Sterrewacht
Leiden

Origins & Evolution of the Universe

an introduction to cosmology — Fall 2018

Lecture 4: Redshift, Horizons, Observable Distances

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Layout of the Course

Sep 24: Introduction and Friedmann Equations
Oct 1: Fluid and Acceleration Equations
Oct 8: Introductory GR, Space Time Metric, Proper Distance
Oct 15: Redshift, Horizonts, Observable Distances, Parameter Constraints, Intro CMB
Oct 17: Problem Class #1
Oct 22: Observable Distances, Parameter Constraints
Oct 29: Thermal History, Early Universe
Nov 5: Early Universe, Inflation
Nov 12: Inflation, Lepton Era
Nov 14: Problem Class #2
Nov 19: Big Bang Nucelosynthesis, Recombination
Nov 26: Introduction to Structure Formation
Dec 3: Cosmic Microwave Background Radiation (I)
Dec 5: Problem Class #3
Dec 10: Cosmic Microwave Background Radiation (II)
Dec 21: Final Exam

You should have received problem set #1 by mail 1.5 weeks ago

Due by Wednesday 13:30 October 17, 2018

First Problem Class,
This Wednesday, October 17,
at 13:30

Review Last Week

Introduction to Concepts Relevant to General Relativity

General Relativity based on the observation that the inertial and gravitational masses are the same

Gravitational Force $F = m_g GM/r^2$

$\updownarrow$ the same

Impact of Force on Mass $F = m_i a$

Assume instead that mass curves space time such that test particles naturally accelerate in a way independent of the mass of the test particle.

In GR, particles move along geodesics (shortest paths)

Space time described by metric $g_{\mu\nu}$ which gives distance ds between events

$\mathbf{x} = (t,x,y,z)$ and $\mathbf{x} + \mathbf{dx} = (t + dt, x + dx, y + dy, z + dz)$

The geometric properties of the surface can be obtained by considering the distance between a close of infinitesimally close points:

$$dl^2 = \Sigma_{i,j}\, g_{ij}(x)\ dx^i\ dx^j$$

$\nearrow$ metric

Space Time

Space time is described by the distance between two events:

$$ds^2 = c^2\, dt^2 - dx^2 - dy^2 - dz^2$$

$$= \eta_{\mu\nu}\, x^\mu\, x^\nu = c^2 dt^2 - \delta_{ij}\, x^i\, x^j$$

where $(x^0, x^1, x^2, x^3) = (ct, x, y, z)$ and $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$

For a general space time: $ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$

In general relativity, the curvature of space time is important:

particles move such that

$$\delta \int_{\text{path}} ds = 0$$

the integral is stationary

In the x-frame, the equation of motion becomes

$$d^2x^\mu / ds^2 = -\Gamma^\mu{}_{\alpha\beta}\, (dx^\alpha/ds)(dx^\beta/ds)$$

Energy-Momentum Tensor, Newtonian Limit → Einstein's Field Equation

For a particle of rest mass m, we can define a four momentum as
$p^\mu = mU^\mu$, with $U^\mu = c\,dx^\mu/ds$

$T_{\mu\nu}$ describes the matter distribution for a perfect fluid (no viscosity, heat flow, or stress) with pressure P and energy density ρ

$$T_{\mu\nu} = (P + \rho c^2)U_\mu U_\nu - Pg_{\mu\nu}$$

One can express the curvature of space time so as to match accelerating particles in a simple Newtonian potential and set it up so as to have the same form as Poisson's equation from Newtonian gravity, therefore setting up a relation between curved space and the mass/energy density of the universe.

Einstein proposed the following expression to encapsulate curvature.

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} R$$

Ricci tensor

Ricci scalar

This tensor is the unique choice:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} / c^4$$

Einstein's Field Equation

Including A Cosmological Constant in Einstein's Field Equation

It is possible to write a modified set of field equations that are consistent with the conservation laws:

$$G_{\mu\nu} + \underset{\uparrow\ \text{cosmological constant}}{\Lambda} g_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$$

This modification allows for a static universe

What is the physical meaning of Λ?

