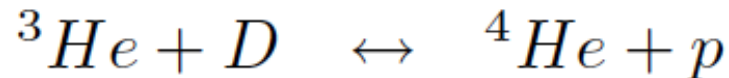
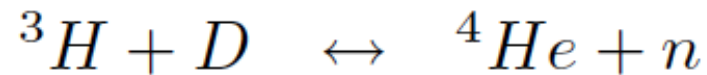
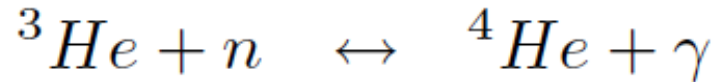
The bottleneck

Helium-3 and Tritium quickly convert to Helium-4

- many pathways

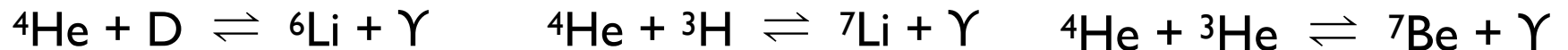


- strong interactions, fast



All strong force interactions so fast.

The binding energy per nucleon is very high for stable nuclei like ${}^4\text{He}$, but there are no stable nuclei with $A = 5$ (${}^5\text{He}$ and ${}^5\text{Li}$ are not stable). Very difficult to make heavier elements.



Synthesis of nuclei with $A > 7$ is hindered by the absence of stable nuclei with $A = 8$

Initially at $T \gg 10^9$ K, all baryons are in the form of protons and neutrons.

As the Deuterium density increases, ${}^3\text{H}$, ${}^3\text{He}$, and ${}^4\text{He}$ are formed.

Note that BBN proceeds from the first 3 min to 10 min of universe ($T \sim 4 \times 10^8$ K)

The bottleneck

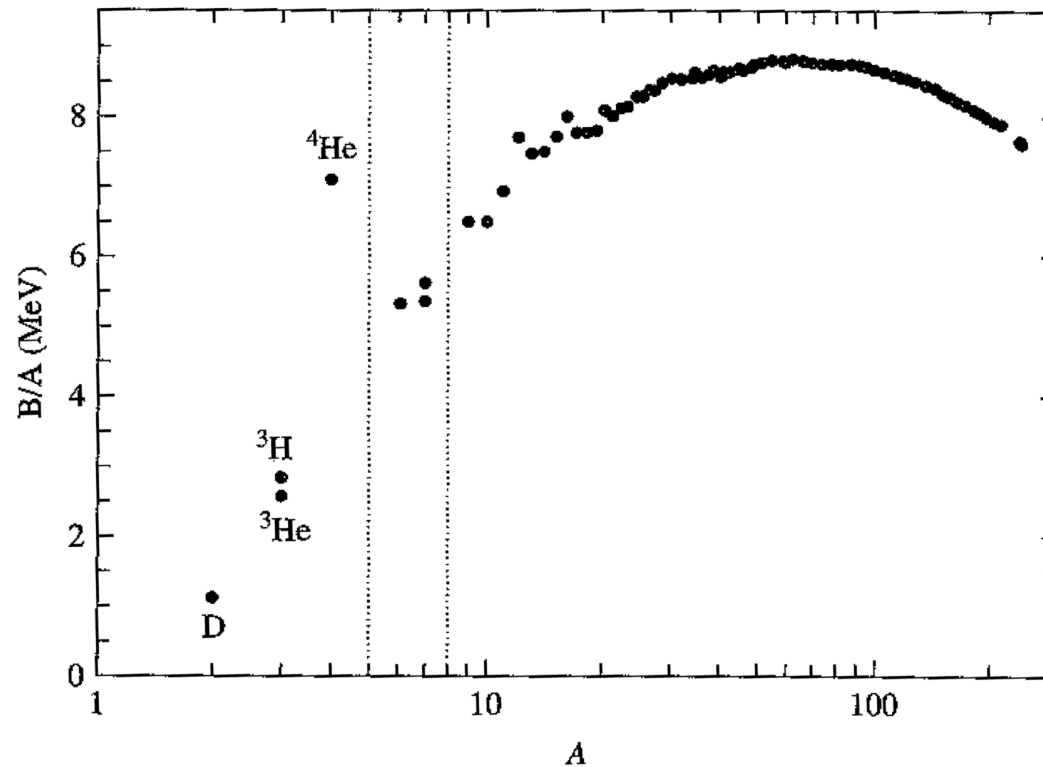
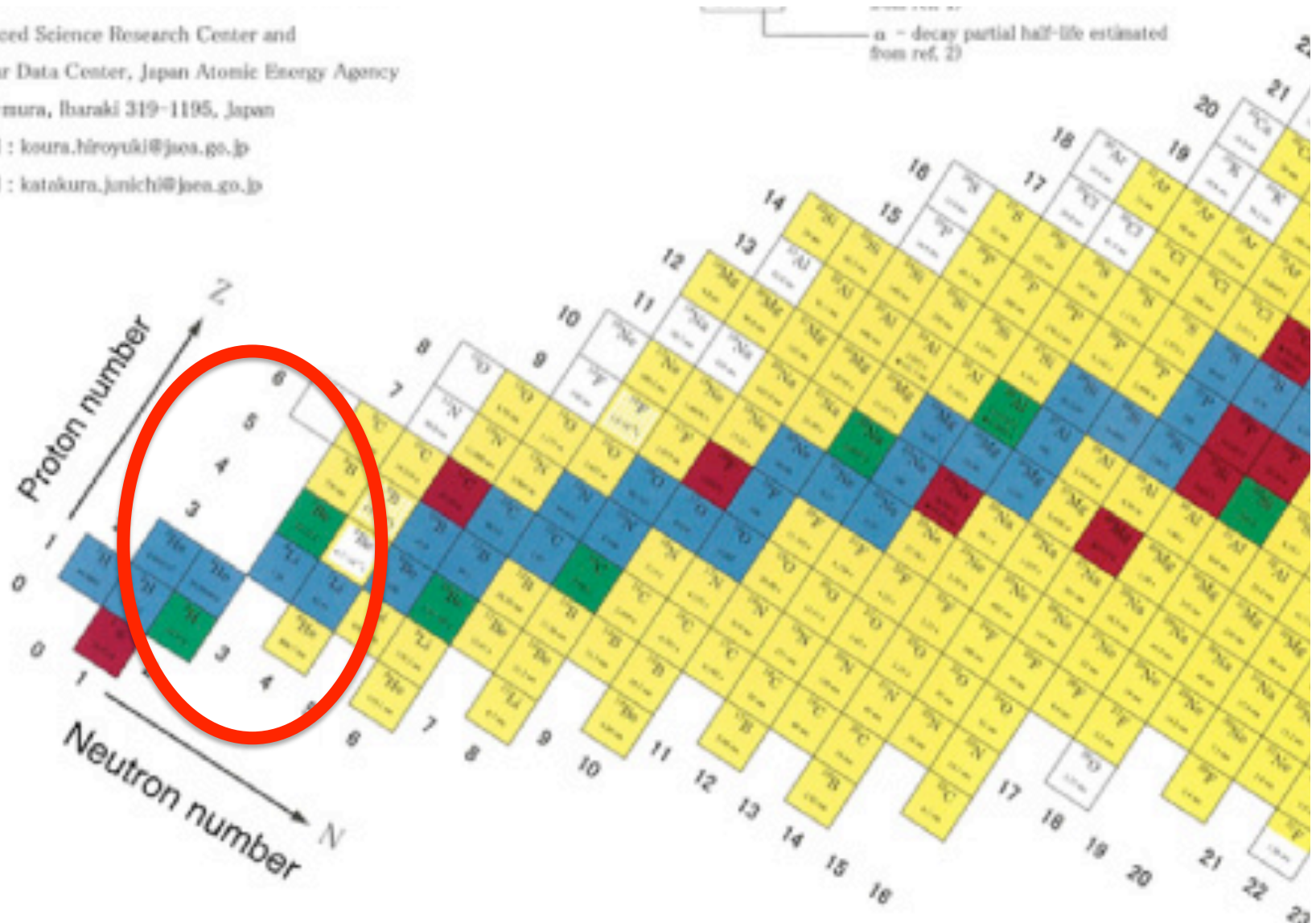


FIGURE 10.1 The binding energy per nucleon (B/A) as a function of the number of nucleons (protons and neutrons) in an atomic nucleus. Note the absence of nuclei at $A = 5$ and $A = 8$.

Binding energy per nucleon = $(m_{\text{nucl}} - Zm_p - Nm_n)c^2/A$
⁵⁶Fe, ⁶²Ni most stable nuclei - ⁴He also stands out

The bottleneck

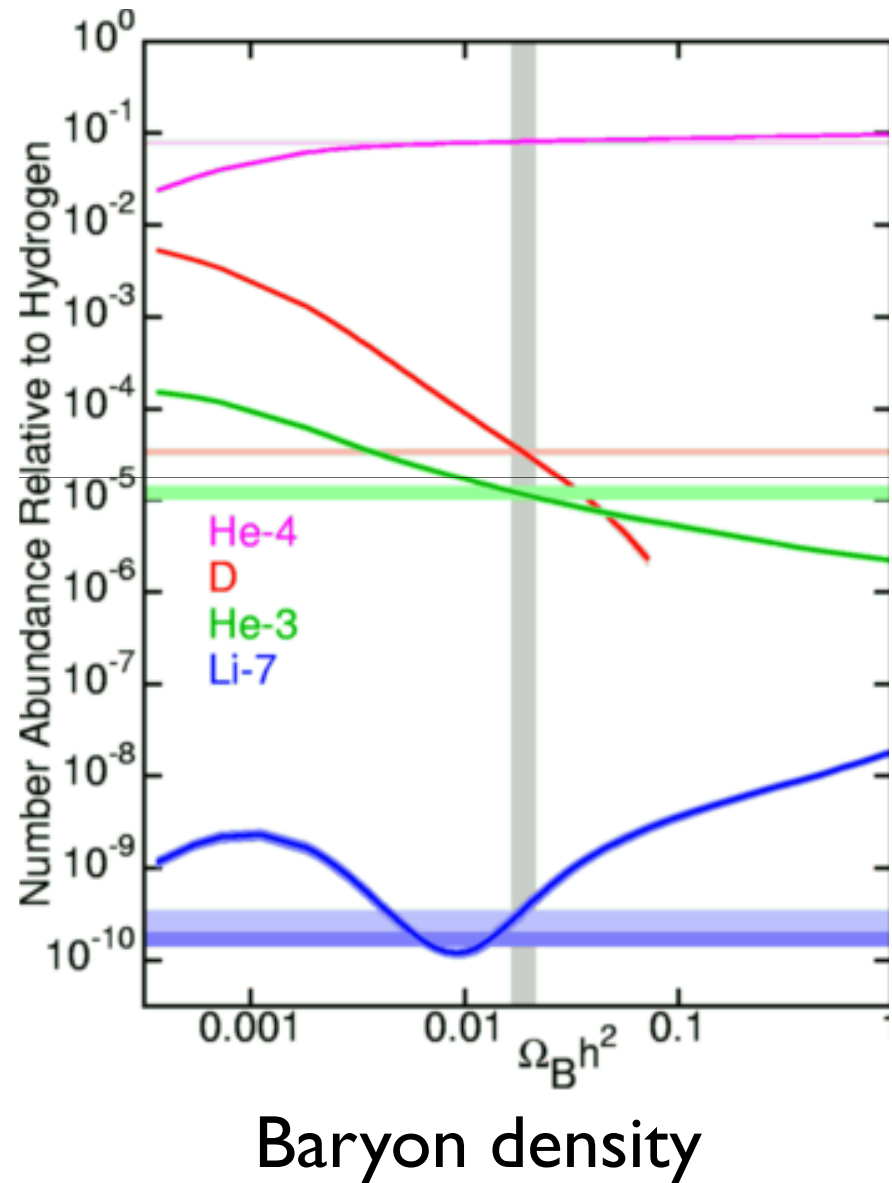
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But what does this teach us
about the matter density in
baryons?

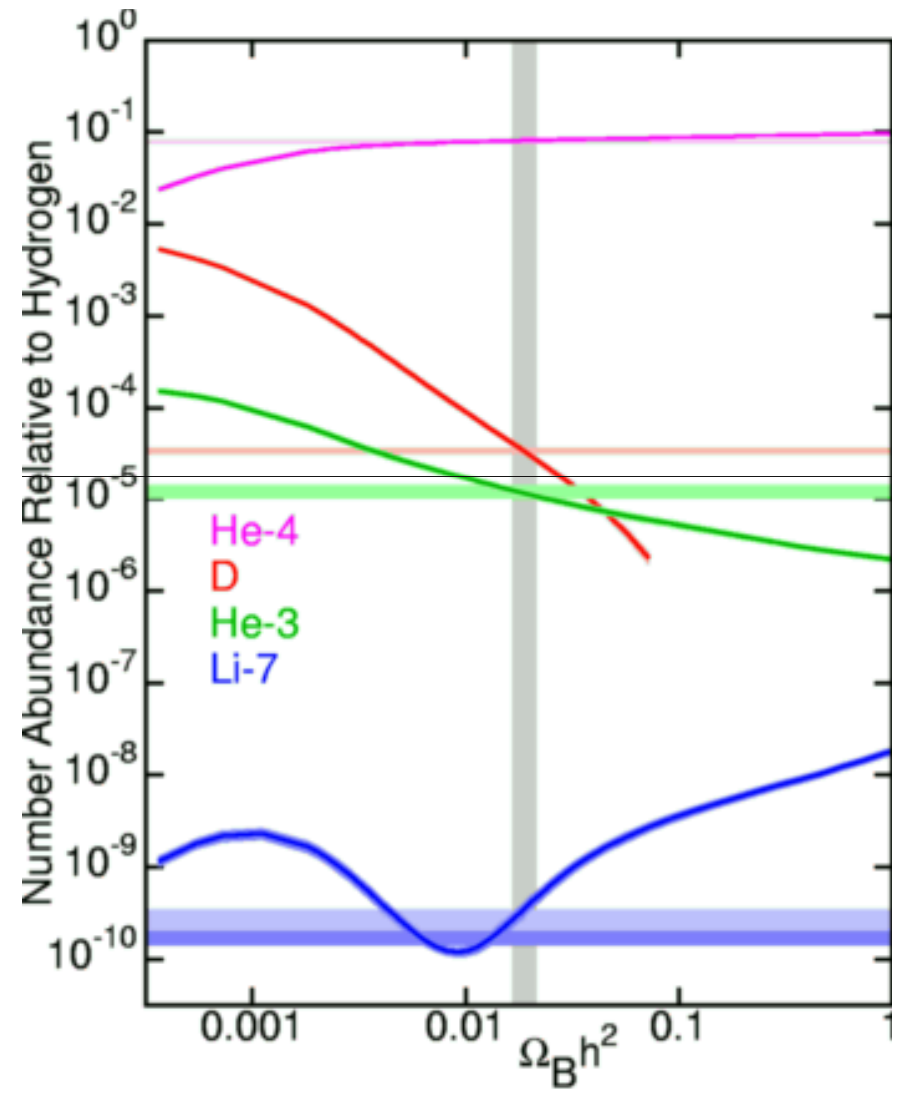
Abundance of heavier elements synthesized depends on baryon density

Reaction Rate \propto Density of Baryons



Abundance of heavier elements synthesized depends on baryon density

IMPLICATION: If we can determine what the abundance of the above elements is relative to hydrogen, we can determine the baryon density in the universe at early times...



