Sterrewacht Leiden

Origins & Evolution of the Universe

an introduction to cosmology — Fall 2018

Lecture 5: Observable Distances, Parameter Constraints

Rychard Bouwens

Layout of the Course

Sep 24: Introduction and Friedmann Equations
Oct 1: Fluid and Acceleration Equations
Oct 8: Introductory GR, Space Time Metric, Proper Distance
Oct 15: Redshift, Horizons, Observable Distances
Oct 17: Problem Class #1
Oct 22: Observable Distances, Parameter Constraints
Oct 29: Thermal History, Early Universe
Nov 5: Early Universe, Inflation
Nov 12: Inflation, Lepton Era
Nov 14: Problem Class #2
Nov 19: Big Bang Nucelosynthesis, Recombination
Nov 26: Introduction to Structure Formation
Dec 3: Cosmic Microwave Background Radiation (I)
Dec 5: Problem Class #3
Dec 10: Cosmic Microwave Background Radiation (II)
Dec 21: Final Exam

Review Last Week

Proper Distance

Imagine we have a distant galaxy at (r, θ, φ)

How far away is a distant galaxy?

(important to specify time since nominal distance depends on when measurement is made since expanding universe)

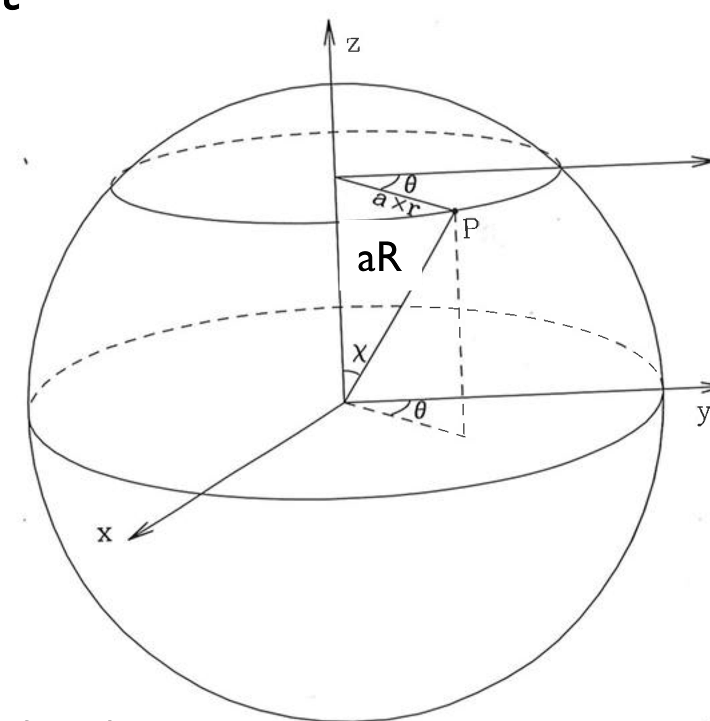
Take case that θ, φ are constant and only r is different

In the flat case (k=0),

Proper Distance: $d_p(t) = \int ds$

"Source-source distance measured based on current topology of the universe and ignoring travel time"

$$ds^2 = a^2(t)\,[dr^2/(1-kr^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)]$$



In general case,

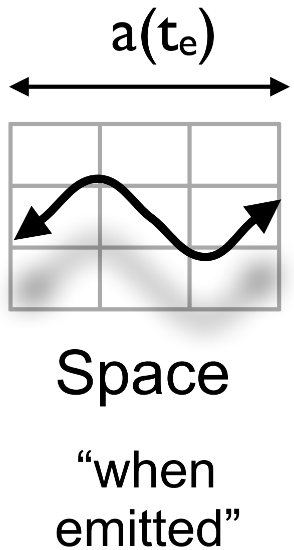
$$\text{Proper Distance: } d_p(t) = a(t) \underbrace{\begin{cases} \arcsin(k^{1/2}r)/k^{1/2}, & \text{if } k>0 \\ r, & \text{if } k=0 \\ \operatorname{arcsinh}((-k)^{1/2}r)/(-k)^{1/2}, & \text{if } k<0 \end{cases}}_{f_k(r)}$$

Not especially practical (since not measurable)!

Distances and Redshifts

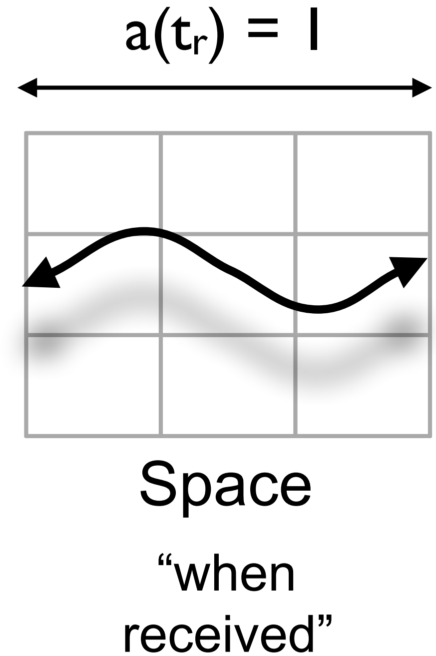
We cannot measure proper distances, but we can measure redshifts.

Redshift — which we denote as z — is directly connected to the scale factor of the universe.



$$\lambda_e\,(1+z) = \lambda_r$$

$$a(t_e) = 1/1+z$$



Redshift — which we denote as z — is directly connected to the scale factor of the universe.

$$c\,dt_e / a(t_e) = c\,dt_r / a(t_r)$$

As such, $dt \propto a(t)$ implies time dilation

This implies $\lambda_r/\lambda_e = a(t_r)/a(t_e)$

Since $a(t_r) = 1$ and $\lambda_r/\lambda_e = 1+z$, $a(t_e) = 1/1+z$

How far can we see?

Light travels at a finite speed — no physical signals travel faster!

Important connection to questions about how the universe became so homogeneous…

Complicated since the universe is expanding!

Horizon Distance

Horizon Distance: The greatest distance one can in principle look — probing back to when the universe had time t=0

$$d_{Hor} = c \int dt/a(t)$$

What is the proper distance to some galaxy which emitted its light at time t_e?

$$d_p(t_0) = c \int_{t_e}^{t_0} dt/a(t) = (c/H_0)(2/(1+3w))[1 - (t_e/t_0)^{(1+3w)/(3+3w)}]$$

What is the proper distance to galaxy who emitted its light at time t_e with redshift z?

$$d_p(z) = ct_0\,(3(1+w)/(1+3w))[1 - (1+z)^{-(1+3w)/2}] \quad = (c/H_0)(2/(1+3w))[1 - (1+z)^{-(1+3w)/2}]$$

if w=0: $d_p(z) = (2c/H_0)[1 - (1+z)^{-1/2}]$

Blows up if $w <= -1/3$

What is the Horizon distance?

$$d_{Hor}(t) = (c/H_0)(2/(1+3w))$$

which is finite if $w > -1/3$

Horizon distance tells us the portion of the universe that is casually connected

w = 0: $d_{Hor}(t) = 2c/H_0 = 3ct_0$

If $w <= -1/3$, then whole universe is casually connected

Particle Horizon

A key question is which parts of the universe can influence or have influenced each other, i.e., which parts are in casual contact.

Particle horizon: boundary of that part of the universe that could have reached us in the age of the universe. At the present epoch, it is the observable universe.

What is this horizon to redshift z_e?

$$d_{Hor}(z_e) = 3ct_0 [1 - (1+z_e)^{-1/2}]$$

$$d_{Hor}(z_e) = (c/H_0) \int_0^{z_e} dz/(1+z)/(\Omega_0(1+z)^{1+3w}+(1-\Omega_0))^{0.5}$$

How did Proper Distance of Particle horizon change with time?

it is proportional to t/t_0 $\quad t/t_0 \propto (a/a_0)^{3/2} \propto (1+z)^{-3/2}$

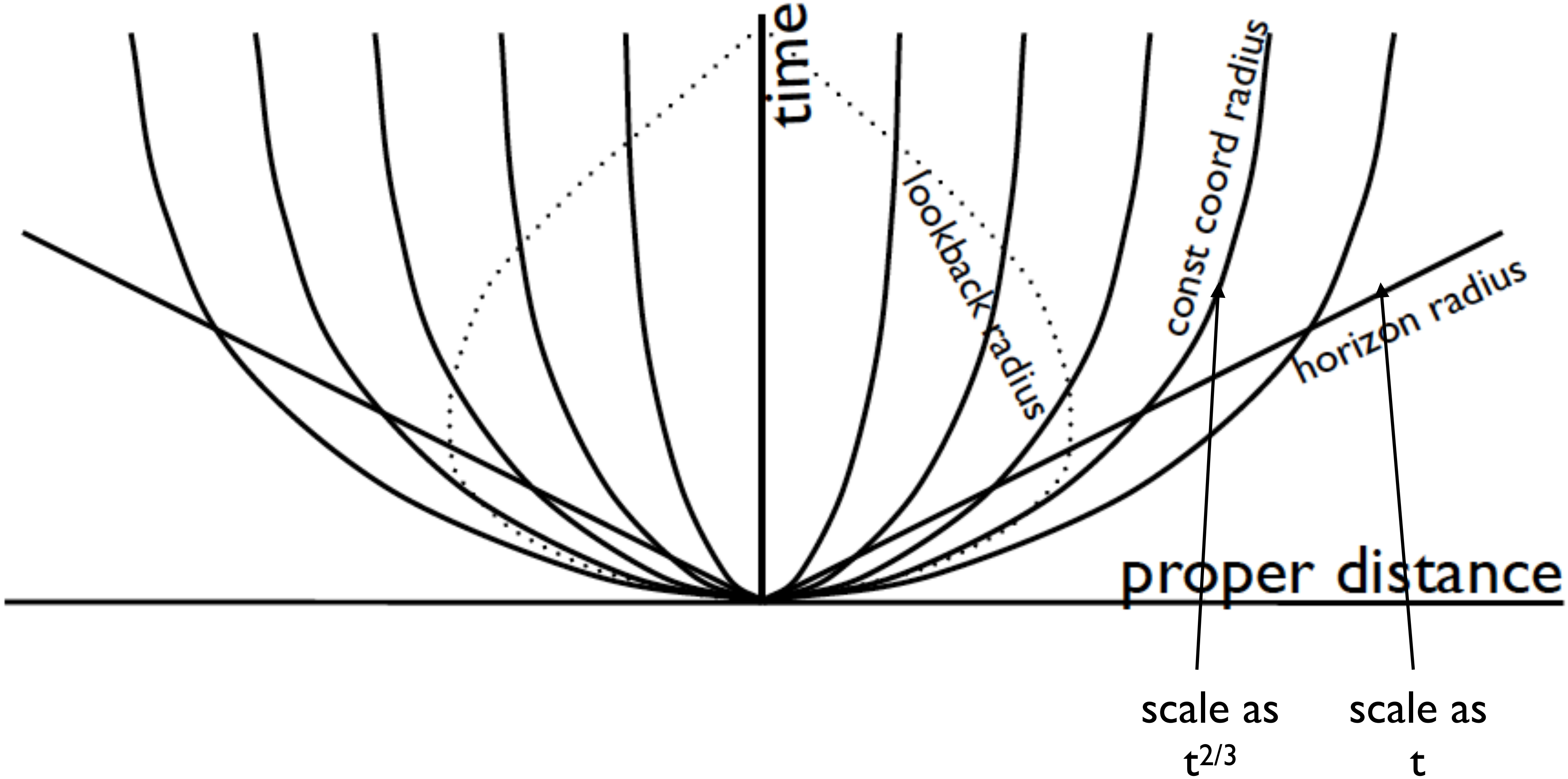
8x smaller at z~3, 30000x smaller at z~1000

