Origins & Evolution of the Universe

an introduction to cosmology — Fall 2018

Lecture 6: Thermal History, Early Universe

Rychard Bouwens

Layout of the Course

Sep 24: Introduction and Friedmann Equations
Oct 1: Fluid and Acceleration Equations
Oct 8: Introductory GR, Space Time Metric, Proper Distance
Oct 15: Redshift, Horizons, Observable Distances
Oct 17: Problem Class #1
Oct 22: Observable Distances, Parameter Constraints
Oct 29: Thermal History, Early Universe
Nov 5: Early Universe, Inflation
Nov 12: Inflation, Lepton Era
Nov 14: Problem Class #2
Nov 19: Big Bang Nucleosynthesis, Recombination
Nov 26: Introduction to Structure Formation
Dec 3: Cosmic Microwave Background Radiation (I)
Dec 5: Problem Class #3
Dec 10: Cosmic Microwave Background Radiation (II)
Dec 21: Final Exam

Expect to receive problem set #2 by mail by Wednesday

Due by Wednesday at 13:30 November 14

Review Last Week

“Classical Cosmology” “Question for Two Numbers”

In the early days of observational cosmology much emphasis was placed on geometric properties.

Sandage: “We need to determine H_0 and q_0”

deceleration parameter

$$a(t) = a(t_0) + \left.\frac{da}{dt}\right|_{t=t_0}(t-t_0) + \frac{1}{2}\left.\frac{d^2a}{dt^2}\right|_{t=t_0}(t-t_0)^2 + \ldots$$

$$\Rightarrow a(t) = 1 + H_0(t-t_0) - (1/2)q_0 H_0^2 (t-t_0)^2$$

where $q_0 = -((d^2a/dt)a / (da/dt)^2)_{t=t0} = -((d^2a/dt) / aH^2)_{t=t0}$

Acceleration Equation

$$(d^2a/dt^2)/a = -(4\pi G/3c^2)\Sigma_w \rho_w (1+3w)$$

$$\Rightarrow \quad q_0 = \Omega_m/2 + \Omega_r - \Omega_\Lambda$$

Proper distance is as follows:

$$d_p(t_0) = c(t_e-t_0) + cH_0/2\ (t_0 - t_e)^2$$

$$d_p(t_0) = cz/H_0(1 - (1+q_0)z/2)$$

Proper distances cannot be directly measured, so use other distances

One way to assign a distance is to use the luminosity

$$d_L = (L/4\pi f)^{1/2}$$

The "luminosity" distance is the proper distance in a static and Euclidean universe

If we consider a FRW metric

$$ds^2 = -c^2\, dt^2 + a^2(t)[dr^2 + S_k(r)\; (d\theta^2 + \sin^2\theta\; d\Phi^2)]$$

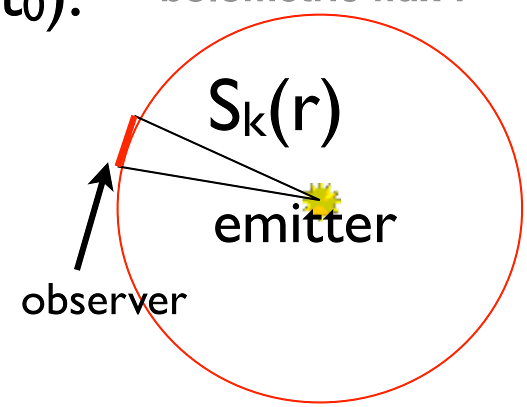
Photons emitted at time t_e spread out over a sphere with radius $d_p(t_0) = r$ and surface area $A_p(t_0)$:

bolometric flux !

$$A_p(t_0) = 4\pi a^2(t_0) S_k^2(r)$$

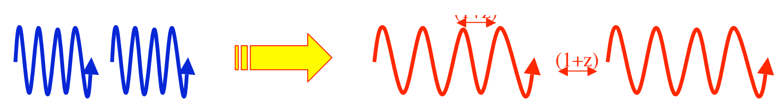
If $k > 0$, $A_p(t_0) < 4\pi r^2$

If $k < 0$, $A_p(t_0) > 4\pi r^2$



In addition to the geometric effects, the expansion of the universe causes the flux to be decreased by a factor $(1+z)^2$

(1)effect of redshifting on energy of photons
(2) time delay between photons



Thus,

$$f = L/(4\pi a^2(t_0) S_k^2(r)(1+z)^2)$$

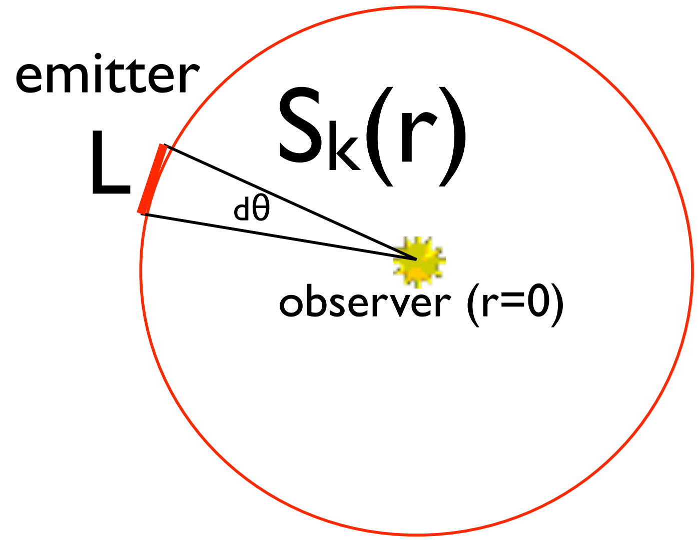
$$d_L = a(t_0) S_k(r)\; (1+z)$$

Another way to assign a distance is to use the angular size $d\theta$

$$d\theta = L / d_A$$

since more distant objects are smaller in general!

Consider FRW metric again where the coordinates of the emitter are as follows:



one side of emitter: (r, θ_1, Φ)

other side of emitter: (r, θ_2, Φ)

$$d\theta = |\theta_1 - \theta_2|$$

Angle subtended by the distant source is set when the source emits its light:

$$ds = a(t_e)S_k(r)\, d\theta = L$$

$$(a(t_0)/(1+z))S_k(r)\, d\theta = a(t_0)S_k(r)d\theta \,/\, (1+z)$$

$$d_A = a(t_0)S_k(r) \,/\, (1+z)$$

Comparison of Luminosity Distance with Angular Diameter Distance Yields the Following:

$d_L = a(t_0)S_k(r)\ (1+z)$ $\qquad$ $d_A = a(t_0)S_k(r)\ /\ (1+z)$

$$d_A = d_L/(1+z)^2$$

For small $z \ll 1$,

$$d_A = (c/H_0)z\ (1 + (-q_0-3)z/2)$$

$$d_L = (c/H_0)z\ (1 + (-q_0+1)z/2)$$

For large $z \to \infty$,

$$d_L = z\ d_{Hor}(t_0)$$

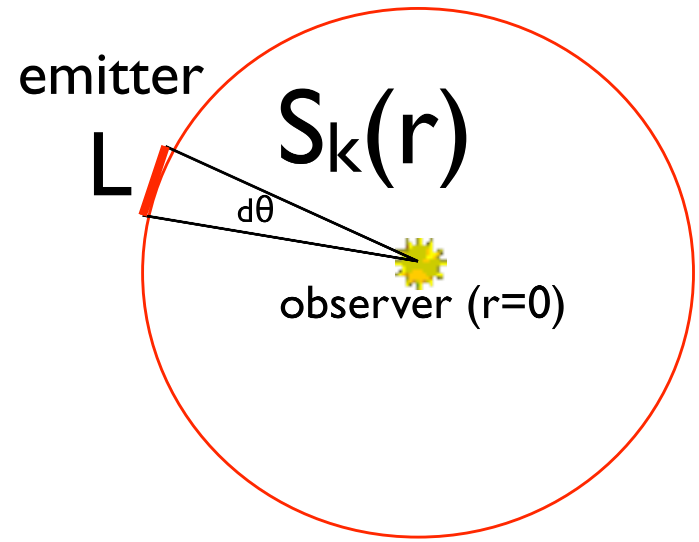
$$d_A = d_{Hor}(t_0)\ /\ z$$

Note that while $d_P(z_1,z_2) = d_p(0,z_2) - d_p(0,z_1)$

but $d_A(z_1,z_2) \neq d_A(0,z_2) - d_A(0,z_1)$

Another way to assign a distance is to use the measured proper motion $d\theta/dt$:

$$d_M = (dL/dt_0) / (d\theta/dt_0)$$



Angle traversed by the distant source is set when the source emits its light:

