

Origins & Evolution of the Universe

an introduction to cosmology — Fall 2018

Rychard Bouwens

This course in the cosmology track

Origins and Evolution of the Universe is the introduction to physical cosmology at Leiden University. The course can be taken as a stand-alone course and requires a minimum of pre-requisites.

Core courses for the MSc cosmology track	
Origin and Evolution of the Universe	6 EC
Particle Physics and Early Universe	6 EC
Large Scale Structure and Galaxy Formation	6 EC
Theory of General Relativity	6 EC
Related courses	
Observational cosmology	3 EC
Theoretical cosmology	3 EC

Although General Relativity is critical to understand our Universe, we can do without for the moment. We will assume a basic BSc level knowledge of astronomy.

Lectures

Rychard Bouwens

Oort 459

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Lecture Hours:

Huygens 414

Monday 1:30-3:15

Course Website:

<http://www.strw.leidenuniv.nl/~bouwens/oeu/>

Layout of the Course

As part of the course, we address some of the most fundamental questions:

What are the basic properties of our Universe (composition, age, origin)?

What is the origin of structure in the Universe?

Sep 24: Introduction and Friedmann Equations

Oct 1: Fluid and Acceleration Equations

Oct 8: Introductory GR, Space Time Metric, Proper Distance

Oct 15: Redshift, Horizons, Observable Distances, Parameter Constraints, Intro CMB

Oct 17: Problem Class #1

Oct 22: Observable Distances, Parameter Constraints

Oct 29: Thermal History, Early Universe

Nov 5: Early Universe, Inflation

Nov 12: Inflation, Lepton Era

Nov 14: Problem Class #2

Nov 19: Big Bang Nucleosynthesis, Recombination

Nov 26: Introduction to Structure Formation

Dec 3: Cosmic Microwave Background Radiation (I)

Dec 5: Problem Class #3

Dec 10: Cosmic Microwave Background Radiation (II)

Dec 21: Final Exam

Examination

A written exam is scheduled for December 21, 2018. The material covered in this class delimits what will be tested in the exam.

Do not forget to register for the exam in uSIS!

Problem Classes

To aid in the preparation for the exam, there are three scheduled problem classes with the homework. This accounts for 20% of the grade, *if* it is higher than the result for the written exam.

The homework needs to be handed in before the start of the problem class.

Oct 17: Problem Class #1

Oct 22: Observable Distances, Parameter Constraints

Oct 29: Thermal History, Early Universe

Nov 5: Early Universe, Inflation

Nov 12: Inflation, Lepton Era

Nov 14: Problem Class #2

Nov 19: Big Bang Nucleosynthesis, Recombination

Nov 26: Introduction to Structure Formation

Dec 3: Cosmic Microwave Background Radiation (I)

Dec 5: Problem Class #3

Dec 10: Cosmic Microwave Background Radiation (II)

Dec 21: Final Exam

Textbook?

Useful Textbooks for the course will be

“Introduction to Cosmology”: Barbara Ryden

“Introduction to Modern Cosmology”: Andrew Liddle

“Cosmology: The Origin and Evolution of Cosmic Structure”:
Peter Coles and Francesco Lucchin

“Cosmological Physics”: John Peacock

The textbooks include a useful discussion of the material, but the course will not be organized to follow the presentation in any of these books.

However, I will advise you as to where you can find the relevant material in one of these textbooks.