Baryon density

So how can we go out and determine what the abundance of these heavier elements is relative to hydrogen?

The challenge is that the universe has not been held in a constant state, there are stars, and they destroy some elements and create others...

Challenge is that:

D → readily destroyed in stars from fusion

He^4 → readily produced from fusion

Li^7 → readily destroyed in stars from fusion

→ may be created by cosmic-ray spallation in the
interstellar medium

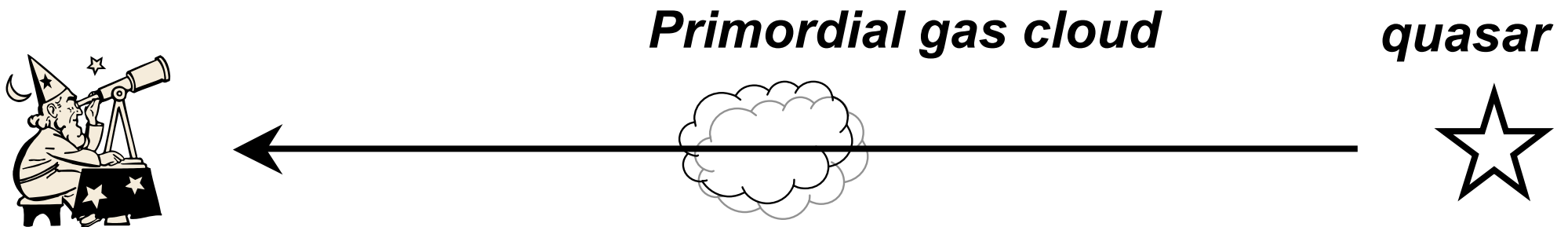
He^3 → produced by burning deuterium

→ destroyed to produce He^4

How might we determine the primordial abundances then?

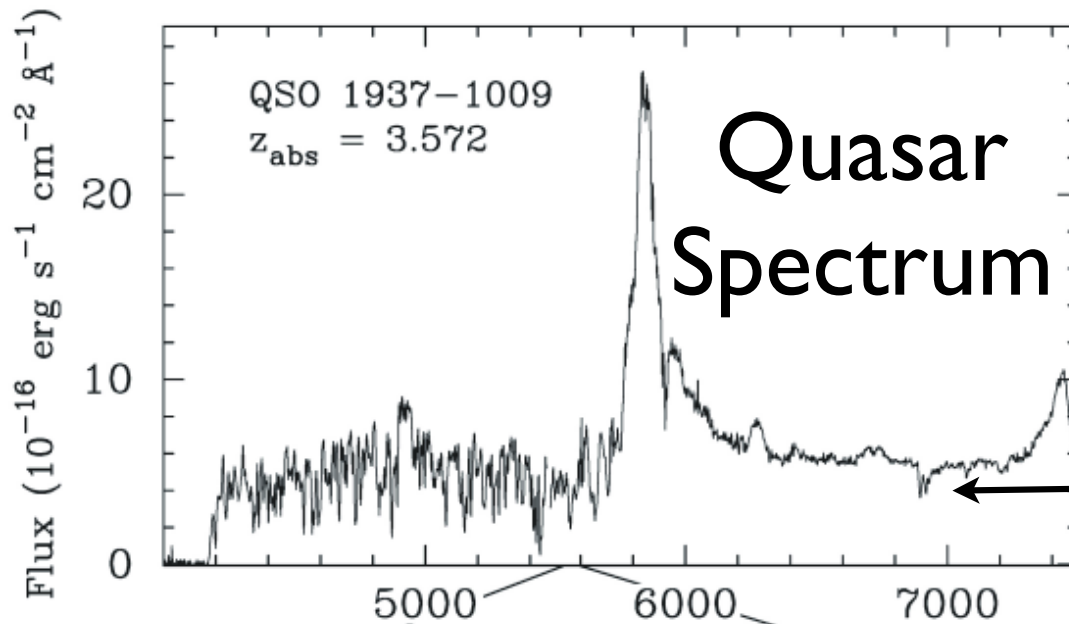
For deuterium:

Observe some gas cloud in early universe where stars have not yet formed and look for absorption by hydrogen and deuterium:

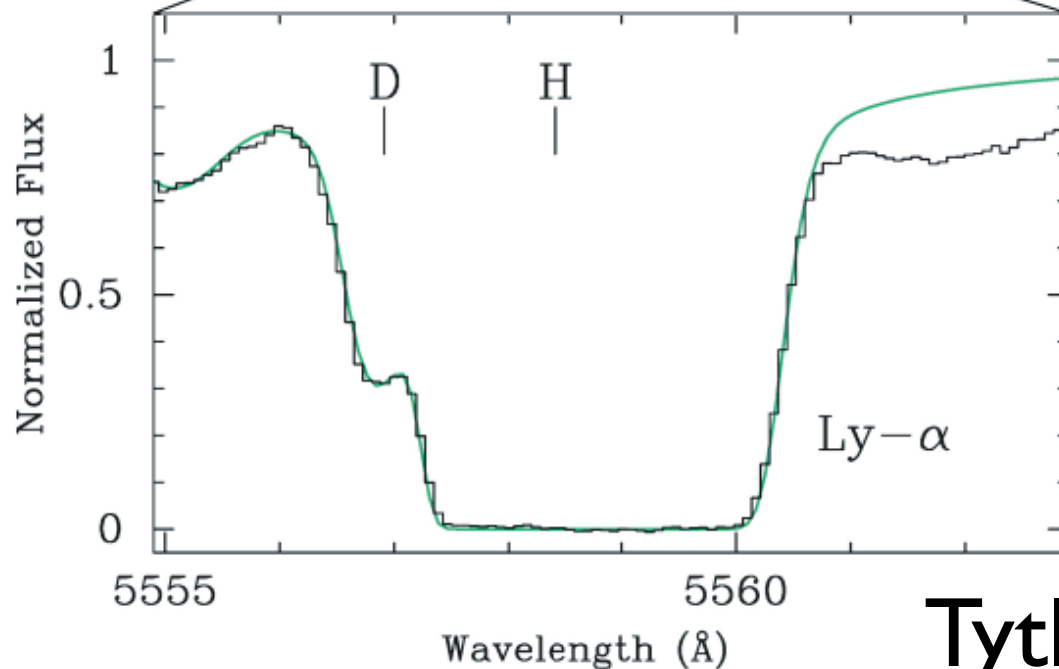


Very bright sources like quasars needed so we infer the presence of rare elements in gas clouds through weak absorption features

What do these spectra look like?

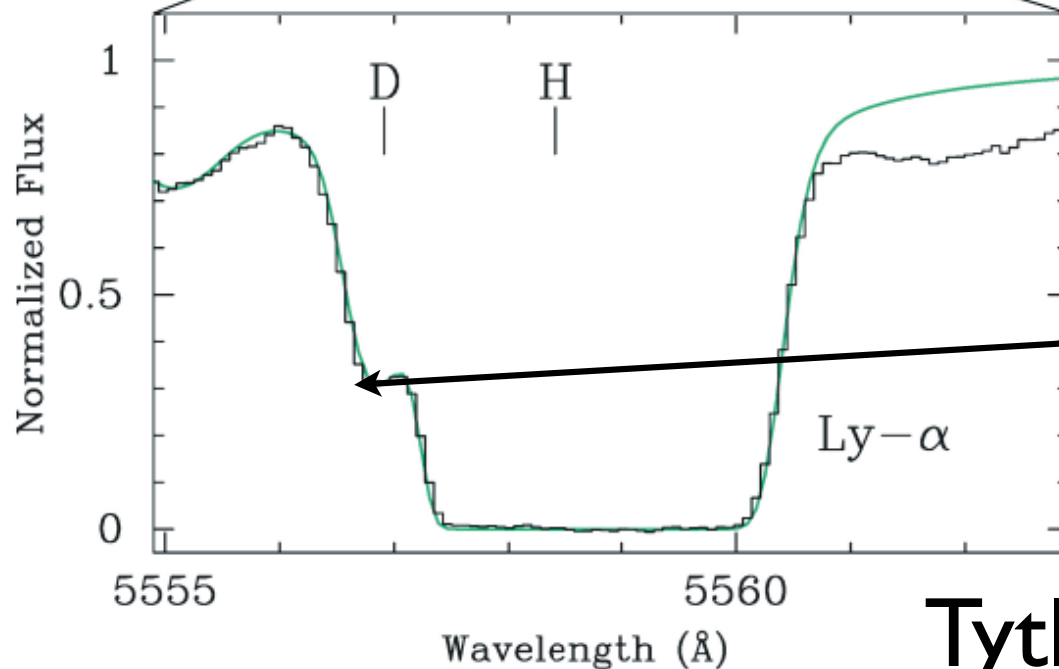
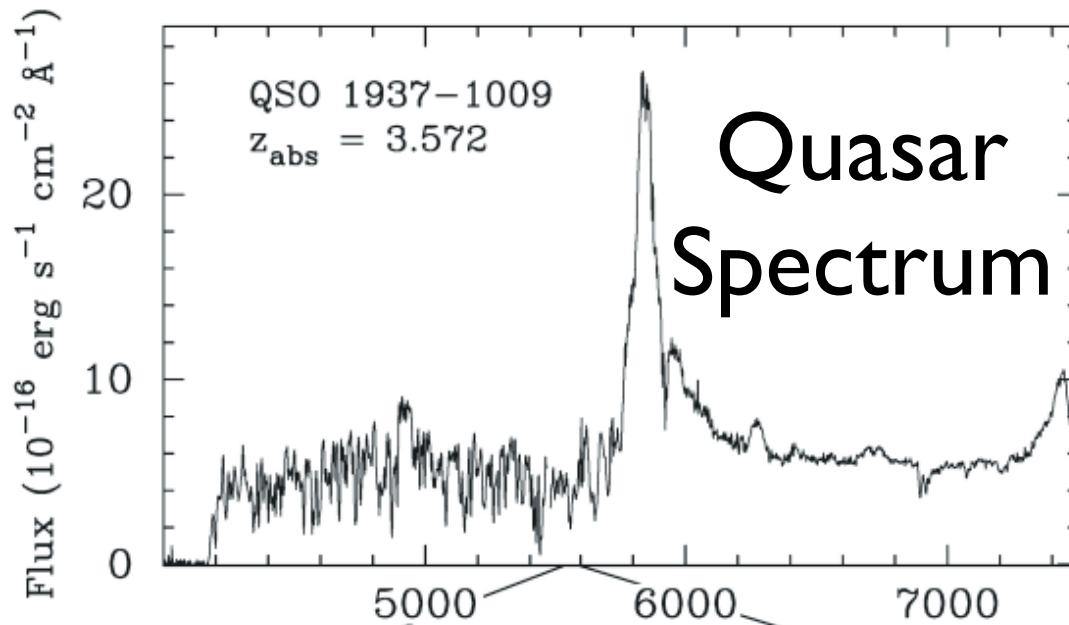


Verify that the gas cloud shows no evidence for being polluted by heavier elements (this ensures deuterium abundance not affected)



Tytler & Burles

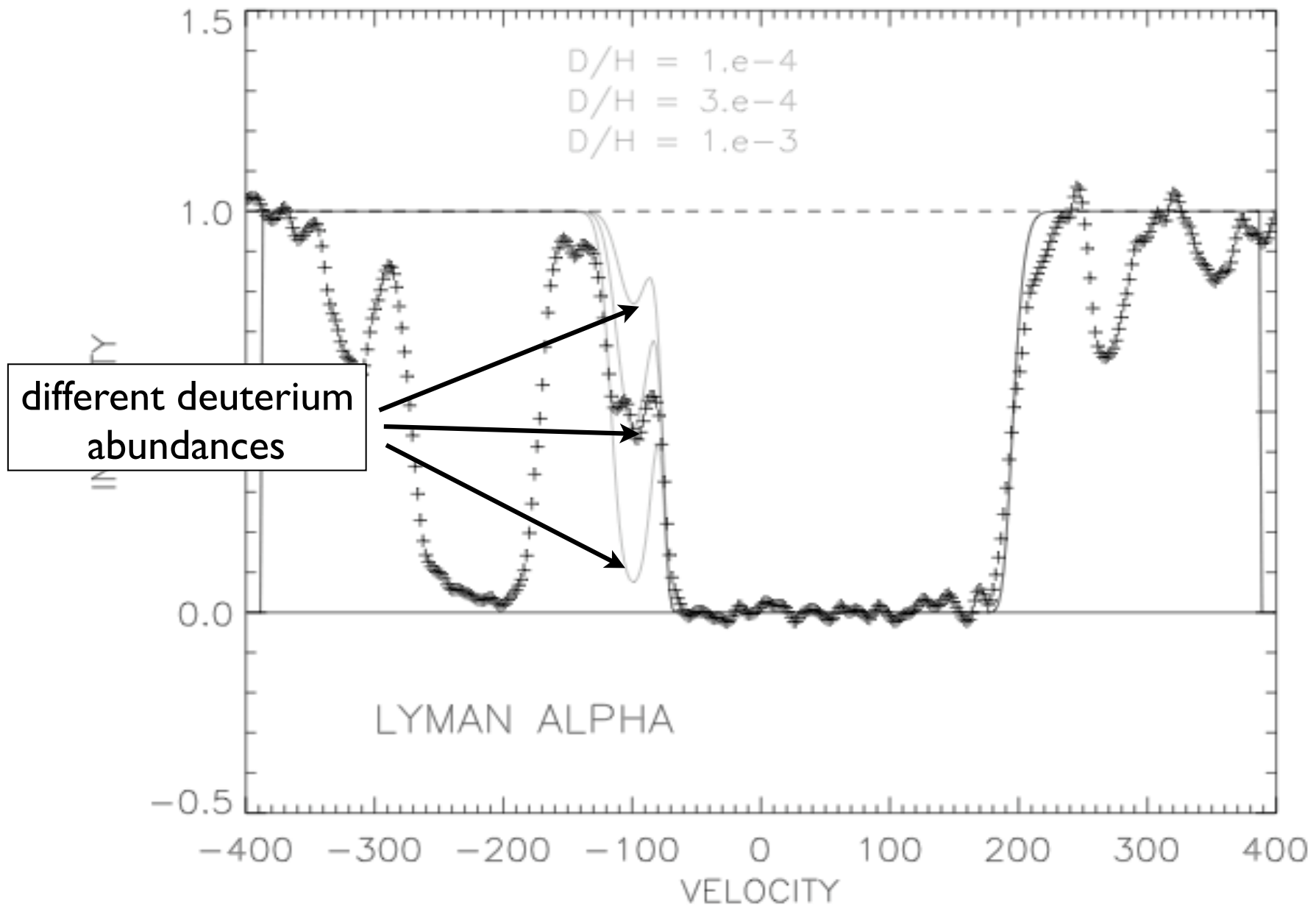
What do these spectra look like?