$$(a(t_e)S_k(r)\, d\theta/dt)\, dt_0 = L$$

$$(a(t_0)S_k(r)(d\theta/dt_e) / (1+z))dt_0 = L$$

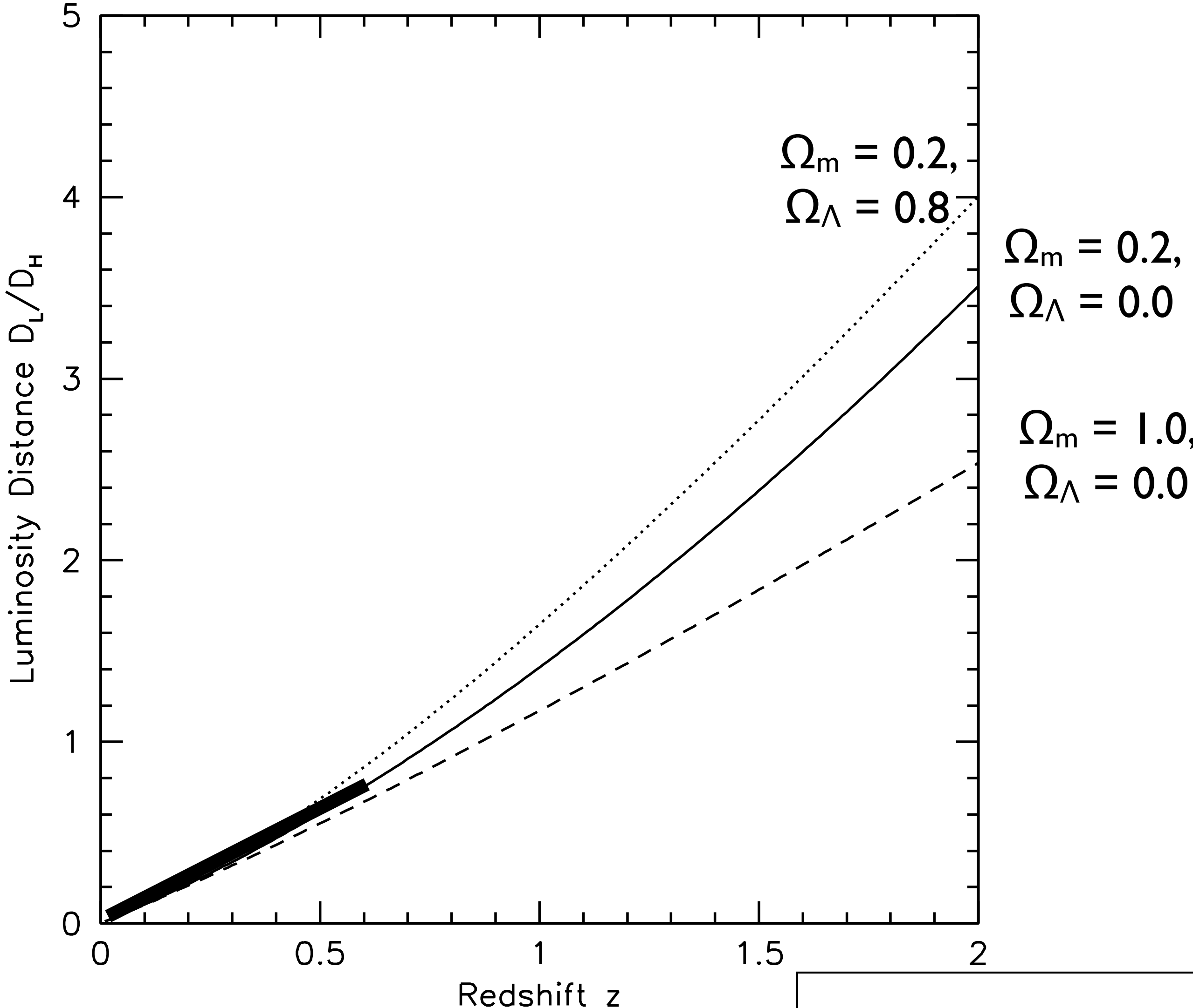
$$a(t_0)S_k(r)d\theta = L$$

Thus,

$$d_M = a(t_0)S_k(r) = d_L / (1+z) = d_A (1+z)$$

Luminosity Distance - Redshift Relation

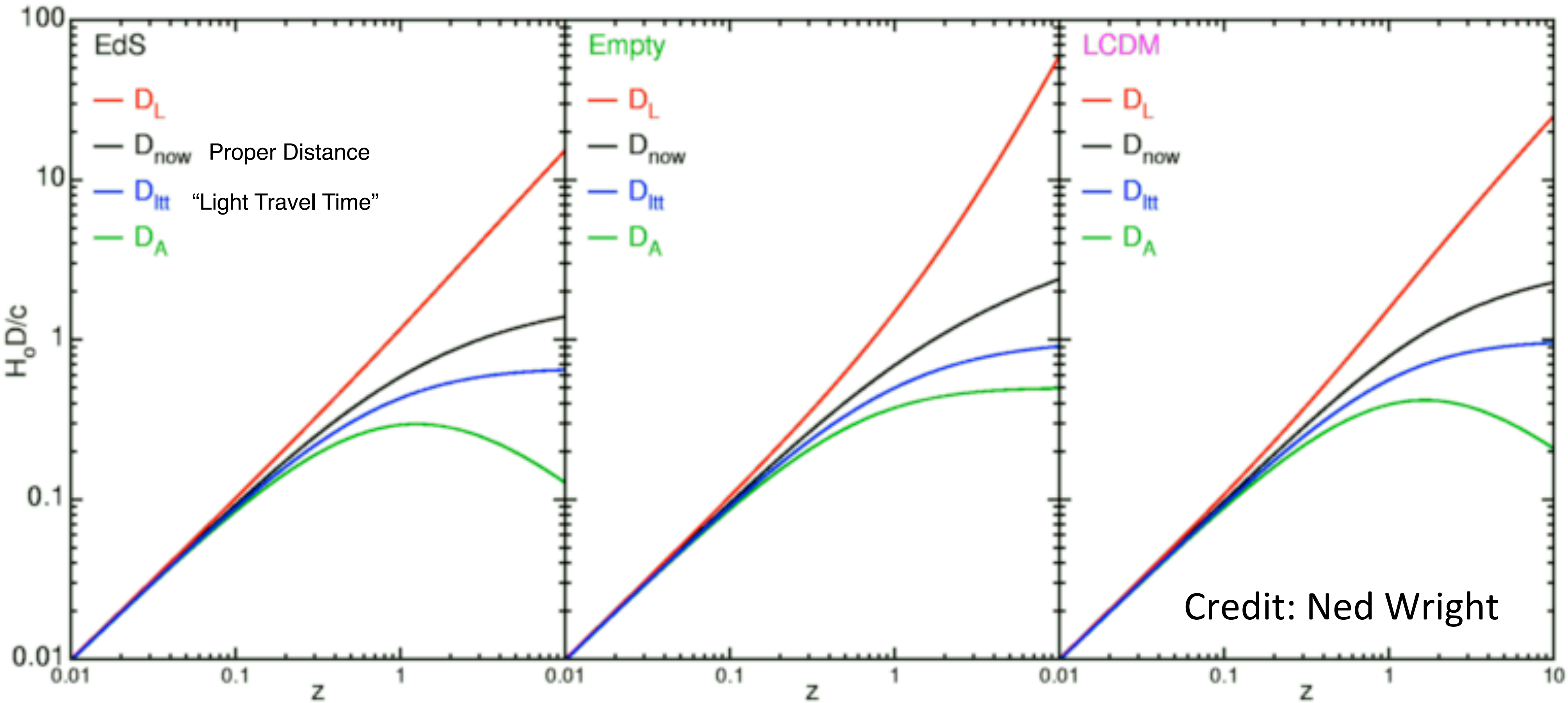
Luminosity Distance



Two different ways of increasing the luminosity distance:

1) Increase Ω_Λ
2) Decrease Ω_m

Distance-redshift relations…



… depend on cosmological parameters!

light travel time distance = c x age of universe

http://www.astro.ucla.edu/~wright/cosmo_02.htm

What are the measured densities in various components of the universe?

What is Ω_{mat}? (density of normal matter relative to critical)

$\Omega_{mat,bary} = 0.04$ Baryonic matter component

0.31

Yields of Helium allow this to be measured + from acoustic oscillations in CMB

0.27 Dark matter component

Measurable from bulk flows / peculiar velocities / and from CMB

What is Ω_{rad}? (energy density of radiations relative to critical)

0.0001

Measurable from temperature of CMB (since it has a black body spectrum)

What is Ω_{Λ}? (energy density of radiations relative to critical)

0.69

Measurable from luminosity distances to SNe

The Flatness Problem

The relation between distance, Hubble constant, and density parameters can be seen by writing the first of Friedmann's equations as follows:

$$H(t)^2 = (8\pi G/3c^2)\varepsilon(t) - \kappa c^2/(R_0^2\, a^2(t))$$

Dividing by $H^2(t)$ and realizing that $(8\pi G/3H^2(t))(\varepsilon(t)/c^2) = \Omega(t)$

$$1 - \Omega(t) = -\kappa c^2/(R_0^2\, a^2(t) H^2(t))$$

Manipulating this expression, one can show

$$1 - \Omega\,(t) = H_0\,(1 - \Omega_0)/(H(t)a(t))^2$$

At early times, one can show the following:

$$1 - \Omega\,(t) = H_0\,(1 - \Omega_0)a^2/(\Omega_{r,0} - a\Omega_{m,0})$$

during radiation domination $\quad |1 - \Omega\,(t)| \propto a^2 \propto t$

during matter domination $\quad |1 - \Omega\,(t)| \propto a \propto t^{2/3}$

deviation grows with time

if $\Omega - 1 \sim$ small now, then $|1 - \Omega| \sim 10^{-60}$ at very early times

as this suggests fine tuning, this is the flatness problem

New Material for This Week

The hot big bang

The Universe started in a very hot phase

Cosmic Microwave Background

There was a time where the Universe was so hot that all the hydrogen was ionized. Photons and plasma were tightly coupled due to frequent interactions.

As the temperature dropped the protons and electrons combined into neutral hydrogen and decoupled from the photons: the Universe became neutral and transparant. This occurred at $z_{rec} \approx 1100$.

