Sterrewacht Leiden

Origins & Evolution of the Universe

an introduction to cosmology — Fall 2018

Lecture 7: Early Universe, Inflation

Rychard Bouwens

Layout of the Course

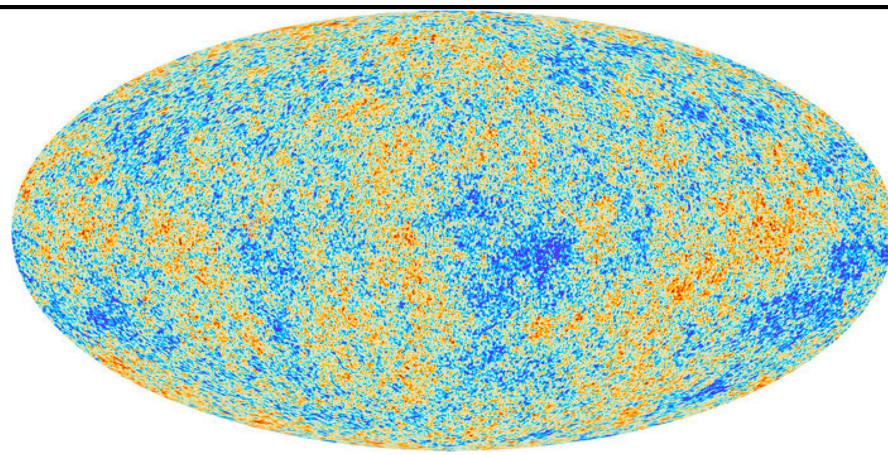
Sep 24: Introduction and Friedmann Equations
Oct 1: Fluid and Acceleration Equations
Oct 8: Introductory GR, Space Time Metric, Proper Distance
Oct 15: Redshift, Horizons, Observable Distances
Oct 17: Problem Class #1
Oct 22: Observable Distances, Parameter Constraints
Oct 29: Thermal History, Early Universe
Nov 5: Early Universe, Inflation
Nov 12: Inflation, Lepton Era
Nov 14: Problem Class #2
Nov 19: Big Bang Nucleosynthesis, Recombination
Nov 26: Introduction to Structure Formation
Dec 3: Cosmic Microwave Background Radiation (I)
Dec 5: Problem Class #3
Dec 10: Cosmic Microwave Background Radiation (II)
Dec 21: Final Exam

Problem set #2 was mailed to you last week

Due by Wednesday 13:30 November 14

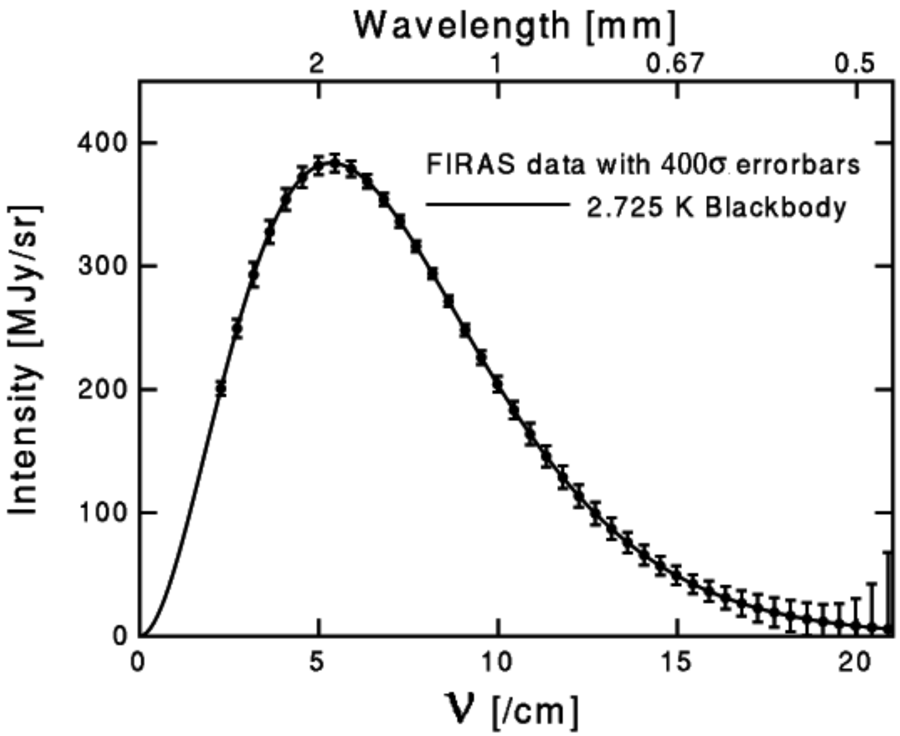
Review Last Week

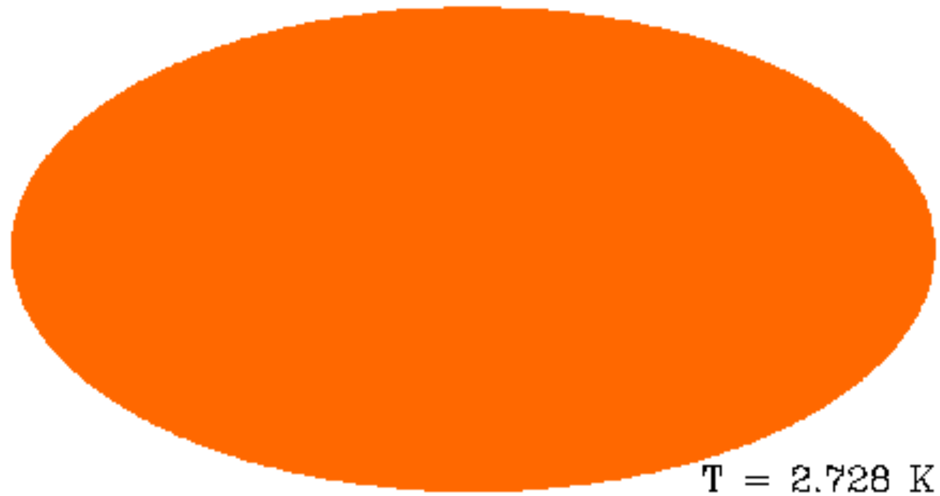
What can we learn from the CMB?