It is useful to move Λ to the right hand side: $T_{\mu\nu}^{vac} = (-c^4\Lambda/8\pi G)g_{\mu\nu}$

$$G_{\mu\nu} = (8\pi G/c^4)(T_{\mu\nu} + T_{\mu\nu}^{vac})$$

If we recall that $T_{\mu\nu} = (P + \rho c^2)U_\mu U_\nu - Pg_{\mu\nu}$, the Λ term can be included as an ideal fluid with

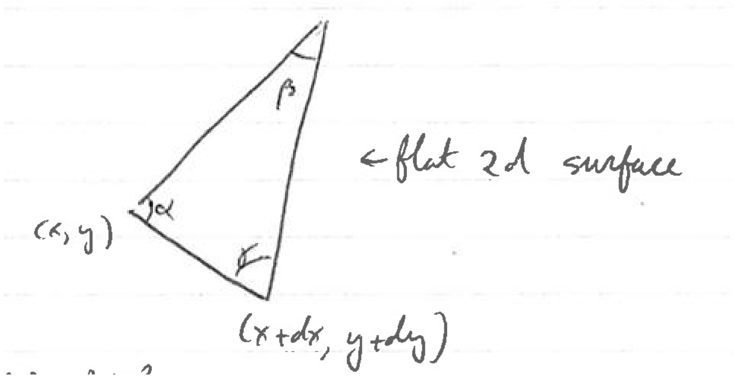
$$\rho = -P/c^2 = (-c^2\Lambda/8\pi G)$$

Curved Space Time

In general relativity, the metric is key, but which one describes the universe and obeys the cosmological principle?

Curvature must be the same everywhere1

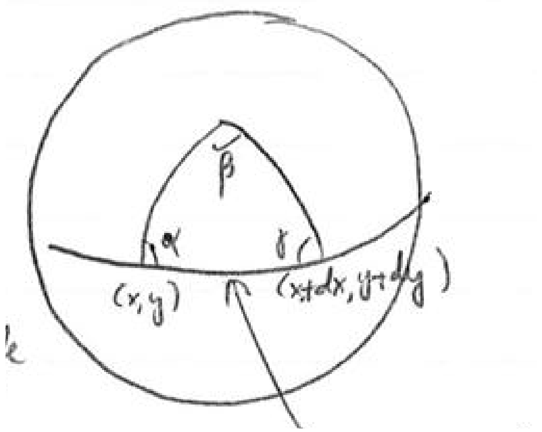
Flat 2D spaces



$\alpha + \beta + \Upsilon = \pi$ (in radians)

we know that $ds^2 = dx^2 + dy^2$

Now curved space - surface of sphere:



Area of Triangle

$\alpha + \beta + \Upsilon = \pi + A/R^2$

Radius of Sphere

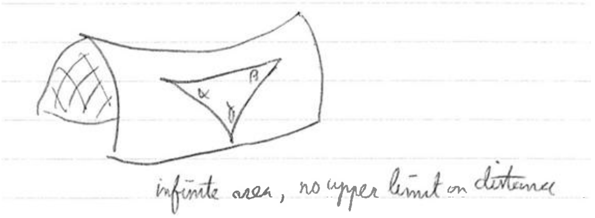
$\alpha + \beta + \Upsilon > \pi$: positively curved

Curvature is homogenous + isotropic

Note maximum distance between points of πR

Volume of $4\pi R^2$

Similarly we can define for negatively curved space



$\alpha + \beta + \Upsilon = \pi - A/R^2$

$\alpha + \beta + \Upsilon < \pi$: negatively curved

Infinite Volume + No Maximum Distances

General Curved Space Time:

Flat Space Time: $ds^2 = dx^2 + dy^2 + dz^2$

$$ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Positively Curved: $ds^2 = dr^2 + R^2\sin^2(r/R)\,(d\theta^2 + \sin^2\theta\, d\varphi^2)$

Negatively Curved: $ds^2 = dr^2 + R^2\sinh^2(r/R)(d\theta^2 + \sin^2\theta\, d\varphi^2)$

