Sterrewacht Leiden

Origins & Evolution of the Universe

an introduction to cosmology — Fall 2018

Lecture 8: Inflation, Lepton Era

Rychard Bouwens

Layout of the Course

Sep 24: Introduction and Friedmann Equations
Oct 1: Fluid and Acceleration Equations
Oct 8: Introductory GR, Space Time Metric, Proper Distance
Oct 15: Redshift, Horizons, Observable Distances
Oct 17: Problem Class #1
Oct 22: Observable Distances, Parameter Constraints
Oct 29: Thermal History, Early Universe
Nov 5: Early Universe, Inflation
Nov 12: Inflation, Lepton Era
Nov 14: Problem Class #2
Nov 19: Big Bang Nucleosynthesis, Recombination
Nov 26: Introduction to Structure Formation
Dec 3: Cosmic Microwave Background Radiation (I)
Dec 5: Problem Class #3
Dec 10: Cosmic Microwave Background Radiation (II)
Dec 21: Final Exam

Problem set #2 was mailed to you 1.5 weeks ago

Due by Wednesday 13:30 November 14

Review Last Week

Timeline up to the radiation era

Unkown physics

Planck time	10^{-43}s	10^{19}GeV 10^{32}K	Quantum Gravity ???
GUT transition	10^{-35}s	10^{15}GeV 10^{28}K	baryon asymmetry frozen in; mag.monopoles
Inflation	???	???	>60 e-foldings monopoles diluted, curvature removed
Electroweak transition	10^{-9}s	100GeV 10^{15}K	separate strong, weak, e/m forces
quark-hadron transition	10^{-4}s	200-300MeV 2-3 10^{12}K	quarks condense into mesons and hadrons. Hadron era starts
pion decay	few 10^{-4}s	130MeV 1.4 10^{12}K	Hadron era ends. Lepton era begins.
muon-antimuon decay	few 10^{-4}s	130MeV 1.4 10^{12}K	leaves electrons, neutrinos, photons, + a few baryons
neutrinos decouple	0.1s	3MeV 3.10^{10}K	Temperatures continue to decrease as a^{-1}
e^+-e^- annihilation	4s	0.5MeV 5.10^{9}K	End of lepton era. Heating of radiation

Phase transitions of the Universe

Between $T \approx 10^{19}$ and 10^{15} GeV, quantum gravity effects decrease in importance and interactions are described by a GUT. Baryon number is not conserved in GUTs, so no asymmetry between matter and antimatter.

Near $T \approx 10^{15}$ GeV ($t=10^{-37}$s) the GUT symmetry breaks leading into the situation described by the standard model of particles; the GUT phase transition typically results in the formation of magnetic monopoles.

For typical GUTs:
- particle mass: $m_M \approx 10^{16}$GeV
- number density: $n_M > 10^{-10} n_\chi$.

$\Longrightarrow \Omega_{monopole} > m_M/m_p \Omega_{bar} \approx 10^{16}$.

This does not match observations: **the monopole problem**

A GUT that unifies the elektroweak interactions with the strong interactions puts leptons and hadrons on the same footing and thus allows processes that do not conserve baryon number: source of matter/anti-matter asymmetry.

As the temperature falls below $T_{GUT} \approx 10^{15}$ GeV the unification of the strong and elektroweak interactions no longer holds. Towards the end of this period (10^{-11}s) the Universe is filled with an ideal gas of leptons and antileptons, the four vector bosons, quarks and anti-quarks.

At $T_{EW} \approx 100$ GeV elektro-weak symmetry is broken and we have separate elektromagnetic and weak forces. All the leptons acquire mass.

When the temperature drops to $T_{QH} \approx 200$-300 MeV (10^{-5}s) we have the final phase transition and the strong interaction leads to the confinement of quarks into hadrons: the quark-hadron phase

Successes of the Big Bang model

- Correctly predicts the abundances of light elements
- Explains the CMB as relic of the hot initial phase
- Naturally accounts for the expansion of the Universe
- Provides a framework to understand the formation of cosmic structure.

Problems with the Big Bang model

- Origin of the Universe
- The horizon problem
- The flatness problem
- Origin of the baryon asymmetry
- Monopole problem
- Origin of primordial density fluctuations
- Nature of dark matter
- Nature of dark energy

In which models does one have a particle horizon?

One has a particle horizon R_H in models where

$$a \propto t^{\beta}$$

$$\text{if } w > -1/3 \quad \Longleftrightarrow \quad 0 < \beta < 1 \quad \Longleftrightarrow \quad d^2a/dt^2 > 0$$

If there is a particle horizon, then one suffers from the horizon problem and it is difficult to understand why regions which were not casually connected all look the same (i.e., are homogeneous).

An early period of inflation (accelerated expansion) solves the flatness, horizon, and monopole problem in an elegant fashion; it also can explain the origin of the density fluctuations

If $w = -1$, then $a \propto e^{t/\tau}$ where $\tau = (a/(da/dt))_{t=ti}$

Inflation

For inflation to be able to casually disconnect regions that were before in causal contact, the expansion must be so rapid that there exists an event horizon at a finite distance from any point (recall cosmological event horizon discussion)

The Hubble radius in comoving coordinates must shrink with time:

$$\frac{d}{dt}\left(\left(\frac{1}{a}\right)\left(\frac{c}{H}\right)\right) < 0 \qquad \text{where } H = (da/dt)/a$$

- $1/a$: scale factor to put in comoving coordinates
- c/H: distance light can travel in Hubble time

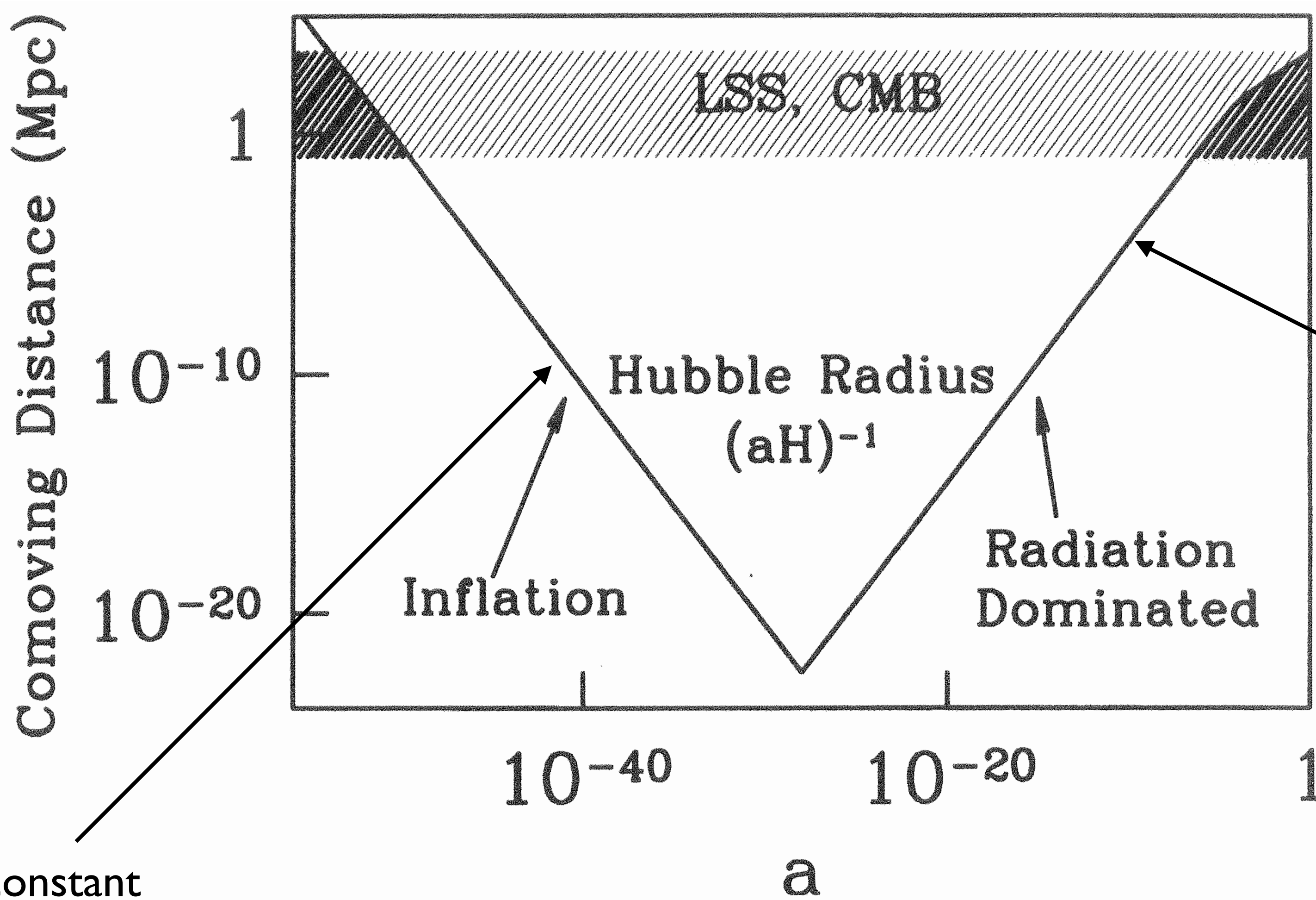
implies $(d/dt)(c/(da/dt)) < 0 \quad \Rightarrow (d^2a/dt^2) > 0$

given the acceleration equation we need a substance with sufficient negative pressure.