View of the Sky from Planck: Blown Up by 10^5

Cosmic Microwave Background is Isotropic





Isotropic to one part in 10^5

Photons from the CMB have a spectral energy distribution which is almost a perfect black body.

Most perfect blackbody spectrum seen anywhere!

The cosmic microwave background radiation has essentially the same spectrum / temperature in every direction!

What happened during the recombination epoch and how did it result in the cosmic microwave background?

Recombination Epoch (z~1100)

Ionized Plasma → Neutral Gas

Ionized Plasma

(z > 1100)

< 380,000 years

Temperature
> 3600 K

Hydrogen ionized

Photons Thomson-scattering off of the ionized hydrogen

Neutral Gas

(z < 1100)

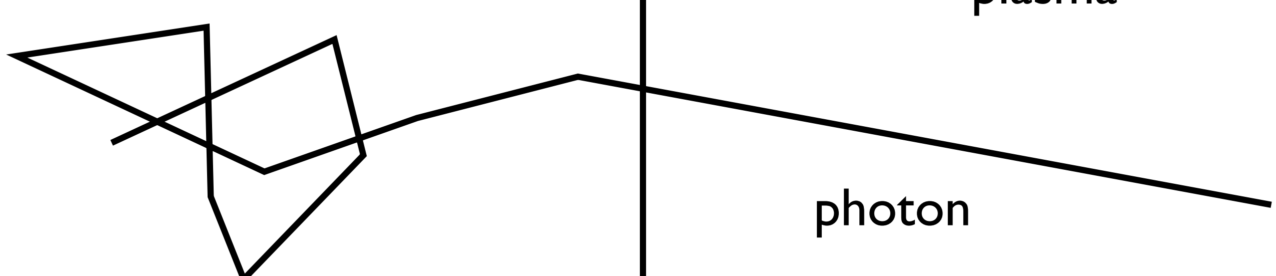
> 380,000 years

Temperature
< 3600 K

Hydrogen neutral

Almost no free electrons

Photons unbound from plasma



Evolution of Temperature

The temperature evolution of matter and radiation is different. We already see see that $\varepsilon_{rad} \propto T^4$ and $T \propto (1+z)$ for the CMB at late times.

For matter, the evolution is as follows: $T \propto 1/a^2 \propto (1+z)^2$
(which follows from $\Upsilon = 5/3$ and $PV^{\Upsilon-1}$ = fixed)

Before t_d (the decoupling time — i.e. similar to last scattering for the CMB), the two temperatures are equal by thermal interactions (again Thomson scattering).

After t_d, the matter density evolves as $(1+z)^3$ and the radiation density as $(1+z)^4$

One can show that from the first law of thermodynamics that

$$dT/T = -(1+ \sigma_{rad})/(1/2 + \sigma_{rad})\ (da/a)$$

where $\sigma_{rad} = 4m_p\, \sigma_r\, T^3 / 3k_B\rho_m$ (entropy in radiation per unit volume per baryon density),

Note that σ_{rad} is unevolving:

$$\sigma_{rad} = \sigma_{rad}(t_0) = 4m_p\sigma_r T^3/3k_B\rho_{m,0} \approx 3.6/\eta_0 \approx 1.35 \times 10^8\, (\Omega_{b,0}\, h^2)^{-1}$$

where η ~ photon / baryon ~ 1.7×10^9

This implies that $dT/T \approx -da/a$

Entropy per Baryon

The high value of σ_{rad} ensures that the temperature and density of the radiation evolve as a pure radiation universe.

σ_{rad} is actually related to the entropy of the radiation per unit volume.

$\sigma_{rad} = S_r/k_B n_b$ where

$S_r = (\rho_{rad}c^2 + p_{rad})/T = (4/3)\rho_{rad}c^2/T = (4/3)\sigma_r T^3$ entropy in the radiation

as $\rho_{rad}c^2 = \sigma_r T^4$

and $\eta^{-1} = n_\gamma/n_b$ $n_b = \rho_m / m_p$ number density of baryons

Baryon asymmetry

Why is there now mostly matter, no anti-matter?

During the hadron era, there must have been many proton-anti-proton pairs; these annihilate as the universe cools, but a small residual of matter remained.

$(n_b - n_{\bar{b}})a^3$ remains constant because baryon number is conserved below $T \sim 10^{15}$ GeV. As we do not see a large Υ-ray background the baryon number per comoving volume is $n_{b,0}$ => $n_{b,0}\, a_0^3$

Above the GUT temperature: $n_b \sim n_{\bar{b}} \sim n_\Upsilon \sim T^3$

$$(n_b - n_{\bar{b}})/(n_b + n_{\bar{b}}) \approx (n_b - n_{\bar{b}})/2n_\Upsilon \approx n_{b,0}/2n_{\Upsilon,0} \propto 1/\sigma_{rad}$$

The asymmetry is very small : for every 10^9 anti-baryons, there are $10^9 + 1$ baryons. => σ_{rad} is large because the asymmetry is so small.

Singularity

Matter or radiation dominated universes decelerate → we expect a finite age for the universe.

at $t = 0$, the density diverges and the proper distance between points goes to 0. This singularity is called the "Big Bang."

It is a consequence of
- the cosmological principle
- Einstein's equations in a cosmological context.
- the expansion of the universe $da/dt/a > 0$
- assumed form of the equation of state $0 < w < 1$

A singularity can be avoided if the equation of state for matter in the early universe is different from a perfect fluid with $P/\rho > -1/3$

Fluids with $w < -1/3$ violate the strong energy condition $\rho + 3P \geq 0$

Current observations show that $\Lambda < (H_0/c)^2 \sim 10^{-55}\ \mathrm{cm}^{-2}$

too small to be relevant in the early universe

If the dynamics of the early universe are dominated by a homogeneous and isotropic scalar field then it may have been important early on

Timeline up to the radiation era

Unkown physics

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neutrinos decouple	0.1s	3MeV 3.10^{10}K	Temperatures continue to decrease as a^{-1}
e^+-e^- annihilation	4s	0.5MeV 5.10^{9}K	End of lepton era. Heating of radiation

Planck Scale

Our definition of the unit of time is arbitrary, but it is possible to derive a time that is "natural" on which everybody in the Universe agrees: there is a unique combination of fundamental constants that yields a time:

$t_P = (\hbar G/c^5) \sim 10^{-43}$ s the Planck time

Similarly we can define $l_P = ct_P = (G\hbar/c^3)^{1/2} \sim 1.7 \times 10^{-35}$ m the Planck length

$m_P = (\hbar c/G)^{1/2} \sim 2.5 \times 10^{-8}$ kg the Planck mass

$E_P = m_P c^2 = (\hbar c^5/G)^{1/2} \sim 1.2 \times 10^{19}$ GeV the Planck Energy

$T_P = E_P/k_B = (\hbar c^5/k_B^2 G)^{1/2} \sim 1.4 \times 10^{34}$ K the Planck Temperature

The first $t_P \sim 10^{-43}$ seconds cannot be described by GR or quantum mechanics.