All these metrics have constant curvature:

$$ds^2 = dr^2 + S^2(r)\,(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

where

$$S(r) = \begin{array}{ll} R\sin(r/R), & \text{if } k = +1 \\ r, & \text{if } k = 0 \\ R\sinh(r/R), & \text{if } k = -1 \end{array}$$

If we change coordinate system such that x = S(r) and replace x with r,

$$ds^2 = dr^2/(1+kr^2) + r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

This is the most general spatial metric with constant curvature. The only change we need to make is to allow space to expand:

$$ds^2 = -\,c^2dt^2 + a^2(t)\,[dr^2/(1+kr^2) + r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2)]$$

New Material for This Week

Proper Distance

Imagine we have a distant galaxy at (r, θ, φ)

How far away is a distant galaxy?

(important to specify time since nominal distance depends on when measurement is made since expanding universe)

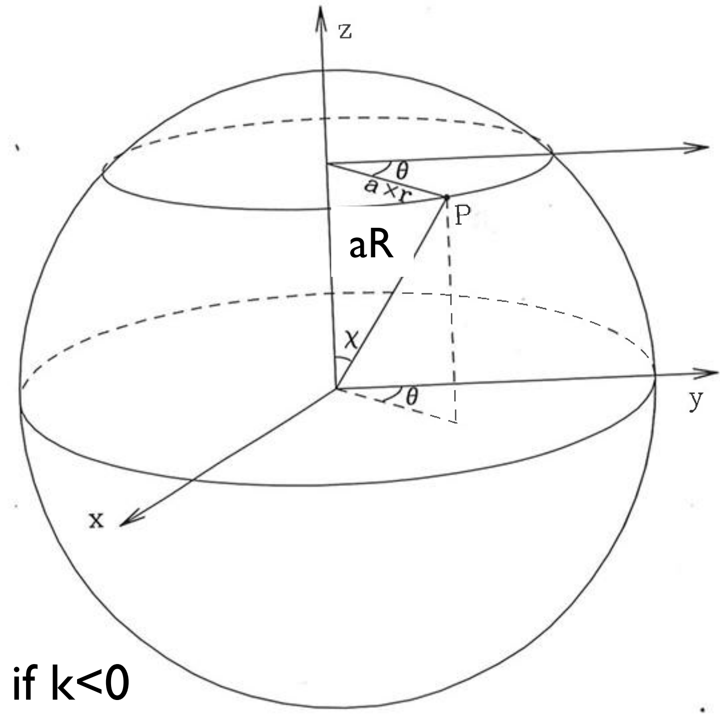
Take case that θ, φ are constant and only r is different

In the flat case (k=0),

$$\text{Proper Distance: } d_p(t) = \int ds$$

"Source-source distance measured based on current topology of the universe and ignoring travel time"

$$ds^2 = a^2(t)\,[dr^2/(1-kr^2) + r^2(d\theta^2 + \sin^2\theta d\varphi^2)]$$



In general case,

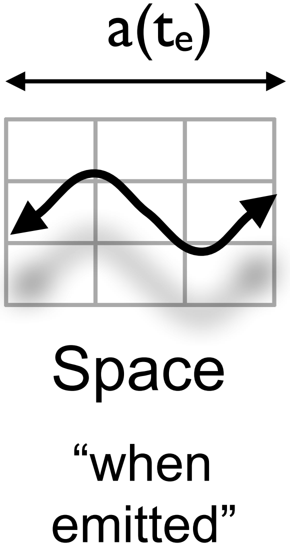
$$\text{Proper Distance: } d_p(t) = a(t)\underbrace{\begin{cases} \arcsin(k^{1/2}r)/k^{1/2}, & \text{if } k>0 \\ r, & \text{if } k=0 \\ \text{arcsinh}((-k)^{1/2}r)/(-k)^{1/2}, & \text{if } k<0 \end{cases}}_{f_k(r)}$$

Not especially practical (since not measurable)!