Inflation

Hubble Radius in comoving coordinates

(c/Ha)



scales as a

H ~ constant
scales as 1/a

Dodelson (2003)

Inflation

Imagine the early universe was filled with a scalar field $\Phi(\mathbf{x},0) = \Phi_0 > 0$, i.e., not in the ground state.

In this case, it may lead to accelerated expansion; after a while the field decays into particles (causing reheating)

The Lagrangian of a scalar field is $L = -(1/2)c^2(\partial_\mu\Phi)(\partial^\mu\Phi) - V(\Phi)$

If we assume homogeneity and isotropy, we can define the effective density and pressure:

$$\rho_\Phi c^2 = (1/2)(d\Phi/dt)^2 + V(\Phi)$$

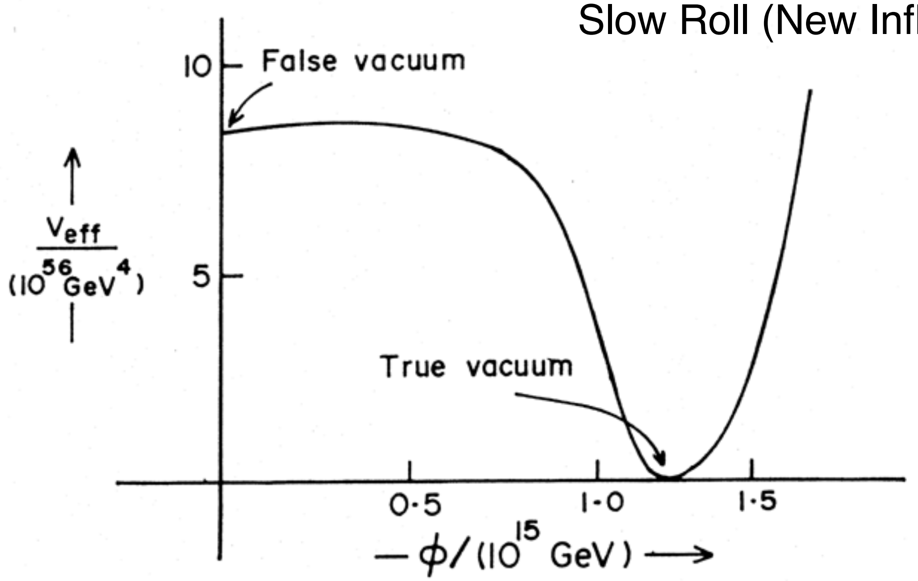
$$P_\Phi = (1/2)(d\Phi/dt)^2 - V(\Phi)$$

Inflation models

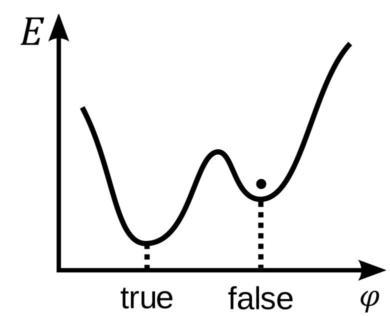
The behavior of the model depends on the potential (or vice versa)

- **Old inflation**: first-order transition (potential barrier) does not work
- **New inflation**: second-order transition, with a slow-roll phase; suffers from fine-tuning the model

Slow Roll (New Inflation)



Old Inflation (Potential barrier)



Inflation

$[V(\Phi)] = \text{erg / cm}^3$ $\quad$ $[(d\Phi/dt)^2] = \text{erg/cm}^3$

$[\Phi] = (\text{erg s}^2 / \text{cm}^3)^{1/2} = (\text{g/cm})^{1/2}$

$\Rightarrow \Phi_{Planck} = (m_{Planck}/\rho_{Planck})^{1/2} = c/G^{1/2}$

The form of the potential depends on the adopted theory. Because we do not have a definitive model, people consider various choices of $V(\Phi)$, e.g.,

$V(\Phi) = (1/2)m^2\Phi^2$ Massive scalar field

Take the Friedmann & Fluid equations and insert the expression for pressure and density of the scalar field:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$d\rho/dt + 3((da/dt)/a)(\rho + P/c^2) = 0$$

$$\Rightarrow H^2 = (8\pi G/3c^2)((1/2)(d\Phi/dt)^2 + V(\Phi)) - kc^2/a^2$$

$$\Rightarrow d^2\Phi/dt^2 + 3H(d\Phi/dt) = -dV(\Phi)/d\Phi$$

If $d\Phi/dt = 0$ or if Φ and $V(\Phi)$ do not change much over the period where a increases exponentially, then we can assume $k \sim 0$.

Inflation

and if $(d\Phi/dt)^2 << V(\Phi)$ $\Rightarrow H^2 = (8\pi G/3c^2) V(\Phi)$ and

if $d^2\Phi/dt^2 << dV(\Phi)/d\Phi$ $\Rightarrow 3H(d\Phi/dt) = -dV(\Phi)/d\Phi$

One can show that the slow roll conditions (needed to obtain a $w < -1/3$ universe)

$$\varepsilon = (c^2/16\pi G)\ (dV'(\Phi)/V(\Phi))^2 << 1$$

and

$$\eta = (c^2/8\pi G)(d^2V(\Phi)/d\Phi^2\ / V(\Phi)) << 1$$

If we consider $V(\Phi) = (1/2)m^2\ \Phi^2$

$\Rightarrow dV/d\Phi\ / V = 2\ /\ \Phi$ and $\Rightarrow d^2V/d\Phi^2\ / V = 2\ /\ \Phi^2$

Slow roll conditions are as follows: i.e. $\Phi >> 1/(4\pi)^{1/2}\ \Phi_{Planck}$

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Inflation continues until Φ drops below $\Phi/(4\pi)^{1/2}$

Inflation

As $\tau_{exp} = 1/H$ and $H^2 = (8\pi G/3c^2)V(\Phi) = (4\pi G/3c^2)\, m^2\, \Phi^2 = (G/3c^2)m^2$ $\Phi_{Planck}{}^2 = m^2/3$

=> $\tau_{exp} = (3)^{1/2}/m$

For successful inflation $\tau_{infl} >> \tau_{exp}$, then $\tau_{infl} >> 1/m$

Right after the end of inflation, the inflation field should, by definition, have an energy density corresponding to the central energy density:

$$\rho c^2 = \rho_{crit}c^2 = (3H^2c^2/8\pi G) = (3H^2\, \Phi_{Planck}{}^2 / 8\pi)$$

If we take $\rho c^2 = V(\Phi) = 1/2\, m^2\, \Phi^2$ and use $\Phi = \Phi_{Planck}/(4\pi)^{1/2}$ we get for the end of inflation:

$\rho c^2 = m^2\, \Phi_{Planck}{}^2 / 8\pi$ ==> $H^2 = m^2/3$

Inflation and Horizon Problem

What about homogeneity? (i.e., the horizon problem)

To solve the horizon problem, we need to make sure there is enough time for inflation

information must have been able to propagate the distance of what is now the observable universe.

the shrinking of the comoving Hubble radius $1/aH$ has to be at least as large as the subsequent increase