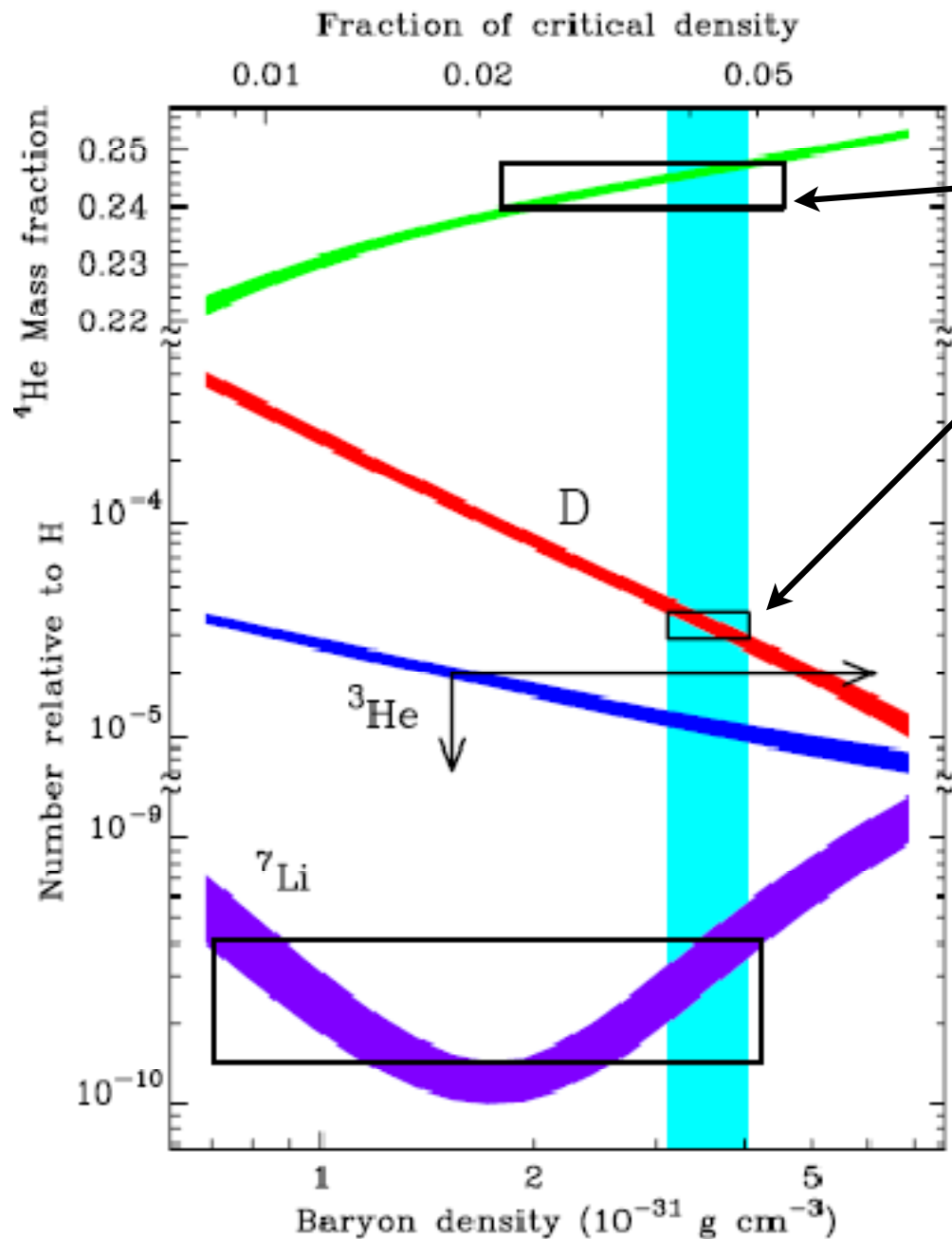
Need to be sure this isn't just another cloud of cold gas at different redshift

Tytler & Burles

How might the observations look different with another deuterium abundance?



What is bottom line putting together constraints?



Boxes show constraints on abundances relative to hydrogen

Best Fit Baryon Density

$$\Omega_B h^2 = 0.019 \pm 0.0024$$

$$\Omega_B = 0.037 \pm 0.009$$

Most useful constraint is from deuterium abundances given steep dependence on $\Omega_b h^2$

What we can learn from BBN about Ω_b

The yields of D, ^3H , ^4He , ^6Li , and ^7Li depend on η :

a high value for η (the baryon to photon ratio) increases the value for T_{nuc}

As BBN is a race against the clock, an earlier start means more ^4He is formed, but less D and ^3He as leftovers.

The deuterium abundance can be used to estimate η :

$$n_{\text{baryon},0} = \eta n_{\gamma,0} = 0.23 \pm 0.02 \text{ m}^{-3} \Rightarrow \Omega_{\text{baryon}} = 0.04 \pm 0.01$$

Also CMB gives a highly consistent constraint on Ω_{baryon}

Radiative Era

The radiative era begins at the moment of the elimination of the electron-positron pairs at $T \sim 5 \times 10^9 \text{ K}$ or $t \sim 10 \text{ s}$

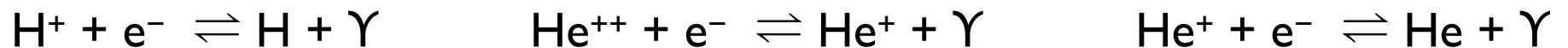
The end of the radiative era occurs when the density of matter coincides with that of the relativistic particles, corresponding to a redshift:

$$1 + z_{\text{eq}} = (\rho_{0,c} \Omega_0) / (K_0 \rho_{0,r}) \sim 4.3 / K_0 \times 10^4 \Omega_0 h^2 \sim 3800$$

where $K_0 = 1 + \varepsilon_{\nu} / \varepsilon_{\gamma} = 1 + (7/8)(4/11)^{4/3} N_{\nu} = 1.68$ if $N_{\nu} = 3$

$$T_{\text{eq}} = T_{0,r} (1 + z_{\text{eq}}) = 10^5 (\Omega_0 h^2 / K_0) \text{ K}$$

At these temperatures, the hydrogen and helium are fully ionized. As the temperature drops, the number of neutral atoms and He^+ atoms grows through the equilibrium reactions:



The number density of the individual components is determined through the Saha equation.

Saha Equation

Let us focus on Hydrogen $p^+ + e^- \rightleftharpoons H + \gamma$, the density of various particles is given by the Boltzmann distribution:

$$n = g (mk_B T / 2\pi\hbar^2)^{3/2} e^{-(\mu - mc^2)/k_B T} \quad \begin{array}{l} g=4 \text{ for H, } g=2 \text{ for } e^-, \\ n_e = n_p, m_p = m_H \end{array}$$

μ is the chemical potential, which is released when a particle is destroyed; the chemical potential is conserved for incoming and outgoing particles, when the reaction is in equilibrium

In this case, $\mu_H = \mu_p + \mu_e$ (photons have $\mu = 0$)

Furthermore, we have $(m_e + m_p - m_H) = 13.6 \text{ eV}$

$$\begin{aligned} n_H / (n_p n_e) &= g_H / (g_p + g_e) (m_H / (m_p m_e))^{3/2} (kT / 2\pi\hbar^2)^{-3/2} e^{[m_p + m_e - m_H]/kT} \\ &= (m_e kT / 2\pi\hbar^2)^{-3/2} e^{Q/kT} \quad \text{where } Q = 13.6 \text{ eV} \end{aligned}$$

If $x = n_p / (n_p + n_H) = n_p / n_{\text{baryon}}$ is the fractional ionization

$$(1-x)/x = n_p (m_e kT / 2\pi\hbar^2)^{-3/2} e^{Q/kT} \quad \text{where } Q = 13.6 \text{ eV}$$

Photon Decoupling / Saha Equation

Recall that $\eta = n_{\text{baryon}} / n_{\gamma}$

if we assume that hydrogen is the only element, then we can write

$$\eta = n_p / x n_{\gamma}$$

But the photons have a blackbody spectrum

$$n_{\gamma} = 2.404/\pi^2 (kT / \hbar c)^3 = 0.243 (kT/\hbar c)^3$$

$$n_p = 0.243 x \eta (kT/\hbar c)^3$$

$$(1-x)/x^2 = 3.84 \eta (kT/m_e c^2)^{3/2} e^{Q/kT}$$

If we define the momentum of recombination as the instant when $x=1/2$ assuming $\eta = 5.5 \times 10^{-10}$, the recombination temperature is

$$kT = 0.323 \text{ eV} = Q/42 \ll 13.6 \text{ eV}$$

This is the consequence of having a large number density of photons in the universe: something similar happens for deuterium formation

This corresponds to the temperature $T_{\text{rec}} \sim 3740 \text{ K}$

$$z_{\text{rec}} \sim 1370,$$

$$t_{\text{rec}} = 240,000 \text{ years}$$

Photon Decoupling / Saha Equation

Recombination is not instantaneous, but quite rapid:

$$x = 0.9 \text{ at } z=1475$$

$$x = 0.1 \text{ at } z=1255$$

$$\Delta t = 70,000 \text{ years}$$

Since the number density of free electrons drops rapidly during the epoch of recombination, the time of photon decoupling comes soon after the time of recombination.

The rate of photon scattering is $\Gamma(z) = n_e(z)\sigma_T c = x(z)(1+z)^3 n_{\text{baryon},0}\sigma_T c$

$$\sigma_T = 6.65 \times 10^{-25} \text{ cm}^2, \text{ using } \Omega_{\text{bary},0} = 0.04$$

$$\Rightarrow \Gamma = 4.4 \times 10^{-21} \text{ s}^{-1} x(z)(1+z)^3$$

$$\text{At } z=0, \quad H_0 = 2.5 \times 10^{-18} \text{ s}^{-1} \Rightarrow \Gamma(z) \ll H$$

$$\text{At } z=1500, \quad \text{expansion rate } H(z) \ll \Gamma(z)$$

so before recombination
photons were well
coupled to electrons

Photon Decoupling

When recombination takes place, the universe is matter dominated, so

$$(H/H_0)^2 = \Omega_{m,0} / a^3 = \Omega_{m,0} (1+z)^3$$

$$\text{If } \Omega_{m,0} = 0.3, H(z) = 1.24 \times 10^{-18} \text{ s}^{-1} (1+z)^{3/2}$$

The redshift for photon decoupling is when the expansion rate equals the scattering rate $H = \Gamma$:

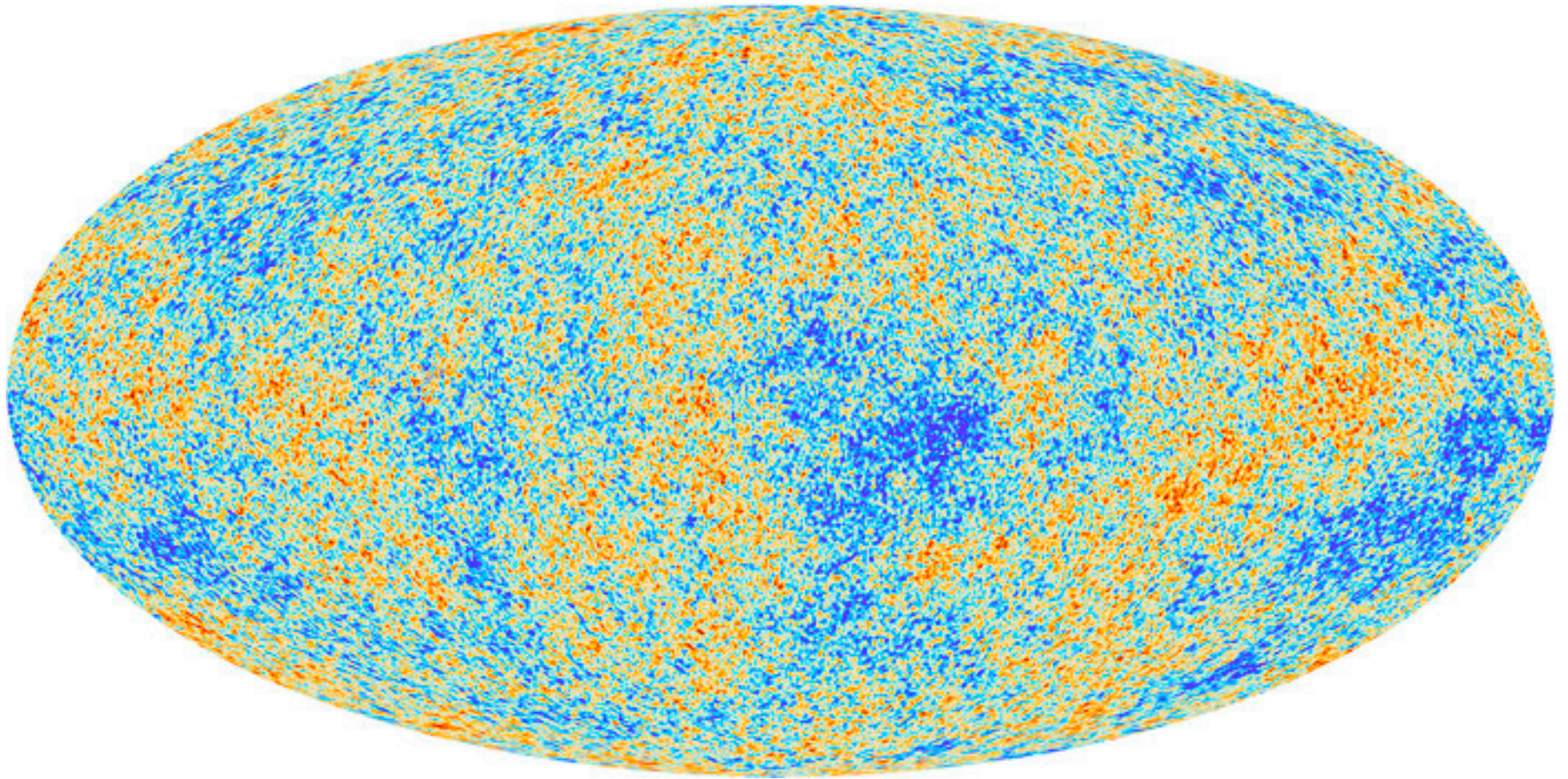
$$\Rightarrow 1+z_{\text{dec}} = 43/x(z_{\text{dec}})^{2/3} \qquad z_{\text{dec}} = 1130$$

The exact redshift is a little lower since the Saha equation assumes that the reaction is in equilibrium, but when Γ drops below H this is no longer the case \Rightarrow below $z \sim 1200$, the Saha equation under predicts the number of free electrons and decoupling is in fact delayed

More detailed calculations yield $z_{\text{dec}} \sim 1100$ or $T_{\text{dec}} \sim 3000 \text{ K}$ or $t_{\text{dec}} \sim 350,000$ years. This is compatible with the redshift of last scattering.