The horizon $ct_P \sim$ Planck length and particle pairs are created which have the Planck mass separated by less than the Planck length → paricles/black holes at once, with quantum effects on the scale of the horizon → we cannot describe this with known physics.

Motivation for Planck Scale

The motivation for the Planck scale comes from the existence of a fundamental limit in our understanding of physics when quantum mechanical effects and strong gravity occur on the same scale. We do not have a theory of quantum gravity.

When does this occur?

We have to define a Compton time for a body of mass m (or energy mc^2) to be

$$t_c = \hbar/mc^2$$

this represents the time to violate energy conservation by $\Delta E = mc^2$

The corresponding Compton length is $l_c = ct_c = \hbar/mc$

Note that t_c and l_c increase as the mass decreases: these scales indicate when quantum mechanics is important

The Schwarzschild radius is $l_s = 2GM/c^2$ and time $t_s = l_s/c = 2GM/c^3$

We need quantum gravity when $l_s = l_c$ $\Rightarrow m = (\hbar c/2G)^{1/2} \approx (\hbar c/G)^{1/2} \equiv m_P$

where m_P is the Planck mass

Is the Early Universe In Thermal Equilibrium?

We need to look at the collision time scale τ_{coll}
and compare this with the Hubble time τ_H (~age of universe)?

To calculate the collision time, we need to know the temperature and density.

We can assume that after the Planck time: $T(t) = T_P\, a(t_P)/a(t)$

$\Longrightarrow$ early on all particles are relativistic!

What is the equilibrium number density of a particle species i?

This depends on whether it is a fermion or a boson, and how many spin or helicity states it possesses, g_i

The number density is given by

$$n_i = g_i\,(k_B T/\hbar c)^3 \int_0^\infty x^2 dx \,/\, (e^x \pm 1) = \begin{pmatrix} 3/4 \\ 1 \end{pmatrix} (g_i/\pi^2)\zeta(3)\,(k_B T/\hbar c)^3$$

(+ for fermions and -1 for bosons)

fermion

boson

Riemann ζ function

$\zeta(3) \sim 1.202$

The energy density is given by

$$\rho_i(T)c^2 = (g_i\, k_B^4 T^4)/(2\pi^2\hbar^3 c^3) \int_0^\infty x^3 dx \,/\, (e^x \pm 1) = \begin{pmatrix} 7/8 \\ 1 \end{pmatrix} (g_i/2)\sigma_r T^4$$

Is the Early Universe In Thermal Equilibrium?

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and compare this with the Hubble time τ_H (~age of universe)?

The cross section of all particles:

$$\sigma = \alpha^2(\hbar c/k_B T)^2 \qquad\qquad \alpha = 1/50$$

The collision time is

$$\tau_{coll} = 1/n\sigma c = \hbar/(g^*(T)\alpha^2 k_B T)$$

This can be compared to the expansion time scale $\tau_H = a/(da/dt)$

$$\tau_H = 2t = (3/32G\pi\rho)^{1/2} = (0.3\hbar T_p/(g^*(T)^{1/2} k_B T^2)$$
$$= (2.42 \times 10^{-6})\ (T/GeV)^{-2}/(g^*(T))^{1/2}\ s$$

$$\tau_{coll}/\tau_H \sim 1/g^{*1/2}\,\alpha^2\,(T/T_p) << 1$$

==> Thermal Equilibrium

Timeline up to the radiation era

Unkown physics

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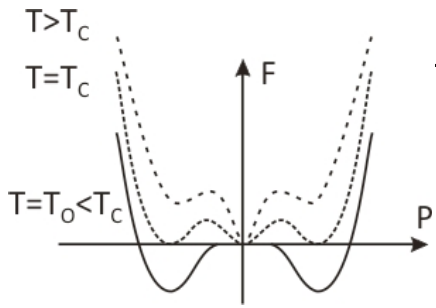
Era of phase transitions

Phase transitions: rearrangement of the microphysics in which a particular symmetry is created or destroyed.

- location of particles: freezing, melting, evaporation
- orientation of particles: ferromagnetism



Example of first order transition

free energy

temperature

$$F=U-T\times S$$

internal energy

entropy

Kibble mechanism

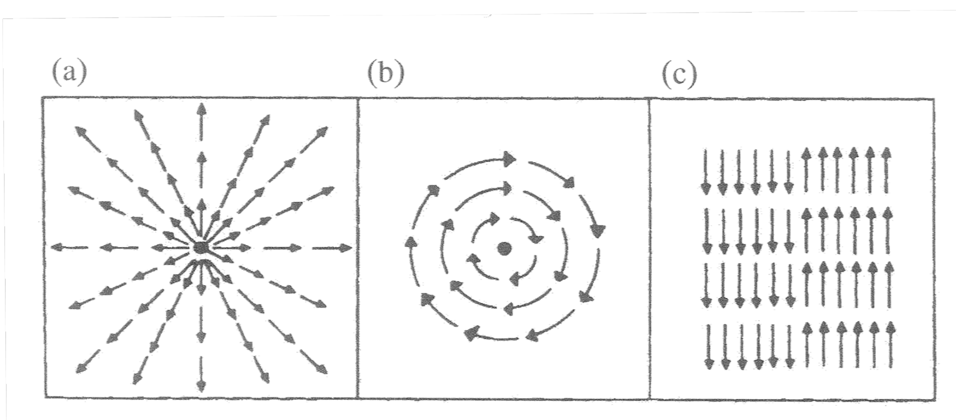
After each phase transition, the effective physics changes.

Phase transitions can leave defects if different regions pick a different state.

Kibble mechanism: different Horizon-sized volumes choose their ground states independently (no causal connections between them).