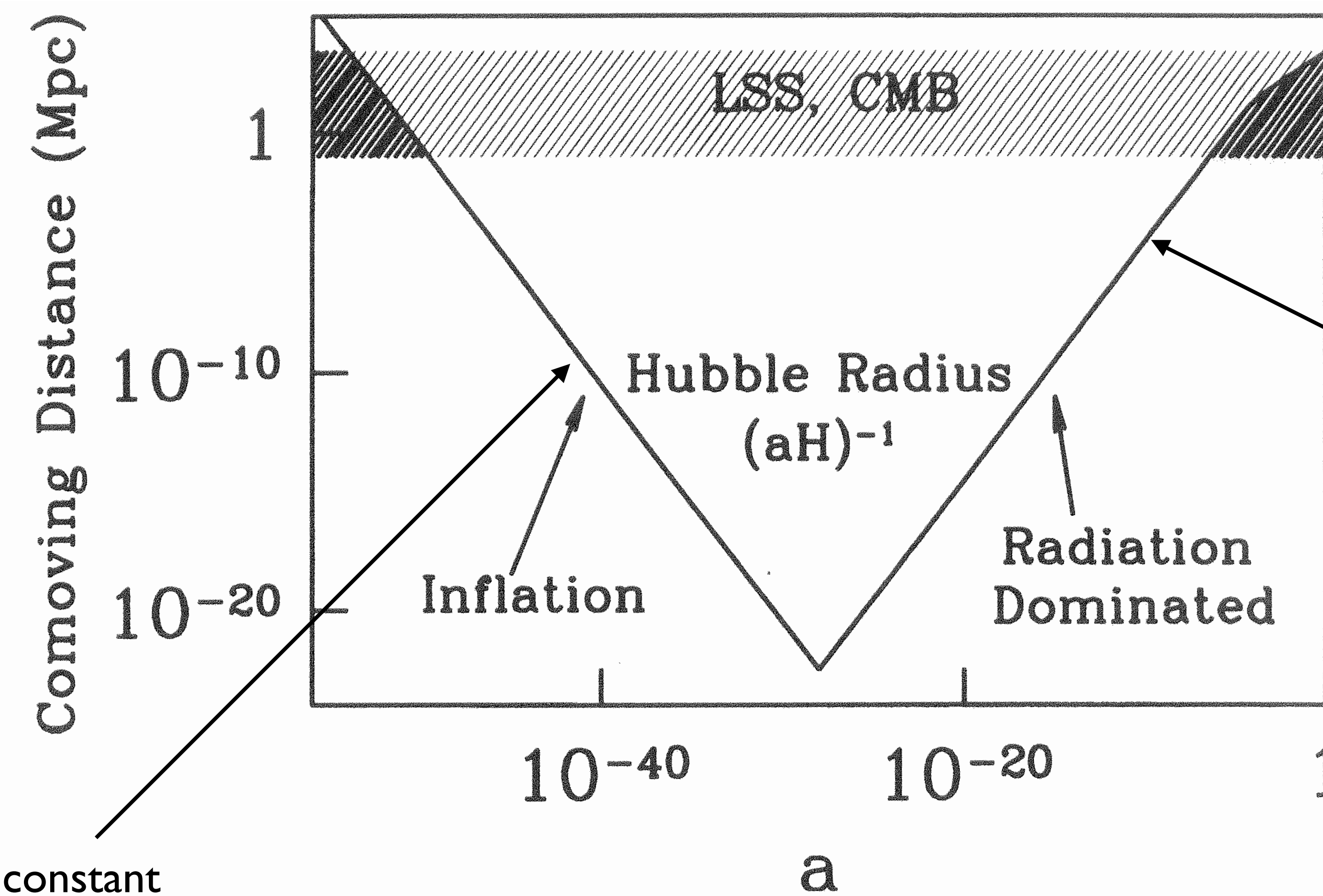
The diagram on the next page illustrates what is needed

Inflation

Hubble Radius in comoving coordinates

(c/Ha)

Region in casual contact

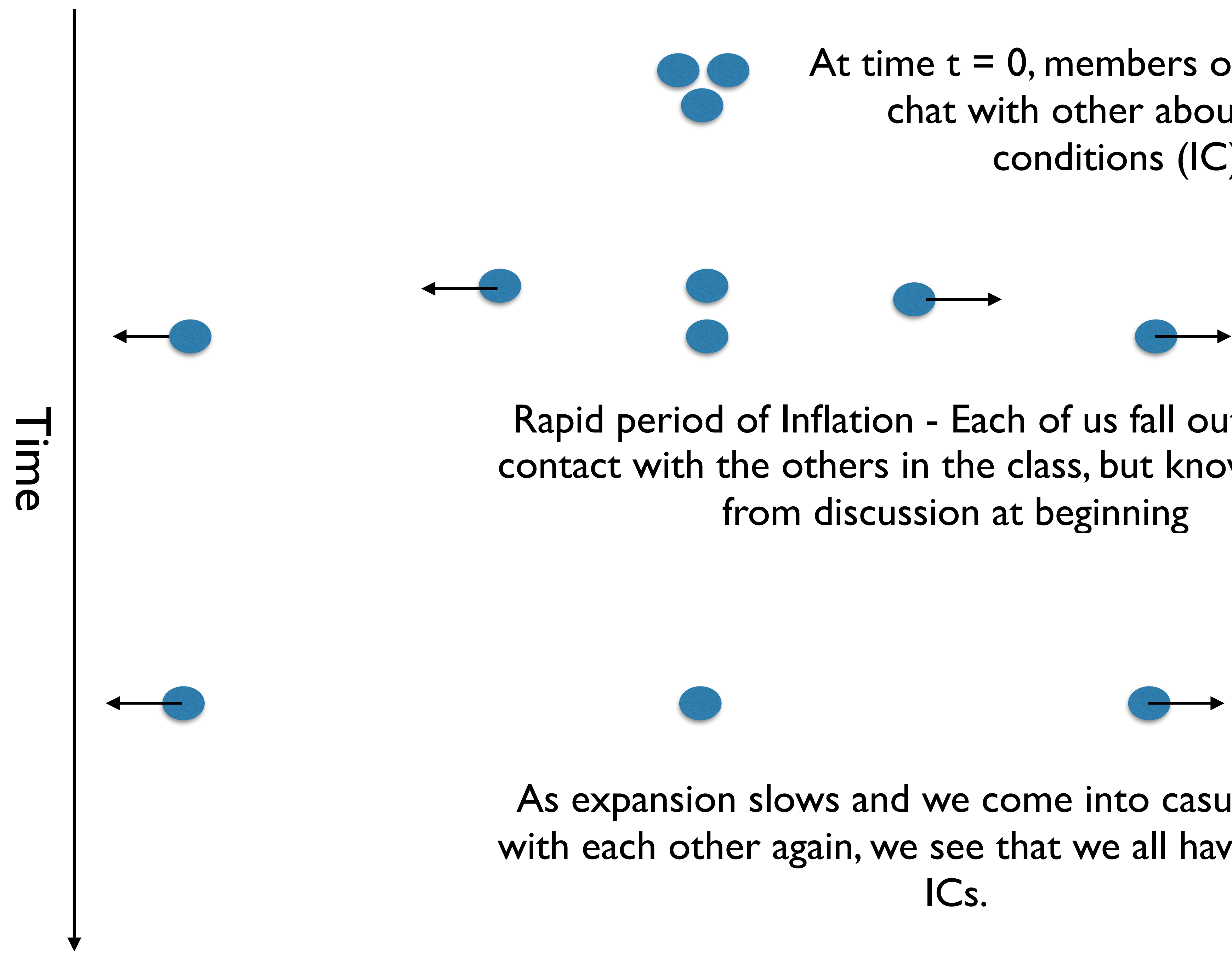


scales as a

H ~ constant
scales as 1/a

Dodelson (2003)

Let’s take as an apology the following situation:



At time t = 0, members of this class - chat with other about initial conditions (IC)

Rapid period of Inflation - Each of us fall out of casual contact with the others in the class, but know about IC from discussion at beginning

As expansion slows and we come into casual contact with each other again, we see that we all have the same ICs.

Inflation and Horizon Problem

By how much has the horizon grown since the beginning during the period when the universe was radiation dominated?

Current age of the universe is ~4×10^{17} s
End of Inflation is ~10^{-32} s

By how much did the horizon grow (in comoving coordinates):

$1/(aH) \sim 1/(a(a^{-2})) \sim a \sim t^{1/2}$

the scale factor grew by $((4 \times 10^{17}\ \mathrm{s})/(10^{-32}))^{1/2} \sim 10^{25}$

Requires >~ 60 e-folding times

If we want ~60 e-foldings in 10^{-32} s, we have $\tau_{exp} \sim 10^{-34}$ s and the time to homogenize is still 10x shorter. => we expect homogeneous universe

Inflation and Flatness Problem

What is the flatness problem?

Now observe $|\Omega_k| << 0.01$ so at $t_{infl} \sim 10^{-32}$ s or $a = 10^{-25}$ and because $\Omega_k \propto a^2$ ever since inflation, then

$|\Omega_k(t_{infl})| << 10^{-52}$.

Does inflation solve the flatness problem?

Inflation causes a to grow exponentially but H^2 and $V(\Phi)$ to remain fixed:

$$H^2 = (8\pi G/3c^2)\,V(\Phi) - kc^2/a^2$$

As a result, the kc^2/a^2 becomes very small very rapidly

Assuming that at the start of inflation Ω_k was ~1 then inflation must have increased by at least $10^{26} = e^{60}$

$\tau_{infl} >= 60\ \tau_{exp}$

New Material for This Week

Inflation and Magnetic Monopole Problem

What is the magnetic monopole problem?

One expects the magnetic monopoles to have a
density of one per horizon volume…

One expects the magnetic monopoles to have a density of
one per horizon volume…

i.e. where the horizon distance is $3\text{x}10^{-27}$ m at $t \sim 10^{-35}$ s

So, potentially high density of magnetic monopoles!

How does inflation solve it?

By blowing up the universe by a factor 10^{26}, ensure that there is less than one magnetic monopole in entire observable universe.