Photon decoupling marks the beginning of structure formation! The gas can evolve without being smoothed by photons, but let's first look at the properties of the CMB radiation and what we can learn from it.

What can we learn from the CMB and how can we learn it?



The excellent black body spectrum and the small temperature fluctuations imply that the early Universe was very simple: easy to model?

How to represent or model anisotropies in the CMB?

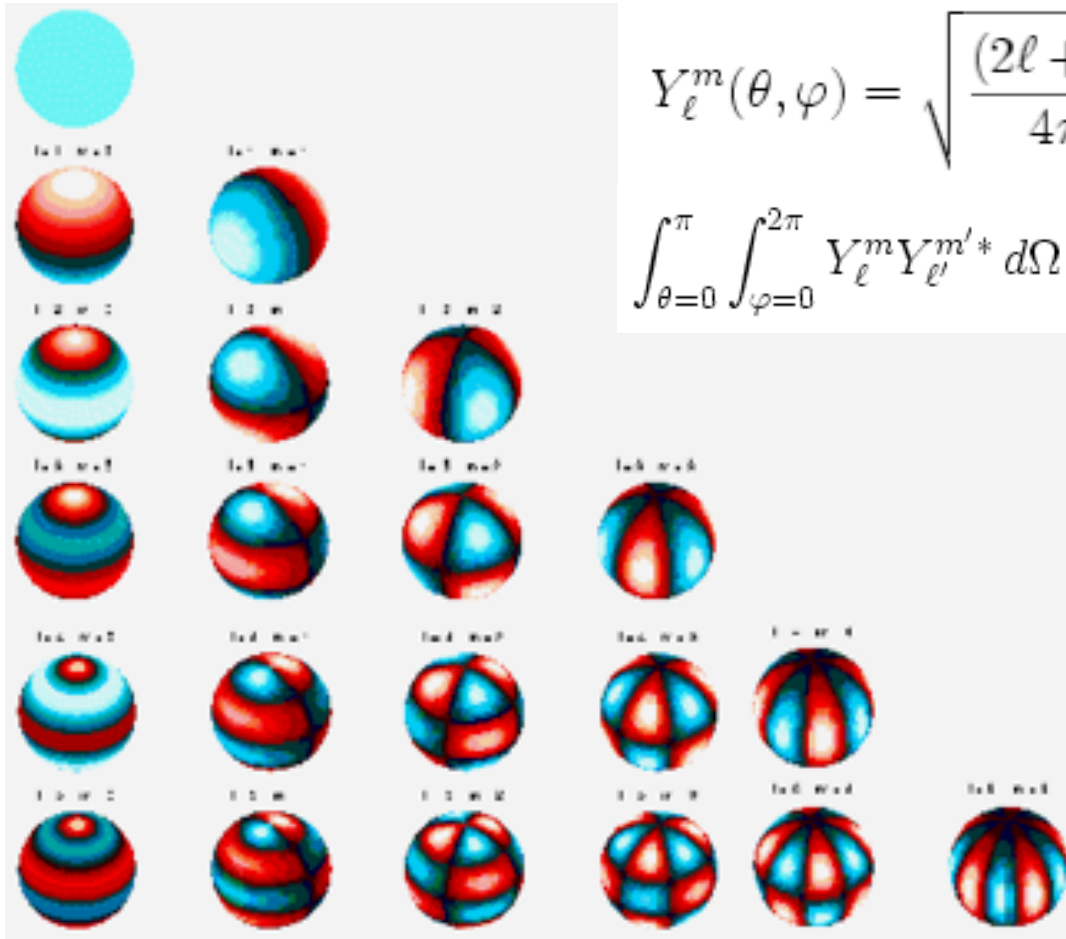
- Since the observed temperature of the CMB as a function of position on sky only differs by a small amount from the mean, represent the anisotropies as a temperature difference

$$\frac{\Delta T}{T}(\theta, \phi) = \frac{T(\theta, \phi) - \bar{T}}{\bar{T}}$$

- Represent this temperature difference as a function of position using an equivalent Fourier series in spherical coordinates -- which are spherical harmonics

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_m^{\ell}(\theta, \phi)$$

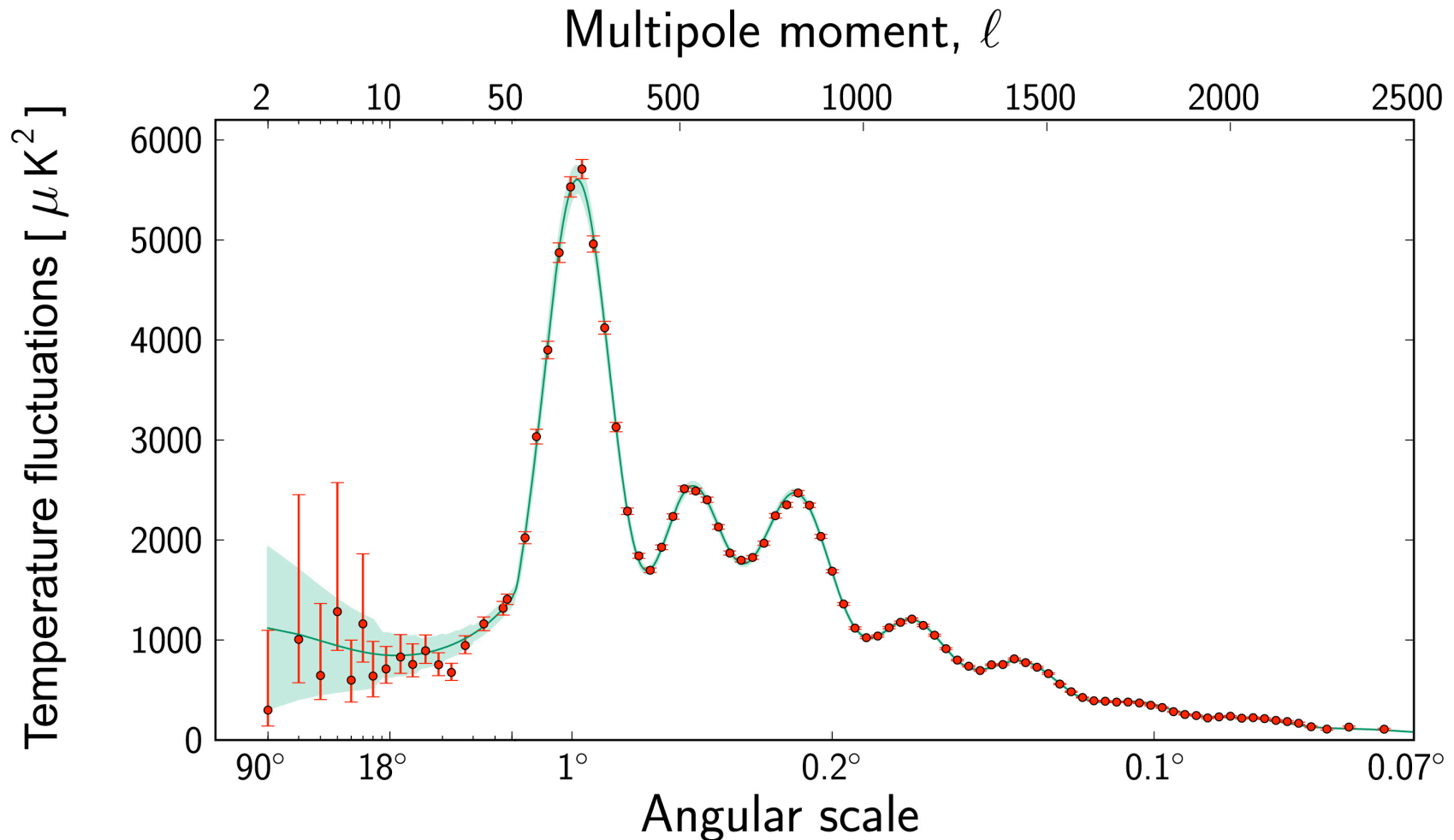
Spherical harmonics (used to represent the anisotropies in the CMB)



$$Y_\ell^m(\theta, \varphi) = \sqrt{\frac{(2\ell + 1)(\ell - m)!}{4\pi(\ell + m)!}} \cdot e^{im\varphi} \cdot P_\ell^m(\cos \theta)$$

$$\int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} Y_\ell^m Y_{\ell'}^{m'*} d\Omega = \delta_{\ell\ell'} \delta_{mm'} \quad d\Omega = \sin \theta d\varphi d\theta$$

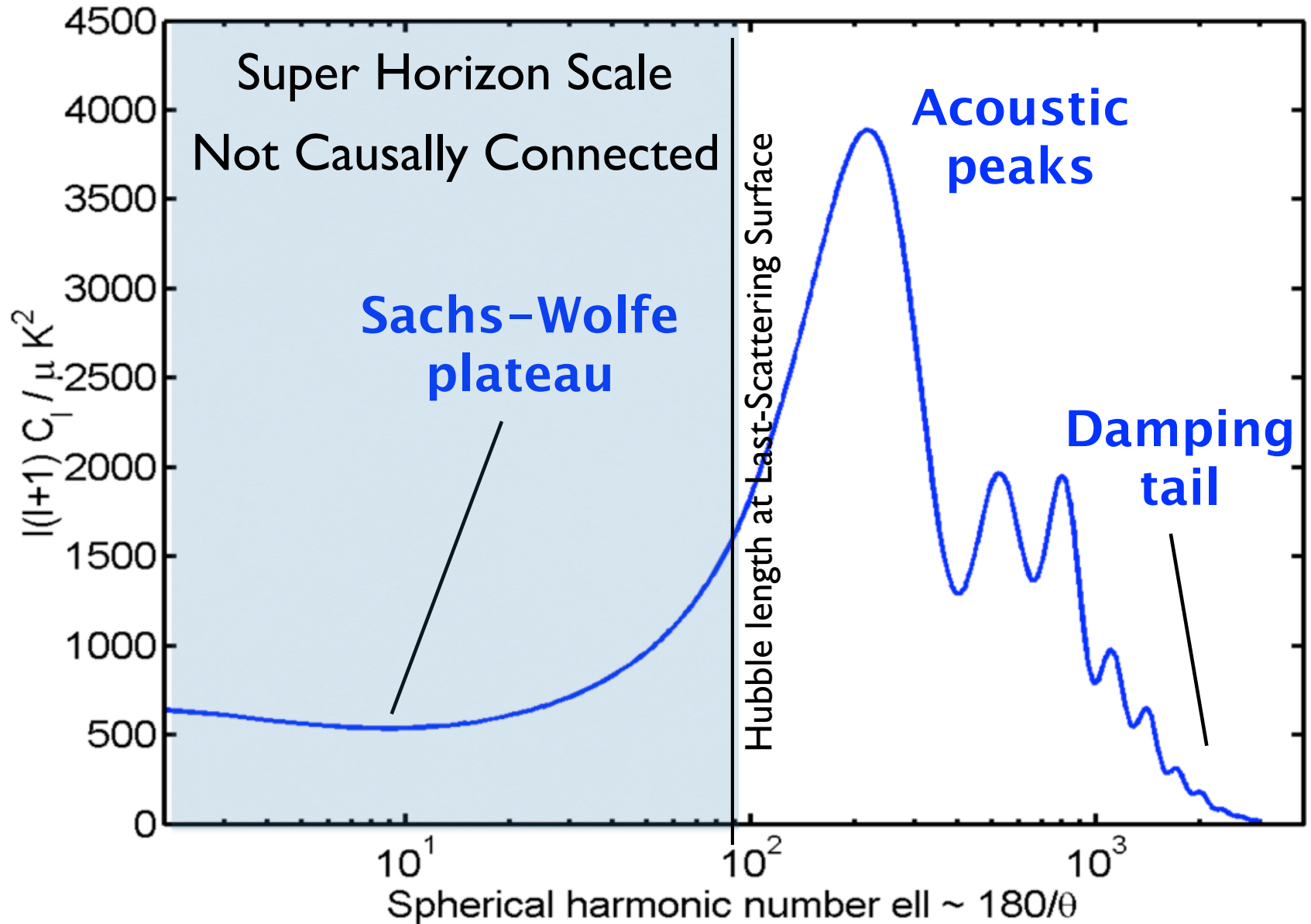
Statistics of fluctuation



We can indeed predict the power spectrum of fluctuations for a set of initial conditions.

What can we learn from the CMB?

Power



Credit: Porciano

Large Angles

Scale

Small Angles

At recombination, non-baryonic dark matter dominated the gravitational potential. If the density of dark matter was not perfectly homogeneous, but varied slightly that

$$\epsilon_{\text{DM}} = \overline{\epsilon_{\text{DM}}} + \delta\epsilon_{\text{DM}}(r)$$

variation in gravitational potential

$$\Rightarrow \nabla^2(\delta\varphi) = (4\pi G/c^2)\delta\epsilon$$

Imagine a photon in a potential well (a minimum of the potential): as it climbs out of the well, it loses energy and is redshifted \Rightarrow cool spot

If the photon rolls down a potential hill \Rightarrow hot spot

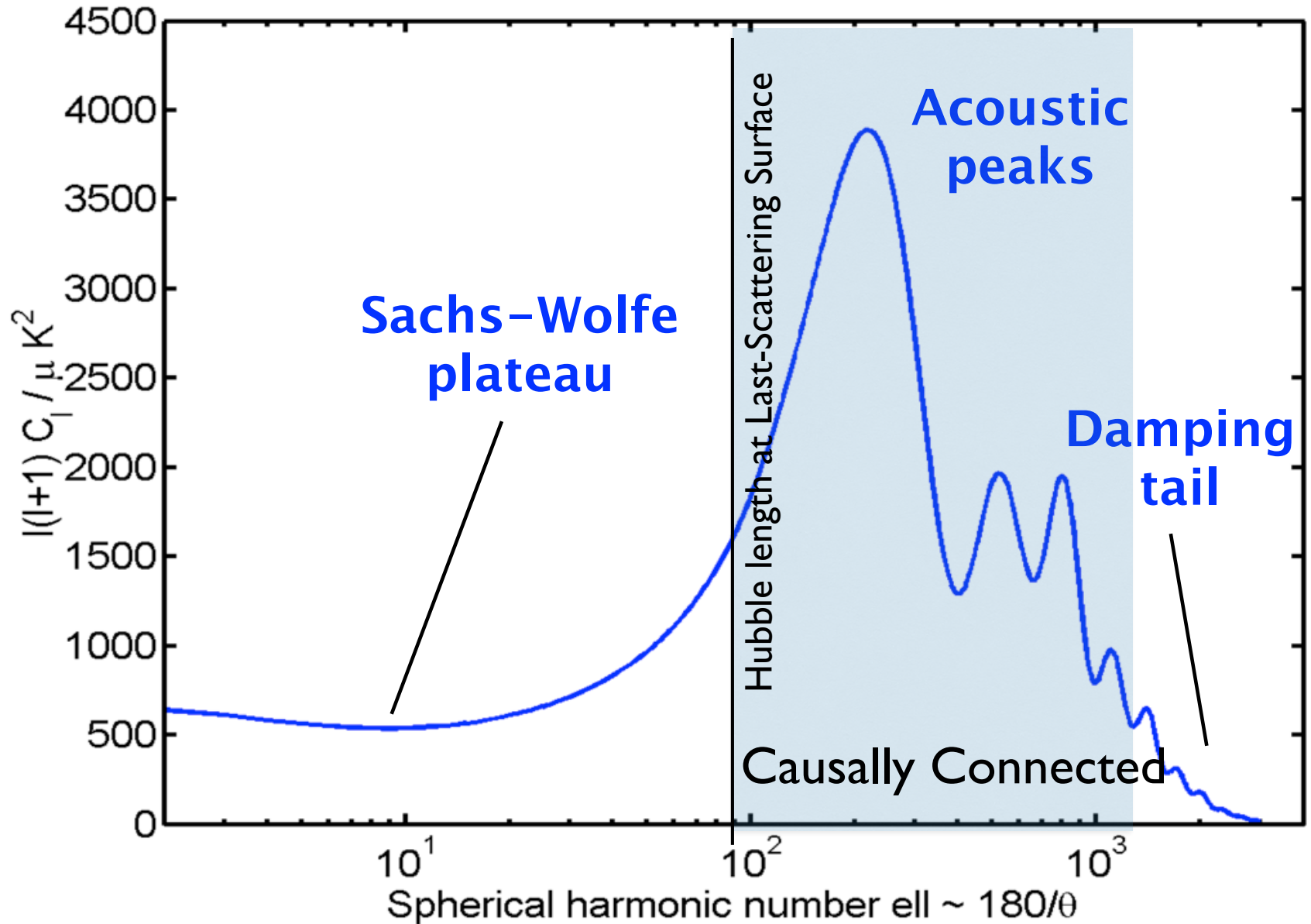
The cool and hot spots in the CMB temperature map correspond to minima and maxima in $\delta\varphi$ at the time of recombination:

$$\delta T/T = (1/3)\delta\varphi/c^2$$

This explains the existence of temperature fluctuations on scales $\theta > \theta_{\text{H}} \sim 1$ degree

What can we learn from the CMB?