How does this horizon change relative to the scale factor of the universe (i.e. divide by $1/(1+z)$)

$$d_{Hor}\,(1+z) = (1+z)^{-1/2}$$

Therefore, with the horizon, we potentially probe further and further in the regions of the universe to which we have not been in casual contact

Horizon in flat Ω_m=1 model



Horizon Problem

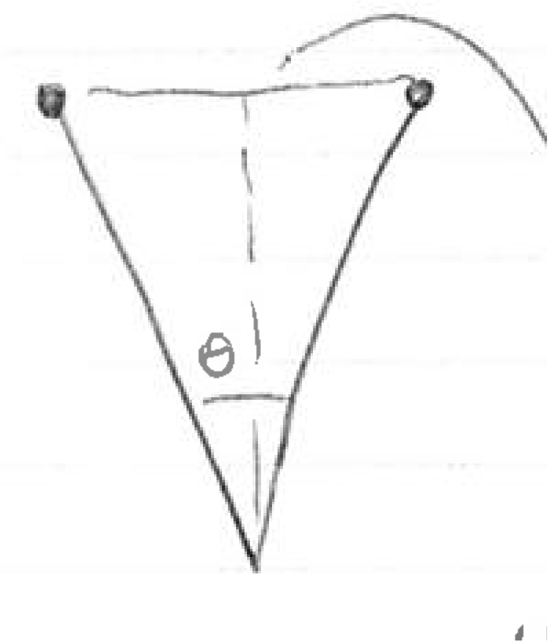
We receive light from galaxies or other sources which may not have been able to communicate with each other, but appear to be completely homogeneous in terms of their properties.

This is called the horizon problem.

Consider the universe where it was ~1100 times smaller than it is today, i.e., at z~1099.

What is the furthest distance that we could expect to be homogeneous due to information propagating from a common source?

$$2\ D_H(z=1099)$$



What is the angular separation between two different points on the sky that corresponds to this distance (at z~1099)?

$$2\ D_H(z=1099) \sim D_A(z=1099)\ \theta$$

$$\theta \sim 3.6 \text{ degrees}$$

Why is z~1100 interesting?

It is when the intense background radiation field of photons decouples from the baryons (i.e., neutral hydrogen atoms) and free streams into the universe.

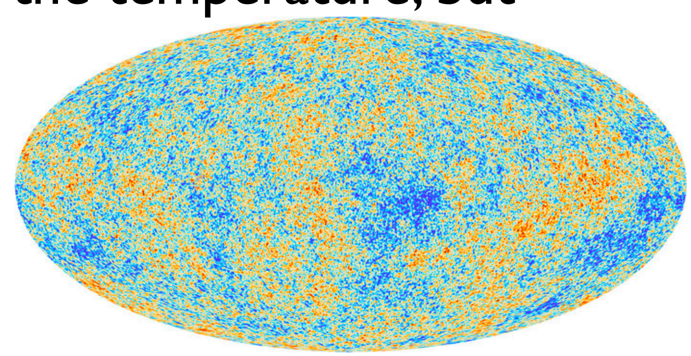
This decoupling occurs due to the decreasing energy / temperature of photons in the background field relative to that needed to ionize hydrogen.

After decoupling this background radiation continues to exist in the universe, but its temperature falls in accordance with the expansion of the universe.

What is the temperature of this relic radiation if we look in all directions on the sky? It is almost entirely homogeneous as we discussed before…

But if we look closely, there are fluctuations in the temperature, but they are small

fluctuations at 1 part in 100000



How can this be, if casual contact is only possible at an angular separation of 3.6 degrees?

Horizon Problem

To resolve this issue, note that the integral $\int c\,dt / a(t)$ depends on the form of a(t) at early times.

we need to change how a(t) scales at early time so that points in casual contact at the earliest times, leave the horizon, and then reenter.

Need $w < -1/3$ accelerated expansion, i.e., inflation, which occurs when. We saw that for $w < -1/3$, there is no particle horizon (if a(t) maintains form to $z \to \infty$)

New Material for This Week

“Classical Cosmology” “Question for Two Numbers”

In the early days of observational cosmology much emphasis was placed on geometric properties.

Sandage: “We need to determine H_0 and q_0”

deceleration parameter

$$a(t) = a(t_0) + \left.\frac{da}{dt}\right|_{t=t_0}(t-t_0) + \frac{1}{2}\left.\frac{d^2a}{dt^2}\right|_{t=t_0}(t-t_0)^2 + \ldots$$

divide by $a(t_0)$

$$a(t) = 1 + H_0(t-t_0) - (1/2)q_0 H_0^2 (t-t_0)^2$$

where $q_0 = -((d^2a/dt)a \,/\, (da/dt)^2)_{t=t0} = -((d^2a/dt) \,/\, aH^2)_{t=t0}$

Make use of acceleration equation:

$$(d^2a/dt^2)/a = -(4\pi G/3)\Sigma_w \rho_w (1+3w)$$

“Classical Cosmology” “Question for Two Numbers”

$$q_0 = -((d^2a/dt)a \,/\, (da/dt)^2)_{t=t0} = -(1/H^2)_{t=t0}\,(-(4\pi G/3c^2)\Sigma_w\, \rho_w\,(1+3w))$$

$$q_0 = (4\pi G/3H^2)\Sigma_w\, \rho_w\,(1+3w))$$

$$q_0 = (1/2)(8\pi G/3H^2)\,\Sigma_w\, \rho_w\,(1+3w))$$

$$q_0 = (1/2\rho_{critical})\,\Sigma_w\, \rho_w\,(1+3w))$$

$$q_0 = \Omega_m/2 + \Omega_r - \Omega_\Lambda$$

Recall that the proper distance is

$$d_p(t_0) = c\int_{t_e}^{t_0} dt/a(t)$$

If we substitute Taylor series expansions of $1/a(t)$, one can show:

$$d_p(t_0) = c(t_e\text{-}t_0) + cH_0/2\,(t_0 - t_e)^2$$

$$d_p(t_0) = cz/H_0(1 - (1+q_0)z/2)$$

But the proper distance is not measurable

One way to assign a distance is to use the luminosity

$$d_L = (L/4\pi f)^{1/2}$$

The "luminosity" distance is the proper distance in a static and Euclidean universe

If we consider a FRW metric

$$ds^2 = -c^2\,dt^2 + a^2(t)[dr^2 + S_k(r)\,d\Omega^2]$$

$$d\theta^2 + \sin^2\theta\,d\Phi^2$$

photons emitted at time t_e spread out over a sphere with radius $d_p(t_0) = r$ and surface area $A_p(t_0)$:

In Euclidean space, $A_p(t_0) = 4\pi d_p(t_0)^2 = 4\pi r^2$

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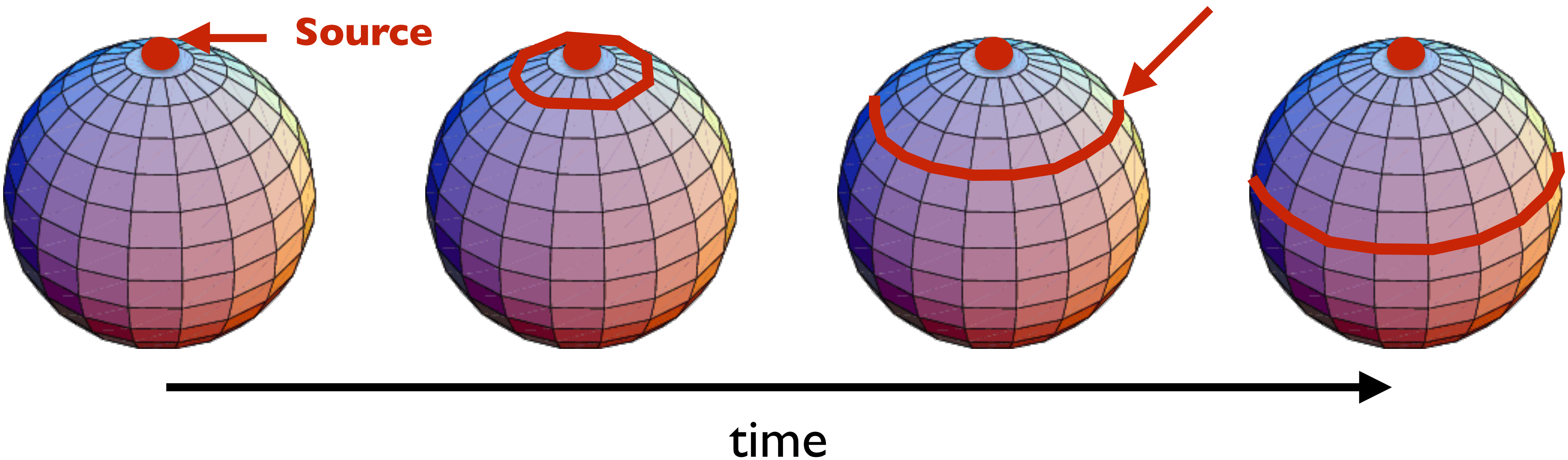
Example using a simple closed space time