Dark Energy throughout Cosmic Time

Both Dark Energy Dominated

	Present Day ($t \sim 10^{18}$ s)	Inflation ($t \sim 10^{-32}$ s)
Energy Density (scales as H^2)	6.4×10^{-30} g/cm^3	6×10^{70} g/cm^3

Universes with Non-Zero Dark Energy

Universes with dark energy have a natural length + time scale.

Dark energy is relevant to current and earliest evolution of the universe.

Consider $((da/dt)/a)^2 = (8\pi G/3)\rho + \Lambda/3$

If $\Lambda > 0$, then eventually

$$((da/dt)/a)^2 = \Lambda/3 \qquad \Rightarrow a = e^{(\Lambda/3)^{1/2}\, t}$$

The horizon is

$$d_{Hor} = c \int_0^{t_0} dt/a(t) = c \int_0^{t_0} dt\, e^{-(\Lambda/3)^{1/2}\, t}$$

$$d_{Hor} = c\, (3/\Lambda)^{1/2}$$

Universes with Non-Zero Dark Energy

The natural time scale in such a dark-energy universe is

$$\tau_\Lambda = (3/\Lambda)^{1/2}$$

while the length scale defined by the horizon distance

$$d_{Hor} = c(3/\Lambda)^{1/2}$$

This is the cosmological event horizon in non-zero dark energy universes and means one eventually cannot see sources further than this proper distance since the accelerating expansion of the universe causes the points to recede faster than the speed of light.

Dark Energy throughout Cosmic Time

Both Dark Energy Dominated

	Present Day ($t \sim 10^{18}$ s)	Inflation ($t \sim 10^{-32}$ s)
Energy Density	6.4×10^{-30} g/cm^3	6.5×10^4 TeV
Horizon Distance	$\sim 5 \times 10^4$ Mpc	3×10^{-26} m
Horizon Time	17 Gyr	10^{-34} s

Dark Energy Problem

The nature of dark energy is not understood, and the observed value is extremely small in natural (i.e., Planck) units. This is one of the largest problems in physics.

The solution may require a better understanding of the quantum mechanical and gravitational properties of the vacuum.

Nominally, we would expect the vacuum energy to be

$$\rho_{\text{vacuum,expected}} = \rho_{\text{zero-point-energies}}$$

Like a harmonic oscillator in the ground state, every mode of free field contributes a zero-point to the energy density of the vacuum.

Energy is $m^4/(\hbar/c)^3$

where the cut-off mass depends on the physics and is 10^{60} $(\text{GeV})^4$ for GUT; alternatively one could choose $m = m_{\text{Planck}}$

We would therefore expect Λ to be much larger than is observed.

This is the cosmological constant problem.

Dark energy problem

Quantum gravity cut-off	$(10^{18}\text{GeV})^4$	fine-tuning to 120 decimal places
SUSY cut-off	$(\text{TeV})^4$	fine-tuning to 60 decimal places
EW phase transition	$(200\text{ GeV})^4$	fine-tuning to 56 decimal places
QCD phase transition	$(0.3\text{ GeV})^4$	
Muon	$(100\text{ MeV})^4$	fine-tuning to 44 decimal places
Electron	$(1\text{ MeV})^4$	fine-tuning to 36 decimal places
	$(\text{meV})^4$	observed value of the effective cosmological constant today…

Dark energy problem

A wide range of "explanations" have been proposed:

- A genuine cosmological constant?
  - the small value is a problem: anthropic arguments?
- It is the result of an evolving scalar field?
  - but why is the mass scale so small?
- Modifications of Einstein gravity
  - Need to obey solar system tests!
- Maybe the Universe is inhomogeneous?
  - Hard to reconcile with large-scale structure data…

Maybe it is something completely different?

Timeline up to the radiation era

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muon-antimuon decay	few 10^{-4}s	130MeV 1.4 10^{12}K	leaves electrons, neutrinos, photons, + a few baryons
neutrinos decouple	0.1s	3MeV 3.10^{10}K	Temperatures continue to decrease as a^{-1}
e^+-e^- annihilation	4s	0.5MeV 5.10^{9}K	End of lepton era. Heating of radiation

Phase transitions of the Universe

At $T_{EW}\approx 100$ GeV elektro-weak symmetry is broken and we have separate elektromagnetic and weak forces. All the leptons acquire mass.

When the temperature drops to $T_{QH}\approx 200\text{-}300$ MeV (10^{-5}s) we have the final phase transition and the strong interaction leads to the confinement of quarks into hadrons: the quark-hadron phase

The horizon is 1km in size

Timeline up to the radiation era

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At T≈200-300 MeV the quark-hadron transition binds quarks into hadrons

- protons/neutrons made of 3 u,d quarks
- mesons in particular π^+, π^0, π^-

In this hadron era pion-pion reactions are very important and the equation-of-state of the hadron fluid is very complicated.

Once the pions disappear (≈10μs after the Big Bang) the lepton era starts. It lasts until the e^+e^- pairs annihilate (≈1s after the Big Bang)

Timeline up to the radiation era

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The lepton era

At the start of the lepton era the Universe comprises

- photons
- small excess of non-relativistic baryons
- leptons & anti-leptons

$e^+ \, e^- \, \nu_e \, \bar{\nu}_e$

$\mu^+ \, \mu^- \, \nu_\mu \, \bar{\nu}_\mu$

$\tau^+ \, \tau^- \, \nu_\tau \, \bar{\nu}_\tau$ most likely already annihilated, but neutrinos remain

The lepton era

Important events during the lepton era are:

- annihilation of muons at $T<10^{12}$K (early-on)
- annihilation of electrons at $T<5x10^9$K (at the end)
- nucleosynthesis

These represent transitions where a particle species disappears. However, entropy is conserved for components still in thermal equilibrium (reactions are reversible):

g^*T^3=constant: as a species annihilate g^* falls and T rises a bit.

Statistical equilibrium distributions

A relativistic particle species in thermal equilibrium follows a Fermi-Direct(+) or Bose-Einstein(-) distribution:

$$n(T) = \int_0^\infty \frac{g}{2\pi^2\hbar^3c^3}\frac{E^2 dE}{e^{E/k_BT} \pm 1} \simeq 0.1216 \binom{3/4}{1} g \left(\frac{k_BT}{\hbar c}\right)^3$$

The energy density is given by:

$$\rho(T)c^2 = \int_0^\infty \frac{g}{2\pi^2\hbar^3c^3}\frac{E^3 dE}{e^{E/k_BT} \pm 1} = \binom{7/8}{1} \frac{g}{2}\sigma_r T^4$$

The total energy density $\rho(T)c^2=(g^*\sigma_r/2)T^4$.

Important Concept in Temperature Evolution of Evolving Universe

We have told you that the temperature of the radiation component of the universe scales as follows:

$$T(t) = T_P \, a(t_P)/a(t)$$

More precisely, it is the entropy which is conserved during the expansion of the universe (since the reactions are in equilibrium):

$$S = (\rho c^2 + P)V / T = 4/3 \ \rho c^2 V / T = 4/3 \ g^*(T) \ (1/2) \ \sigma T^3 V$$

where g^* gives the statistical weight (~ degrees of freedom)

=> $S \propto g^* T^3 V$ is conserved

=> $T \propto (g^*)^{-1/3}/a$

Lepton Era

At the start of the Lepton era, $g^*(T<T_\pi) = 4 \times 2 \times 7/8 + N_\nu \times 2 \times 7/8 + 2 = 14.25$

if $N_\nu = 3$

$e^+ e^- \mu^+ \mu^-$: $g = 2$ neutrinos: $g = 1$

photons: $g = 2$

How does the conservation of $g^*T^3a^3$ impact the evolution of the temperature as various species annihilate?

The first instance in the lepton era is when muons annihilate at $T \sim 10^{12}$ K

g^* falls from $6 \times (7/8) + 4 \times 7/8 \times 2 + 2 = 14.25$ to $6 \times (7/8) + 2 \times 7/8 \times 2 + 2 = 10.75$

T of the constituent radiative components of the universe of rises by $(14.25/10.75)^{1/3} - 1 \sim 9.8\%$

Decoupling of Neutrinos During Lepton Era

The particles are still in thermal equilibrium because the relevant collision time scale is much smaller than the Hubble time.

Note that at the start of the lepton era, the neutrinos are still in thermal equilibrium through weak force reactions such as

$\nu_e + \mu^- \rightleftharpoons \nu_\mu + e^-$ $\qquad$ $\nu_\mu + \mu^+ \rightleftharpoons \nu_e + e^+$

The cross section for these weak interactions is T^2: decreases quickly as T decreases. When the rate for the interactions falls below the expansion rate they cannot maintain equilibrium and the neutrinos become decoupled

The key quantity we need to consider for freeze out is τ_H/τ_{coll} and when it becomes less than 1.