This is called *recombination*. Note that the Universe is currently ionized again…

Recombination Epoch (z~1100)

Ionized Plasma → Neutral Gas

Ionized Plasma

(z > 1100)

< 380,000 years

Temperature
> 3600 K

Hydrogen ionized

Photons Thomson-scattering off of the ionized hydrogen

Neutral Gas

(z < 1100)

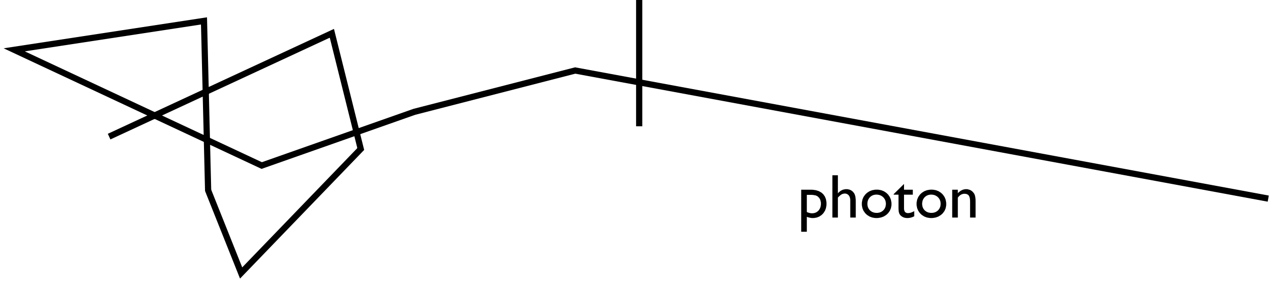
> 380,000 years

Temperature
< 3600 K

Hydrogen neutral

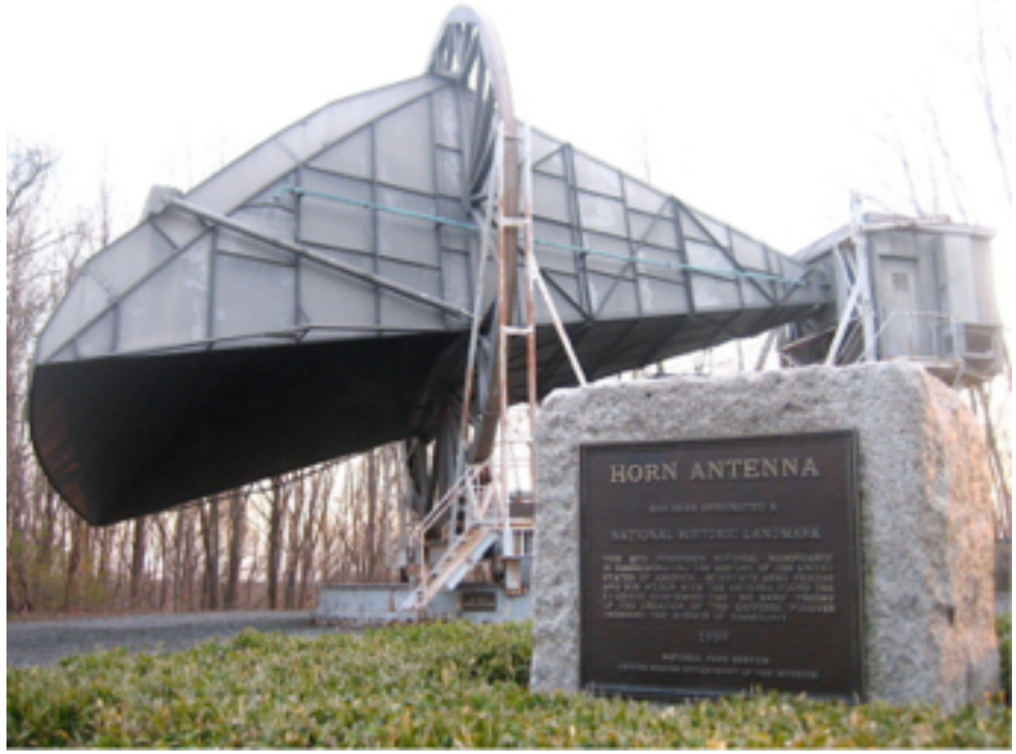
Almost no free electrons

Photons unbound from plasma



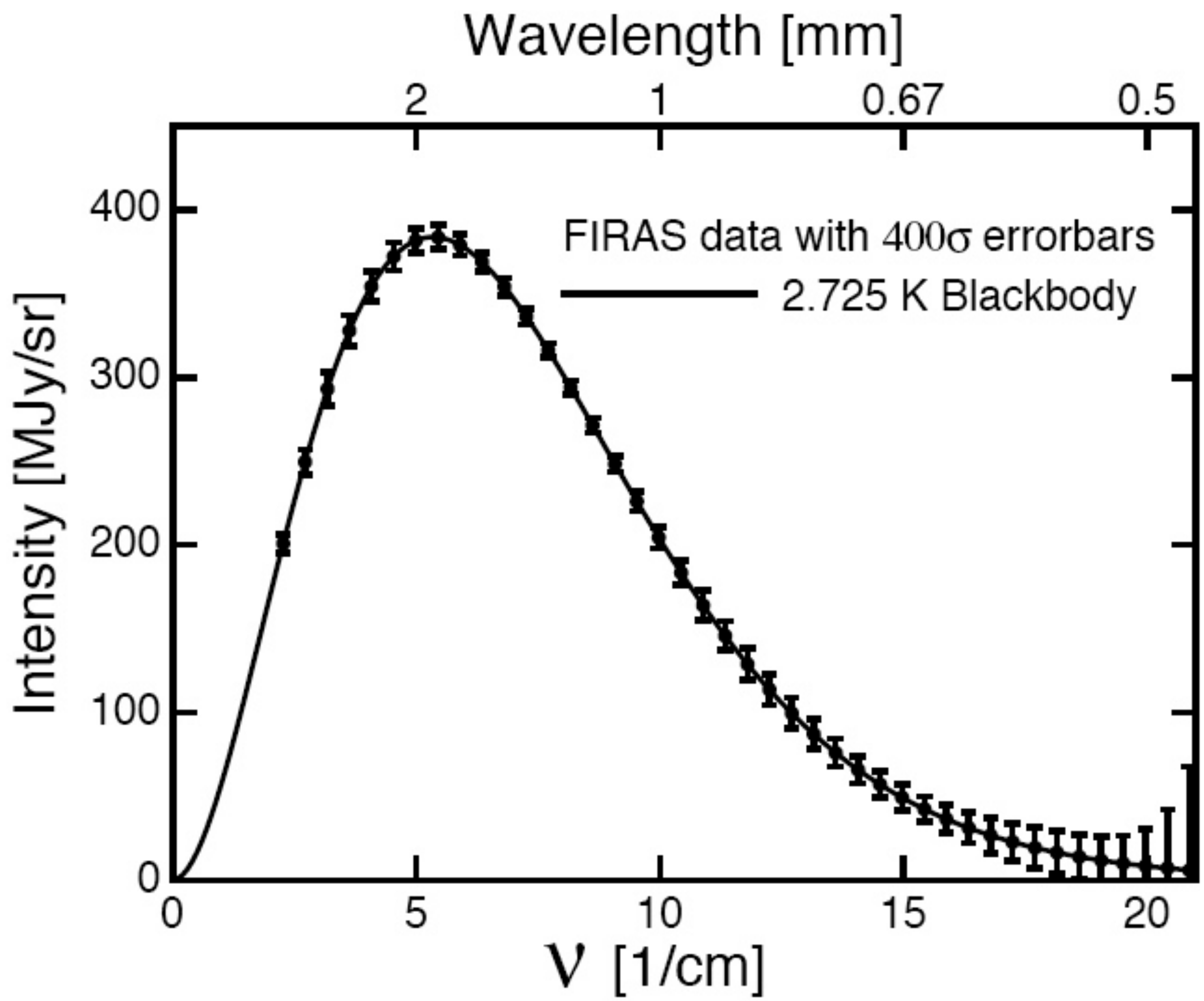
photon

Cosmic Microwave Background



In 1964 Penzias & Wilson detected radio noise that came from all directions. This discovery made cosmology a genuine science topic! It has developed into the prime tool for precision cosmology and the measurements keep improving .

Perfect Black Body?



The CMBR emission peaks in mm-wave part of spectrum and is (still) the most precisely measured perfect black body with T = 2.72548±0.00057 K.

How does the temperature of the CMB evolve with time?

We already discussed how the energy density in the CMB blackbody radiation is as follows:

$\varepsilon_{rad} = \alpha T_{rad}^4$ $\qquad\qquad$ $\alpha = \pi^2 k_B^4 / (15\hbar^3 c^3)$

Given that $\rho_R \propto a^{-4}$, it would seem that $T \propto a^{-1}$

If the temperature is changing then the thermal distribution must change as well:

$$\varepsilon(\nu) = (8\pi h/c^3)\nu^3 d\nu/(e^{h\nu/KT} - 1)$$

If we scale $\nu \propto 1/a$, this expression retains its black body form with $T \propto 1/a$.

Therefore, as long as early stage interactions were frequent enough to set up a black body energy distribution, it will persist to later times when interactions become less frequent.

How does the temperature of the CMB evolve with time?

Note that the temperature evolution of matter and radiation is different. We already see see that $\varepsilon_{rad} \propto T^4$ and $T \propto (1+z)$.

This can also be seen by considering adiabatic expansion $TV^{\gamma-1}$ = constant

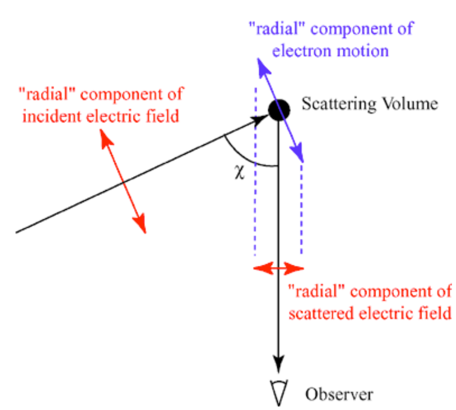
For relativistic gas $\gamma = 4/3$ (and $V \propto a^3$) => $T \propto 1/a \propto (1+z)$

For relativistic gas $\gamma = 5/3$ => $T \propto 1/a^2 \propto (1+z)^2$

Back in time the matter temperature rises faster than the radiation temperature until t_d when matter and radiation were coupled. Before t_d, the temperatures are equal by thermal interactions (Thomson scattering).