As the universe expands and cools, the fields decay to their ground state over most of space, but trapped energy domains remain as defects: ***this is a generic prediction!***

Topological defects

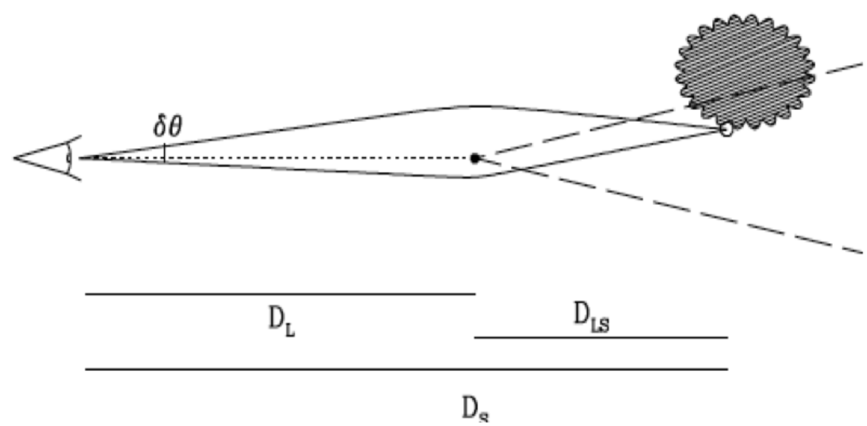


monopole string domain wall

From: Coles & Lucchin (2002)

Effect of a cosmic string

Cosmic strings split images - angle of splitting proportional to mass/unit length.



New Material for This Week

Phase transitions of the Universe

Between $T \approx 10^{19}$ and 10^{15} GeV, quantum gravity effects decrease in importance and interactions are described by a GUT. Baryon number is not conserved in GUTs, so no asymmetry between matter and antimatter.

Near $T \approx 10^{15}$ GeV ($t = 10^{-37}$s) the GUT symmetry breaks leading into the situation described by the standard model of particles; the GUT phase transition typically results in the formation of magnetic monopoles.

For typical GUTs:
- particle mass: $m_M \approx 10^{16}$GeV
- number density: $n_M > 10^{-10} n_{\gamma}$.

$\Longrightarrow \Omega_{monopole} > m_M / m_p \Omega_{bar} \approx 10^{16}$.

This does not match observations: **the monopole problem**

Phase transitions of the Universe

A GUT that unifies the elektroweak interactions with the strong interactions puts leptons and hadrons on the same footing and thus allows processes that do not conserve baryon number: source of matter/anti-matter asymmetry.

As the temperature falls below $T_{GUT} \approx 10^{15}$ GeV the unification of the strong and elektroweak interactions no longer holds. Towards the end of this period (10^{-11}s) the Universe is filled with an ideal gas of leptons and antileptons, the four vector bosons, quarks and anti-quarks.

The horizon is 1cm and contains $\approx 10^{19}$ particles!

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Successes of the Big Bang model

- Correctly predicts the abundances of light elements
- Explains the CMB as relic of the hot initial phase
- Naturally accounts for the expansion of the Universe
- Provides a framework to understand the formation of cosmic structure.

There are also several problems (some of which can be addressed by incorporating "new physics")

Problems with the Big Bang model

- Origin of the Universe
- The horizon problem
- The flatness problem
- Origin of the baryon asymmetry
- Monopole problem
- Origin of primordial density fluctuations
- Nature of dark matter
- Nature of dark energy

Problems with the Big Bang model

- Origin of the Universe
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How we can solve these issues with the Big Bang model?

Consider the horizon problem…

As the universe ages, we are continually probing regions of the universe which were not yet in casual contact, but appear to be homogeneous.

In the lecture on horizons, we showed the size of the particle horizon in comoving coordinates evolves as $a^{3/2}/a \sim a^{1/2}$.

We need to propagate information within a casually connected volume to great comoving distances, i.e., for the particle horizon to be plausibly infinite.

If the scale factor tends to 0 at early times as t^{β} then the particle horizon at time t is as follows:

$$R_H(t) = a(t) \int_0^t c\,dt'/a(t')$$

This integral diverges if $\beta >= 1$

What does $\beta >= 1$ imply regarding other quantities of importance?

What does this imply regarding d^2a/dt^2?

Use Friedmann's second equation:

$$d^2a/dt^2 = -(4/3)\pi G(\rho+3P/c^2)a = \beta(\beta-1)a/t^2$$

$$-(4/3)\pi G(\rho+3P/c^2)t^2 = \beta(\beta-1) \propto d^2a/dt^2$$

$$-(4/3)\pi G\rho(1+3w)t^2 = \beta(\beta-1) \propto d^2a/dt^2$$

If $\beta >= 1$, then $d^2a/dt^2 > 0$

$\beta >= 1 \iff d^2a/dt^2 >= 0 \iff$ If $w <= -1/3 \iff$ there is no particle horizon

If there is no particle horizon, then information from a small region can propagate to the entire universe.

But if not, there is a particle horizon and the universe will not be in casual contact. is is hard to reconcile with the cosmological principle.

Propagating Information from Small Region to Large Volume in Universe (Inflation)

We need to a mechanism to disconnect regions that were before in causal contact, the expansion must be so rapid that there exists an event horizon at a finite distance from any point

How can we formalize this?

The Hubble radius in comoving coordinates must shrink with time:

$$\frac{d}{dt}\left(\underbrace{(1/a)}_{\text{scale factor to put in comoving coordinates}}\ \underbrace{(c/H)}_{\text{distance light can travel in Hubble time}}\right) < 0 \qquad \text{where } H = (da/dt)/a$$

implies $(d/dt)(c/(da/dt)) < 0 \quad \Rightarrow \quad (d^2a/dt^2) > 0$

given the acceleration equation we need a substance with sufficient negative pressure.