$\tau_H/\tau_{coll} = (T/3\times10^{10}\ K)^3 < 1$ ==> neutrino decoupling occurs at $T \sim 3 \times 10^{10}$ K. They do remain relativistic at $mc^2 << k_BT$

Annihilation of Electrons-Positrons / Ratio of T_γ / T_ν

At the time of decoupling, the temperature of the neutrinos coincides with the temperature of the other constituents. After decoupling, the temperature T of neutrinos evolve as $1/a$, such that ρ evolves as $1/a^4$.

The e^+, e^-, Υ component follows the same evolution, but at e^+, e^- annihilation (5 x 10^9 K: 4 seconds after Big Bang) the temperature is increased again; this ratio persists until T_γ / T_ν = constant > 1

What is T_γ / T_ν ?

g^* falls from 2 x 7/8 x 2 + 2 = 5.5 to 2 = 2

Therefore T rises by $(5.5/2)^{1/3} - 1 = (11/4)^{1/3} - 1 = \sim 40\%$

The ratio of temperature T_γ / T_ν is $(11/4)^{1/3}$.

Once e^+, e^- annihilation is over in the universe, the universe is dominated by a radiation background with thermal black-body spectrum.

Big Bang Nucleosynthesis

The following is the elemental abundance in the universe:
^{4}He: Y = 0.25
^{3}He: ~10^{-3}Y
^{2}H or D: ~2×10^{-2} Y

It is the ratio of the mass of a particular element to the total mass in baryons.

Observational challenge — dealing with late-time affects that can impact abundance of different elements (production in stars, cosmic-ray bombardment)

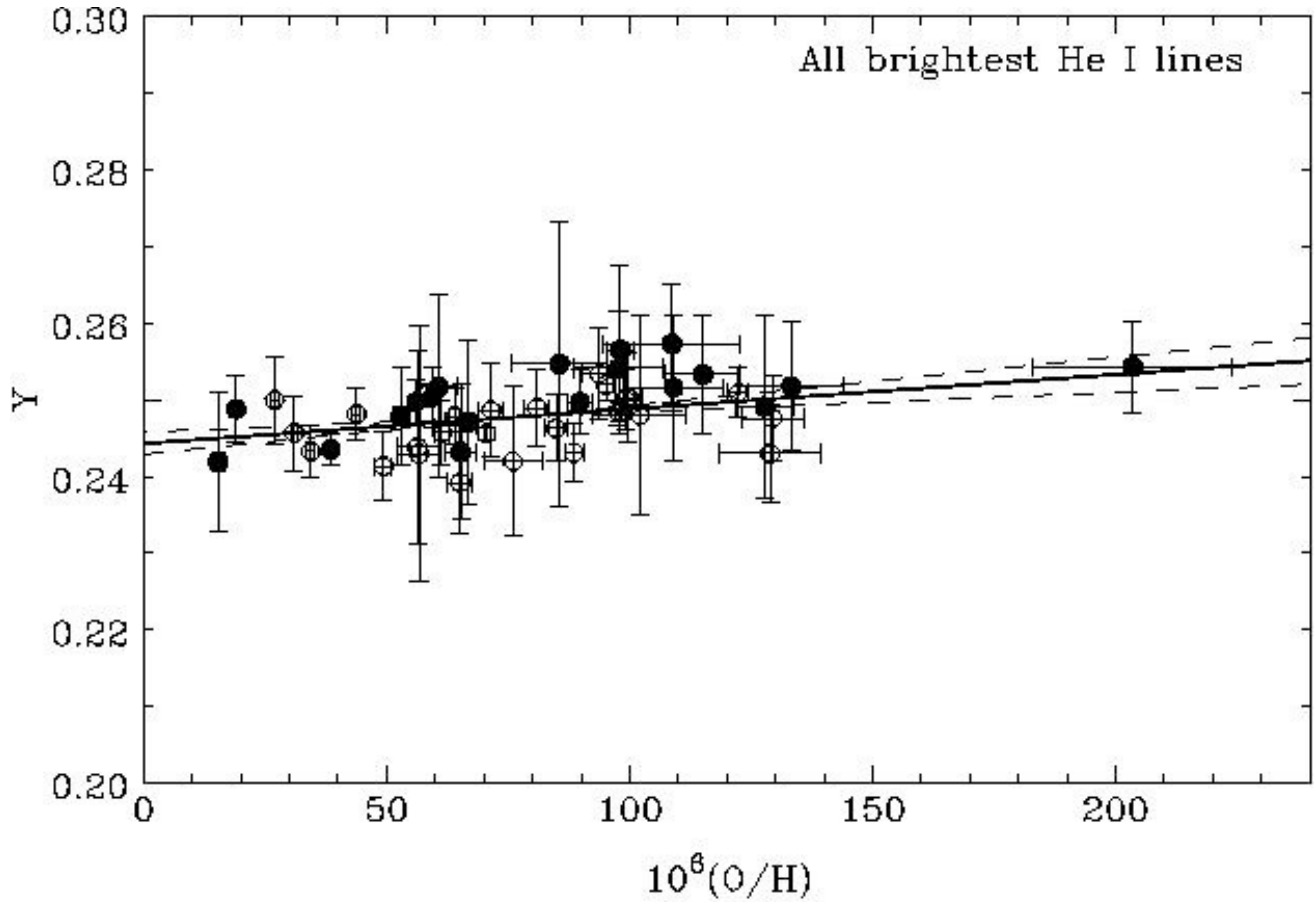
D is dissociated in stars.

Could the helium be produced in the stars? No! Just 1% of the nucleons would undergo fusion if galaxies remains at fixed luminosity for 10^{10} years.

Atmospheres of main sequence stars consist of ~25% helium with only weak trends with age or metallicity. This He is therefore not made in the stars.

Note that essentially all elements heavier than ^{7}Li are made in stars.

Big Bang Nucleosynthesis



Big Bang Nucleosynthesis

Towards the end of the lepton era, nuclear physics begins to take place, ultimately resulting in H, He, and traces of D and Li. This phase is not important for the thermal history of the universe, but clearly important for our existence.

How many neutrons are bound in nuclei?

1. When does the neutron / proton equilbration interaction freeze out, i.e., fixing the ratio (except for neutron decay)?

Neutron - Proton Ratio

The ratio of neutron to proton number densities n_n / n_p is $e^{-Q/kT} = e^{-(1.5e10/T)}$ as long as the neutrons and protons are in thermal equilibrium.

The equilibrium is maintained by weak nuclear reactions:

$$n + \nu_e \rightleftharpoons p + e^-$$

Because of the mass difference, there are more protons than neutrons.

The equilibrium is maintained for > 10^{10} K when the neutrinos decouple.

When this happens:

$$x_n = n/(n+p) = n / n_{tot} = (1 + e^{-1.5})^{-1} = 0.17 = x_n(0)$$

After this point, neutrons decay via beta decay.