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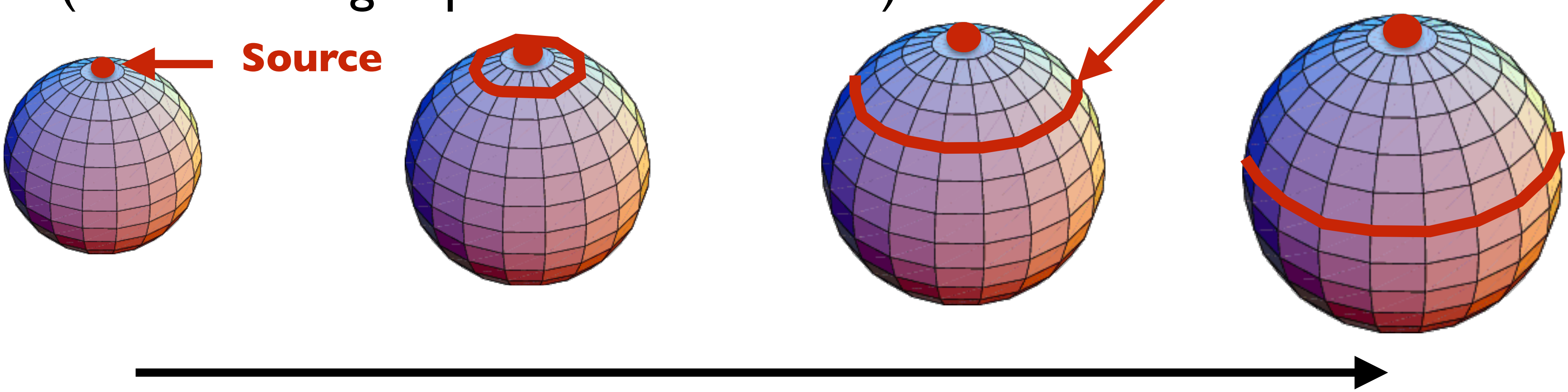
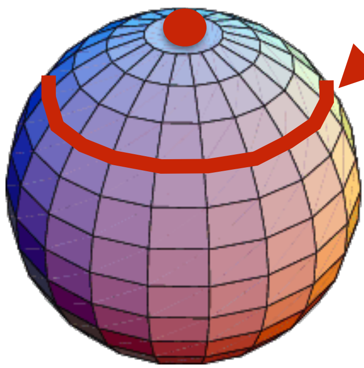
Example using a simple closed space time (now showing expansion of universe)

Light cone eminating from source

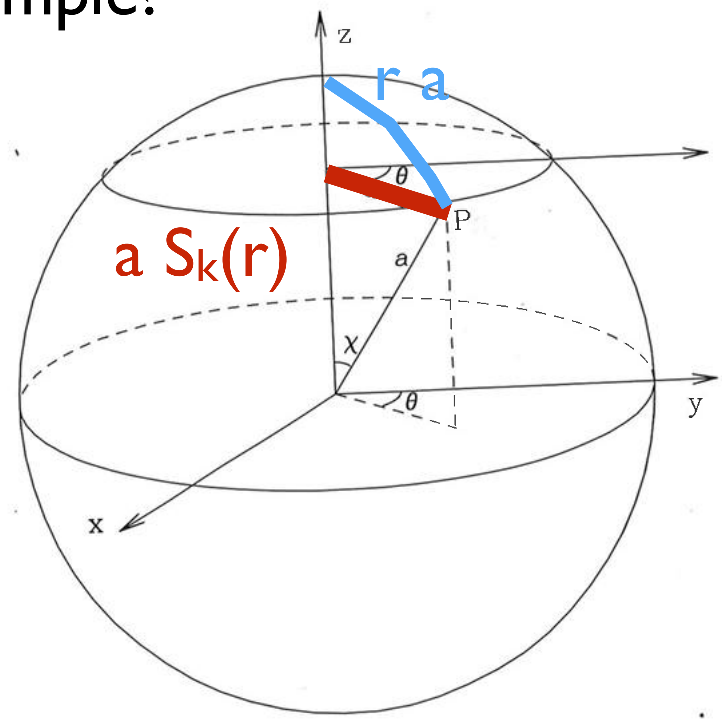


time

What are $d_P(t_0)$ and $A_P(t_0)$ for this example?



Light cone eminating from source (i.e., where we observe it)

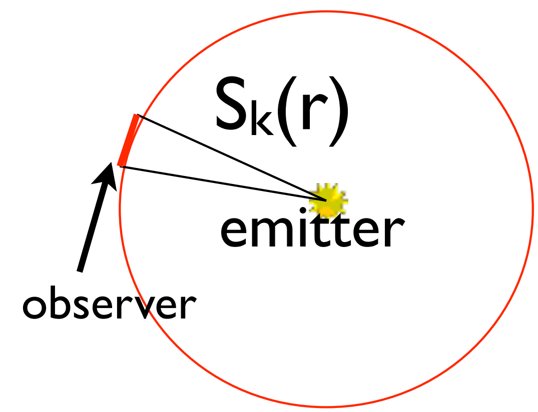


In the general case,

$$d_P(t_0) = a(t_0)r \qquad A_P(t_0) = 4\pi a^2(t_0)S_k(r)^2$$

View from above

bolometric flux !



As in the above example,

If $k > 0$, $S_k(r) < r \rightarrow A_p(t_0) < 4\pi r^2$

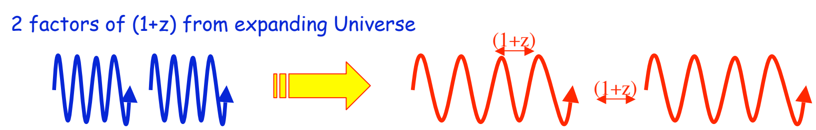
In an open geometry:

If $k < 0$, $S_k(r) > r \rightarrow A_p(t_0) > 4\pi r^2$

In addition to the geometric effects, the expansion of the universe causes the flux to be decreased by a factor $(1+z)^{-2}$

(1) effect of redshifting on energy of photons

(2) time delay between photons



Thus,

$$f = L/(4\pi a^2(t_0) S_k^2(r)(1+z)^2)$$

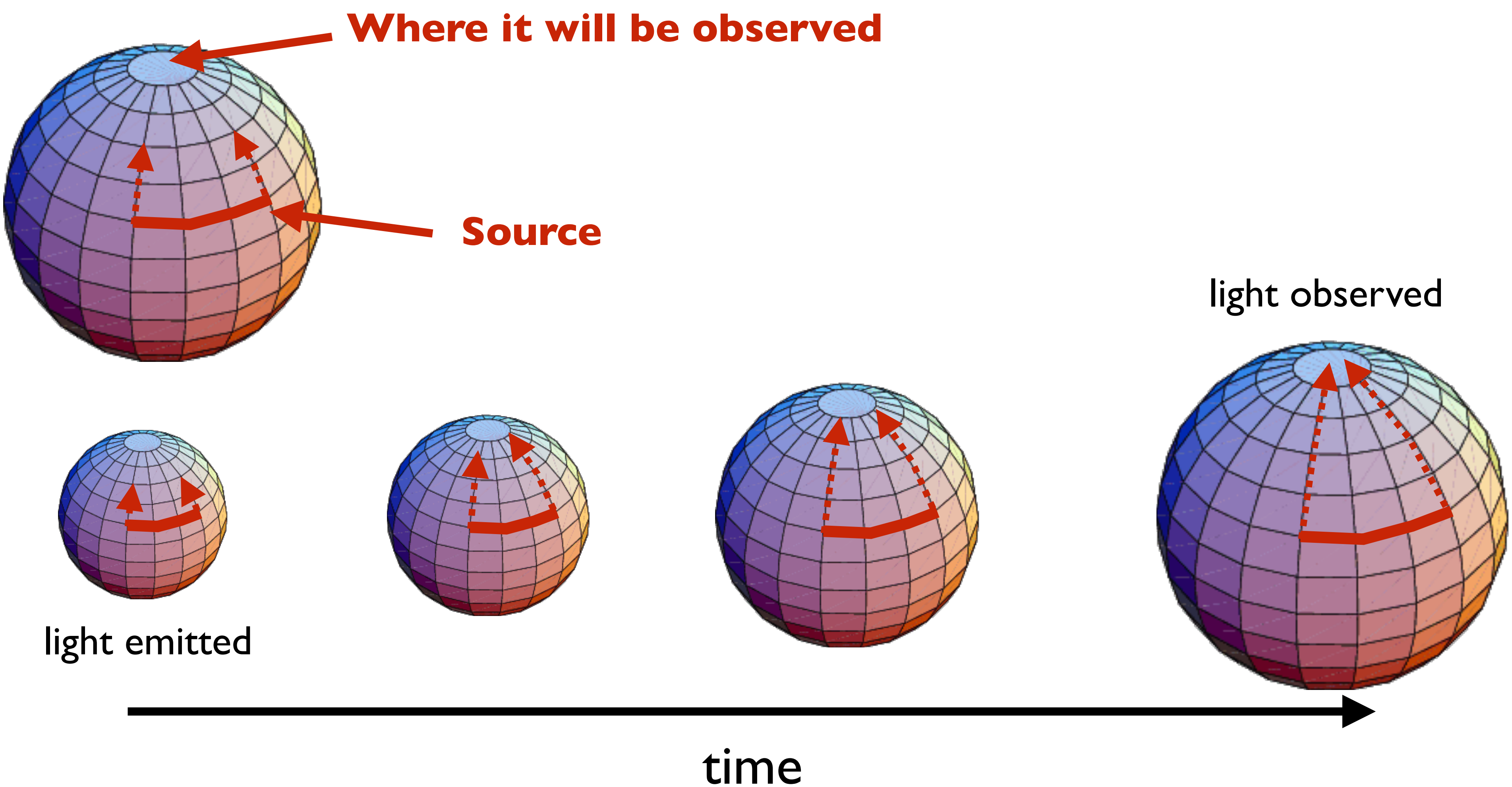
$$d_L = a(t_0) S_k(r)\ (1+z)$$

Another way to assign a distance is to use the angular size $d\theta$

$$d\theta = L / d_A$$

since more distant objects are smaller in general!