Thomson scattering

Photons and charged particles in a plasma interact. The dominant process is the elastic scattering of radiation by a charged particle. At low energies this is called Thomson scattering.



The cross section is independent of wavelength and largest for electrons.

After t_d, the matter density evolves as $(1+z)^3$ and the radiation density as $(1+z)^4$

Before t_d, they evolve as $(1+z)^{4+\varepsilon}$, where $\varepsilon(z)$ is due to the exchange of energy with matter

$\varepsilon(z)$ is small due to the high photon-to-baryon ratio

Adiabatic gas + radiation in a coming volume evokes with $dE + pdV = 0$ (no heat flow):

$$d[(\rho_m c^2 + 3\rho_m kT/2m_p + \sigma_r T^4)a^3] = -(\rho_m kT/m_p + \sigma_r T^4/3)\, d(a^3)$$

we assume that the matter component has the equation of state of a perfect gas $P = \rho_m kT/m_p$ and $\rho_m a^3$ = constant because of mass conservation

If we define $\sigma_{rad} = 4m_p\, \sigma_r\, T^3 / 3k\rho_m$ (entropy per baryon), one can show that

$$dT/T = -(1+ \sigma_{rad})/(1/2 + \sigma_{rad})\ (da/a)$$

Because σ_{rad} depends on the unknown $T(a)$, we cannot integrate this analytically

If we interpret T as T_{rad}, then $\sigma_{rad}(T)$ does not depend on the scale factor a

$$\sigma_{rad}(t_0) = 4m_p\sigma_r T^3/3kB\rho_{m,0} \approx 3.6/\eta_0$$

where $\eta_0^{-1} = n_{\gamma,0}/n_{b,0}$

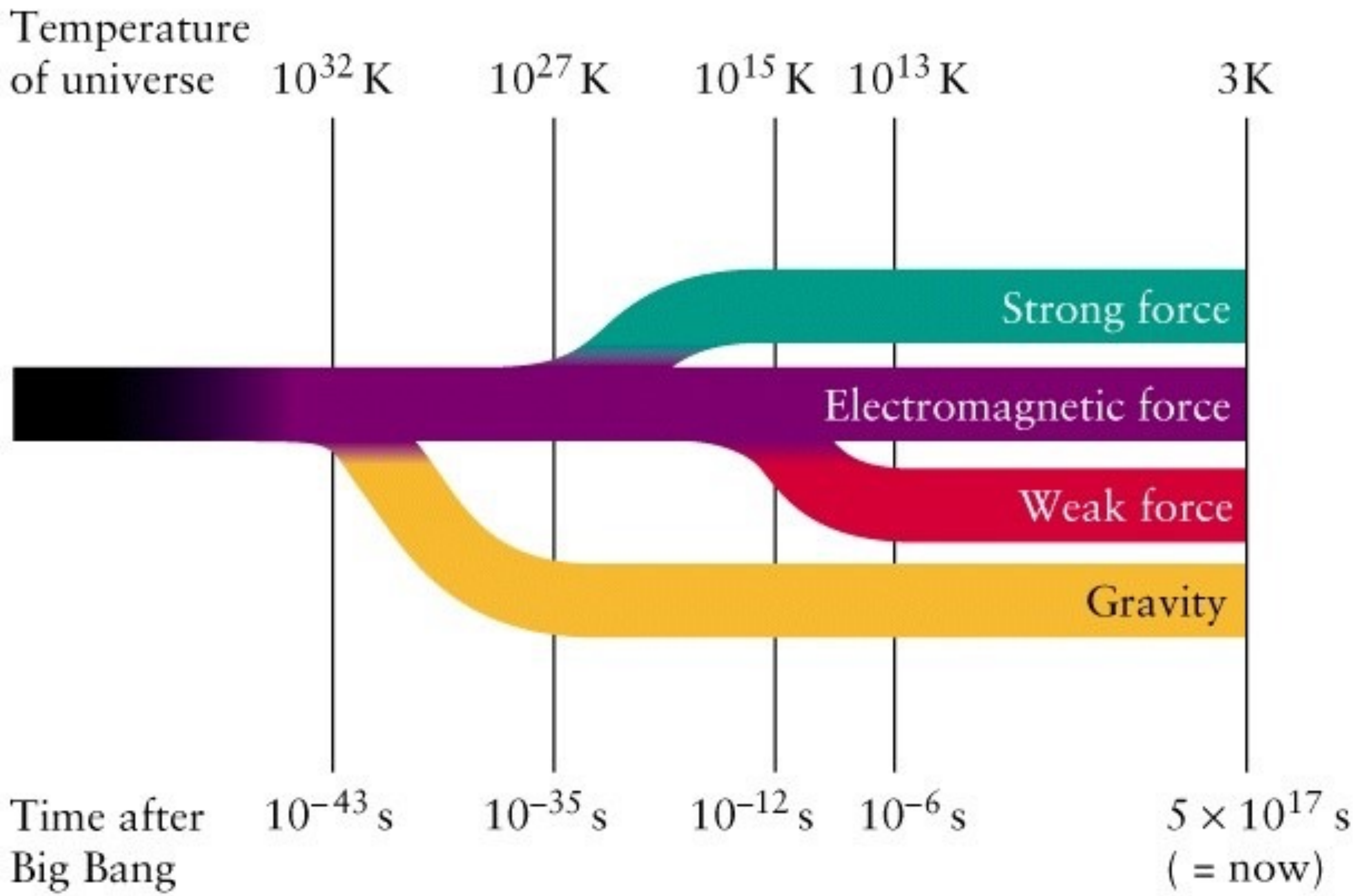
In the case that σ_{rad} is large (as we will show is the case), $dT/T \approx -da/a$

$\Rightarrow \varepsilon \approx 0$ because of the large $1/\eta$

At higher temperatures, matter also becomes relativistic and the total equation of state $w \sim 1/3$ and $T \propto (1+z)$

the temperature keeps rising towards early time

Epoch	Time (s)	Density (kg/m^3)	Temp (K)	Events
Planck	$0 - 10^{-43}$	∞	∞	•Quantum gravity
GUT	10^{-43} - 10^{-35}	10^{95} - 10^{75}	10^{32} - 10^{27}	•Gravity splits off •Strong, weak, EM combined
Quark	$10^{-35} - 10^{-4}$	10^{75} - 10^{16}	10^{27} - 10^{12}	•Strong force frozen out •Particles in thermal equilibrium •EW freezes out at 10^{15} K
Lepton	$10^{-4} - 10^{2}$	10^{16} - 10^{4}	10^{12} - 10^{9}	•Low-mass particles in thermal equilibrium •Neutrinos decouple at 10^{10}K
Nuclear	$10^{2} - 10^{12}$	10^{4} - 10^{-15}	10^{9} - 10^{5}	•Deuterium and helium formed by fusion during first 1000 sec


Temperature of universe
10^32 K
10^27 K
10^15 K
10^13 K
3K
Strong force
Electromagnetic force
Weak force
Gravity
Time after Big Bang
10^-43 s
10^-35 s
10^-12 s
10^-6 s
5 × 10^17 s
(= now)

What is the current density of photons in the universe from the CMB?

$\varepsilon_{rad}(t_0) = 4.2 \times 10^{-14}$ J m^{-3} Mean Energy = 3 $k_B T$ = 7.05×10^{-4} eV

$\Rightarrow \eta_\gamma = 3.7 \times 10^{8}$ / m^3

It is interesting to compare this to the # density of baryons in the universe:

$\Omega_b = 0.02\ h^{-2} \sim 0.04$

It is interesting to compare this to the # density of baryons in the universe:

$\varepsilon_{bar} = \rho_{bar}\, c^2 = \Omega_b\, \rho_{crit}\, c^2 \sim 3.4 \times 10^{-11}$ J / m^3

The rest-mass of a proton ~ 939 MeV $n_p = 0.22$ / m^3

photon / baryon ~ 1.7×10^{9} (this needs to be explained!)

Entropy per Baryon

How large is σ_{rad}, i.e., the entropy per baryon?

$$\sigma_{rad}(t_0) = 4m_p\sigma_r T^3/3kB\rho_{m,0} \approx 3.6/\eta_0 \approx 1.35 \times 10^8 (\Omega_{b,0} h^2)^{-1}$$

which is a very large number as $\Omega_b \sim 0.04$

The high value of σ_{rad} ensures that the temperature and density of the radiation evolve as a pure radiation universe.

σ_{rad} is actually related to the entropy of the radiation per unit volume.

$$S_r = (\rho_{rad}c^2 + P_{rad})/T = (4/3)\rho_{rad}c^2/T = (4/3)\sigma_r T^3$$

$\sigma_{rad} = S_r/k_B n_b$ where $n_b = \rho_m / m_p$ number density of baryons

as $\rho_{rad}c^2 = \sigma_r T^4$

and $\eta^{-1} = n_\gamma/n_b$

$\sigma_{rad} = 3.6\eta^{-1}$

σ_{rad} is also proportional to the ratio of the heat capacities C of radiation and plasma:

radiation $\rho_{rad}C_{rad} = dE/dT = 4\sigma_r T^3$ (per unit volume)

plasma $\rho_m C_m = dE/dT = 3/2\ \rho k_B/m_p$ (per unit volume)

$(\rho_{rad}C_{rad})/(\rho_m C_m) = 2\sigma_{rad}$

radiation dominates the heat budget during matter-radiation coupling

Baryon asymmetry

Why is there now mostly matter, no anti-matter?