Distances and Redshifts

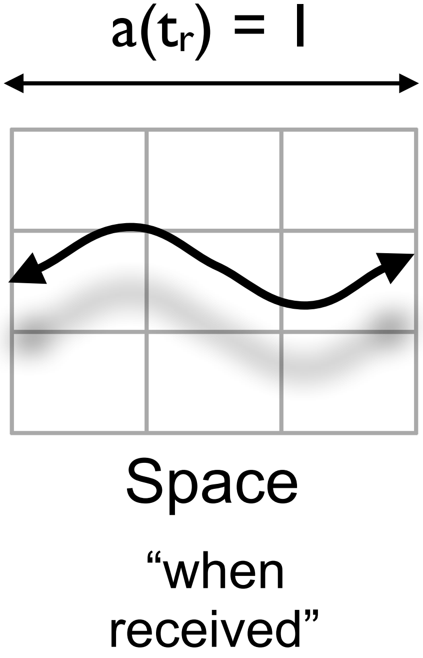
We cannot measure proper distances, but we can measure redshifts.

Redshift — which we denote as z — is directly connected to the scale factor of the universe.



$$\lambda_e\ (1+z) = \lambda_r$$

$$a(t_e) = 1/1+z$$



Redshift — which we denote as z — is directly connected to the scale factor of the universe.

Distances and Redshifts

Now let us to demonstrate how redshift relates to scale factor using the metric.

(Again take the case that θ, φ are constant and only r is different)

Since light travels on geodesics, ds = 0 and so

$$c\,dt / a(t) = dr / (1 - kr^2)^{0.5}$$

Consider the crest of a wave and consider it traveling from coordinate r = 0 to r_{obs}:

$$\int_0^{r_{obs}} dr / (1 - kr^2)^{0.5} = \int_{t_e}^{t_r} c\,dt / a(t)$$

It will make this trip from some time the t_e some later time t_r

Imagine some short time later… $t_e + dt_e$, $t_r + dt_r$ that the next wavefront moves between the same coordinates

$$\int_0^{r_{obs}} dr / (1 - kr^2)^{0.5} = \int_{t_e+dt_e}^{t_r+dt_r} c\,dt / a(t) \ = \text{above expression}$$

We can subtract $\int_{t_e+dt_e}^{t_r} c\,dt / a(t)$ from both

$$\Longrightarrow \int_{t_e}^{t_e+dt_e} c\,dt / a(t) = \int_{t_r}^{t_r+dt_r} c\,dt / a(t)$$

Distances and Redshifts

We can subtract $\int_{t_e+dt_e}^{t_r} c\ dt\ /\ a(t)$ from both $\Longrightarrow \int_{t_e}^{t_e+dt_e} c\ dt\ /\ a(t) = \int_{t_r}^{t_r+dt_r} c\ dt\ /\ a(t)$