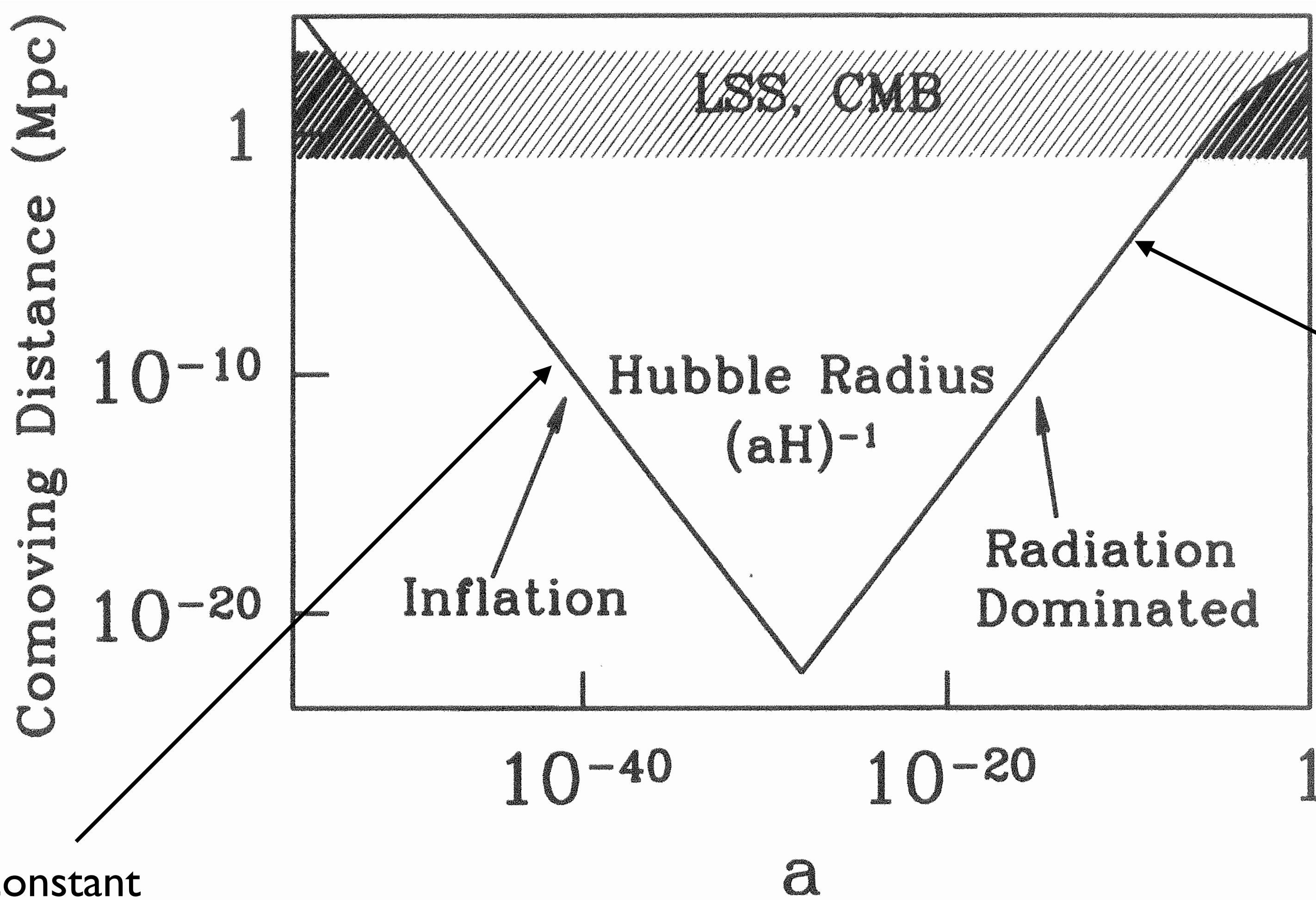
How can we implement this?

The inflation field; in physics we encounter scalar fields to describe the potential energy with a particular force; the force is the gradient of the potential energy scalar field. Other examples are the temperature or pressure field. In quantum field theory a scalar field is associated with a spin-0 particles. The Higgs field is an example.

Inflation

Hubble Radius in comoving coordinates

(c/Ha)