Example using a simple closed space time



Note angle maintained as universe expands!

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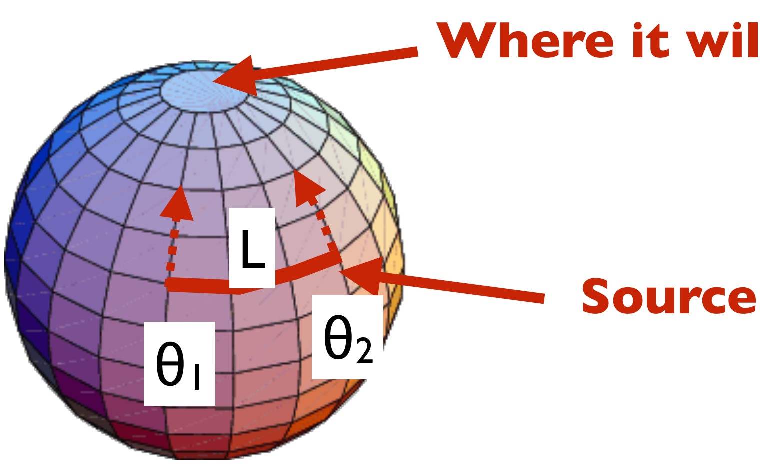
Example using a simple closed space time



Consider FRW metric again where the coordinates of the emitter are as follows:

one side of emitter: (r, θ_1, Φ)

other side of emitter: (r, θ_2, Φ)

$$d\theta = |\theta_1 - \theta_2|$$

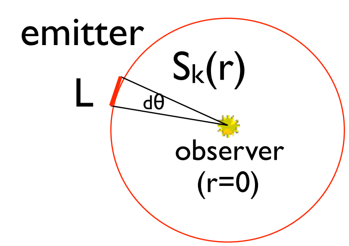
What is $d\theta$?

$$d\theta = L / (a(t_e)S_k(r))$$

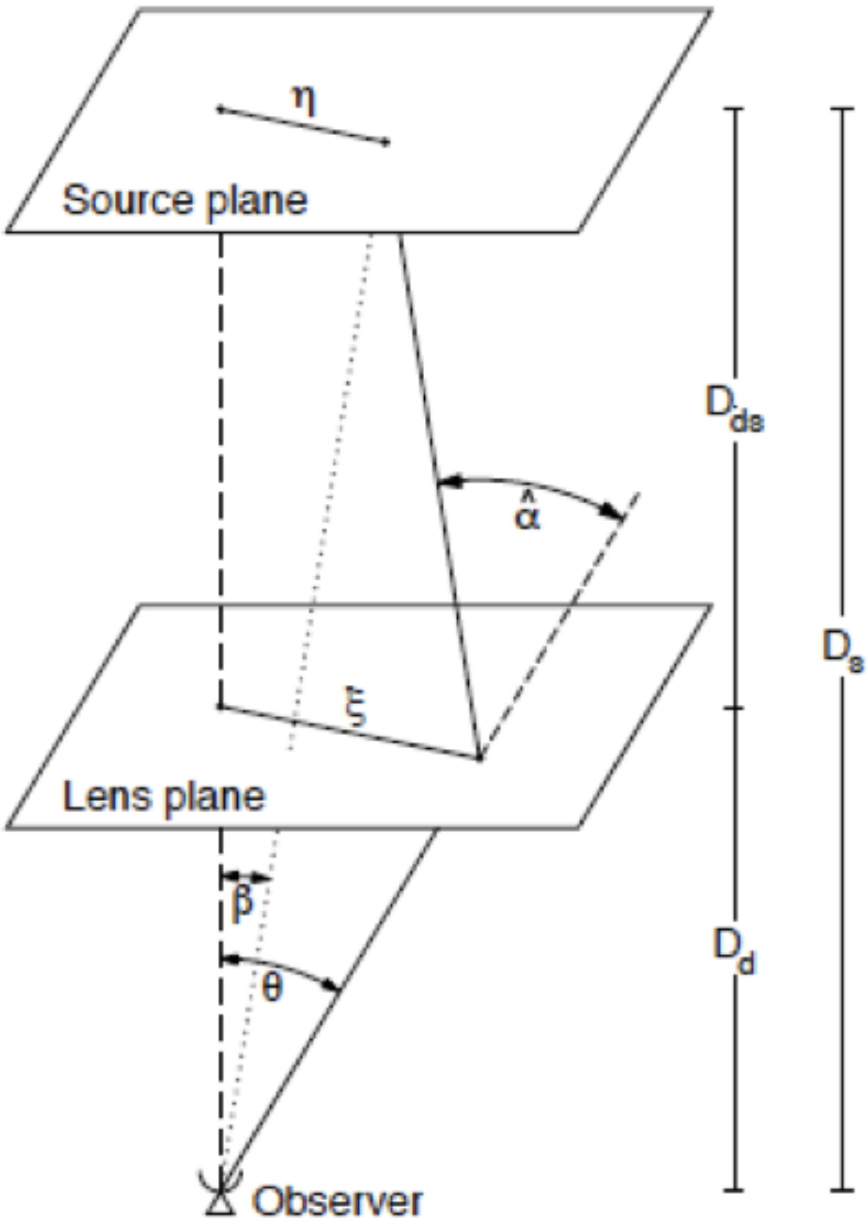
$$d_A = a(t_e)S_k(r)$$

$$d_A = (a(t_0)/(1+z))\, S_k(r)$$

View from above



Gravitational lensing



angular diameter distances!

$$\boldsymbol{\eta} = \frac{D_\mathrm{s}}{D_\mathrm{d}}\boldsymbol{\xi} - D_\mathrm{ds}\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi})$$

Another distance measure is a proper motion distance $d\theta/dt$, i.e., angle on sky per unit time.

$$d_M = (dL/dt_0) / (d\theta/dt_0)$$

As for angular diameter distance, angle on sky is determined by when a source emits its light… but then there is time delay…

So

$$d_M = d_A(t_0)\ (1+z)$$

Overall, there is a nice relation between all 3 distance measures:

$$d_A = d_M / (1+z) = d_L / (1+z)^2$$

What happens to these distance measures in the limit of small or large redshifts?

For small $z << 1$,

$$d_A = (c/H_0)z\,(1 + (-q_0-3)z/2)$$

$$d_L = (c/H_0)z\,(1 + (-q_0+1)z/2)$$

For large $z \rightarrow \infty$,

$$d_L = z\, d_{Hor}\,(t_0)$$

$$d_A = d_{Hor}\,(t_0)\,/\,z$$

Note that while $d_P\,(z_1,z_2) = d_p(0,z_2) - d_p(0,z_1)$

but $d_A\,(z_1,z_2) \neq d_A(0,z_2) - d_A(0,z_1)$

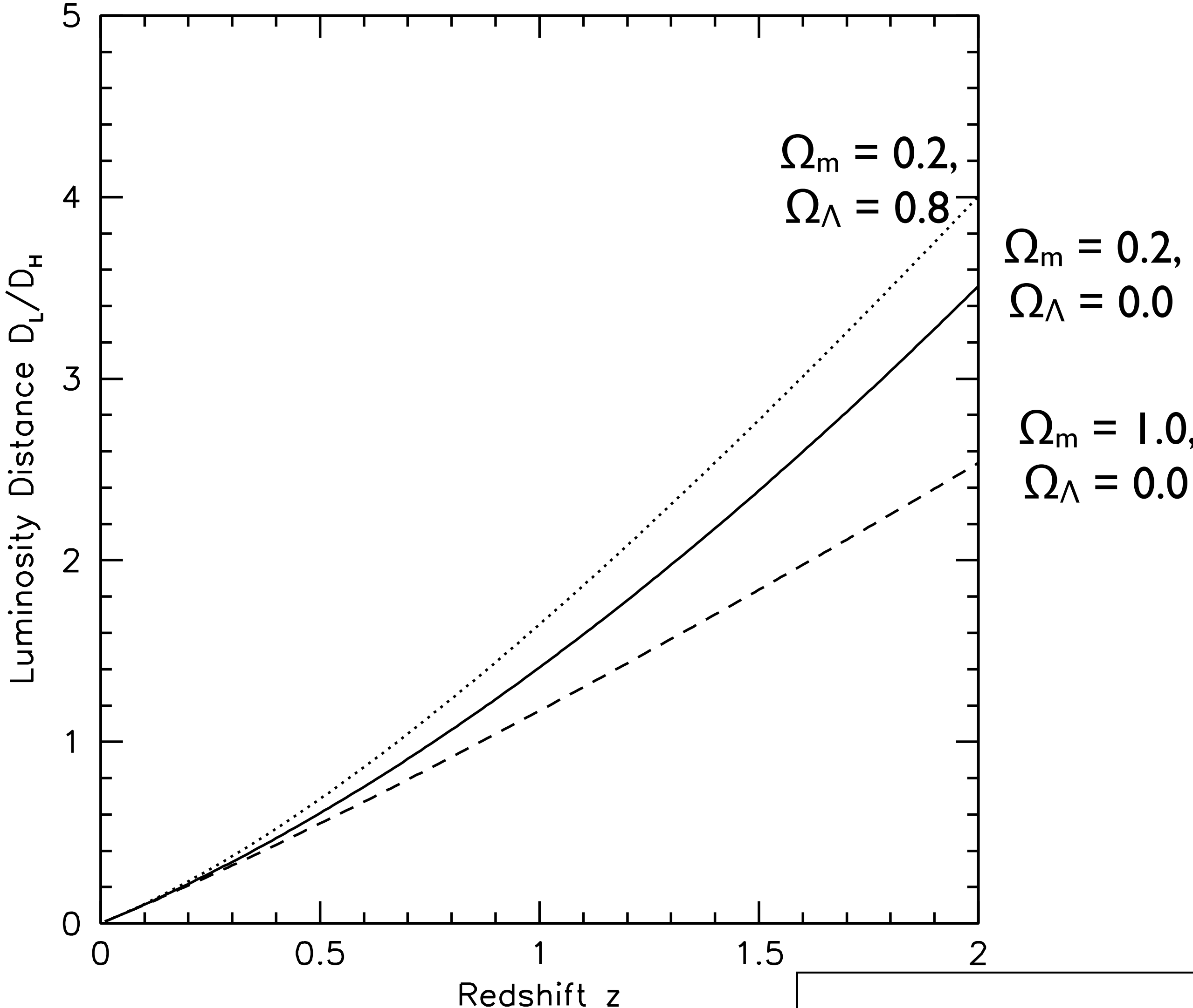
Do the distance measurements depend on the values of the cosmological parameters, i.e., Ω_m or Ω_Λ?