Power



Credit: Porciano

Large Angles

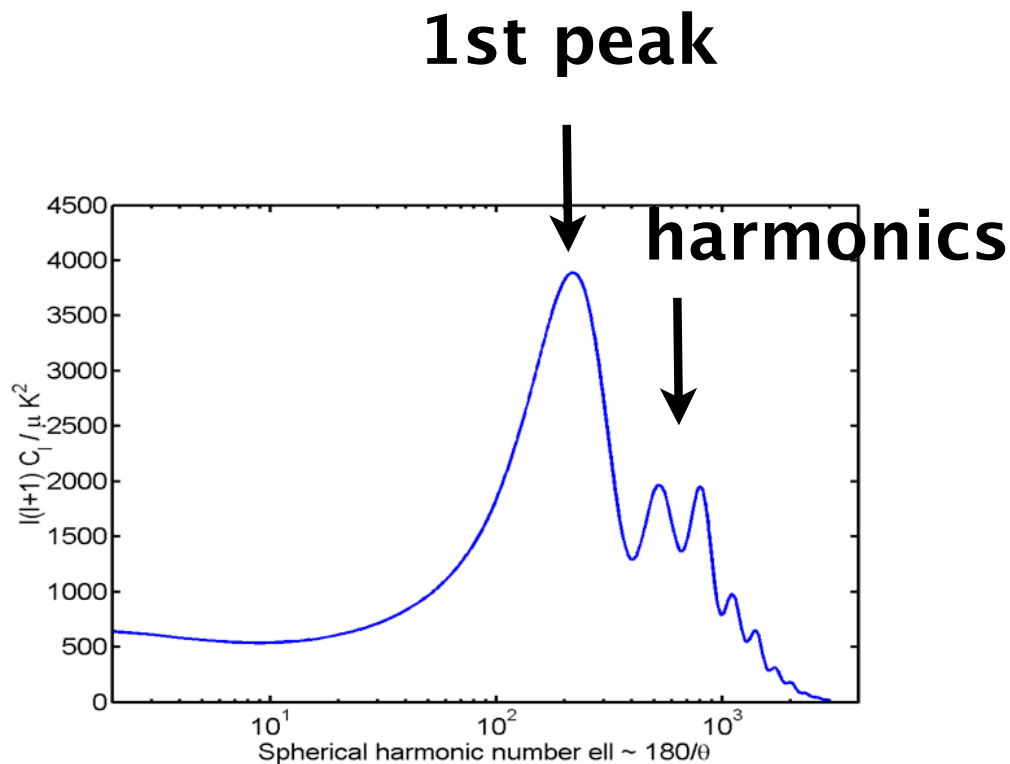
Scale

Small Angles

Acoustic Oscillations:

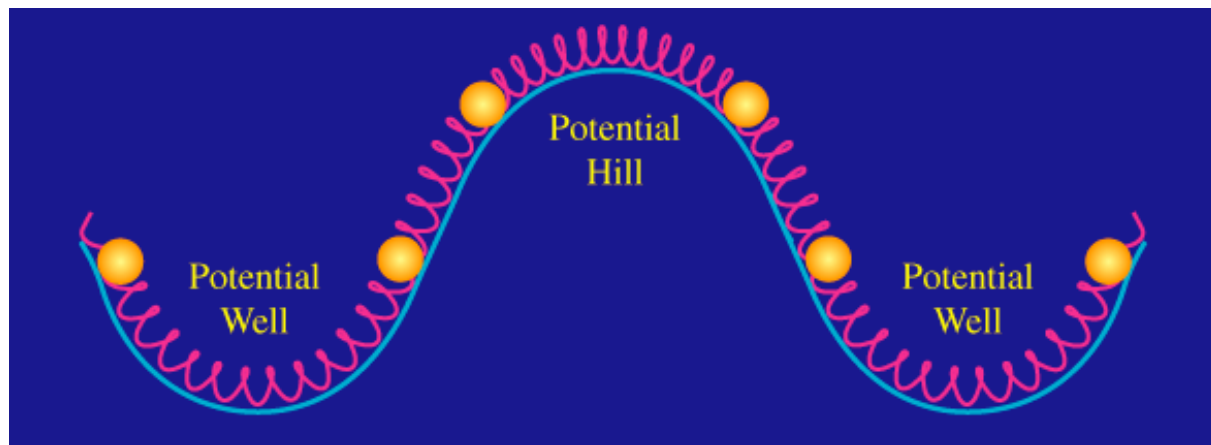
- First peak is a compression mode
- Second peak is a rarefaction mode
- Third peak is a compression mode

(Similar to harmonics on a musical instrument/string/pipe!)



Acoustic Oscillations:

- Universe filled with slight dark matter overdensities on all scales
- Baryons will fall onto these overdensities due to the force of gravity heating the fluid up
- Large number of baryons falling onto overdensity causes an increase in pressure due to baryon-photon coupling -- which resists gravitational forces and causes it to expand (cooling the fluid down)
- An oscillation is set up and continues until decoupling



-- credit: Wayne Hu

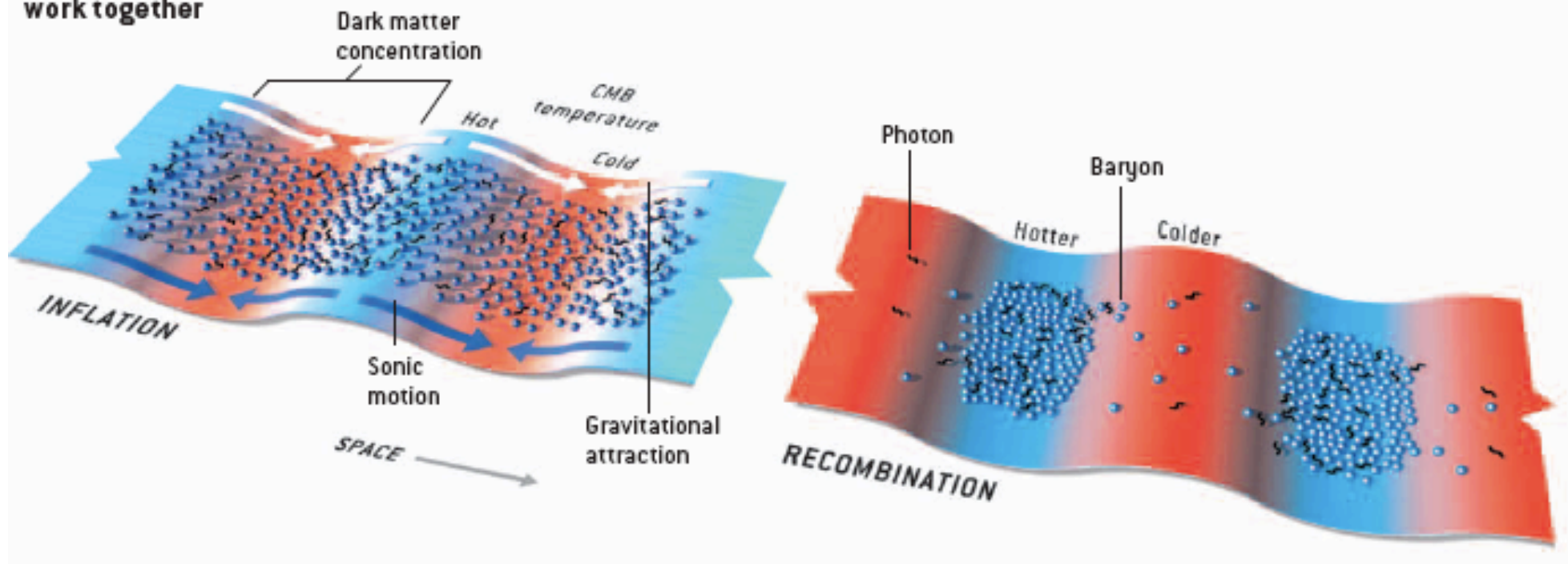
First Peak: Illustration

INFLUENCE OF DARK MATTER modulates the acoustic signals in the CMB. After inflation, denser regions of dark matter that have the same scale as the fundamental wave (represented as troughs in this potential-energy diagram) pull in baryons and photons by gravitational attraction. (The troughs are shown in

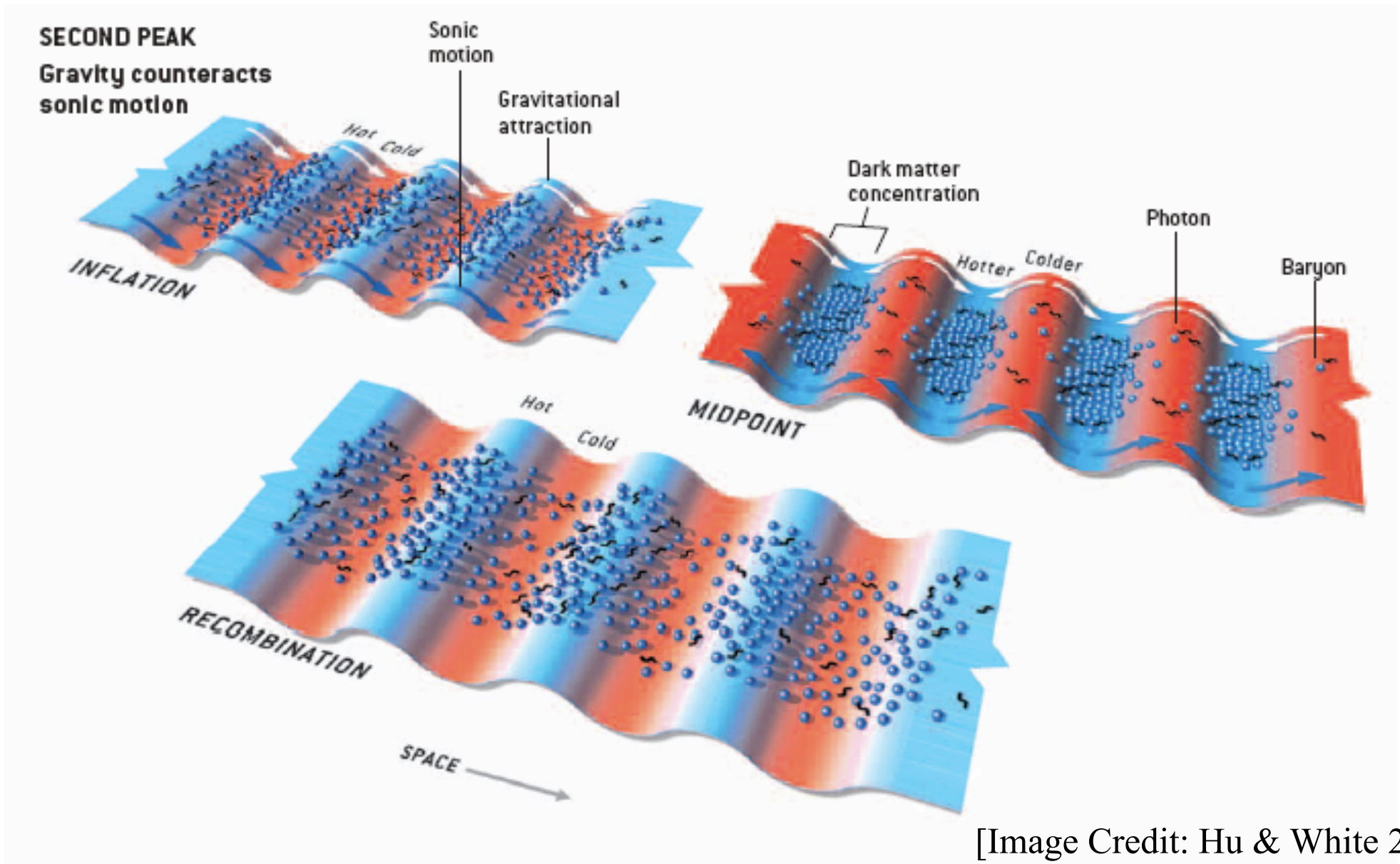
red because gravity also reduces the temperature of any escaping photons.) By the time of recombination, about 380,000 years later, gravity and sonic motion have worked together to raise the radiation temperature in the troughs (blue) and lower the temperature at the peaks (red).