During the hadron era, there must have been many proton-anti-proton pairs; these annihilate as the universe cools, but a small residual of matter remained.

Above the GUT temperature: $\eta_b \sim \eta_{\bar{b}} \sim \eta_\Upsilon \sim T^3$

$(\eta_b - \eta_{\bar{b}})a^3$ remains constant because baryon number is conserved below $T \sim 10^{15}$ GeV.

What is the baryon asymmetry and can we relate it to σ_{rad}?

$$(\eta_b - \eta_{\bar{b}})/(\eta_b + \eta_{\bar{b}}) \approx (\eta_b - \eta_{\bar{b}})/2\eta_\Upsilon \approx \eta_{b,0}/2\eta_{\Upsilon,0} \propto 1/\sigma_{rad}$$

The asymmetry is very small : for every 10^9 anti-baryons, there are $10^9 + 1$ baryons. Or σ_{rad} is large because the asymmetry is so small.

Singularity

Matter or radiation dominated universes decelerate → we expect a finite age for the universe.

at $t = 0$, the density diverges and the proper distance between points goes to 0. This singularity is called the "Big Bang."

It is a consequence of
- the cosmological principle
- Einstein's equations in a cosmological context.
- the expansion of the universe $\frac{da/dt}{a} > 0$
- assumed form of the equation of state $0 < w < 1$

Current observations show that $\Lambda < (H_0/c)^2 \sim 10^{-55}\ \mathrm{cm}^{-2}$

too small to be relevant in the early universe

If the dynamics of the early universe are dominated by a homogeneous and isotropic scalar field then it may have been important early on

Singularity

If the dynamics of the early universe are dominated by a homogeneous and isotropic scalar field then it may have been important early on.

Such a field has a Lagrangian $L = \underbrace{(1/2)(d\Phi/dt)^2}_{\text{kinetic term}} \underbrace{-V(\Phi)}_{\text{effective potential}}$

One can define the effective density and pressure (it is not a fluid) as

$\rho_\Phi c^2 = (1/2)(d\Phi/dt)^2 + V(\Phi)$ $\qquad$ $P_\Phi = (1/2)(d\Phi/dt)^2 - V(\Phi)$

If the kinetic term is negligible compared to the potential term, then

$$P_\Phi = -\rho_\Phi c^2$$

It behaves like a fluid with $w = -1$ (thus violating the strong energy condition) or as an effective cosmological constant $\Lambda = (8\pi G/c^2)\rho_\Phi$

This could happen in a false vacuum at $T > 10^{12}$ K when quantum effects become important

Whether or not the singularity can be avoided is an open question: we do not understand the origin of the Universe.

Planck Scale

There is a fundamental limit in our understanding of physics when quantum mechanical effects and strong gravity occur on the same scale. We do not have a theory of quantum gravity.

When does this occur?

We have to define a Compton time for a body of mass m (or energy mc^2) to be

$t_c = \hbar/mc^2$

this represents the time to violate energy conservation by $\Delta E = mc^2$

The corresponding Compton length is $l_c = ct_c = \hbar/mc$

Note that t_c and l_c increase as the mass decreases: these scales indicate when quantum mechanics is important

The Schwarzschild radius is $l_s = 2GM/c^2$ and time $t_s = l_s/c = 2GM/c^3$

We need quantum gravity when $l_s = l_c$ => $m = (\hbar c/2G)^{1/2} \approx (\hbar c/G)^{1/2} \equiv m_P$

where m_P is the Planck mass

Planck Scale

Our definition of the unit of time is arbitrary, but it is possible to derive a time that is "natural" on which everybody in the Universe agrees: there is a unique combination of fundamental constants that yields a time:

$t_P = (\hbar G/c^5) \sim 10^{-43}$ s — the Planck time

Similarly we can define $l_P = ct_P = (G\hbar/c^3)^{1/2} \sim 1.7 \times 10^{-35}$ m — the Planck length

$m_P = (\hbar c/G)^{1/2} \sim 2.5 \times 10^{-8}$ kg — the Planck mass

$E_P = m_P c^2 = (\hbar c^5/G)^{1/2} \sim 1.2 \times 10^{19}$ GeV — the Planck Energy

$T_P = E_P/k_B = (\hbar c^5/k_B^2 G)^{1/2} \sim 1.4 \times 10^{34}$ K — the Planck Temperature

The first $t_P \sim 10^{-43}$ seconds cannot be described by GR or quantum mechanics.

The horizon $ct_P \sim$ Planck length and particle pairs are created which have the Planck mass separated by less than the Planck length → particles/black holes at once, with quantum effects on the scale of the horizon → we cannot describe this with known physics.

Experimental constraints

quantum gravity: $\sim 10^9$J

GUT: $\sim 10^6$J

Good to keep in mind that
1 eV = 1.6×10^{-19} J = $k_B(1.16 \times 10^4$ K)

cosmic rays: ~ 50J

LHC: $\sim 10^{-6}$J

nuclear reactions: $\sim 10^{-12}$J

atoms: $\sim 10^{-18}$J

ENERGY

Is the Early Universe In Thermal Equilibrium?

We need to look at the collision time scale τ_{coll}
and compare this with the Hubble time τ_H (~age of universe)?

To calculate the collision time, we need to know the temperature and density.

We can assume that after the Planck time: $T(t) = T_P\, a(t_P)/a(t)$

$\Longrightarrow$ early on all particles are relativistic!

What is the equilibrium number density of a particle species i?

This depends on whether it is a fermion or a boson, and how many spin or helicity states it possesses, g_i

The number density is given by

$$n_i = g_i (k_B T/\hbar c)^3 \int_0^\infty x^2 dx \,/\, (e^x \pm 1) = \begin{pmatrix} 3/4 \\ 1 \end{pmatrix} (g_i/\pi^2)\zeta(3)\,(k_B T/\hbar c)^3$$

fermion

(+ for fermions and −1 for bosons)

boson

Riemann ζ function

$\zeta(3) \sim 1.202$

The energy density is given by

$$\rho_i(T)c^2 = (g_i k_B^4 T^4)/(2\pi^2\hbar^3 c^3) \int_0^\infty x^3 dx \,/\, (e^x \pm 1) = \begin{pmatrix} 7/8 \\ 1 \end{pmatrix} (g_i/2)\sigma_r T^4$$

Is the Early Universe In Thermal Equilibrium?

the total energy density is

$$\rho_i(T)c^2 = (\Sigma_B\, g_{iB} + (7/8)\Sigma_F\, g_{iF})\sigma_r T^4/2 = g^*(T)\sigma_r T^4/2$$

effective degrees of freedom:
$g^*(T) < 200$ or so

To get the total energy density one should add particles that have decoupled and are no longer in thermal equilibrium, or no longer relativistic; but this is negligible in the early universe

The average separation between particles is $d = [g^*(T)n_B]^{-1/3} \sim n_B^{-1/3} \sim \hbar c/k_B T$

The cross section of all particles:

$$\sigma = \alpha^2(\hbar c/k_B T)^2 \qquad \alpha = 1/50$$

The collision time is

$$\tau_{coll} = 1/(n\sigma c) = \hbar/(g^*(T)\alpha^2 k_B T)$$

Is the Early Universe In Thermal Equilibrium?

The collision time is

$$\tau_{coll} = 1/n\sigma c = \hbar/(g^*(T)\alpha^2 k_B T)$$

This can be compared to the expansion time scale $\tau_H = a/(da/dt)$

$$\tau_H = 2t = (3/32G\pi\rho)^{1/2} = (0.3\hbar T_p/(g^*(T)^{1/2} k_B T^2)$$
$$= (2.42 \times 10^{-6})\ (T/\mathrm{GeV})^{-2}/(g^*(T))^{1/2}\ \mathrm{s}$$

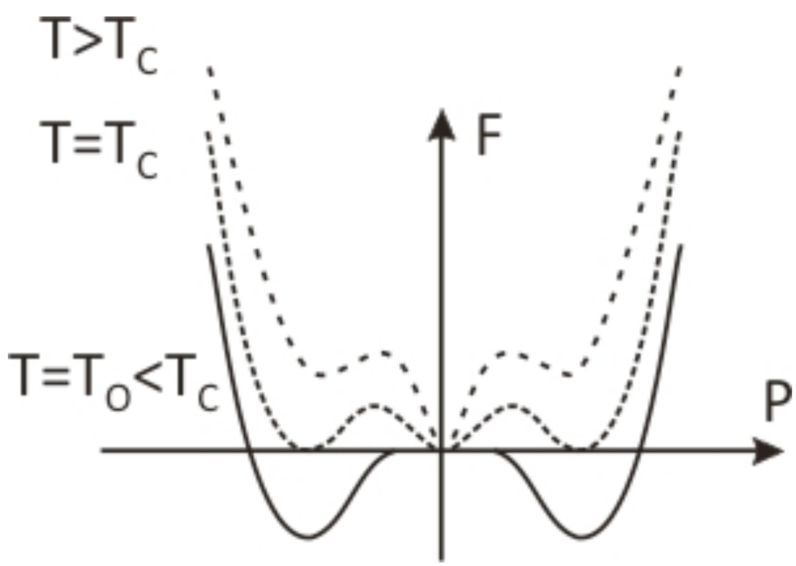
$$\tau_{coll}/\tau_H \sim (1/g^*)^{1/2}\ \alpha^2\ (T/T_p) << 1$$

==> Thermal Equilibrium

Era of phase transitions

Phase transitions: rearrangement of the microphysics in which a particular symmetry is created or destroyed.