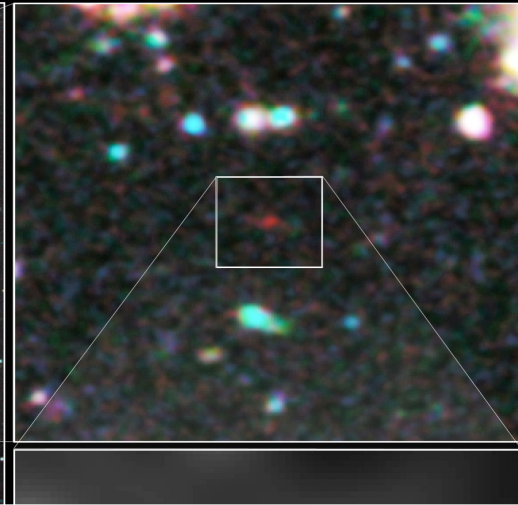
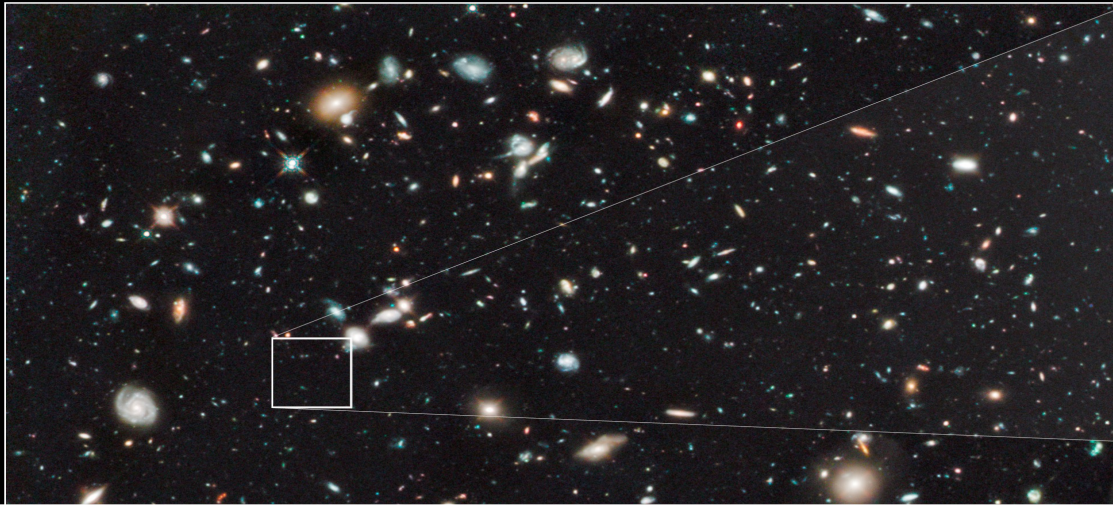
Who am I?

My name is Rychard Bouwens
(studied in the United States: Berkeley & Santa Cruz)

I study the most distant galaxies in the universe

A good understanding of galaxies is very important
for my research

Discovery of Plausible Galaxy just ~400-450 Myr after Big Bang



Candidate for the most distant galaxy ever discovered...



UDFj-39546284

Hubble Ultra Deep Field 2009–2010
Hubble Space Telescope • WFC3/IR

NASA, ESA, G. Illingworth (University of California, Santa Cruz),
R. Bouwens (University of California, Santa Cruz and Leiden University), and the HUDF09 Team

STScI-PRC11-05

Most distant galaxy candidates discovered to date

Redshift (68% CL)	age of universe	object	F160W AB magnitude	Flux (nJy)	reference	field / survey
11.9 or emission line galaxy	370 Myr	UDF12-3954-6284	29.3 ± 0.2	7	Ellis13, Bouwens11	UDF12
11.09	414 Myr	GN-z11	25.9	140	Oesch16	CANDELS
10.8 ± 0.3	420 Myr	MACS0647-JD	25.9, 26.1, 27.3	162, 132, 42	Coe13	CLASH
9.8 ± 0.6	480 Myr	XDFj-3812-6243	29.9 ± 0.4	4	Oesch13	UDF12
9.8	480 Myr	Abell2744-JD	27.0	70	Zitrin14	Abell2744
9.6 ± 0.2	490 Myr	MACS1149-JD	25.7 ± 0.07	194	Zheng12	CLASH
9.5 ^{+0.4} _{-0.8} or star spike	500 Myr	UDF12-4106-7304	29.7 ± 0.3	5	Ellis13	UDF12
9.5 ^{+0.4} _{-0.7}	500 Myr	UDF12-4265-7049	29.7 ± 0.4	5	Ellis13	UDF12
9.2 ^{+0.4} _{-0.6}	520 Myr	MACS1115-JD	26.2 ± 0.2	115	Bouwens13	CLASH
9.0 ^{+0.3} _{-0.8}	540 Myr	MACS1720-JD	26.9 ± 0.3	66	Bouwens13	CLASH