FIRST PEAK

Gravity and sonic motion work together



Second Peak: Illustration

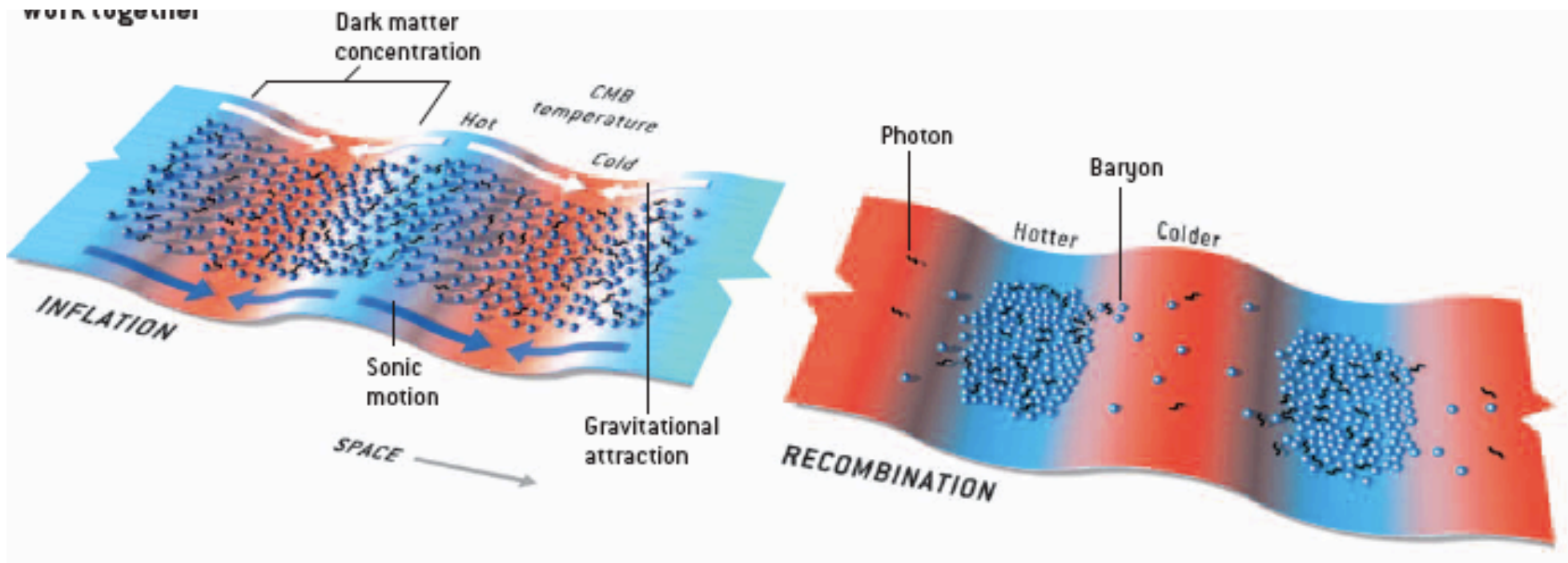


What can we learn from the
properties of these acoustic
peaks?

Let's examine acoustic peak #1

(What can we learn from the angular scale at which is observed?)
(they give us standard rods to measure geometry of universe)

-- For this peak, baryonic matter would be falling onto this pattern of overdensities for the first time



Let's examine acoustic peak #1

(What can we learn from the angular scale at which is observed?)
(they give us standard rods to measure geometry of universe)

- For this peak, baryonic matter would be falling onto these overdensities for the first time
- Length scale spanned by peak is comoving length transversed by a sound wave to the point of last scattering:

$$L_S(t_R) = a(t_R) \int_0^{t_R} \frac{c_S dt}{a(t)} \quad \text{where} \quad \text{sound speed}$$
$$c_S \approx \frac{c}{\sqrt{3}}$$
$$\approx 110 \left(\frac{0.7}{h} \right) \left(\frac{0.3}{\Omega_M} \right)^{1/2} \text{ kpc}$$

- This length scale acts as a standard rod

Let's examine acoustic peak #1

(What can we learn from the angular scale at which is observed?)

-- For this peak, baryonic matter would be falling onto these overdensities for the first time

-- Key Question: What is the angle of the peak on the sky?

$$\theta = \frac{L_S(z)}{D_A(z)}$$

length scale traversed by matter

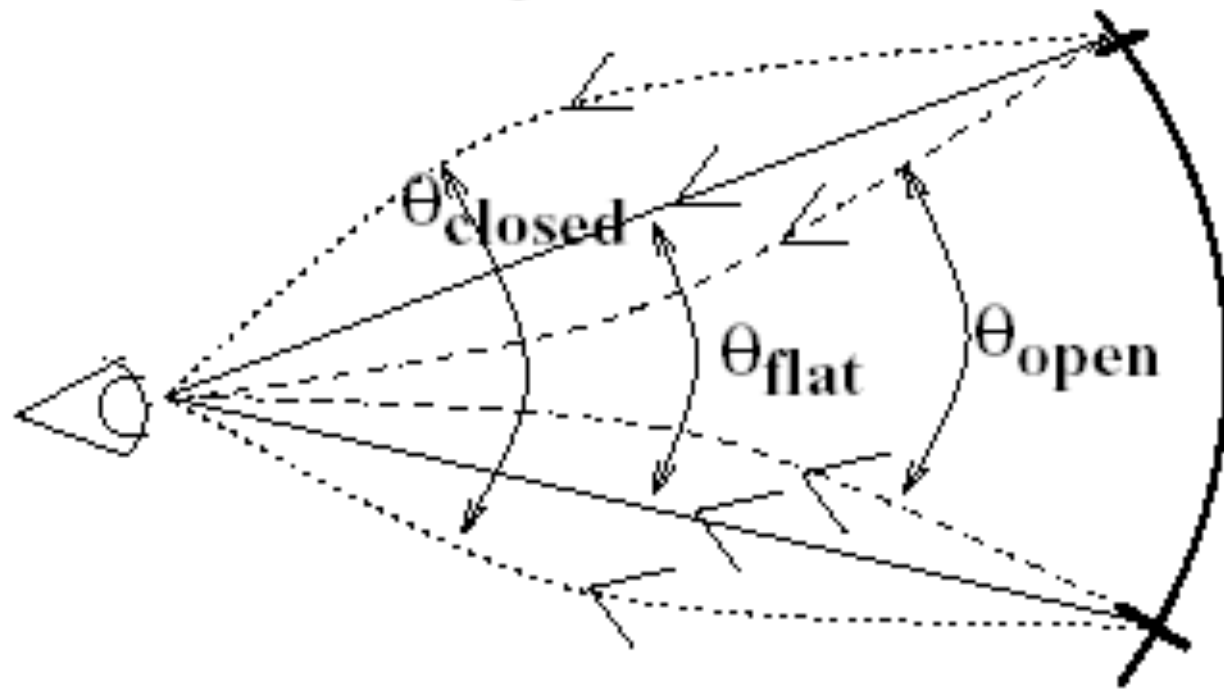
angular diameter distance

-- Can compute $L_S(z)$ and can measure θ

-- Can solve for $D_A(z)$ and use to constrain geometry of universe

How does the angular diameter distance depend on the cosmological parameters?

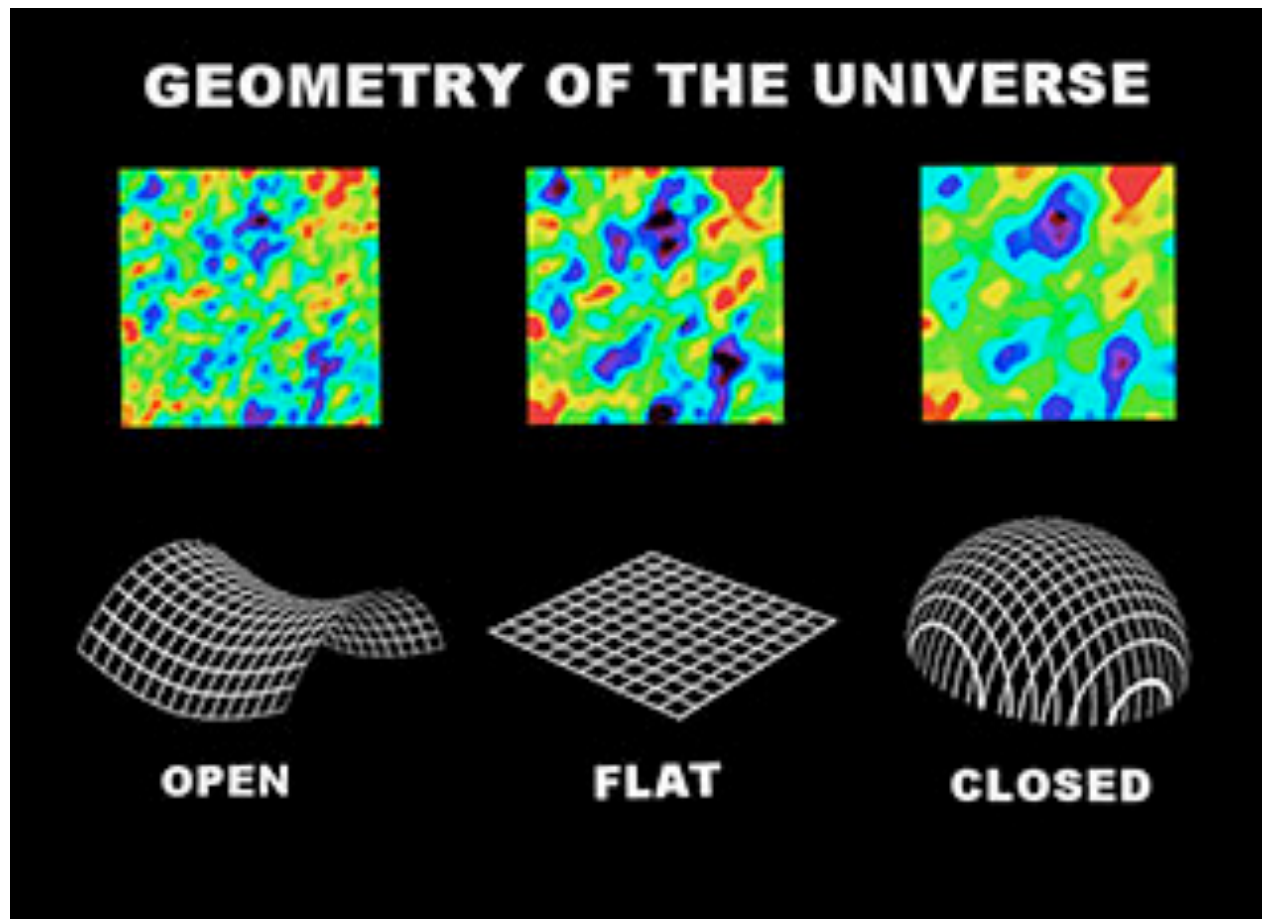
Geometry of the Universe



Fixed Distance
Traversed by
Baryons in First
Acoustic Peak

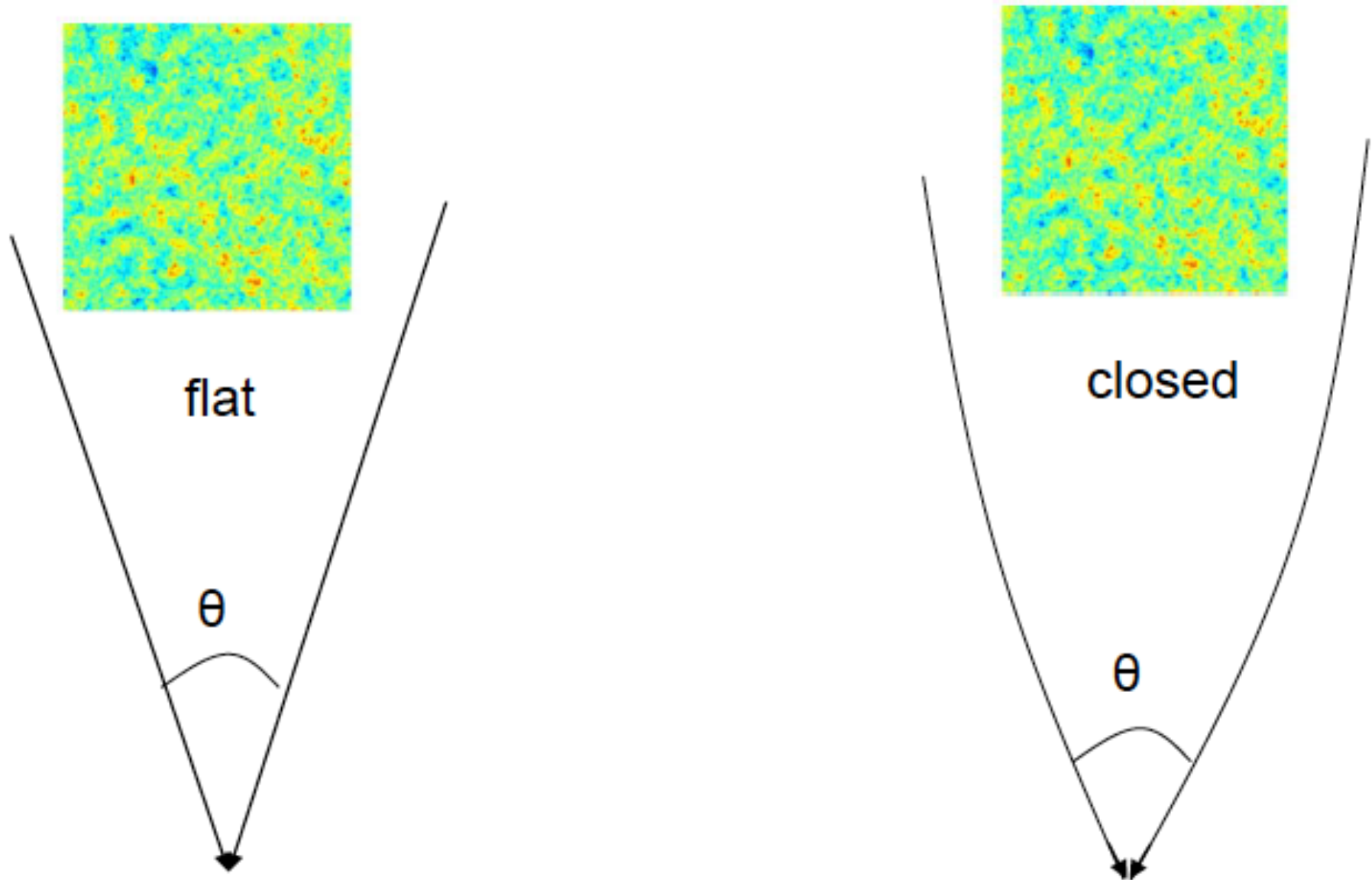
$$\theta_{\text{closed}} > \theta_{\text{flat}} > \theta_{\text{open}}$$