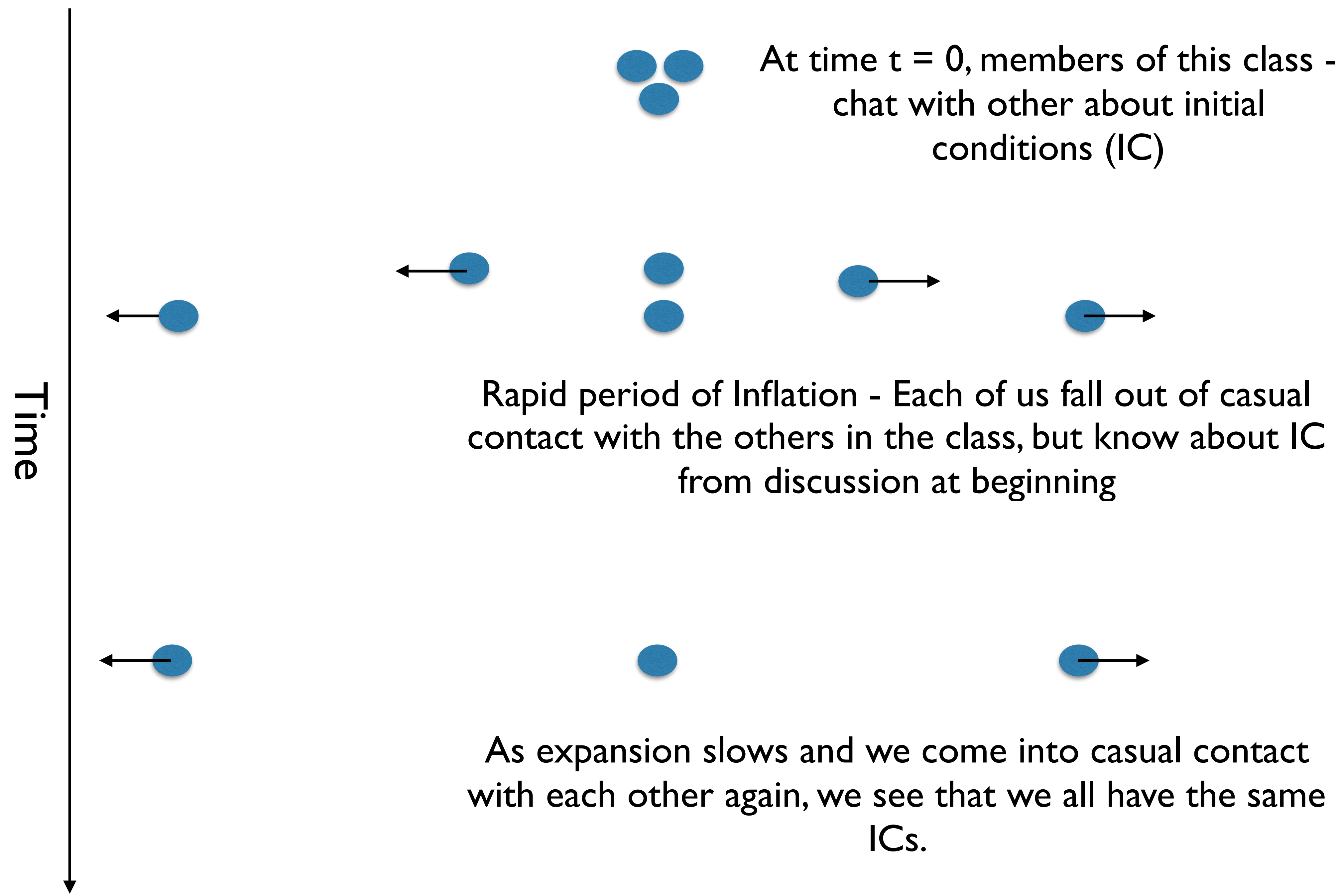


scales as a

H ~ constant
scales as 1/a

Dodelson (2003)

Let's take as an apology the following situation:



Inflation

Imagine the early universe was filled with a scalar field $\Phi(\mathbf{x},0) = \Phi_0 > 0$, i.e., not in the ground state.

In this case, it may lead to accelerated expansion; after a while the field decays into particles (causing reheating)

The Lagrangian of a scalar field is $L = -(1/2)c^2(\partial_\mu\Phi)(\partial^\mu\Phi) - V(\Phi)$

If we assume homogeneity and isotropy, we can define effective density and pressure:

$$\rho_\Phi c^2 = (1/2)(d\Phi/dt)^2 + V(\Phi)$$

$$P_\Phi = (1/2)(d\Phi/dt)^2 - V(\Phi)$$

To get $P_\Phi < -\rho_\Phi c^2/3$

$$(1/2)(d\Phi/dt)^2 - V(\Phi) < -(1/3)((1/2)(d\Phi/dt)^2 + V(\Phi))$$

$\Longrightarrow (d\Phi/dt)^2 < V(\Phi)$ slow roll condition

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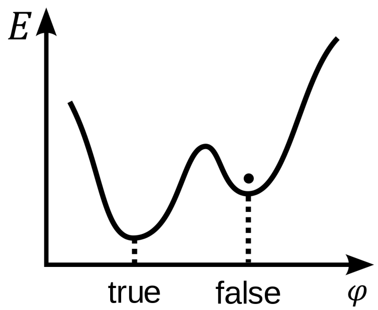
Inflation models

The behavior of the model depends on the potential (or vice versa)

- **Old inflation**: first-order transition (potential barrier) does not work



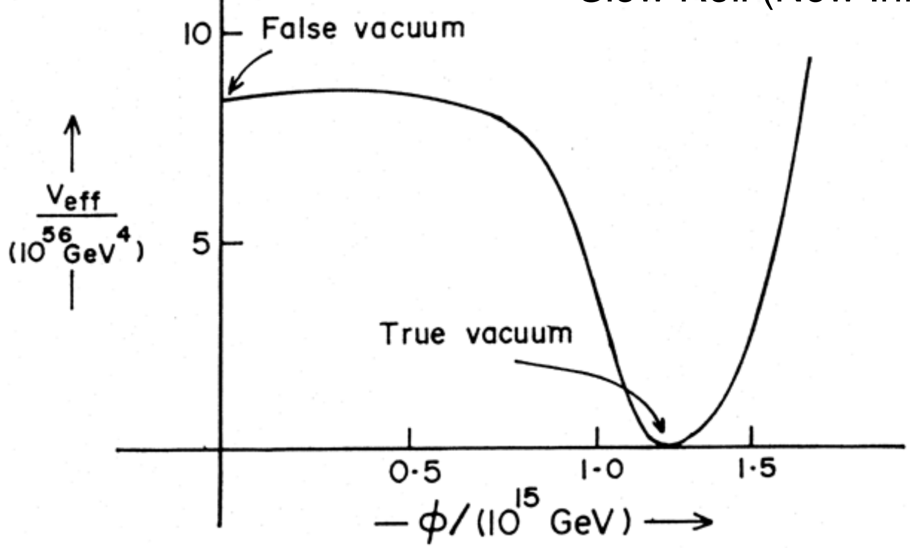
original idea does not work because
the false vacuum areas grow too fast

Inflation models

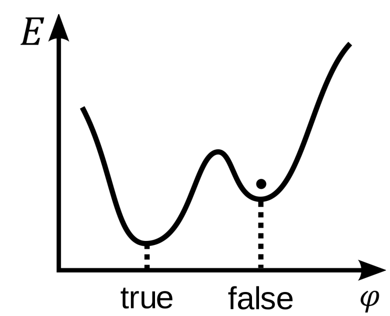
The behavior of the model depends on the potential (or vice versa)

- **Old inflation**: first-order transition (potential barrier) does not work
- **New inflation**: second-order transition, with a slow-roll phase; suffers from fine-tuning the model

Slow Roll (New Inflation)



Old Inflation (Potential barrier)



Inflation models

The behavior of the model depends on the potential (or vice versa)

- **Old inflation**: first-order transition (potential barrier) does not work
- **New inflation**: second-order transition, with a slow-roll phase; suffers from fine-tuning the model
- **Chaotic inflation**: the scalar field varies from place-to-place; inflation occurs if conditions are favorable and "take-over" the Universe
- **Stochastic inflation**: many instances of chaotic inflation spawning new macro-universe; the multi-verse may also solve the cosmological constant problem.

In the end we lack a solid connection to particle physics. This is needed to really make progress.