Highest redshifts spectroscopically confirmed

Redshift	age of universe	object	AB magnitude	reference
11.09	414 Myr	GN-z11	25.9	Oesch16
8.68	580 Myr	EGS-zs8-1	25.2	Zitrin15
7.73	680 Myr	EGS-zs7-1	25.0	Oesch15

Highlighted = more robust

Credit: Dan Coe

Teaching Assistant

Anna de Graaff

Oort 532

graaff@strw.leidenuniv.nl

Anna should also be available by appointment to answer your questions.

Who are you?

Why don't we go around the class and introduce ourselves briefly?

Name

Program -- Physics or Astronomy?

Master's Student?

First or Second Year?

Why interested in course?

What's your background?

How many of you have taken Huub's or Marijn's course "Galaxies and Cosmology" when you were a bachelor student?

How many of you are from the physics program?

Please ask questions

This is **your** course. It is your opportunity to learn.

By asking questions, you allow me to clarify issues

Standard Model of cosmology

We start with the reviewing the status of observational cosmology over the past century and how we arrived at the current “standard model” of cosmology.

<http://arxiv.org/abs/1502.01589>

Astronomy & Astrophysics manuscript no. planck_parameters_2015
February 9, 2015

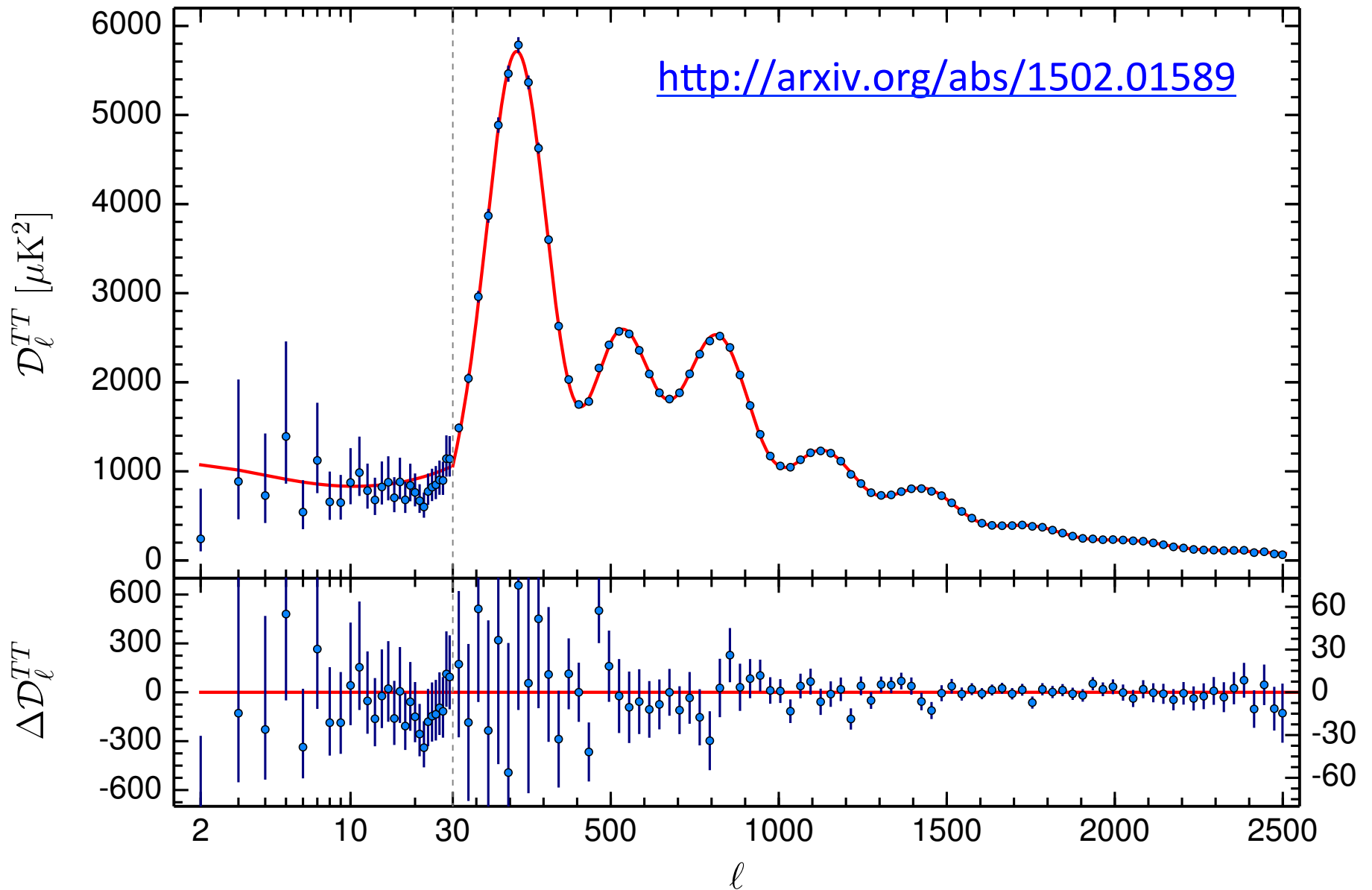
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Planck 2015 results. XIII. Cosmological parameters