- location of particles: freezing, melting, evaporation
- orientation of particles: ferromagnetism



Example of first order transition

temperature

$$F=U-T\times S$$

internal energy

entropy

At high T, an increase in entropy leads to decrease in free energy. After the phase transition there is spontaneous symmetry breaking and the system has to choose a new state

Kibble mechanism

After each phase transition, the effective physics changes.

Phase transitions can leave defects if different regions pick a different state.

Kibble mechanism: different Horizon-sized volumes choose their ground states independently (no causal connections between them).

As the universe expands and cools, the fields decay to their ground state over most of space, but trapped energy domains remain as defects: ***this is a generic prediction!***

Topological defects

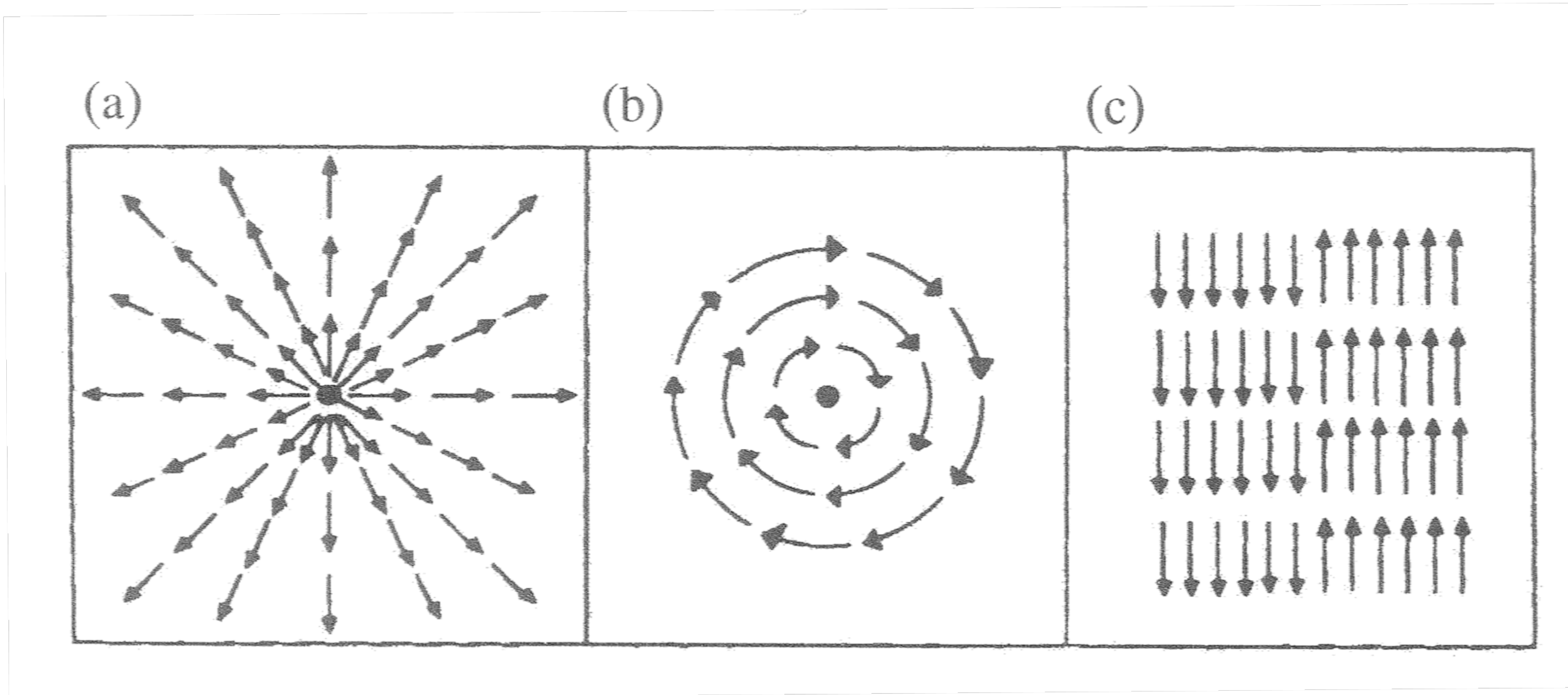
Monopole (point defect): defects that form when a spherical symmetry is broken, are predicted to have magnetic charge.

Cosmic string (line defect): one-dimensional lines that form when an axial or cylindrical symmetry is broken

Domain wall (surface defect): two-dimensional membranes that form when a discrete symmetry is broken at a phase transition.

Textures (higher dimensional defects): form when larger, more complicated symmetries are completely broken. They are not as localized as the other defects, and are unstable.

Topological defects

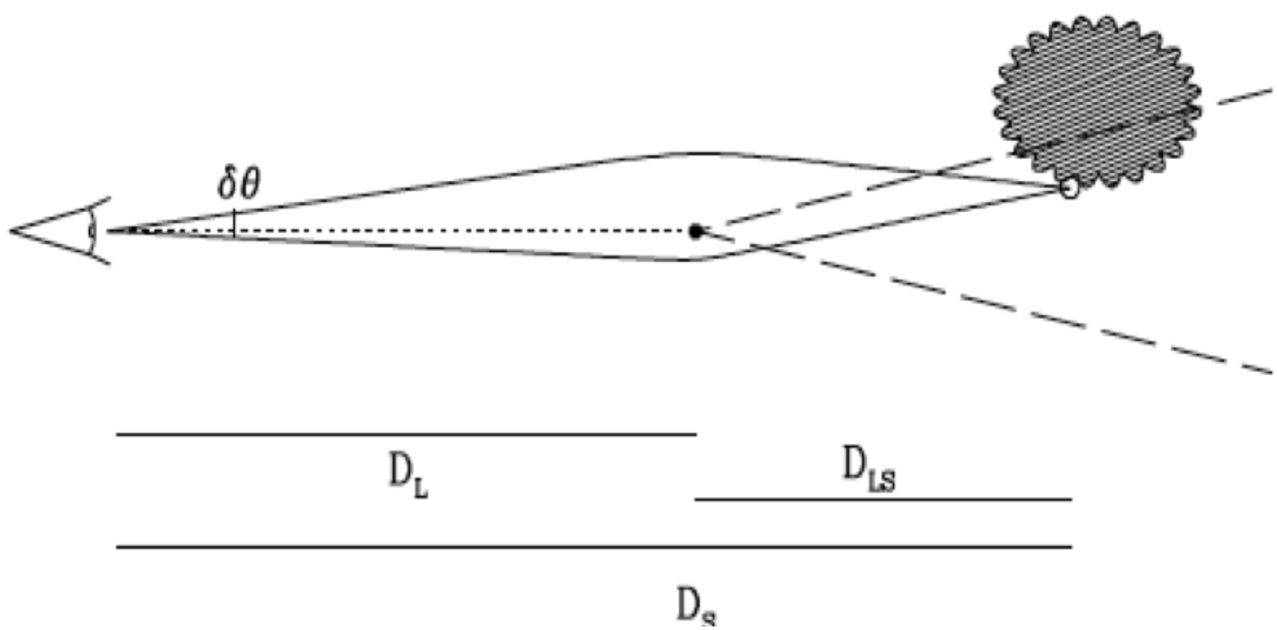


monopole string domain wall

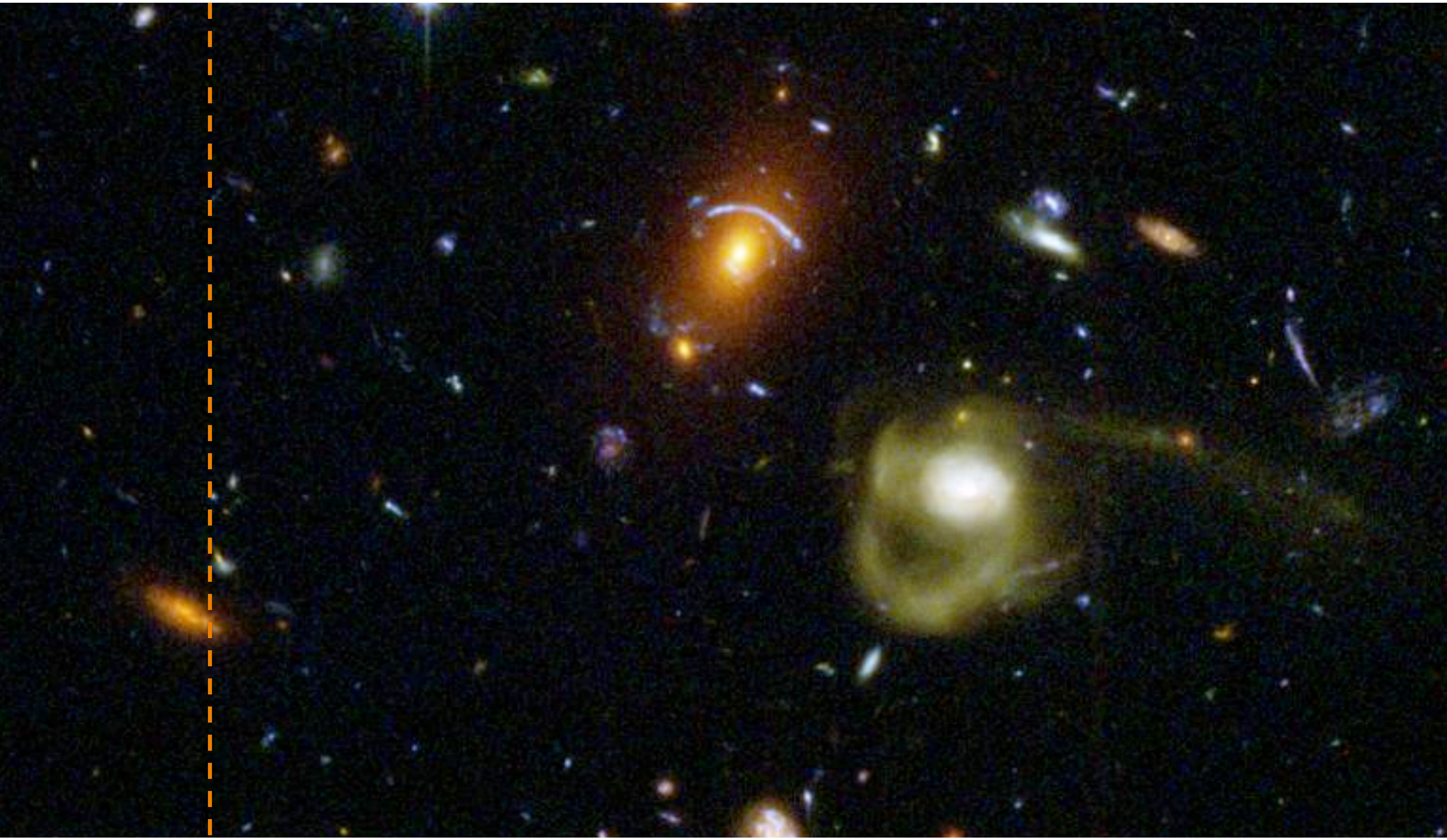
From: Coles & Lucchin (2002)