$$\Longrightarrow c\ dt_e\ /\ a(t_e) = c\ dt_r\ /\ a(t_r)$$

As such, $dt \propto a(t)$ implies time dilation

Since the delta interval correspond to different crests of a wave,

$$\lambda \propto dt \propto a(t)$$

This implies

$$\lambda_r/\lambda_e = a(t_r)/a(t_e)$$

Since $a(t_r) = 1$ and $\lambda_r/\lambda_e = 1+z$,

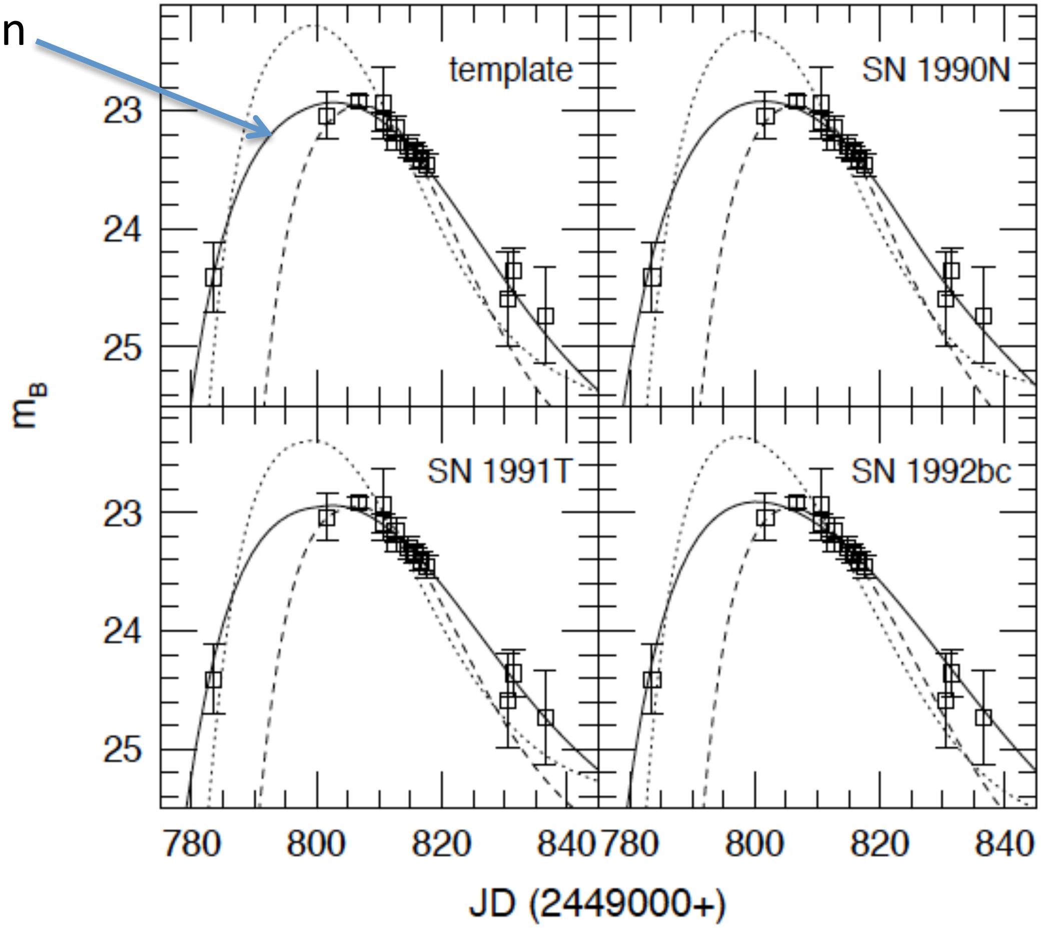
$$a(t_e) = 1/1+z$$

from lecture 3

Observation of time dilation

SN 1995K @z=0.479

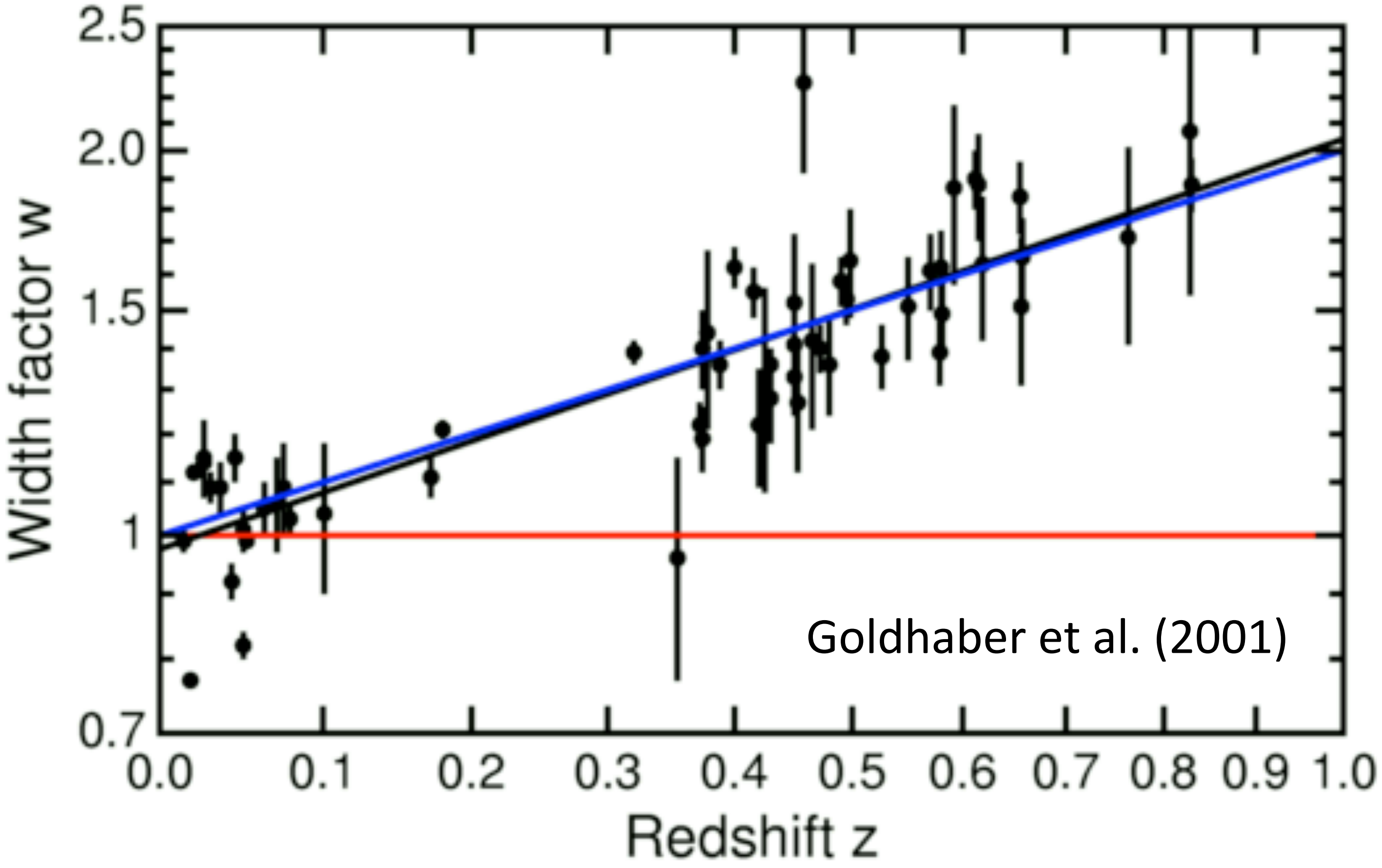
with time dilation



Leibundgut et al. (1996)

from lecture 3

Interpretation of redshift



Supernovae at greater redshifts are seen to take longer to decay: it scales linearly with the redshift: the redshifts are true reflections of Doppler shift.

How far can we see?

Light travels at a finite speed — no physical signals travel faster!

Complicated since the universe is expanding!

Horizon Distance

Horizon Distance: The greatest distance one can in principle look — probing back to when the universe had time t=0

$$d_{Hor} = c \int dt/a(t)$$

Let's first derive an expression for proper distance to a galaxy for a single component universe with equation of state parameter w:

How does a(t) evolve for an arbitrary value of w (equation of state)?

$$a(t) = (t/t_0)^{(2/(3+3w))}$$

What is the age of the universe in terms of H_0 ?

$$t_0 = (2/(3+3w))(1/H_0)$$

Can we write the ratio of t_0/t_e in terms of redshift?

$$1+z = (t_0/t_e)^{2/(3+3w)}$$

What is the proper distance to some galaxy which emitted its light at time t_e?

$$d_p(t_0) = c \int_{t_e}^{t_0} dt/a(t) = (c/H_0)(2/(1+3w))[1 - (t_e/t_0)^{(1+3w)/(3+3w)}]$$

Horizon Distance

What is the proper distance to galaxy who emitted its light at time t_e with redshift z?

$$d_p(z) = ct_0\,(3(1+w)/(1+3w))[1 - (1+z)^{-(1+3w)/2}]$$

$$d_p(z) = (c/H_0)(2/(1+3w))[1 - (1+z)^{-(1+3w)/2}]$$

Blows up if $w <= -1/3$

if $w=0$: $d_p(z) = (2c/H_0)[1 - (1+z)^{-1/2}]$

What is the Horizon distance?

$$d_{Hor}(t) = (c/H_0)(2/(1+3w))$$

which is finite if $w > -1/3$

Horizon distance tells us the portion of the universe that is casually connected

$w = 0$: $d_{Hor}(t) = 2c/H_0 = 3ct_0$

Particle Horizon

A key question is which parts of the universe can influence or have influenced each other, i.e., which parts are in casual contact.