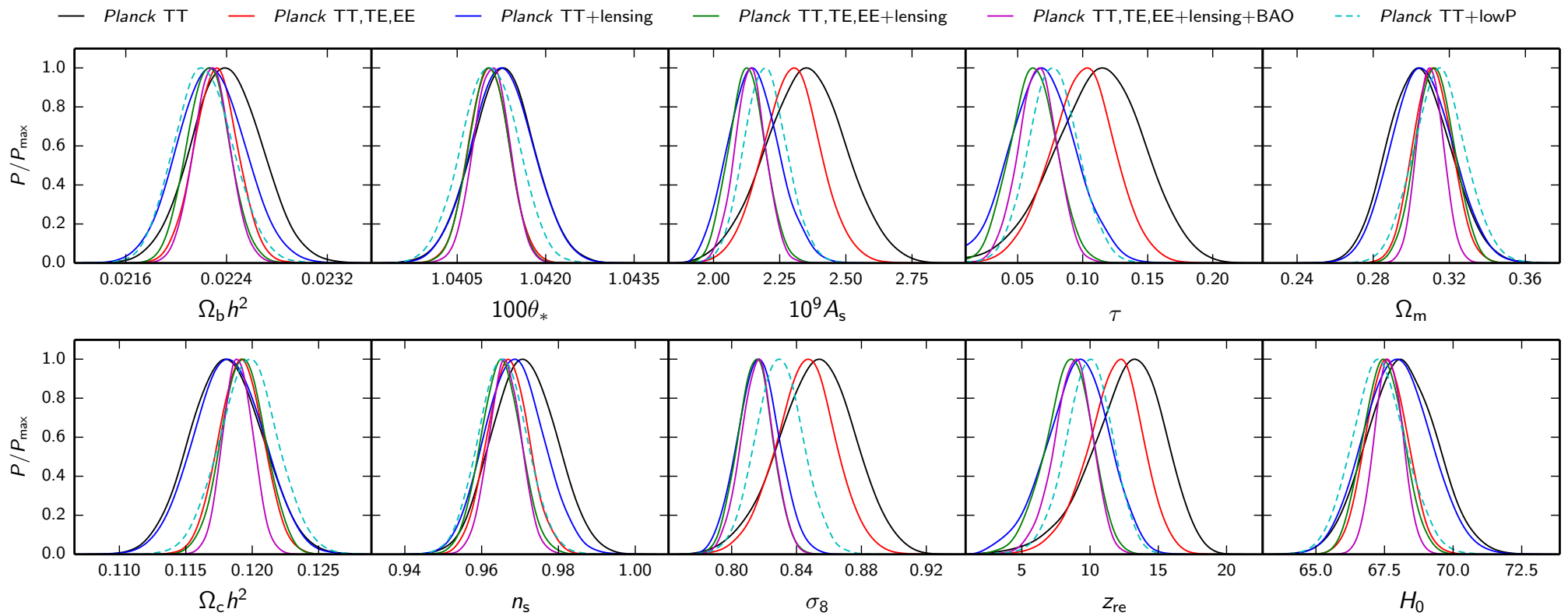
[astro-ph.CO] 6 Feb 2015

Planck Collaboration: P. A. R. Ade¹⁰⁰, N. Aghanim⁷⁰, M. Arnaud⁸⁴, M. Ashdown^{80,7}, J. Aumont⁷⁰, C. Baccigalupi⁹⁹, A. J. Banday^{111,11}, R. B. Barreiro⁷⁶, J. G. Bartlett^{1,78}, N. Bartolo^{36,77}, E. Battaner^{114,115}, R. Battye⁷⁹, K. Benabed^{71,110}, A. Benoît⁶⁸, A. Benoit-Lévy^{27,71,110}, J.-P. Bernard^{111,11}, M. Bersanelli^{39,58}, P. Bielewicz^{111,11,99}, A. Bonaldi⁷⁹, L. Bonavera⁷⁶, J. R. Bond¹⁰, J. Borrill^{16,104}, F. R. Bouchet^{71,102}, F. Boulanger⁷⁰, M. Bucher¹, C. Burigana^{57,37,59}, R. C. Butler⁵⁷, E. Calabrese¹⁰⁷, J.-F. Cardoso^{85,1,71}, A. Catalano^{86,83}, A. Challinor^{73,80,14}, A. Chambaullu^{84,18,70}, R.-R. Chary⁶⁷, H. C. Chiang^{31,8}, J. Chluba^{26,80}, P. R. Christensen^{94,43}, S. Church¹⁰⁶, D. L. Clements⁶⁶, R. J. Davis⁷⁹, P. de Bernardis³⁸, A. de Rosa⁵⁷, G. de Zotti^{54,99}, J. Delabrouille¹, F.-X. Désert⁶³, E. Di Valentino³⁸, C. Dickinson⁷⁹, J. M. Diego⁷⁶, K. Dolag^{13,91}, L. P. L. Colombo^{25,78}, C. Combet⁸⁶, A. Couvais⁸³, B. P. Crill^{78,95}, A. Curio^{7,76}, A. Cuttaia⁵⁷, L. Danese⁹⁹, R. D. Davies⁷⁹, R. J. Davis⁷⁹, P. de