Effect of a cosmic string

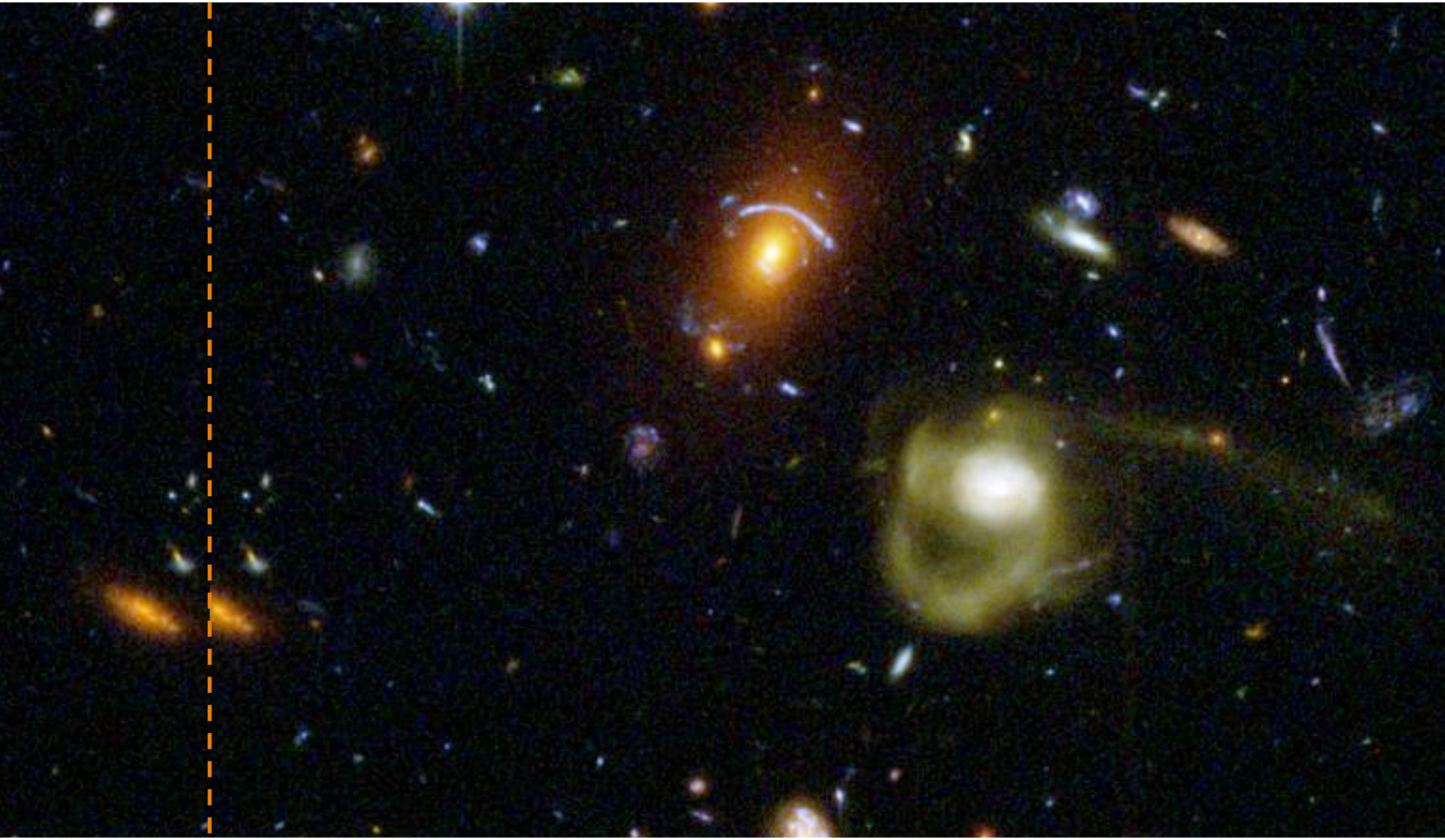
Cosmic strings split images - angle of splitting proportional to mass/unit length.



Effect of a cosmic string



Effect of a cosmic string



Phase transitions of the Universe

Between $T \approx 10^{19}$ and 10^{15} GeV, quantum gravity effects decrease in importance and interactions are described by a GUT. Baryon number is not conserved in GUTs, so no asymmetry between matter and antimatter.

Near $T \approx 10^{15}$ GeV ($t = 10^{-37}$s) the GUT symmetry breaks leading into the situation described by the standard model of particles; the GUT phase transition typically results in the formation of magnetic monopoles.

For typical GUTs:
- particle mass: $m_M \approx 10^{16}$GeV
- number density: $n_M > 10^{-10} n_\gamma$.

$\Longrightarrow \Omega_{monopole} > m_M/m_p \Omega_{bar} \approx 10^{16}$.

This does not match observations: **the monopole problem**

Phase transitions of the Universe

A GUT that unifies the elektroweak interactions with the strong interactions puts leptons and hadrons on the same footing and thus allows processes that do not conserve baryon number: source of matter/anti-matter asymmetry.

As the temperature falls below $T_{GUT} \approx 10^{15}$ GeV the unification of the strong and elektroweak interactions no longer holds. Towards the end of this period (10^{-11}s) the Universe is filled with an ideal gas of leptons and antileptons, the four vector bosons, quarks and anti-quarks.

The horizon is 1cm and contains $\approx 10^{19}$ particles!

Phase transitions of the Universe

At $T_{EW} \approx 100$ GeV elektro-weak symmetry is broken and we have separate elektromagnetic and weak forces. All the leptons acquire mass.

When the temperature drops to $T_{QH} \approx 200$-300 MeV (10^{-5}s) we have the final phase transition and the strong interaction leads to the confinement of quarks into hadrons: the quark-hadron phase

The horizon is 1km in size

Successes of the Big Bang model

- Correctly predicts the abundances of light elements
- Explains the CMB as relic of the hot initial phase
- Naturally accounts for the expansion of the Universe
- Provides a framework to understand the formation of cosmic structure.

There are also several problems (some of which can be addressed by incorporating "new physics")

Problems with the Big Bang model

- Origin of the Universe
- The horizon problem
- The flatness problem
- Origin of the baryon asymmetry
- Monopole problem
- Origin of primordial density fluctuations
- Nature of dark matter
- Nature of dark energy

Problems with the Big Bang model

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How we can solve these issues with the Big Bang model?

Consider the horizon problem…

As the universe ages, we are continually probing regions of the universe which were not yet in casual contact, but appear to be homogeneous.

In the lecture on horizons, we showed the size of the particle horizon in comoving coordinates evolves as $a^{3/2}/a \sim a^{1/2}$.

We need to propagate information within a casually connected volume to great comoving distances, i.e., for the particle horizon to be plausibly infinite.

If the scale factor tends to 0 at early times as t^{β} then the particle horizon at time t:

$$R_H(t) = a(t) \int_0^t c\,dt'/a(t')$$

This integral diverges if $\beta >= 1$

What does $\beta >= 1$ imply regarding other quantities of importance?

What does this imply regarding d^2a/dt^2?

Use Friedmann's second equation:

$$d^2a/dt^2 = -(4/3)\pi G(\rho+3P/c^2)a = \beta(\beta-1)a/t^2$$

$$-(4/3)\pi G(\rho+3P/c^2)t^2 = \beta(\beta-1) \propto d^2a/dt^2$$

$$-(4/3)\pi G\rho(1+3w)t^2 = \beta(\beta-1) \propto d^2a/dt^2$$

If $\beta >= 1$, then $d^2a/dt^2 > 0$

$\beta >= 1 \iff d^2a/dt^2 >= 0 \iff$ If $w <= -1/3 \iff$ there is no particle horizon

If there is no particle horizon, then information from a small region can propagate to the entire universe.

But if not, there is a particle horizon and the universe will not be in casual contact. is is hard to reconcile with the cosmological principle.

Propagating Information from Small Region to Large Volume in Universe (Inflation)

We need to a mechanism to disconnect regions that were before in causal contact, the expansion must be so rapid that there exists an event horizon at a finite distance from any point

How can we formalize this?