Particle horizon: boundary of that part of the universe that could have reached us in the age of the universe. At the present epoch, it is the observable universe.

What is this horizon to redshift z_e?

$$d_{Hor}(z_e) = 3ct_0 [1 - (1+z_e)^{-1/2}]$$

In the more general case, can derive for an arbitrary geometry of the universe and arbitrary fluid component with arbitrary w:

$$d_{Hor}(z_e) = (c/H_0) \int_0^{z_e} dz/(1+z)/(\Omega_0(1+z)^{1+3w}+(1 - \Omega_0))^{0.5}$$

Note Ω_0 represents the density of fluid relative to critical, such that $1 - \Omega_0 = \Omega_k$ is the critical density.

Particle Horizon

How does the Particle horizon change with time?

It is proportional to t/t_0

$$t/t_0 \propto (a/a_0)^{3/2} \propto (1+z)^{-3/2}$$

8x smaller at z~3, 30000x smaller at z~1000

Particle Horizon

How did Proper Distance of Particle horizon change with time?
it is proportional to t/t_0

$$t/t_0 \propto (a/a_0)^{3/2} \propto (1+z)^{-3/2}$$

8x smaller at z~3, 30000x smaller at z~1000

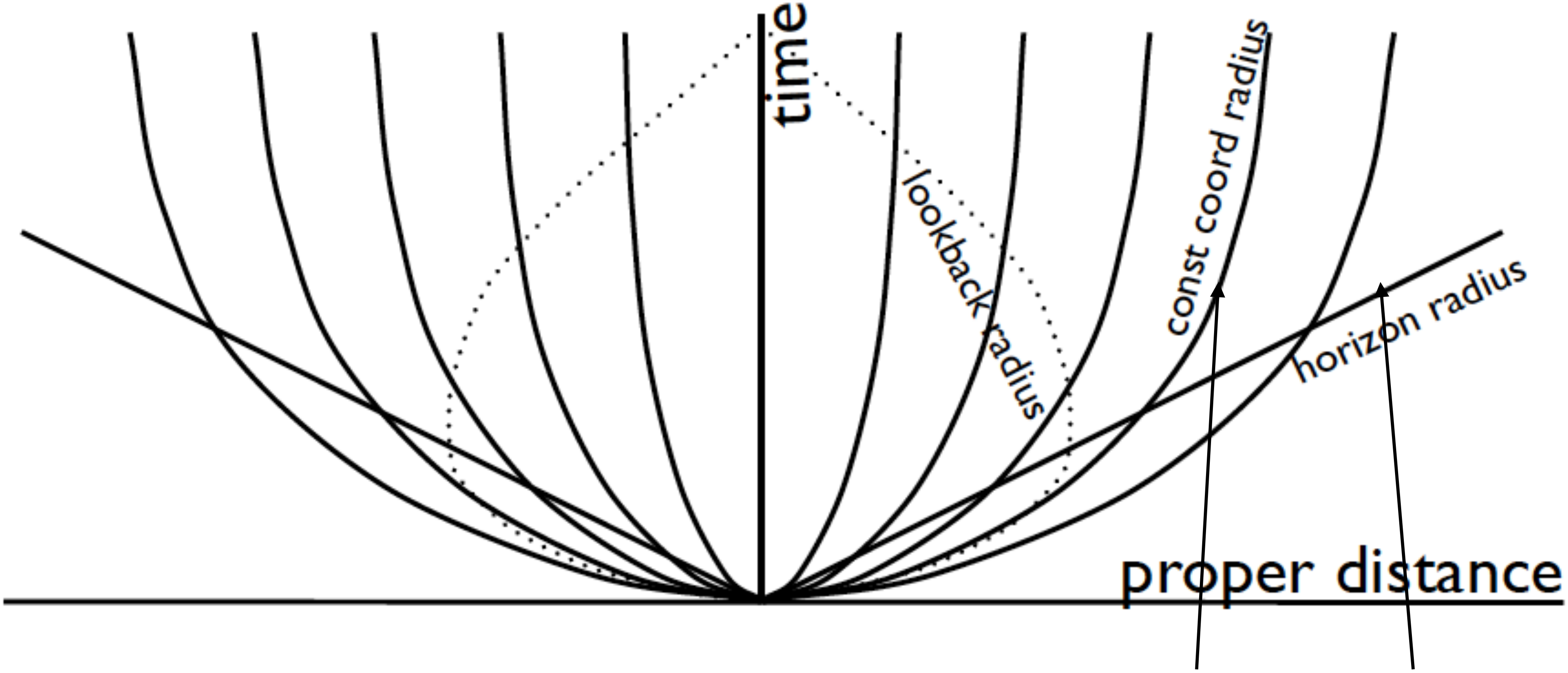
How does this horizon change relative to the scale factor of the universe (i.e. divide by 1/(1+z))