Bernardis³⁸, S. Donzelli⁵⁸, O. Doré^{78,13}, M. Douspis⁷⁰, A. Ducout^{71,66}, J. Dunkley¹⁰⁷, X. Dupac⁴⁶, G. Efstathiou^{80,73}, F. Elsner^{27,71,110}, H. O. Dole^{70,69}, S. H. Eriksen⁷⁴, M. Farhang^{10,97}, J. Ferguson¹⁴, F. Finelli^{57,59}, O. Forni^{111,11}, M. Frailis⁵⁶, A. A. Fraisse³¹, E. Franceschi⁵⁷, T. A. Enßlin⁹¹, H. K. Eriksen⁷⁴, K. Ganga¹, C. Gauthier^{1,90}, M. Gerbino³⁸, T. Ghosh⁷⁰, M. Giard^{111,11}, Y. Giraud-Héraud¹, E. Giusarma³⁸, A. Freise⁹⁴, S. Galeotta⁵⁶, S. Galli⁷¹, K. Ganga¹, C. Gauthier^{1,90}, M. Gerbino³⁸, T. Ghosh⁷⁰, M. Giard^{111,11}, Y. Giraud-Héraud¹, E. Giusarma³⁸, J. Hamann^{109,108}, F. K. Hansen⁷⁴, D. Hanson^{92,78,10}, E. Hivon^{71,110}, M. Hobson⁷, W. A. Holmes⁷⁸, A. Hornstrup¹⁹, W. Hovest⁹¹, R. Kesikitalo¹⁶, T. S. Kisner⁸⁸, D. Herranz⁷⁶, S. R. Hildebrandt^{78,13}, A. H. Jaffe⁶⁶, T. R. Jaffe^{111,11}, W. C. Jones³¹, M. Juvela³⁰, E. Keihänen³⁰, J.-M. Lamarre⁸³, A. Lasenby^{7,80}, K. M. Huffenberger²⁹, G. Hurier⁷⁰, A. Kneissl^{45,9}, J. Knoche⁹¹, L. Knox³³, M. Kunz^{20,70,3}, S. Gratton^{80,73}, G. Helou¹³, S. Henrot-Versillé⁸¹, C. Hernández-Monteagudo^{15,91}, M. Lattanzi³⁷, C. R. Lawrence⁷⁸, J. P. Leahy⁷⁹, R. Leonardi⁴⁶, J. Lesgourgues^{109,98,82}, F. Levrier⁸³, A. Lewis²⁸, M. Liguori^{36,77