$$n \rightarrow p + e^- + \overline{\nu}_e$$

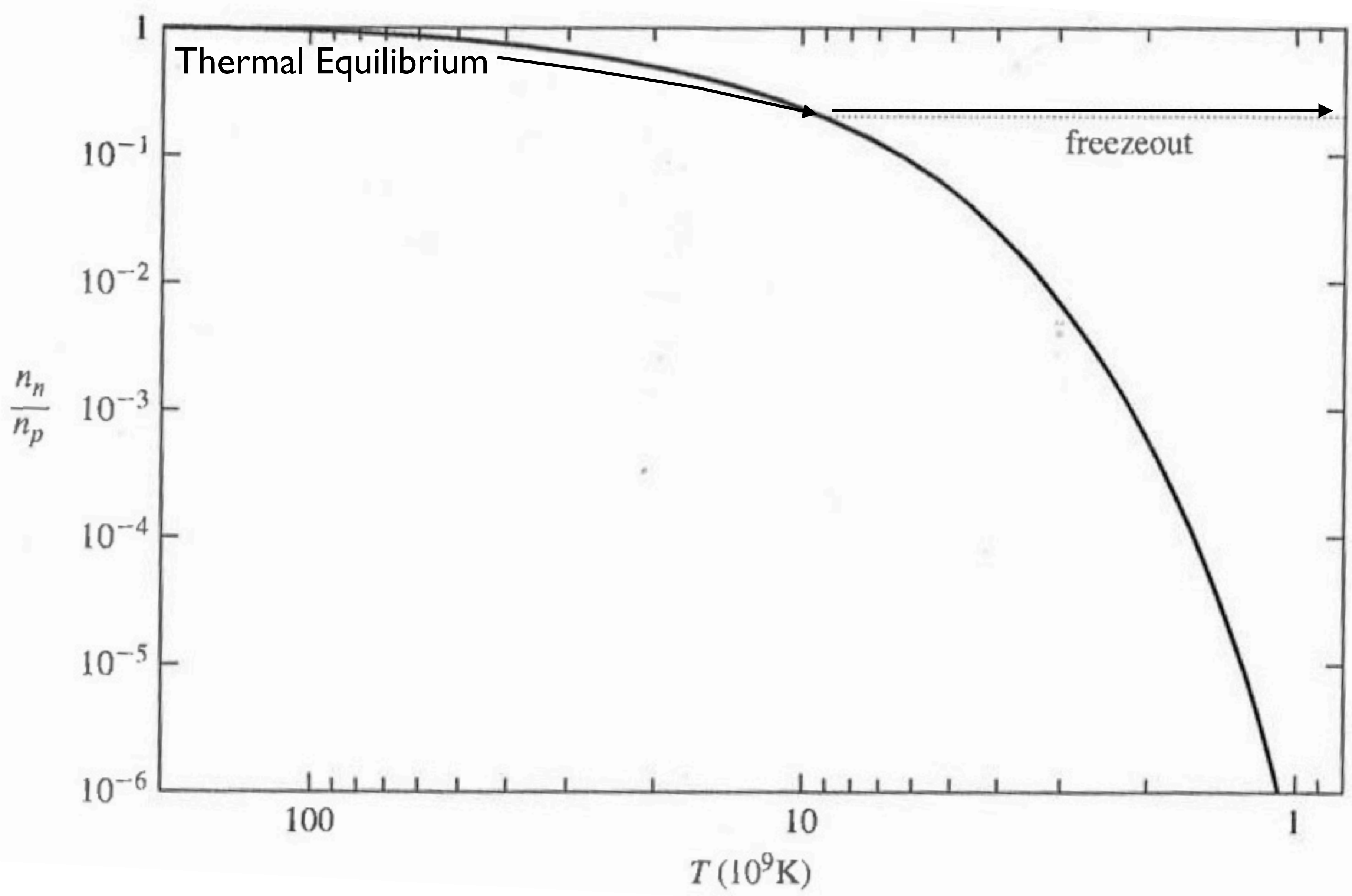
Neutrons have a mean lifetime of 900 s

$$x_n(t) = x_n(0)e^{-t/(900s)}$$

$$m_n - m_p = 1.3 \text{ MeV} \sim kT_{ew}$$

if T_{ew} were much lower, then only protons would form and then there would be no nuclear reactions.

Neutron - Proton Ratio



Theoretical Maximum Production of ^{4}Helium Allowed

Still the scarcity of neutrons relative to protons explains why BBN is so incomplete, leaving 75% of baryons unfused.

$p + p \rightleftharpoons D + e^+ + \nu_e$ which is a weak force interaction (so $\sigma_w = 10^{-47}\ m^2\ (T/MeV)^2$)

whereas $p + n \rightleftharpoons D + \Upsilon$ is a strong force interaction

As $p + n \rightleftharpoons D + \Upsilon$ is so much more efficient ($n + n \rightleftharpoons D + e^- + \nu_e$ is also a weak interaction), BBN proceeds until all neutrons are bonded into nuclei.

This allows us to compute the maximum value for Y if we take $n_n/n_p = 0.2$ and consider a group of 2 neutrons and 10 protons $\rightarrow$ 1 ^{4}He + 8 p

$Y_{max} = 4/12 = 1/3$ generally if $f = n_n/n_p \Rightarrow Y_{max} = 2f/(1+f)$

The observed value is indeed smaller. This is because nucleosynthesis takes a while and some neutrons will decay; some will end up in ^{3}He or D or heavier nuclei such as Li.

To compute Y accurately, as well as other abundances, all reactions must be considered.

How many neutrons are bound in nuclei?

1. When does the neutron / proton equilbration interaction freeze out, i.e., fixing the ratio (except for neutron decay)?

2. At which point does the temperature favor baryons being bound in nuclei than remaining unbound (at high temperatures, nucleosynthesis is not favored)?

3. At the point when nucleosynthesis begins in ernest, which fraction of the neutrons have decayed?

When Thermodynamics Favors Nucleosynthesis

At t ~ 2s, the proton-neutron freeze-out occurs, the neutrinos are decoupled, but the photons are still strongly coupled. To build heavier nuclei, we need a series of 2-particle interactions:

$$p + n \rightleftharpoons D + \Upsilon$$

the binding energy $B_D = (m_p + m_n - m_D)c^2 = 2.22$ MeV

Υ with $E > B_D$ can destroy D

Around the time of D formation, the relative number densities are given by a Saha-like equation:

$$n_D / (n_p n_n) = g_D / (g_p + g_n)\ (m_D / (m_p\ m_n))^{3/2}\ (kT / 2\pi\hbar^2)^{-3/2} e^{B_D/kT}$$

Note that $n_D = g_D\ (2\pi\hbar^2 m_D / kT)^{3/2}\ e^{-m_D/kT}$

As $m_p = m_n = m_D/2$ and $g_D = 3, g_p = 2, g_n = 2$

$$n_D / (n_p n_n) = 6\ (m_p\ kT / \pi\hbar^2)^{-3/2} e^{B_D/kT}$$

Deuterium is expected in the limit $T \rightarrow 0$, where p^+, n are favored when $T \rightarrow \infty$

When Thermodynamics Favors Nucleosynthesis

Define T_{nuc} is the temperature where $n_D / n_n = 1$ (half neutrons fused)

$$n_D / n_n = 6\, n_p\, (m_n kT / \pi\hbar^2)^{-3/2} e^{B_D/kT}$$

Currently 75% of the baryons are in the form of H and before deuterium synthesis, 83% of them were

$$n_p = 0.8\, n_{baryon} = 0.8\, n_\gamma\, \eta = 0.8\eta\, (0.243\, (kT/\hbar c)^3)$$

where $\eta = n_{baryon} / n_\gamma$

$$n_D/n_n = 6.5\eta\, (kT/m_n c^2)^{3/2}\, e^{B_D/kT}$$

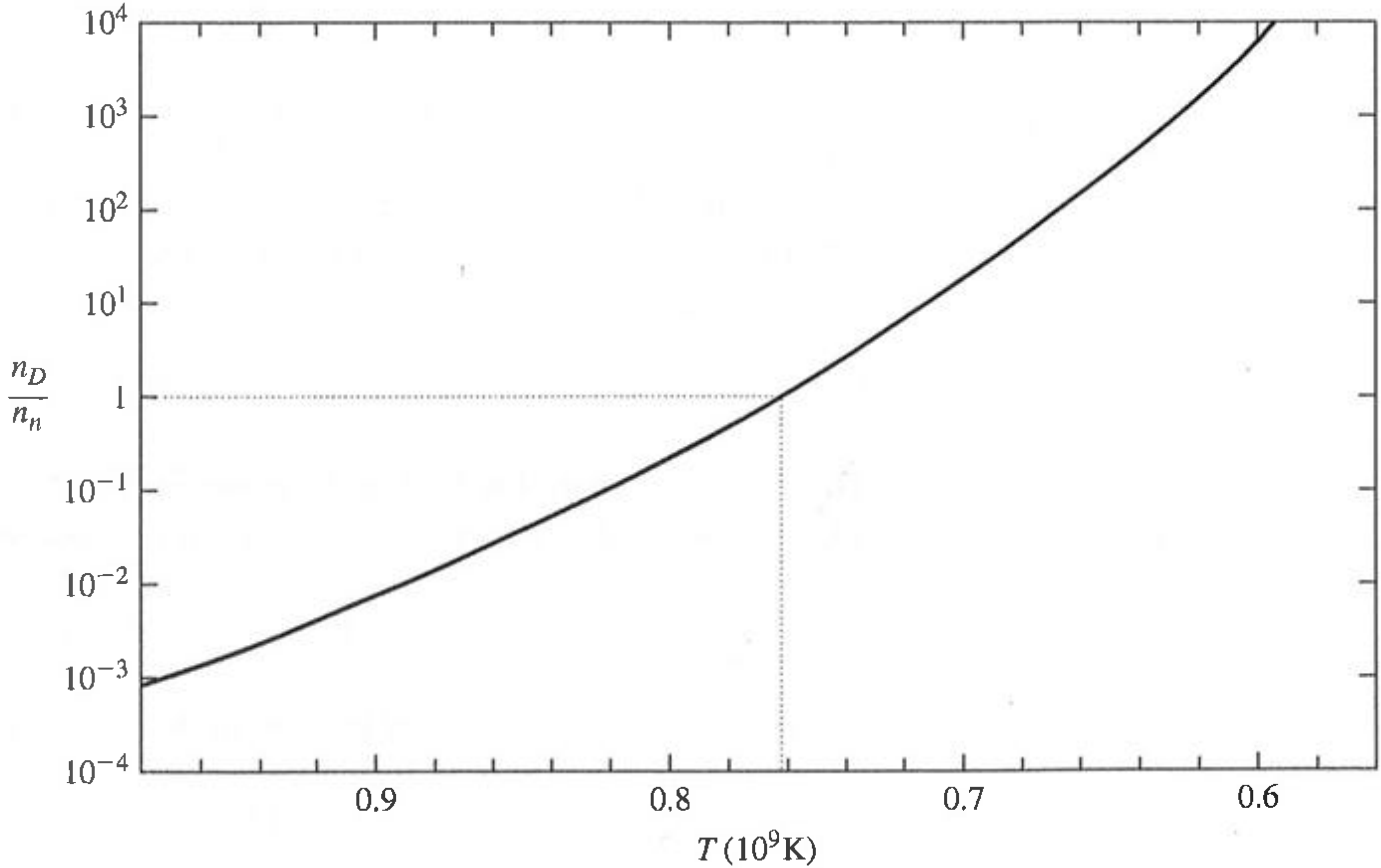
The ratio is unity for $T_{nuc} \sim 8 \times 10^8$ K => 200 s after Big Bang

The time delay until the start of nucleosynthesis is non-negligible compared to the neutron decay time (900 s).