Angular diameter distance

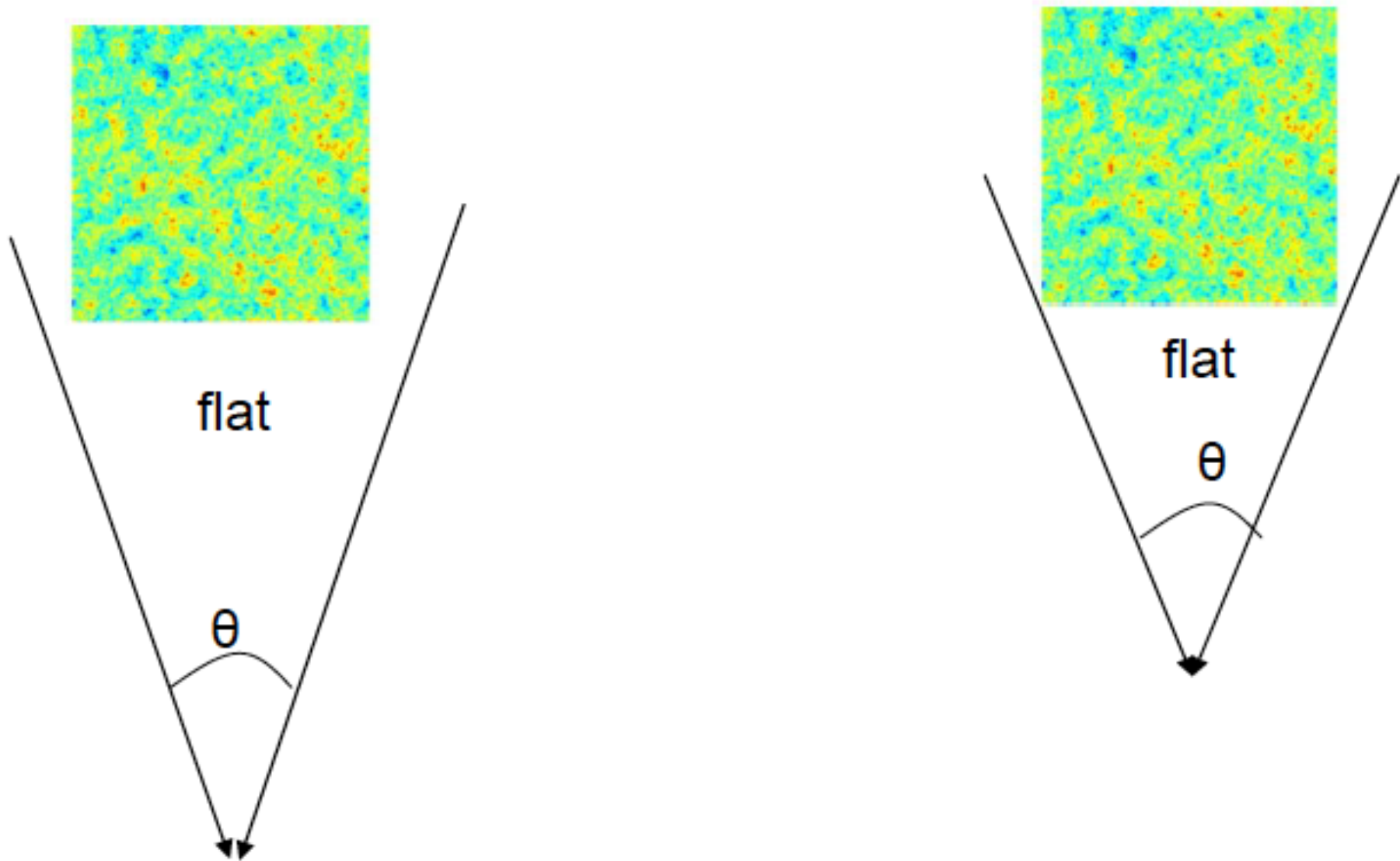


The observed angles depend on geometry *and* the distance to the surface of last scattering.

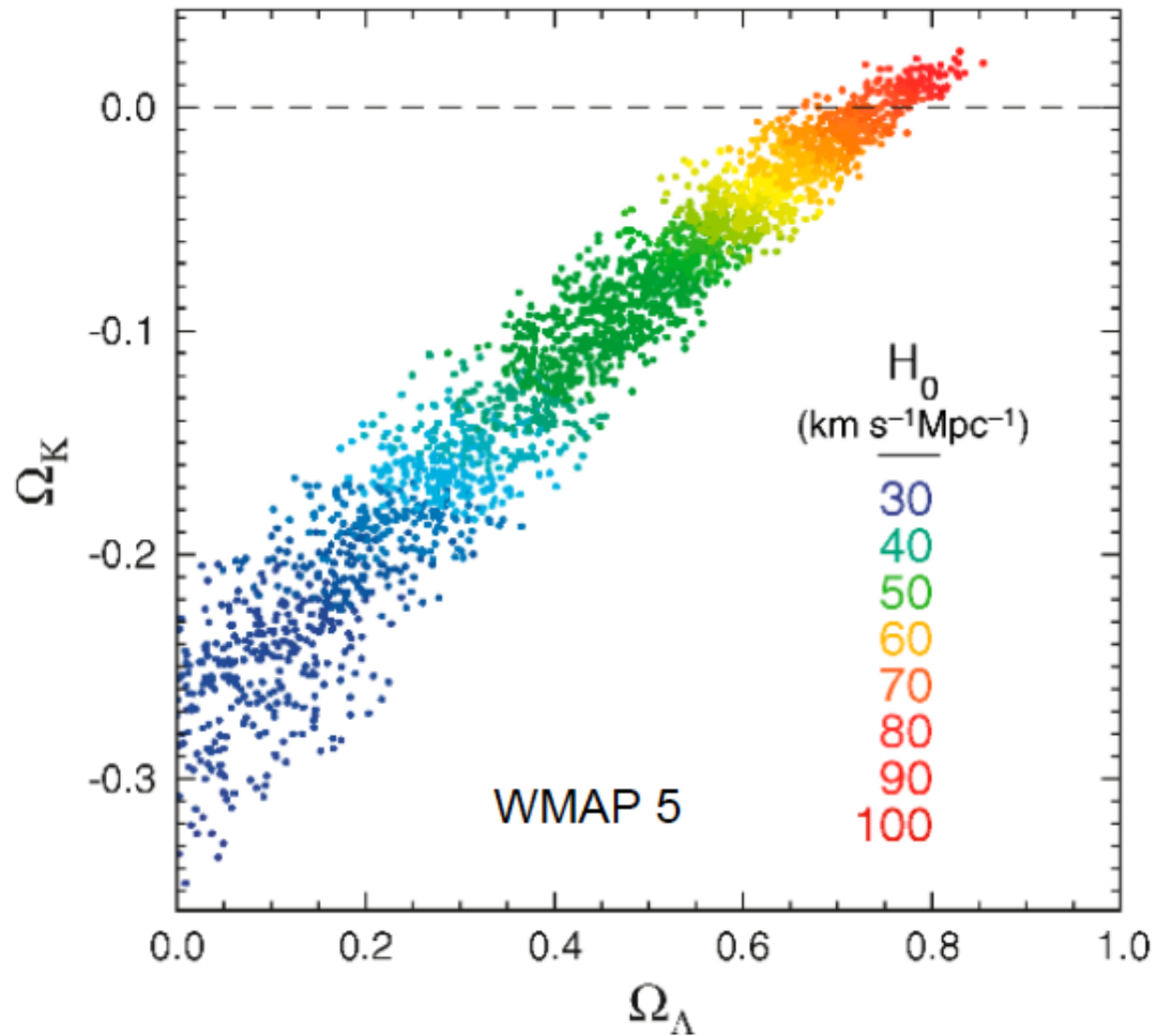
Distance to recombination



Distance to recombination



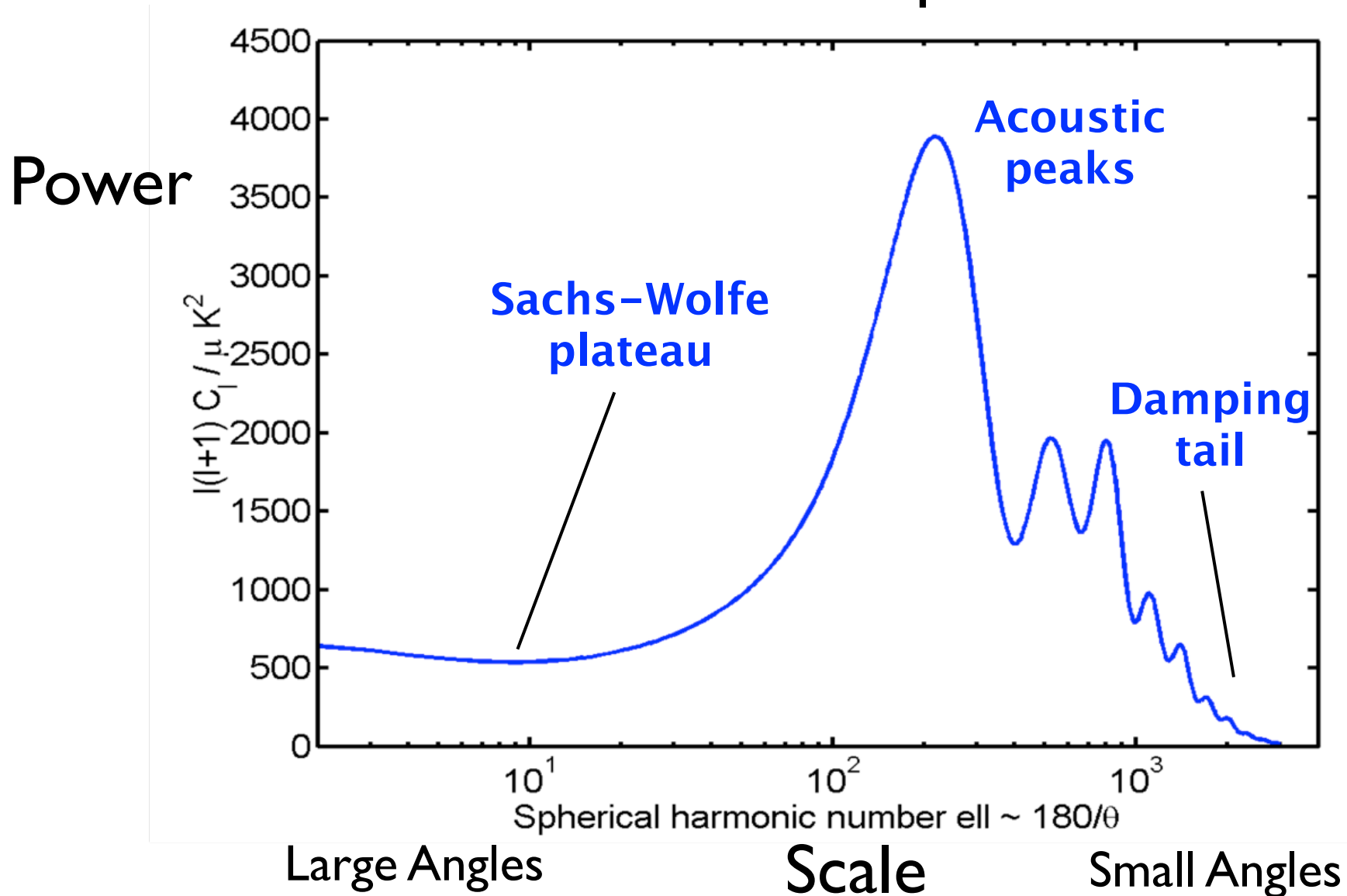
Parameter degeneracies



What can we learn from the
other peaks?

What can we learn from the other peaks?

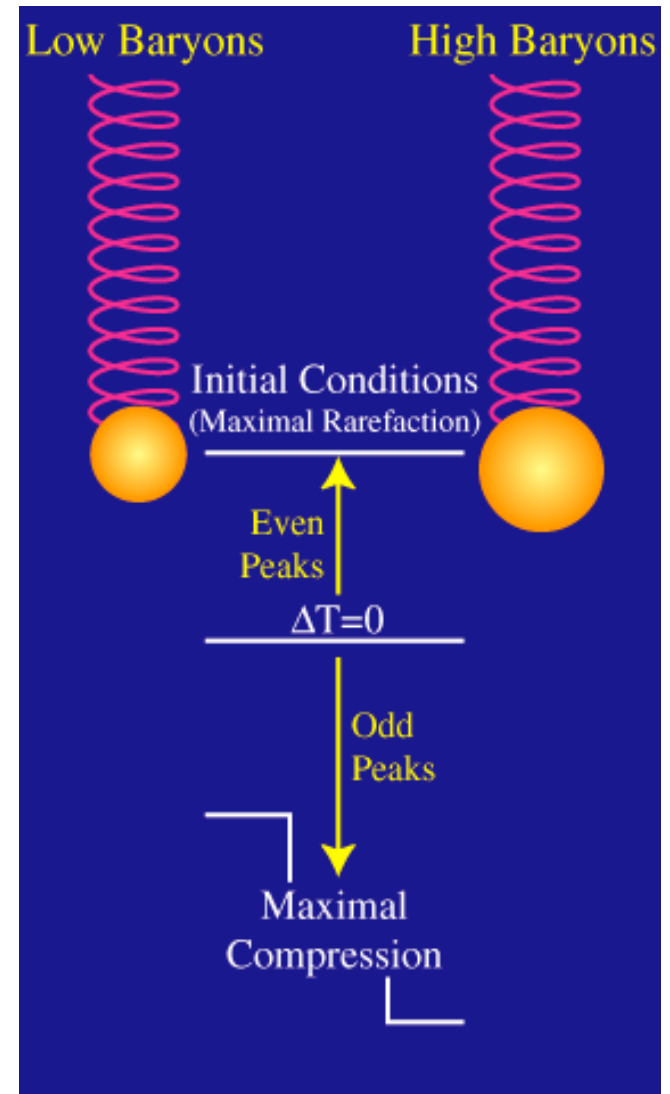
Here is such a spectrum:



What can we learn from the other peaks?

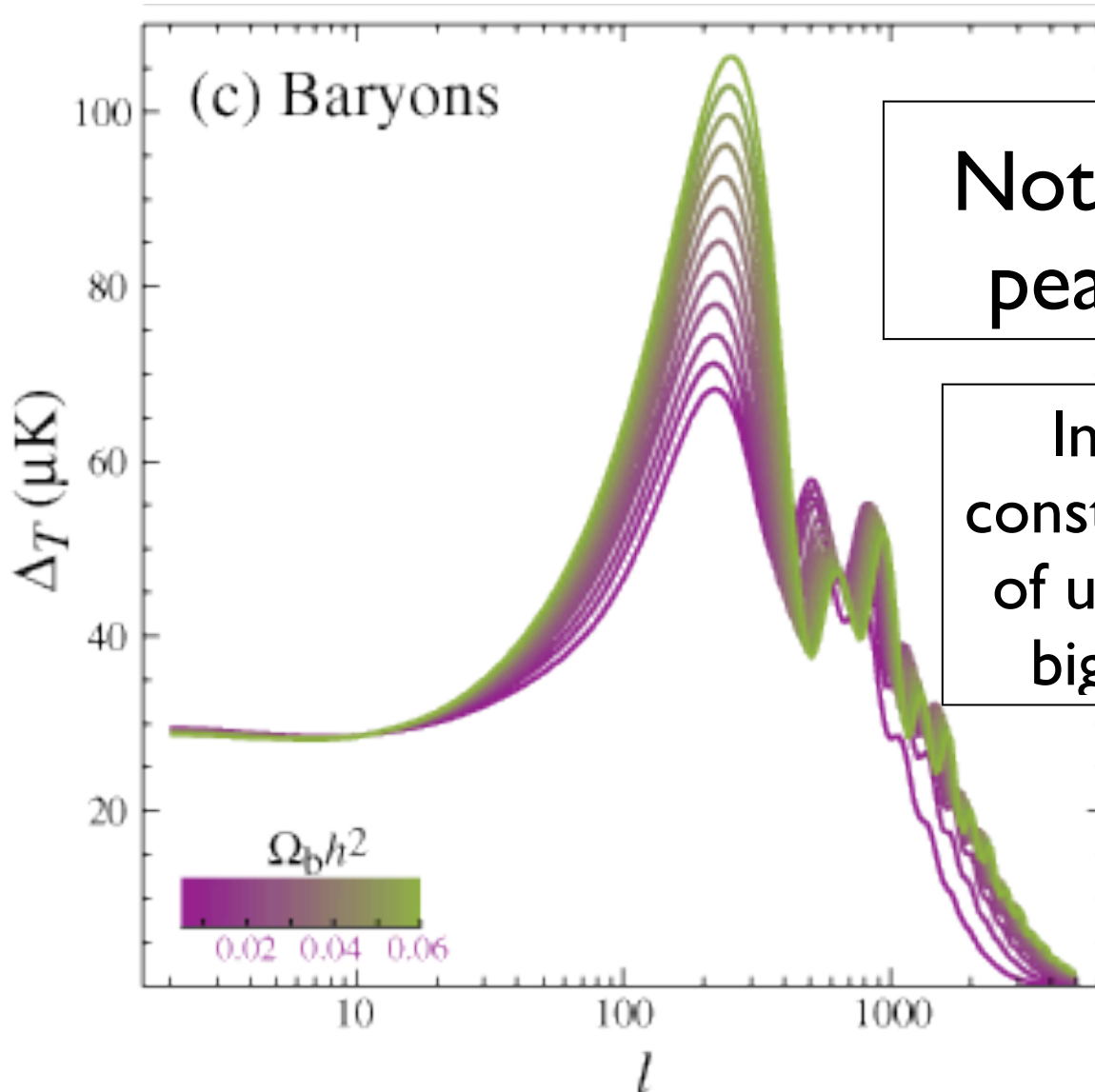
Learn about baryon content

- The presence of more baryons increases the amplitude of the oscillations
- As a result, the fluid is compressed more before photon pressure can resist the compression
- This results in an asymmetry between the even and odd peaks



What can we learn from the other peaks?