Yes — In fact, we can use the dependence on the distance measures on the cosmological parameters to "weight" the universe

What are the dependencies?

Luminosity Distance - Redshift Relation

Luminosity Distance

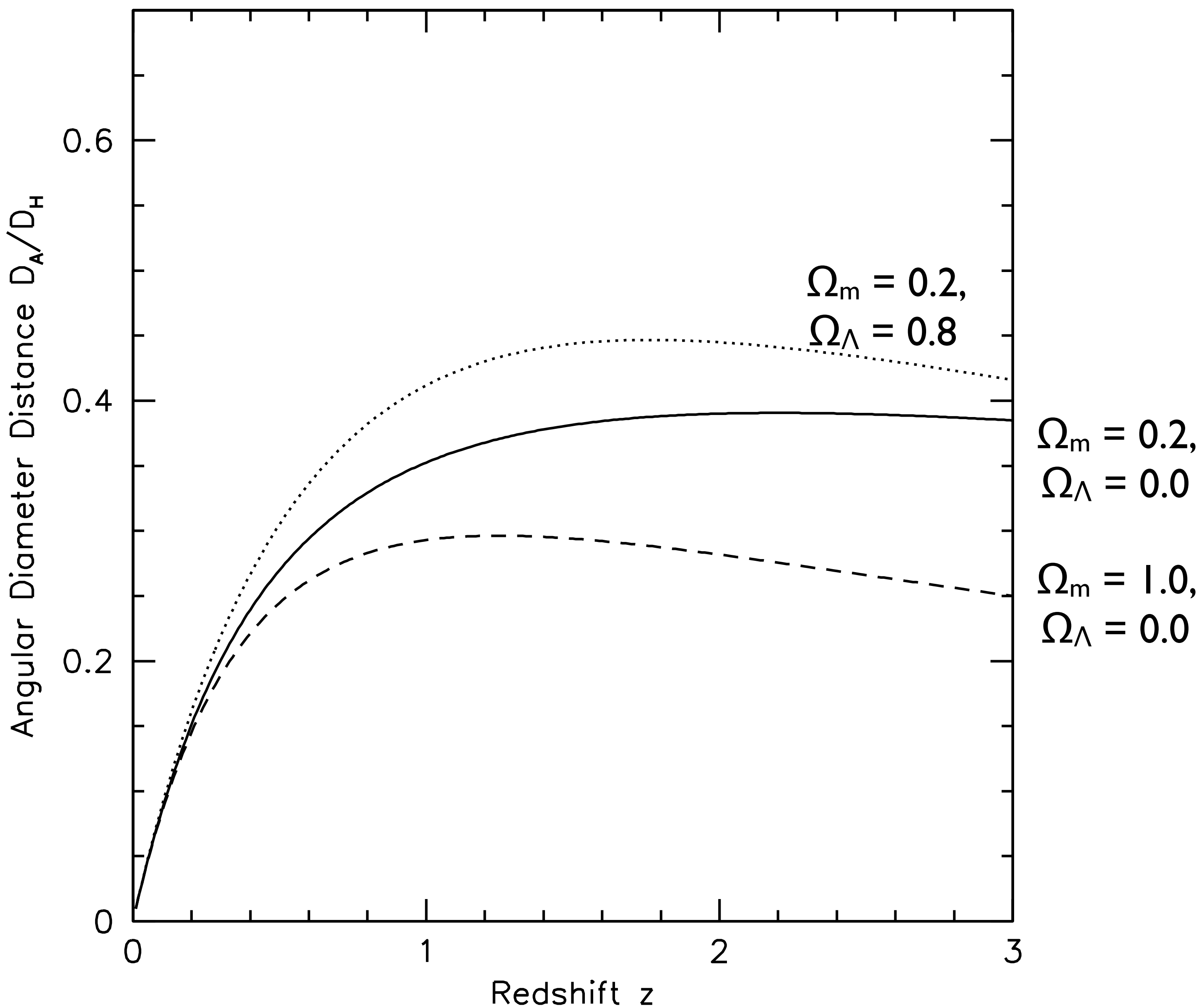


Two different ways of increasing the luminosity distance:

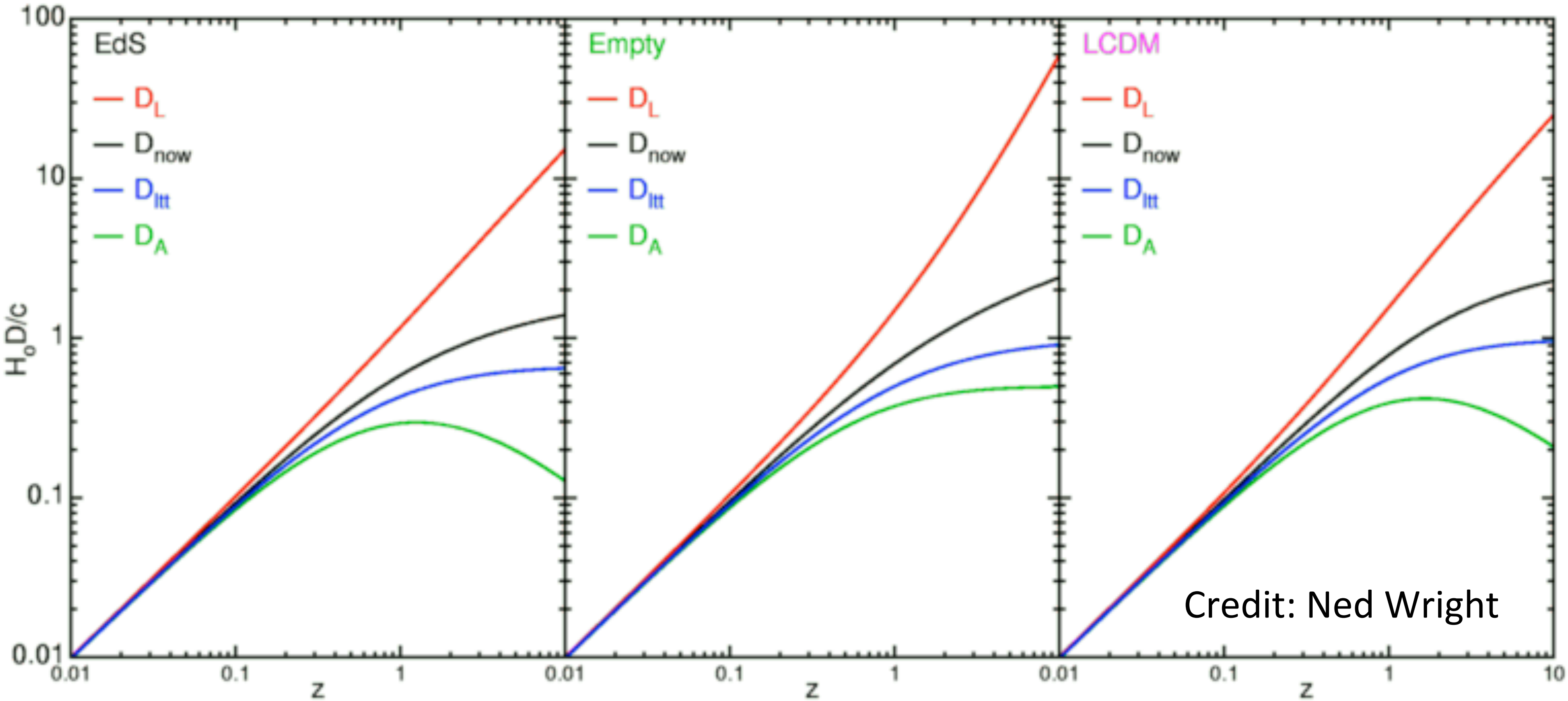
1) Increase Ω_Λ
2) Decrease Ω_m

Angular Diameter Distance - Redshift Relation

Angular
Diameter
Distance

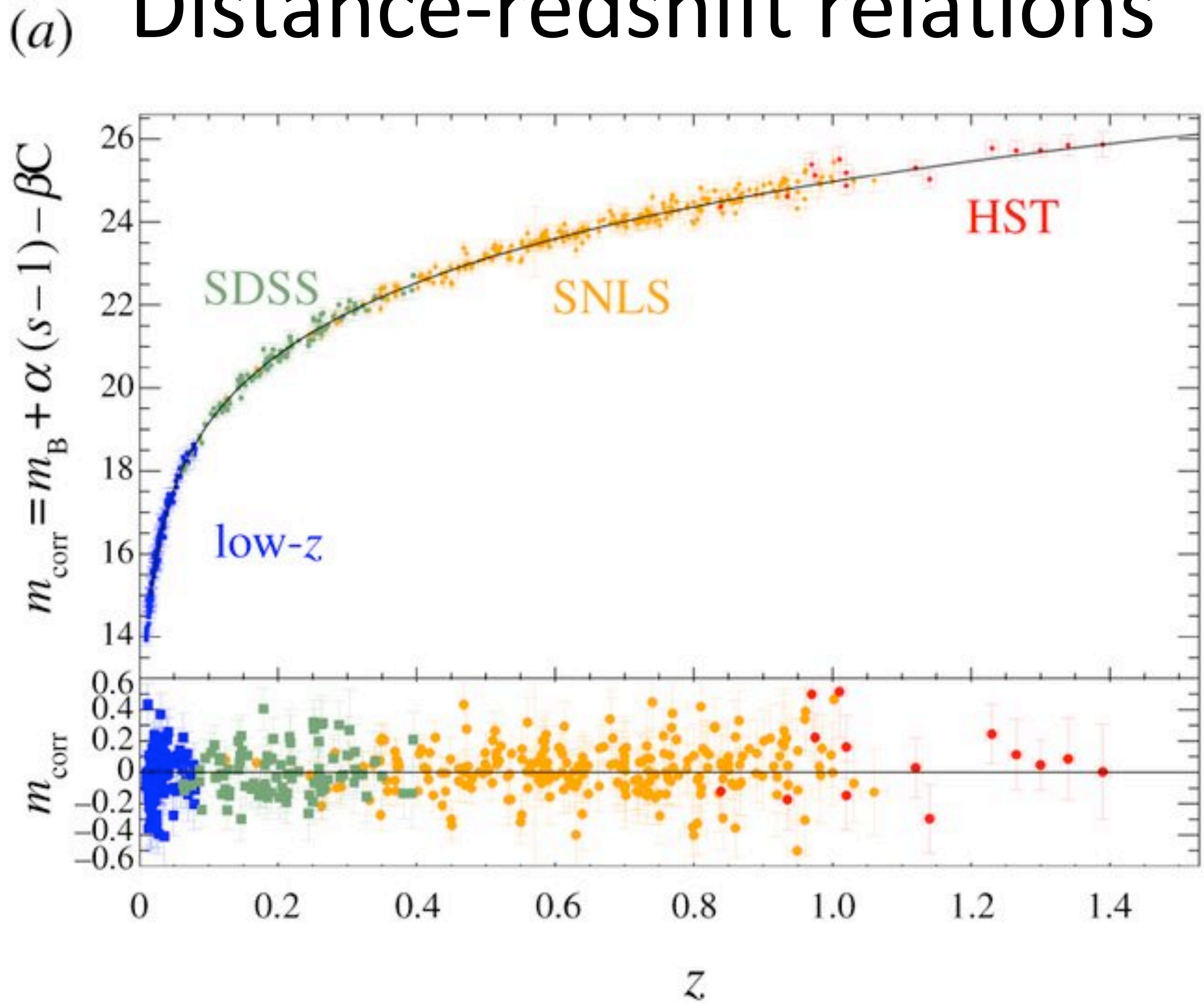


Distance-redshift relations…



… depend on cosmological parameters!

Distance-redshift relations



What are the measured densities in various components of the universe?

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What is Ω_{mat}? (density of normal matter relative to critical)

$\Omega_{mat,bary} = 0.04$ Baryonic matter component

0.31

Yields of Helium allow this to be measured + from acoustic oscillations in CMB

0.27 Dark matter component

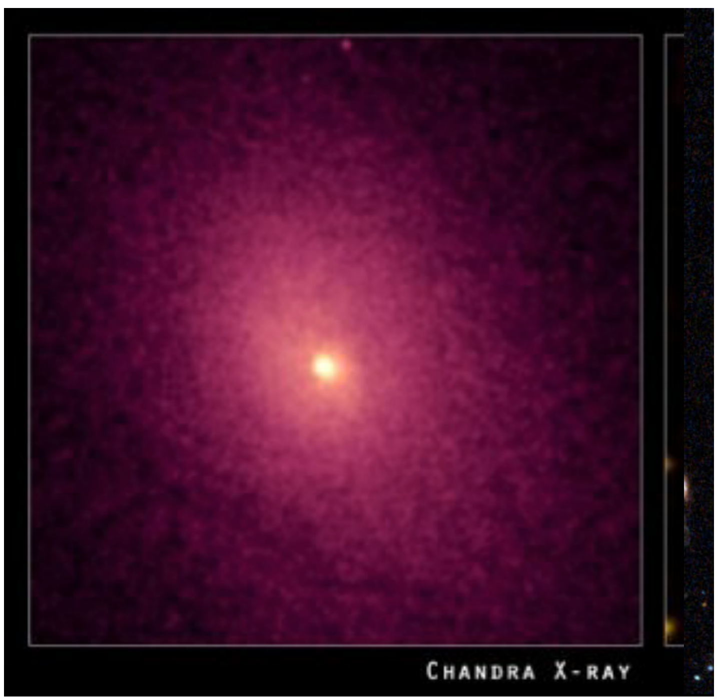
Measurable from bulk flows / peculiar velocities / and from CMB

Evidence for Dark Matter from the Observations of Colliding Galaxy Clusters

Evidence for Dark Matter from Observations of Colliding Clusters

First a few words to orient you a little more about what a galaxy cluster is…

Galaxy clusters are regions of the universe that have collapsed (due to gravity)



Approximate mass budget:

~2% galaxies

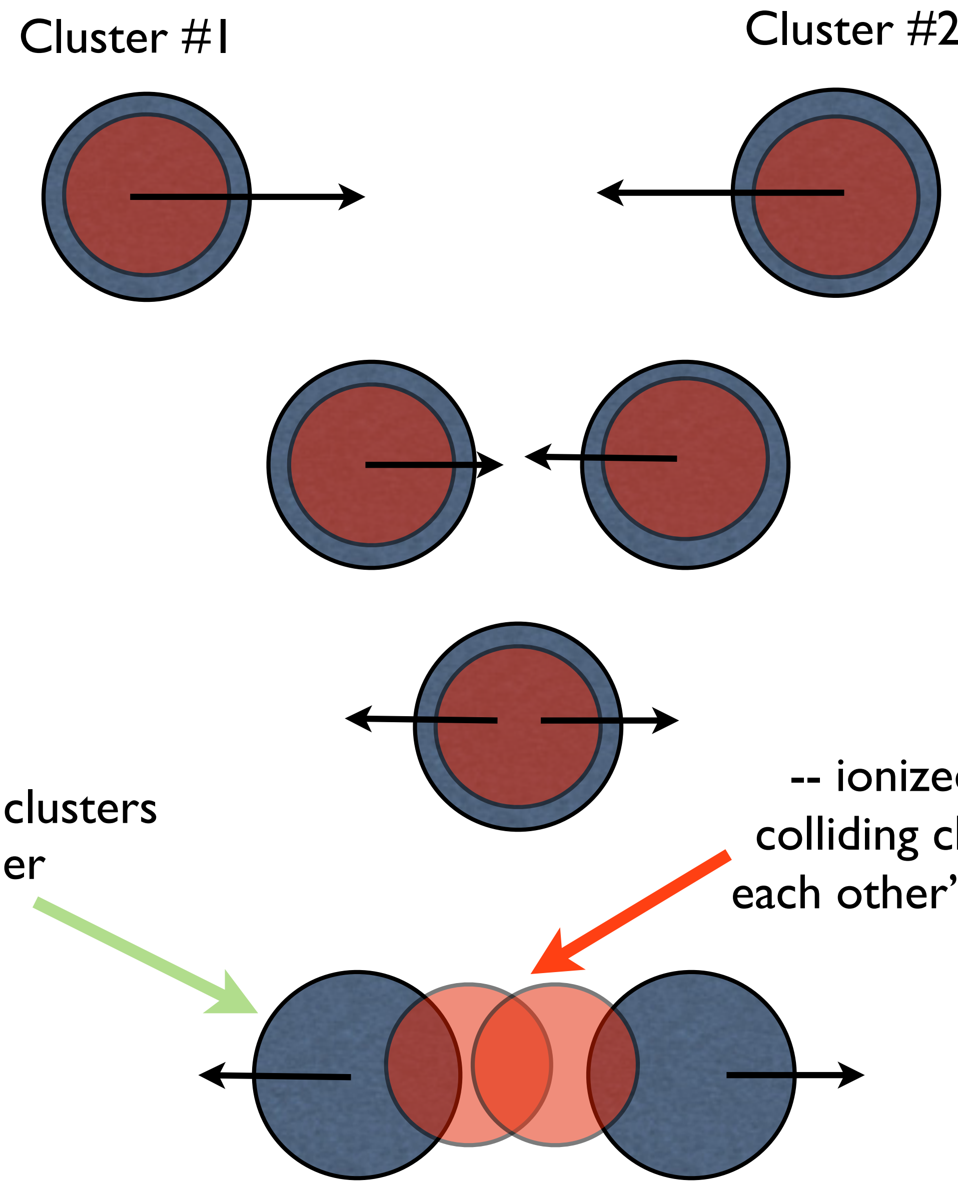
~13% in a very hot ionized gas

~85% in dark matter

Most of the baryons are in the ionized gas!

Evidence from the Observations of Colliding Clusters

Reason it is useful can be seen from the following “simulation”:



this presents us with a situation where the light (from baryons) and mass (from dark matter) are in different places

Evidence from the Observations of Colliding Clusters

-- how can we use the observations to see that baryons do not provide most of the mass

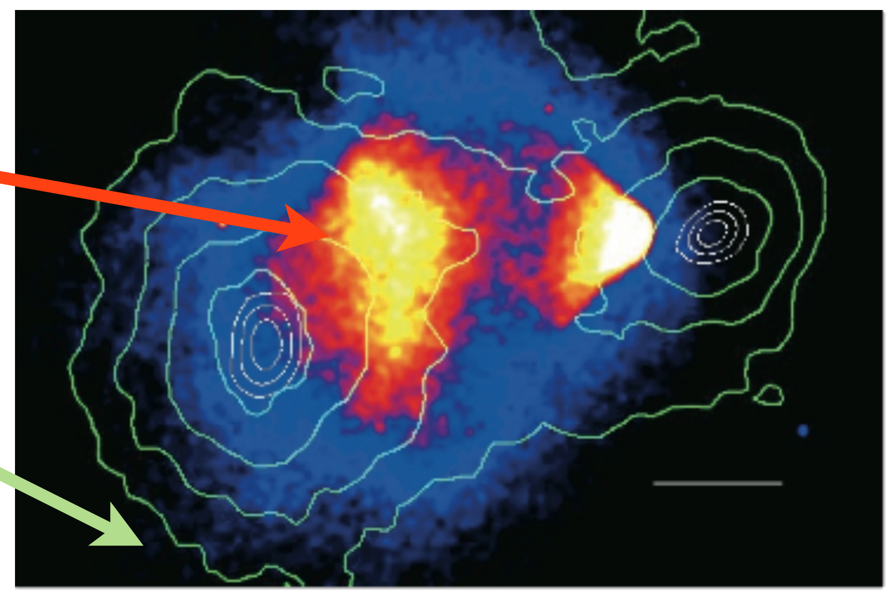
-- x-ray light shows us where the ionized gas (i.e., baryons) is