Inflation

$[V(\Phi)] = \text{erg} / \text{cm}^3$ $\qquad$ $[(d\Phi/dt)^2] = \text{erg/cm}^3$

$$[\Phi] = (\text{erg s}^2 / \text{cm}^3)^{1/2} = (\text{g/cm})^{1/2}$$

$$\Rightarrow \Phi_{Planck} = (m_{Planck}/\rho_{Planck})^{1/2} = c/G^{1/2}$$

The form of the potential depends on the adopted theory. Because we do not have a definitive model, people consider various choices of $V(\Phi)$:

$V(\Phi) = \lambda(\Phi^2 - M^2)^2$	Higgs potential
$V(\Phi) = (1/2)m^2\Phi^2$	Massive scalar field
$V(\Phi) = \lambda\Phi^2$	Self-interacting scalar field

Take the Friedmann & Fluid equations and insert the expression for pressure and density of the scalar field:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$d\rho/dt + 3((da/dt)/a)(\rho + P/c^2) = 0$$

Inflation

Take the Friedmann & Fluid equations and insert the expression for pressure and density of the scalar field:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$d\rho/dt + 3((da/dt)/a)(\rho+P/c^2) = 0$$

$$\Rightarrow H^2 = (8\pi G/3c^2)((1/2)(d\Phi/dt)^2 + V(\Phi)) - kc^2/a^2$$

$$\Rightarrow d^2\Phi/dt^2 + 3H(d\Phi/dt) = -dV(\Phi)/d\Phi$$

If $d\Phi/dt = 0$ or if Φ and $V(\Phi)$ do not change much over the period where a increases exponentially, then we can assume $k \sim 0$.

Inflation

and if $(d\Phi/dt)^2 << V(\Phi)$ => $H^2 = (8\pi G/3c^2)\, V(\Phi)$ and

if $d^2\Phi/dt^2 << dV(\Phi)/d\Phi$ => $3H(d\Phi/dt) = -dV(\Phi)/d\Phi$

One can show that the slow roll conditions are

$$\varepsilon = (c^2/16\pi G)\ (dV'(\Phi)/V(\Phi))^2 << 1$$

and

$$\eta = (c^2/8\pi G)(d^2V(\Phi)/d\Phi^2\ / V(\Phi)) << 1$$

The potential must be very flat

$\varepsilon < 1$ needed for inflation

$\eta < 1$ needed for prolonged inflation

If we consider $V(\Phi) = (1/2)m^2\, \Phi^2$

=> $dV/d\Phi\ / V = 2 / \Phi$ and => $d^2V/d\Phi^2\ / V = 2 / \Phi^2$

Slow roll conditions are as follows:

$\Phi >> 1/(4\pi)^{1/2}\ \Phi_{Planck}$ $\Phi >> 1/(4\pi)^{1/2}\ \Phi_{Planck}$ i.e. $\Phi >> 1/(4\pi)^{1/2}\ \Phi_{Planck}$

Inflation continues until Φ drops below $\Phi/(4\pi)^{1/2}$

Inflation

What is the form of the expansion of the universe?

We want the time scale for variations in expansion time scale to be slow:

$$\tau_{vary} \sim (d\ln H / dt)^{-1} >> t_{exp} = 1/H$$

Equivalent to the following:
$(dH/dt)^2 << H^2$

This implies that $a(t) \propto e^{Ht}$

Let's consider Friedmann equation:

$$=> H^2 = (8\pi G/3c^2)V(\Phi) = (4\pi G/3c^2)\, m^2\, \Phi^2 = (G/3c^2)m^2\, \Phi_{Planck}^2 = m^2/3$$

since $(d\Phi/dt)^2 << V(\Phi)$

As such, the expansion time scale is $\tau_{exp} = 1/H = (3)^{1/2}/m$

For successful inflation $\tau_{infl} >> \tau_{exp}$

$$\tau_{infl} >> (3)^{1/2}/m$$

$$\tau_{infl} >> 1/m$$

Inflation

Right after the end of inflation, the inflation field should, by definition, have an energy density corresponding to the central energy density:

$$\rho c^2 = \rho_{crit} c^2 = (3H^2c^2/8\pi G) = (3H^2\,\Phi_{Planck}{}^2\,/\,8\pi)$$

If we take $\rho c^2 = V(\Phi) = 1/2\ m^2\,\Phi^2$ and use $\Phi = \Phi_{Planck}/(4\pi)^{1/2}$

we get for the end of inflation:

$$\rho c^2 = m^2\,\Phi_{Planck}{}^2\,/\,8\pi$$

$$(3H^2\,\Phi_{Planck}{}^2\,/\,8\pi) = m^2\,\Phi_{Planck}{}^2\,/\,8\pi$$

$\Longrightarrow H^2 = m^2/3$ (consistent with the previous estimate)

If we assume that inflation ends at $\tau_{infl} = 10^{-32}$ s

this gives a mass scale (including $\hbar$ correctly)

$$mc^2 \sim 6.5 \times 10^4 \text{ TeV}$$

Inflation and Horizon Problem

What about homogeneity? (i.e., the horizon problem)