The Hubble radius in comoving coordinates must shrink with time:

$$\frac{d}{dt}\left(\left(1/a\right)\left(c/H\right)\right) < 0 \qquad \text{where } H = (da/dt)/a$$

scale factor to put in comoving coordinates

distance light can travel in Hubble time

implies $(d/dt)(c/(da/dt)) < 0 \quad \Rightarrow \quad (d^2a/dt^2) > 0$

given the acceleration equation we need a substance with sufficient negative pressure.

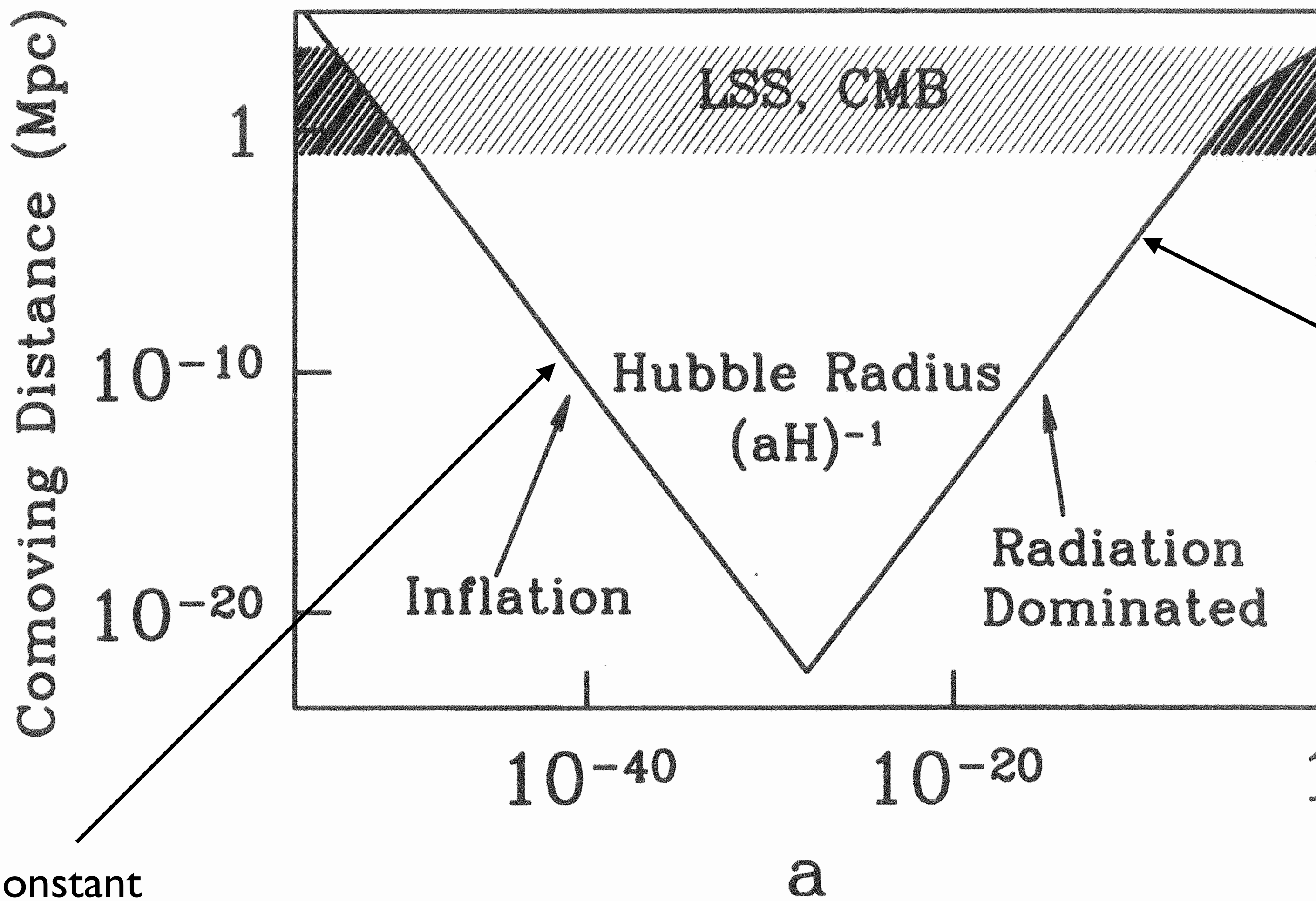
How can we implement this?

The inflation field; in physics we encounter scalar fields to describe the potential energy with a particular force; the force is the gradient of the potential energy scalar field. Other examples are the temperature or pressure field. In quantum field theory a scalar field is associated with a spin-0 particles. The Higgs field is an example.

Inflation

Hubble Radius in comoving coordinates

(c/Ha)

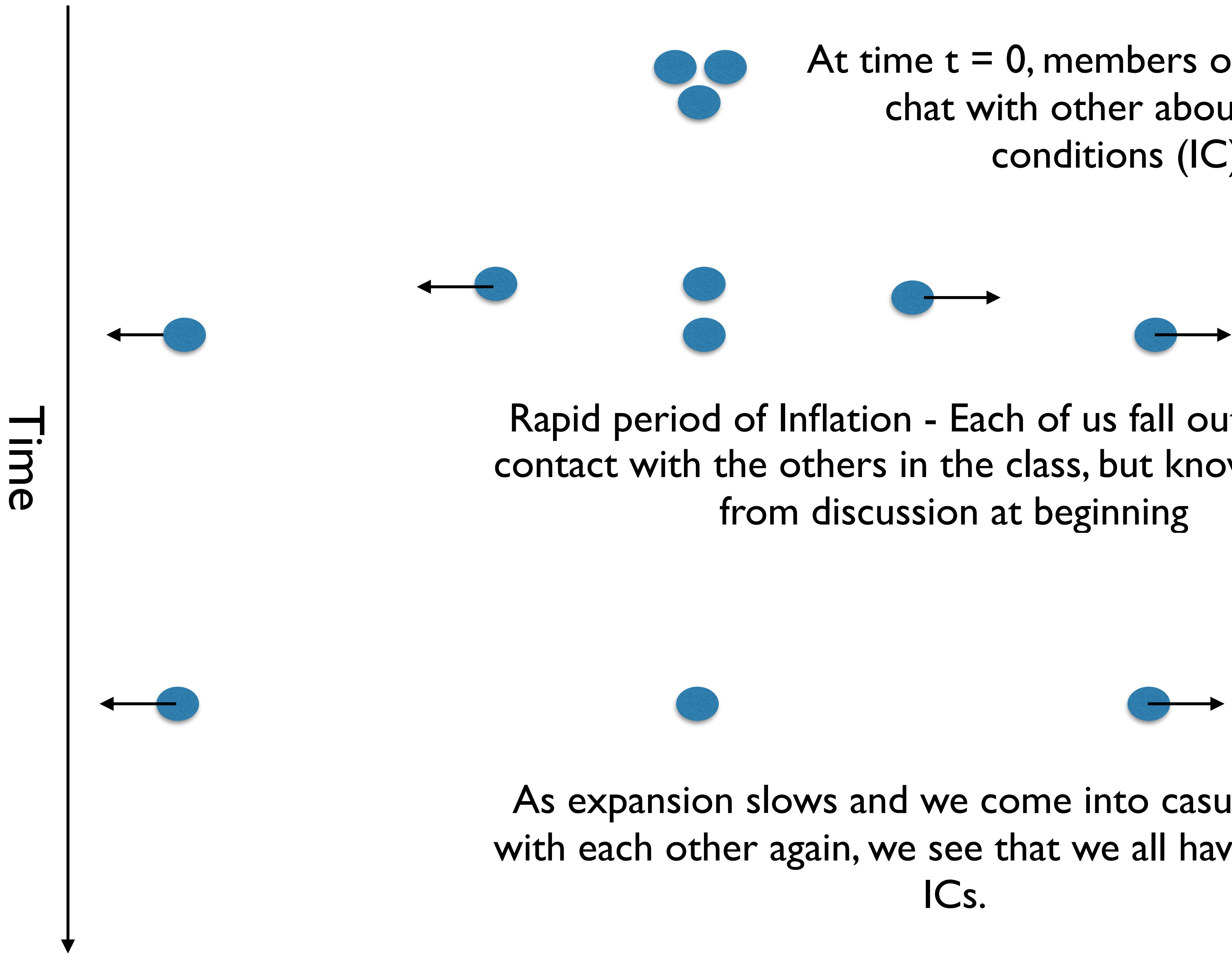


scales as a

H ~ constant
scales as 1/a

Dodelson (2003)

Let’s take as an apology the following situation:



At time t = 0, members of this class - chat with other about initial conditions (IC)

Rapid period of Inflation - Each of us fall out of casual contact with the others in the class, but know about IC from discussion at beginning

As expansion slows and we come into casual contact with each other again, we see that we all have the same ICs.

Inflation

Imagine the early universe was filled with a scalar field $\Phi(\mathbf{x},0) = \Phi_0 > 0$, i.e., not in the ground state.

In this case, it may lead to accelerated expansion; after a while the field decays into particles (causing reheating)

The Lagrangian of a scalar field is $L = -(1/2)c^2(\partial_\mu\Phi)(\partial^\mu\Phi) - V(\Phi)$

If we assume homogeneity and isotropy, we can define effective density and pressure:

$$\rho_\Phi c^2 = (1/2)(d\Phi/dt)^2 + V(\Phi)$$

$$P_\Phi = (1/2)(d\Phi/dt)^2 - V(\Phi)$$

To get $P_\Phi < -\rho_\Phi c^2/3$

$$(1/2)(d\Phi/dt)^2 - V(\Phi) < -(1/3)((1/2)(d\Phi/dt)^2 + V(\Phi))$$

$\Longrightarrow (d\Phi/dt)^2 < V(\Phi)$ slow roll condition