At the start of nucleosynthesis, $n_n / n_p = e^{-(200/900)}/(5+(1-e^{-200/900})) = 0.8/5.2 \sim 0.15$

This lowers Y_{max} to 0.27

When Thermodynamics Favors Nucleosynthesis



Neutron - Proton Ratio

The ratio n_D / n_n does not remain at the equilibrium values, once there is enough D we get

- proton capture $D + p \leftrightarrow {}^3He + \gamma$
- neutron capture $D + n \leftrightarrow {}^3H + \gamma$
- formation of α particle (rare) $D + D \leftrightarrow {}^4He + \gamma$
- Tritium formation $D + D \leftrightarrow {}^3H + p$
- ^{3}He formation $D + D \leftrightarrow {}^3He + n$

^{3}H decays into ^{3}He + e^- + ν_e , but has a decay time of 18 years, i.e. stable during BBN

The bottleneck

Helium-3 and Tritium quickly convert to Helium-4

- many pathways
- strong interactions, fast

$$^3H + p \leftrightarrow {}^4He + \gamma$$
$$^3He + n \leftrightarrow {}^4He + \gamma$$
$$^3H + D \leftrightarrow {}^4He + n$$
$$^3He + D \leftrightarrow {}^4He + p$$

All strong force interactions so fast.

The binding energy per nucleon is very high for stable nuclei like ^{4}He, but there are no stable nuclei with A = 5 (^{5}He and ^{5}Li are not stable). Very difficult to make heavier elements.

$^4He + D \rightleftharpoons {}^6Li + \gamma$ $\quad$ $^4He + {}^3H \rightleftharpoons {}^7Li + \gamma$ $\quad$ $^4He + {}^3He \rightleftharpoons {}^7Be + \gamma$

Synthesis of nuclei with A > 7 is hindered by the absence of stable nuclei with A = 8

Initially at T >> 10^9 K, all baryons are in the form of protons and neutrons.

As the Deuterium density increases, ^{3}H, ^{3}He, and ^{4}He are formed.

At t~10 min (T ~ 4 x 10^8 K), BBN is essentially over.

The bottleneck

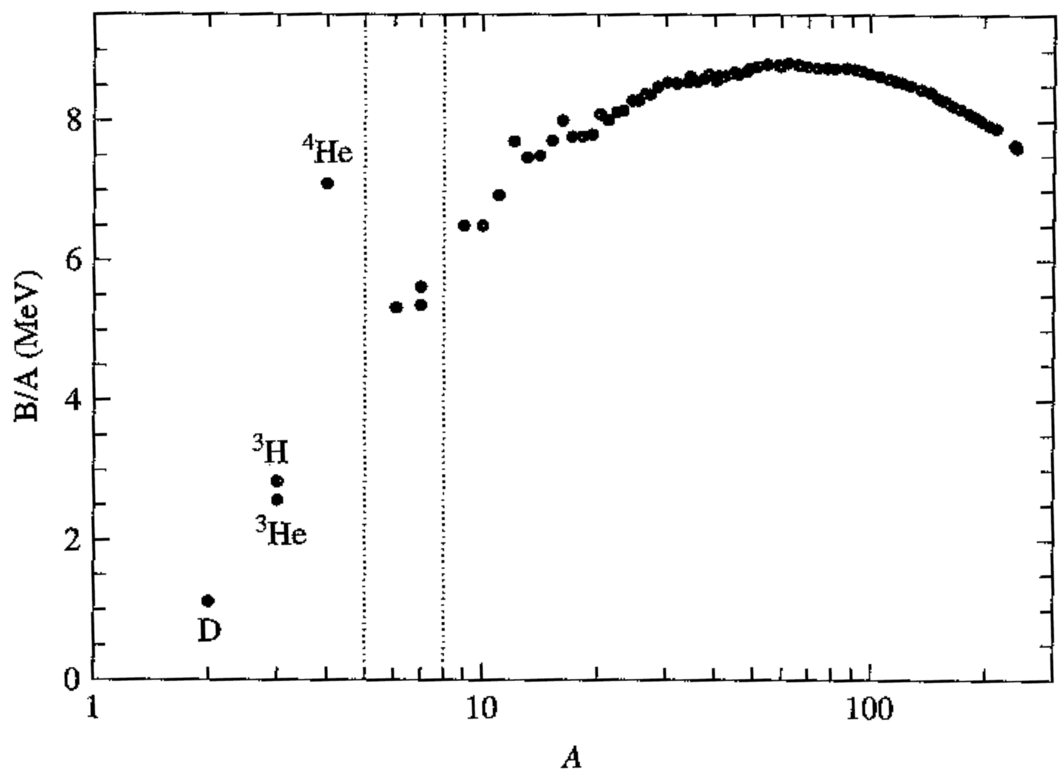
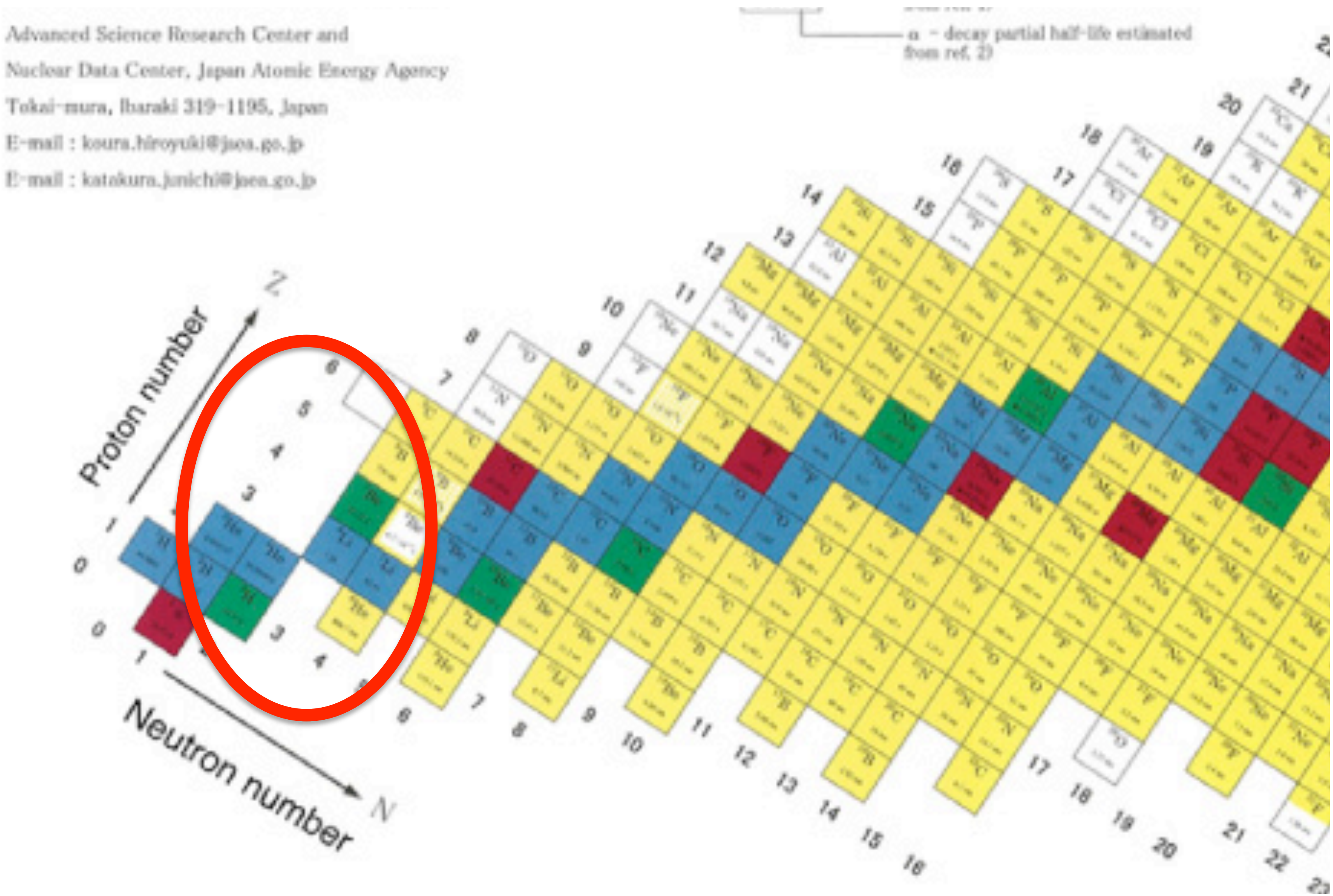


FIGURE 10.1 The binding energy per nucleon (B/A) as a function of the number of nucleons (protons and neutrons) in an atomic nucleus. Note the absence of nuclei at $A = 5$ and $A = 8$.