-- gravitational lensing shows us where the mass is (mostly dark matter)

“Bullet Cluster” Clowe et al. 2006

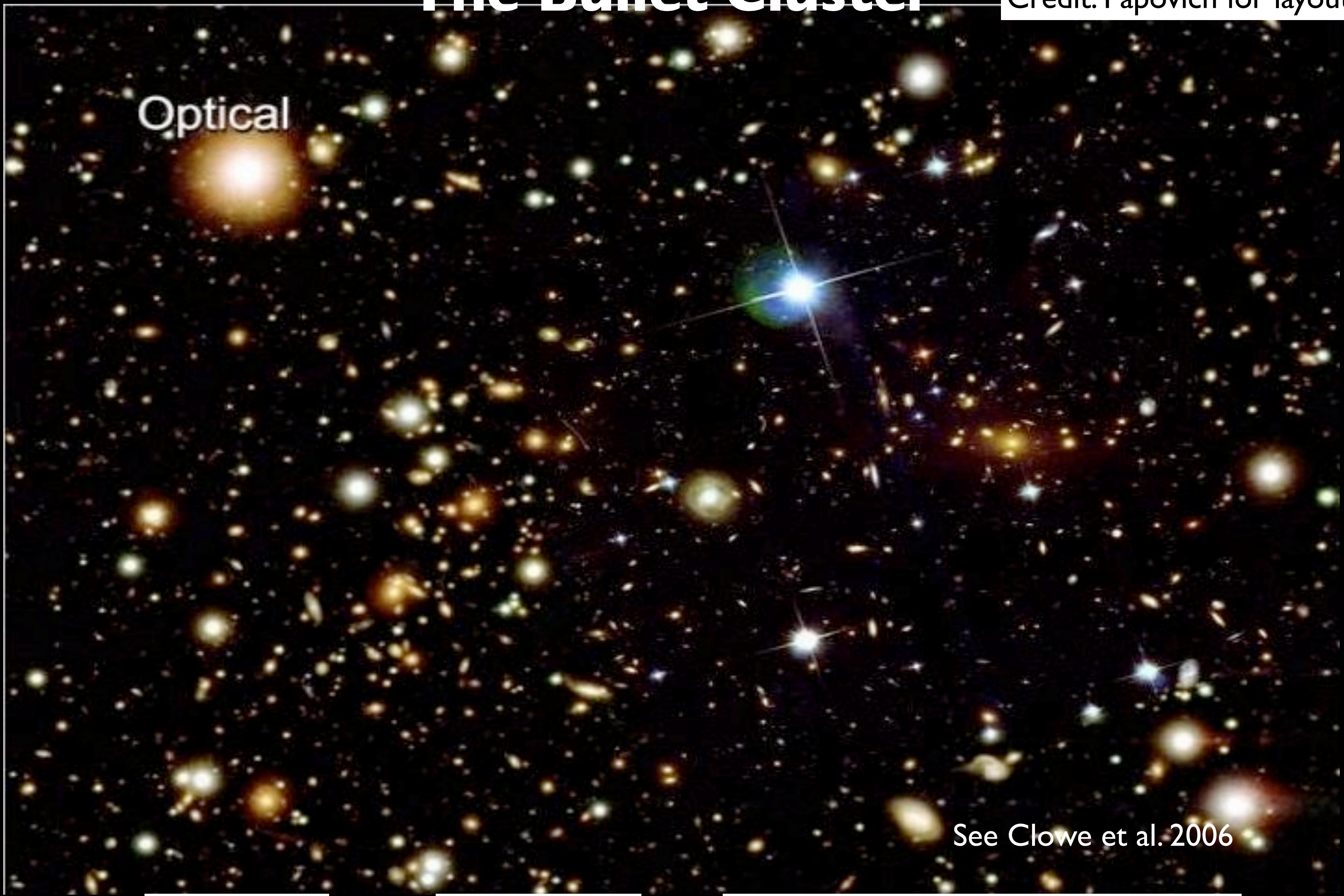
-- ionized gas from the colliding clusters “run into each other” forming a shock

-- dark matter from the colliding clusters pass right through each other



The Bullet Cluster

Credit: Papovich for layout



Orange: stars

Red : X-ray gas

Blue : Mass from lensing measurements

The Bullet Cluster

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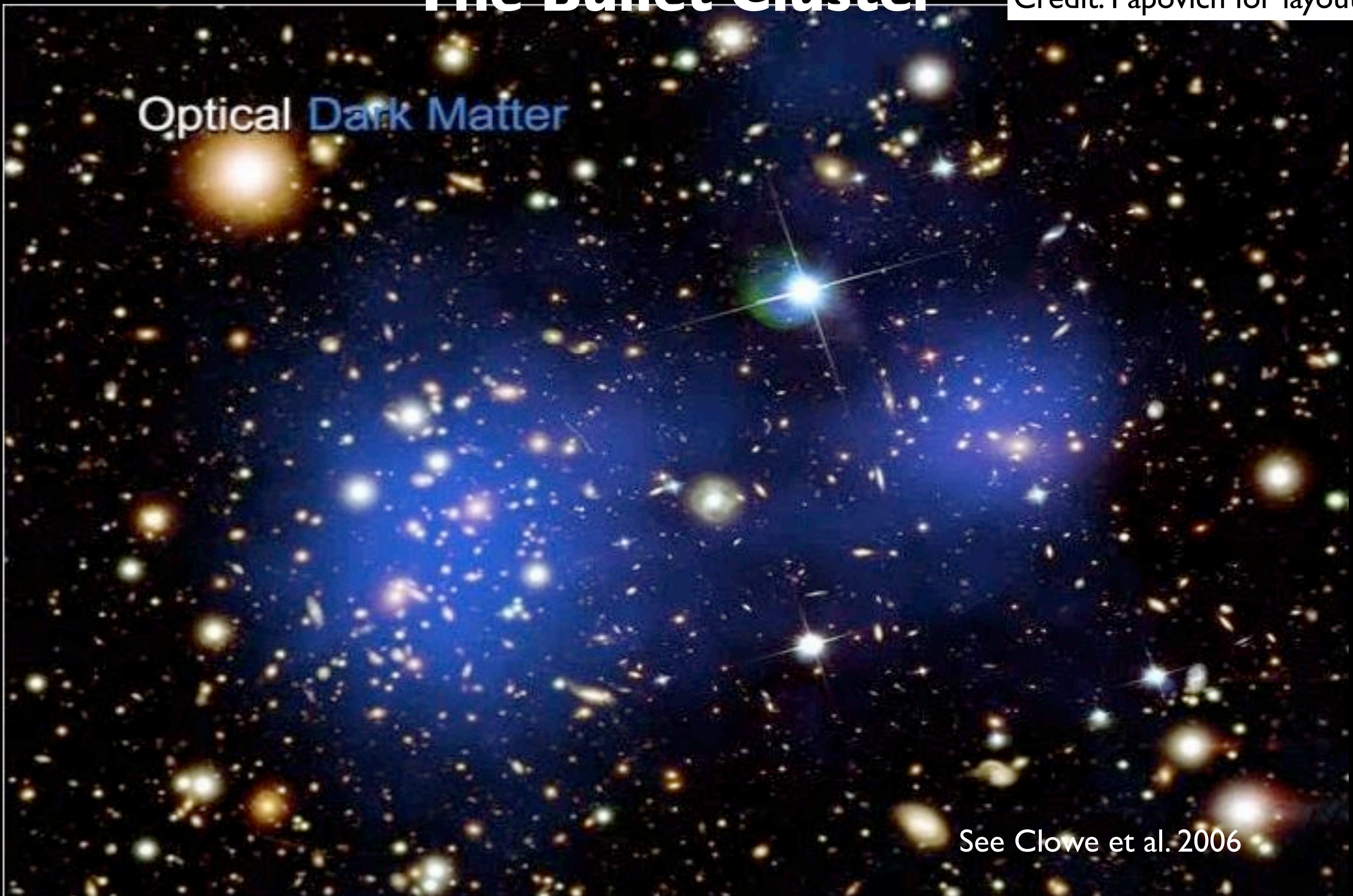

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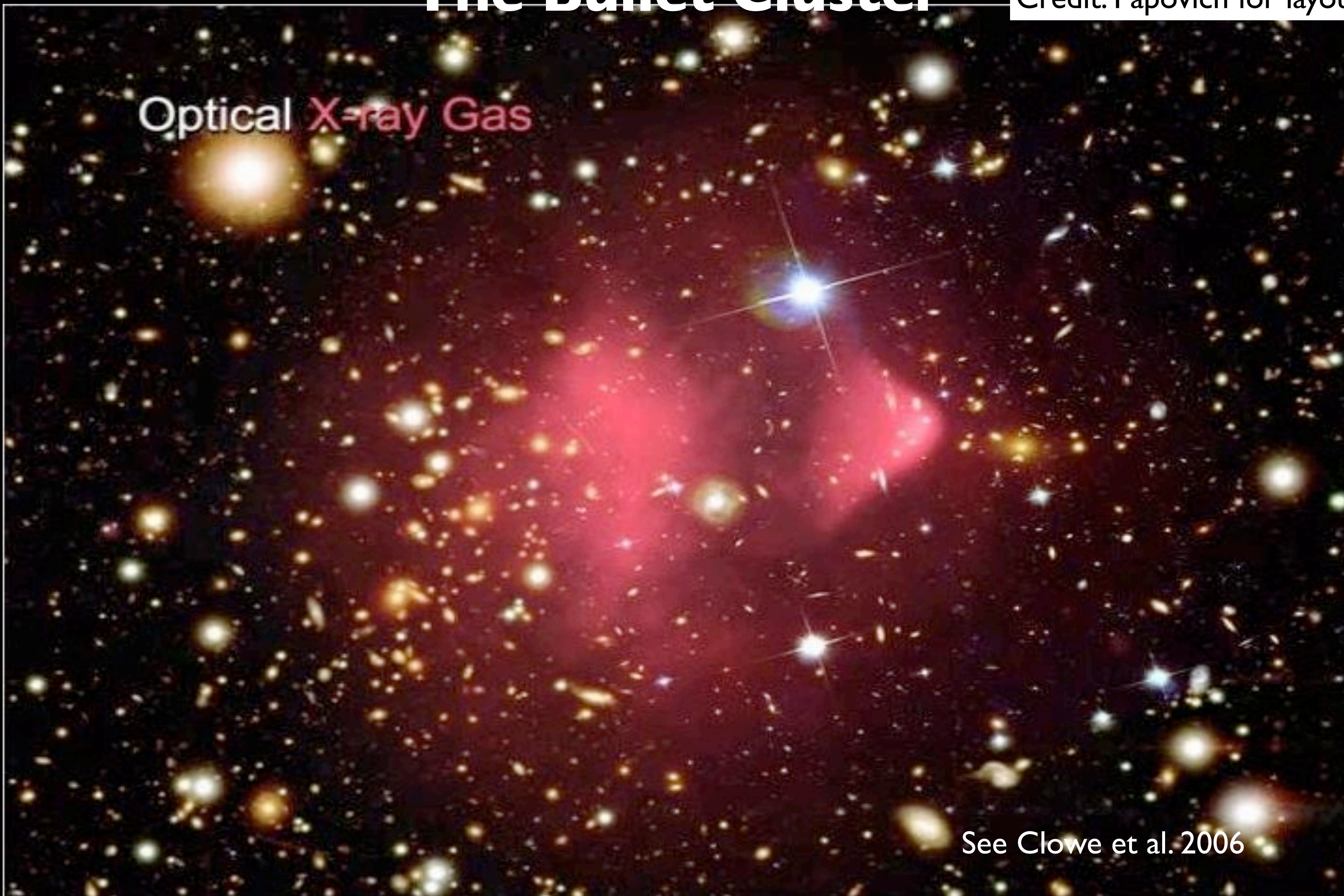