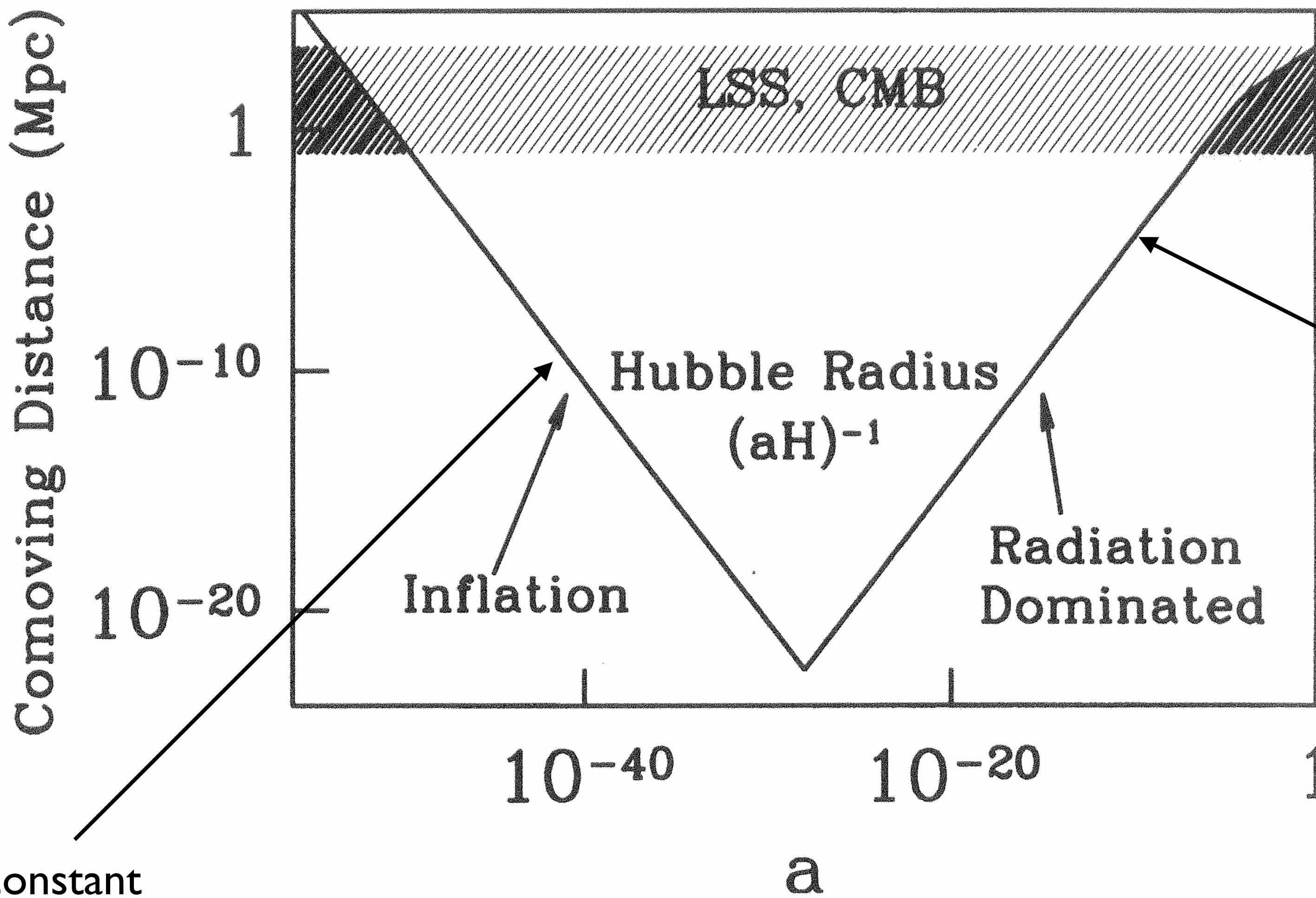
To solve the horizon problem, we need to make sure there is enough time for inflation

information must have been able to propagate the distance of what is now the observable universe.

the shrinking of the comoving Hubble radius 1/aH has to be at least as large as the subsequent increase

The diagram on the next page illustrates what is needed

Inflation

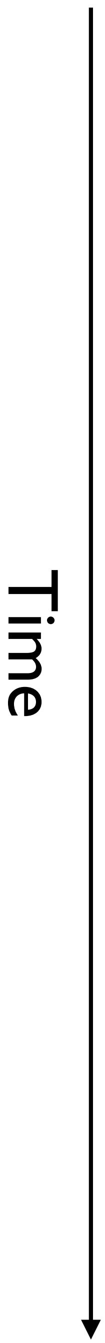


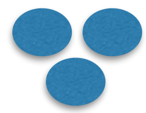
scales as a

H ~ constant
scales as 1/a

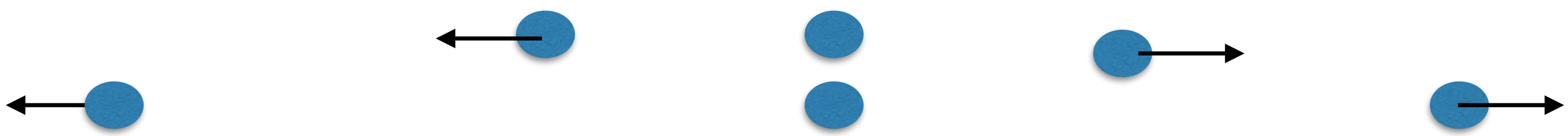
Dodelson (2003)

Let's take as an apology the following situation:





At time t = 0, members of this class - chat with other about initial conditions (IC)



Rapid period of Inflation - Each of us fall out of casual contact with the others in the class, but know about IC from discussion at beginning

As expansion slows and we come into casual contact with each other again, we see that we all have the same ICs.

Inflation and Horizon Problem

By how much has the horizon grown since the beginning during the period when the universe was radiation dominated?

Current age of the universe is ~4×10^{17} s
End of Inflation is ~10^{-32} s

By how much did the horizon grow (in comoving coordinates):

$$1/(aH) \sim 1/(a(a^{-2})) \sim a \sim t^{1/2}$$

the scale factor grew by $((4 \times 10^{17}\ \text{s})/(10^{-32}))^{1/2} \sim 10^{25}$

Requires >~ 60 e-folding times

If we want ~60 e-foldings in 10^{-32} s, we have $\tau_{exp} \sim 10^{-34}$ s and the time to homogenize is still 10x shorter. => we expect homogeneous universe

Inflation and Flatness Problem

What is the flatness problem?

Now observe $|\Omega_k| << 0.01$ so at $t_{infl} \sim 10^{-32}$ s or $a = 10^{-25}$ and because $\Omega_k \propto a^2$ ever since inflation, then
$|\Omega_k(t_{infl})| << 10^{-52}$.

Does inflation solve the flatness problem?

Inflation causes a to grow exponentially but H^2 and $V(\Phi)$ to remain fixed:

$$H^2 = (8\pi G/3c^2)\, V(\Phi) - kc^2/a^2$$

As a result, the kc^2/a^2 becomes very small very rapidly

Assuming that at the start of inflation Ω_k was ~1 then inflation must have increased by at least $10^{26} = e^{60}$

$\tau_{infl} >= 60\ \tau_{exp}$

Note that after inflation $V(\Phi)$ the field decays into ρc^2 of matter which behaves relativistically $\propto 1/a^4$

Inflation and Magnetic Monopole Problem

What is the magnetic monopole problem?

One expects the magnetic monopoles to have a density of one per horizon volume…

One expects the magnetic monopoles to have a density of one per horizon volume…

i.e. where the horizon distance is 3×10^{-27} m at $t \sim 10^{-35}$ s

So, potentially high density of magnetic monopoles!

How does inflation solve it?

By blowing up the universe by a factor 10^{26}, ensure that there is less than one magnetic monopole in entire observable universe.