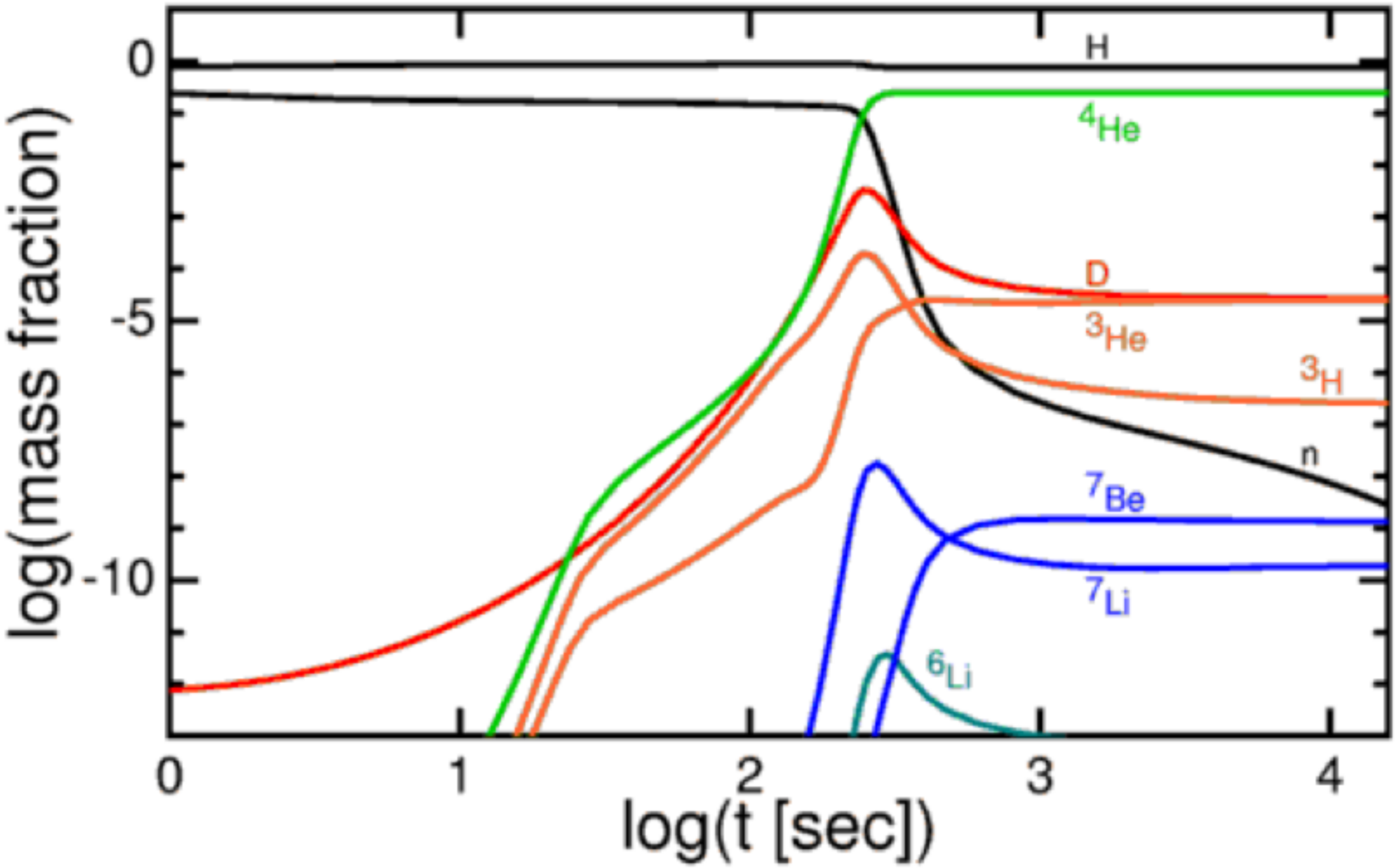
Binding energy per nucleon = $(m_{nucl} - Zm_p - Nm_n)c^2/A$

^{56}Fe, ^{62}Ni most stable nuclei - ^{4}He also stands out

The bottleneck



BBN calculations



Observational Constraints on the Primordial Abundances of Different Elements

- ^4He: extrapolating abundance in old, low-metallicity stars to zero metallicity gives Y=23.4%.

What we can learn from BBN about Ω_b

The yields of D, ^3H, ^4He, ^6Li, and ^7Li depend on η:

a high value for η(the baryon to photon ratio) increases the value for T_{nuc}

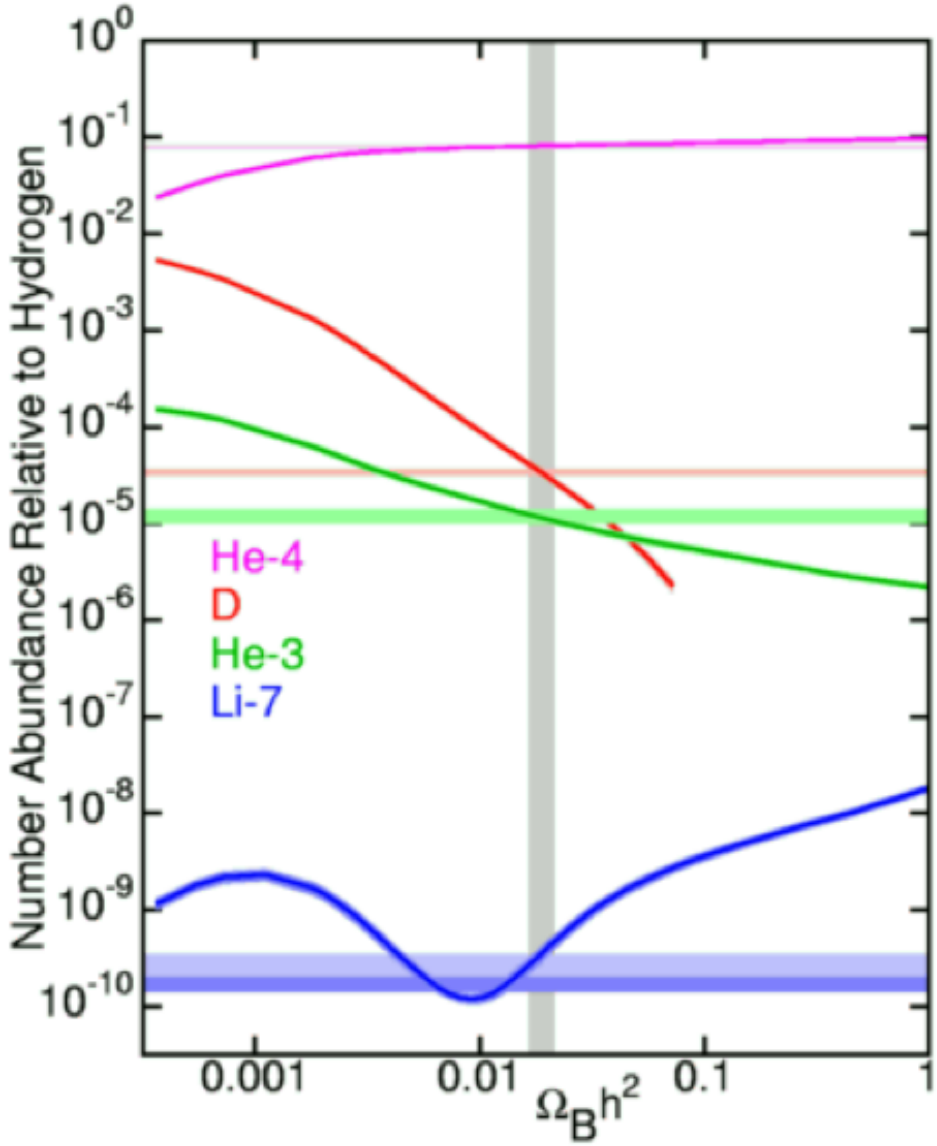
As BBN is a race against the clock, an earlier start means more ^4He is formed, but less D and ^3He as leftovers.

The deuterium abundance can be used to estimate η:

$n_{baryon,0} = \eta \, n_{\Upsilon,0} = 0.23 \pm 0.02 \text{ m}^{-3}$ => $\Omega_{baryon} = 0.04 \pm 0.01$

Also CMB gives a highly consistent constraint on Ω_{baryon}

Consistency with observations!



Find $\Omega_b h^2$ where observed abundance matched predicted abundance from BBN