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The Bullet Cluster

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What are the measured densities in various components of the universe?

What is Ω_{mat}? (density of normal matter relative to critical)

$\Omega_{mat,bary} = 0.04$ Baryonic matter component

0.31

Yields of Helium allow this to be measured + from acoustic oscillations in CMB

0.27 Dark matter component

Measurable from bulk flows / peculiar velocities / and from CMB

What is Ω_{rad}? (energy density of radiations relative to critical)

0.0001

Measurable from temperature of CMB (since it has a black body spectrum)

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Measurable from bulk flows / peculiar velocities / and from CMB

What is Ω_{rad}? (energy density of radiations relative to critical)

0.0001

Measurable from temperature of CMB (since it has a black body spectrum)

What is Ω_{Λ}? (energy density of radiations relative to critical)

0.69

Measurable from luminosity distances to SNe

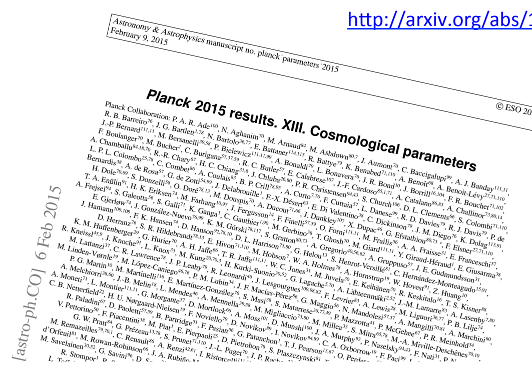
What are the measured densities in various components of the universe?

[1] Parameter	[5] 2015F(CHM)	[6] 2015F(CHM) (Plik)
$100\theta_{\mathrm{MC}}$	1.04094 ± 0.00048	1.04086 ± 0.00048
$\Omega_b h^2$	0.02225 ± 0.00023	0.02222 ± 0.00023
$\Omega_c h^2$	0.1194 ± 0.0022	0.1199 ± 0.0022
H_0	67.48 ± 0.98	67.26 ± 0.98
n_{s}	0.9682 ± 0.0062	0.9652 ± 0.0062
Ω_{m}	0.313 ± 0.013	0.316 ± 0.014
σ_8	0.829 ± 0.015	0.830 ± 0.015
τ	0.079 ± 0.019	0.078 ± 0.019
$10^9 A_{\mathrm{s}} e^{-2\tau}$	1.875 ± 0.014	1.881 ± 0.014

Standard Model of cosmology

We start with the reviewing the status of observational cosmology over the past century and how we arrived at the current "standard model" of cosmology.

http://arxiv.org/abs/1502.01589


Astronomy & Astrophysics manuscript no. planck parameters 2015
February 9, 2015
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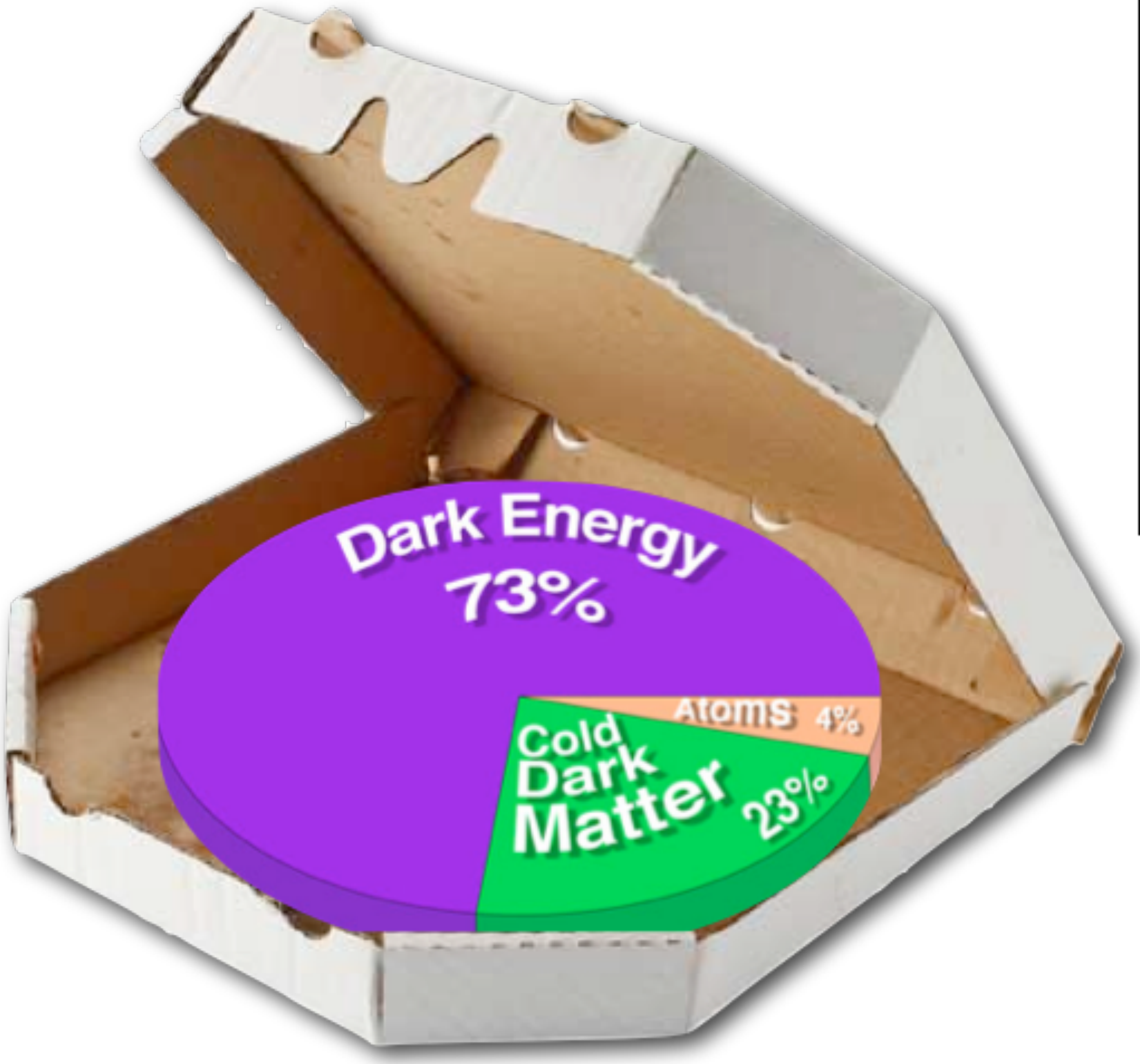
Planck 2015 results. XIII. Cosmological parameters

[astro-ph.CO] 6 Feb 2015

What are the measured densities in various components of the universe?



Ingredients
(per Universe)

DE: 73%
CDM: 23%
H: 3%
He: 1%
neutrinos: 0.3%

Note: slightly different than Planck constraints

Importantly, what do the different densities add up to equal?

$$\Omega_m + \Omega_r + \Omega_\Lambda = 1.000 \pm 0.005$$

What does this imply regarding Ω_k ?

$$\Omega_m + \Omega_r + \Omega_\Lambda + \Omega_k = 1 \longrightarrow \Omega_k = 1 - \Omega_m - \Omega_r - \Omega_\Lambda$$

$$|\Omega_k| < 0.005$$

Why might this be unexpected?! (and therefore it is important!)

The relation between the Hubble and density parameters can be seen by writing the Friedmann equation as follows:

$$H(t)^2 = (8\pi G/3c^2)\varepsilon(t) - \kappa c^2/(R_0^2\, a^2(t))$$

Dividing by $H^2(t)$ and realizing that $(8\pi G/3H^2(t))(\varepsilon(t)/c^2) = \Omega(t)$

$$1 - \Omega(t) = -\kappa c^2/(R_0^2\, a^2(t) H^2(t))$$

Manipulating this expression, one can show

$$1 - \Omega(t) = H_0\,(1 - \Omega_0)/(H(t)a(t))^2$$

How does this expression vary with a or t at early times?

$$1 - \Omega(t) = H_0\,(1 - \Omega_0)a^2/(\Omega_{r,0} - a\Omega_{m,0})$$

during radiation domination $|1 - \Omega(t)| \propto a^2 \propto t$

during matter domination $|1 - \Omega(t)| \propto a \propto t^{2/3}$

deviation grows with time

if $|1 - \Omega| \sim 0.005$ now, then $|1-\Omega| \sim 10^{-62}$

as this suggests fine tuning, this is the flatness problem