Learn about baryon content

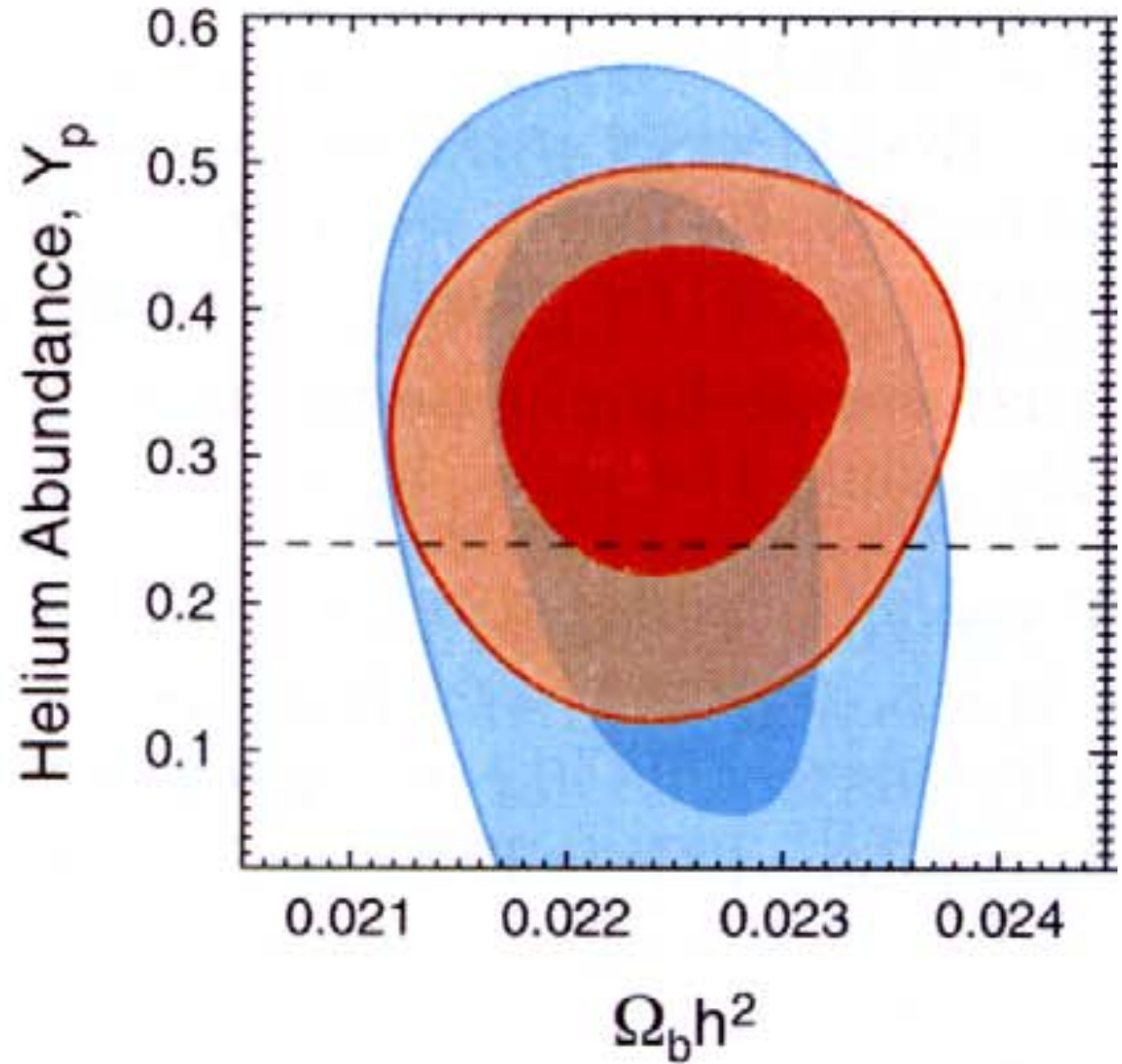
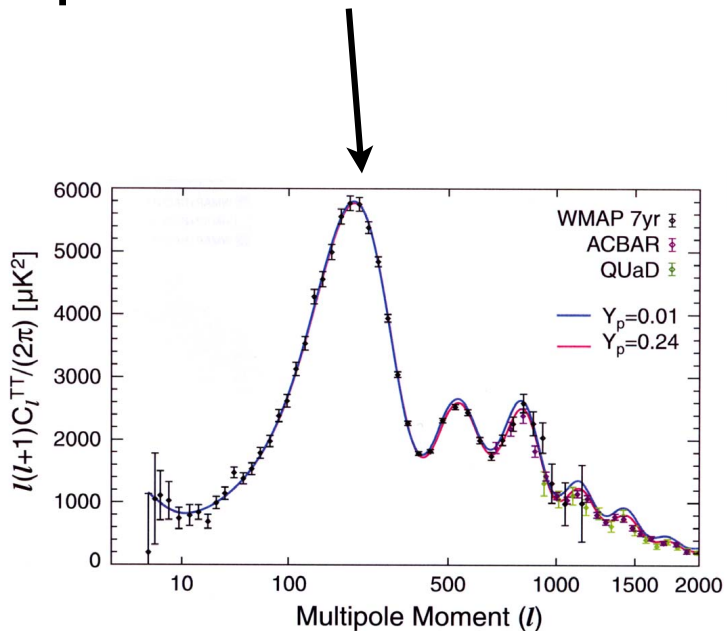


Note how 1st and 3rd peaks are enhanced!

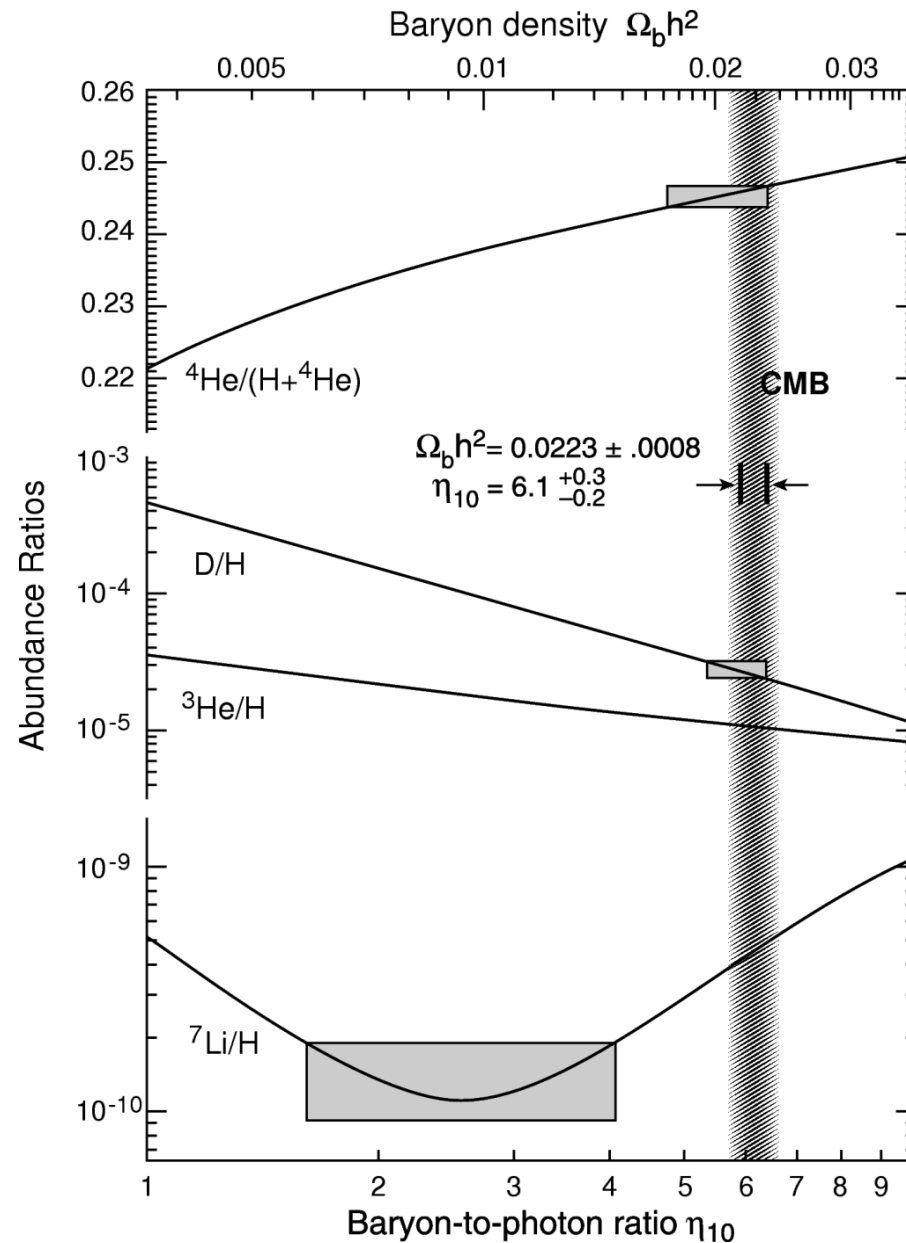
In fact, this provides best constraint on baryonic content of universe (even better than big-bang nucleosynthesis)

What type of constraints can we set on Ω_{baryon} ?

Amazingly, one can even weakly constrain the abundance of primordial helium

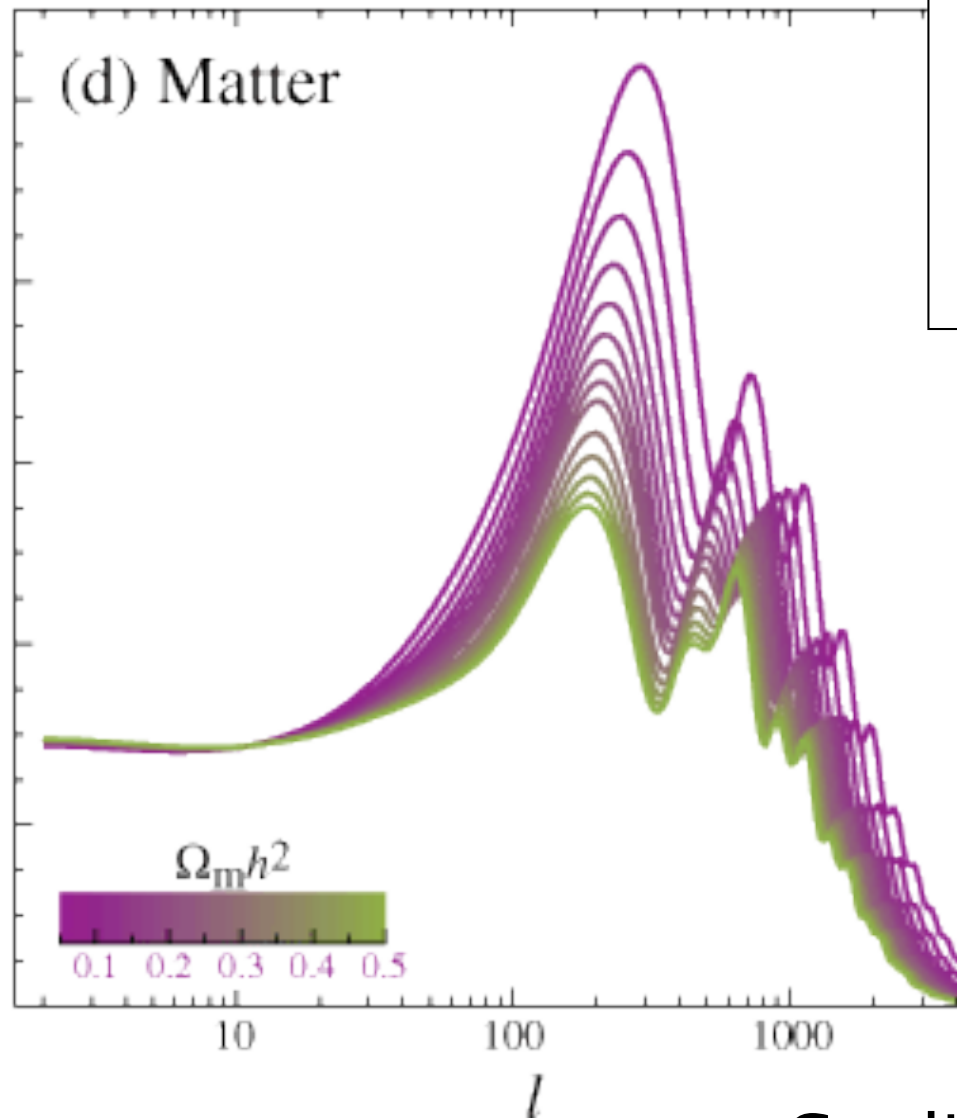


Constraints on Ω_{baryon} from CMB in perfect agreement with Big Bang Nucleosynthesis



What can we learn from the other peaks?

Learn about dark matter content



Note how 3rd peak is enhanced when dark matter density higher!

To ensure this peak is prominent, necessary to have a relatively high dark matter content earlier in universe. Otherwise, the universe will have a longer radiation dominated phase -- inhibiting the growth of fluctuations

Credit: '