$$d_{Hor}\ (1+z) = (1+z)^{-1/2}$$

Therefore, with the horizon, we potentially probe further and further in the regions of the universe to which we have never been in casual contact

We illustrate this on the next page…

Horizon in flat Ω_m=1 model



scale as $t^{2/3}$

scale as t

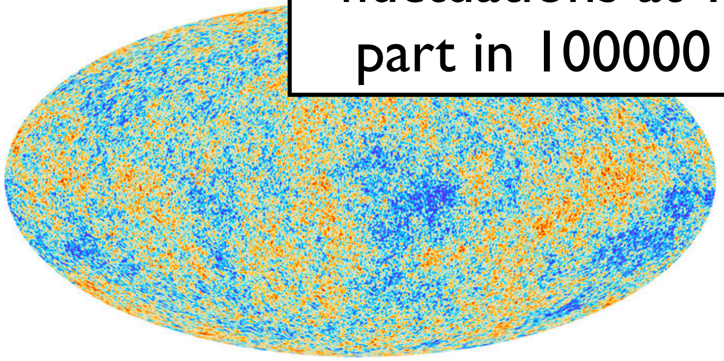
Horizon Problem

We receive light from galaxies or other sources which appear never to have been able to communicate with each other or us before.

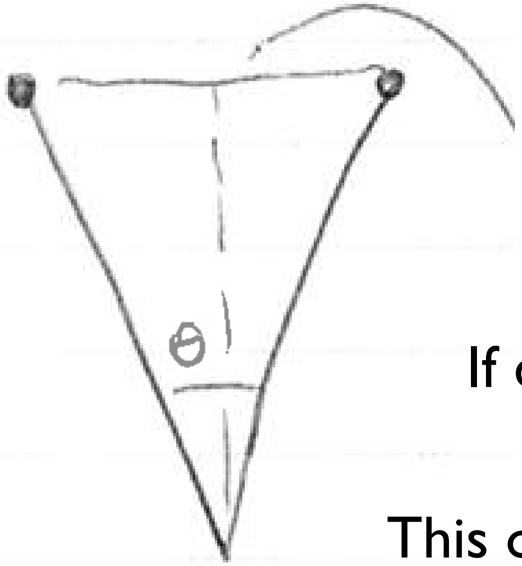
$\Longrightarrow$ So, why do these different regions of the universe appear homogeneous?

This is called the horizon problem.

Consider two different regions in the CMB



fluctuations at 1 part in 100000



angular diameter distance
$2 \sin(\theta/2)\, D_A(z)$

If distance is larger than $2D_H(z)$, then the points are not casually connected

This occurs when $\theta > 3.6$ degrees

but almost all points on sky are separated by a larger distance than this, so how can the points have similar temperatures?

Horizon Problem

but almost all points on sky are separated by a larger distance than this, so how can the points have similar temperatures?

Note that the integral $\int$ cdt / a(t) depends on the form of a(t) at early times.

we need to change how a(t) scales at early time so that points in casual contact at the earliest times, leave the horizon, and then reenter.

Need accelerated expansion, i.e., inflation, which occurs when $w < -1/3$. We saw that for $w < -1/3$, there is no particle horizon
(if a(t) maintains form to $z \rightarrow \infty$)