}, P. B. Lilje⁷⁴, M. Linden-Vørnle¹⁹, M. López-Caniego^{46,76}, P. M. Lubin³⁴, J. F. Macías-Pérez⁸⁶, G. Maggio⁵⁶, N. Mandolesi^{57,37}, A. Mangilli^{70,81}, A. Marchini⁶⁰, P. G. Martin¹⁰, M. Martinelli¹¹⁶, E. Martínez-González⁷⁶, S. Masi³⁸, S. Matarrese^{36,77,49}, P. Mazzotta⁴¹, P. McGehee⁶⁷, P. R. Meinhold³⁴, A. Melchiorri^{38,60}, J.-B. Melin¹⁸, L. Mendes⁴⁶, A. Mennella^{39,58}, M. Migliaccio^{73,80}, M. Millea³³, S. Mitra^{65,78}, M.-A. Miville-Deschênes^{70,10}, A. Moneti⁷¹, L. Montier^{111,11}, G. Morgante⁵⁷, D. Mortlock⁶⁶, A. Moss¹⁰¹, D. Munshi¹⁰⁰, J. A. Murphy⁹³, P. Naselsky^{94,43}, F. Nati³¹, P. Nataraj⁷¹, R. Natterfield²², H. U. Nørgaard-Nielsen¹⁹, F. Novello⁷⁹, D. Novikov⁸⁹, I. Novikov^{94,89}, C. A. Oxborrow¹⁹, O. Perdereca⁷¹, R. Paladini⁶⁷, D. Paoletti^{57,59}, B. Partridge⁵¹, F. Pasian⁵⁶, G. Patanchon¹, T. J. Pearson^{13,67}, M. A. Perotto⁷⁰, V. Pettorino⁵⁰, F. Piacentini³⁸, M. Piat¹, E. Pierpaoli²⁵, D. Pietrobon⁷⁸, S. Plaszczyński⁸¹, J. P. Rachen³⁰, L. Remazeilles^{79,70,1}, G. Prézeau^{13,78}, S. Prunet^{71,110}, J.-L. Puget⁷⁰, J. P. Rachen³⁰, R. Stompor¹, P. S. Tanzi^{111,11}, L. Toffi⁷⁰, M. Tomasi

Standard Model of cosmology



Standard Model of cosmology

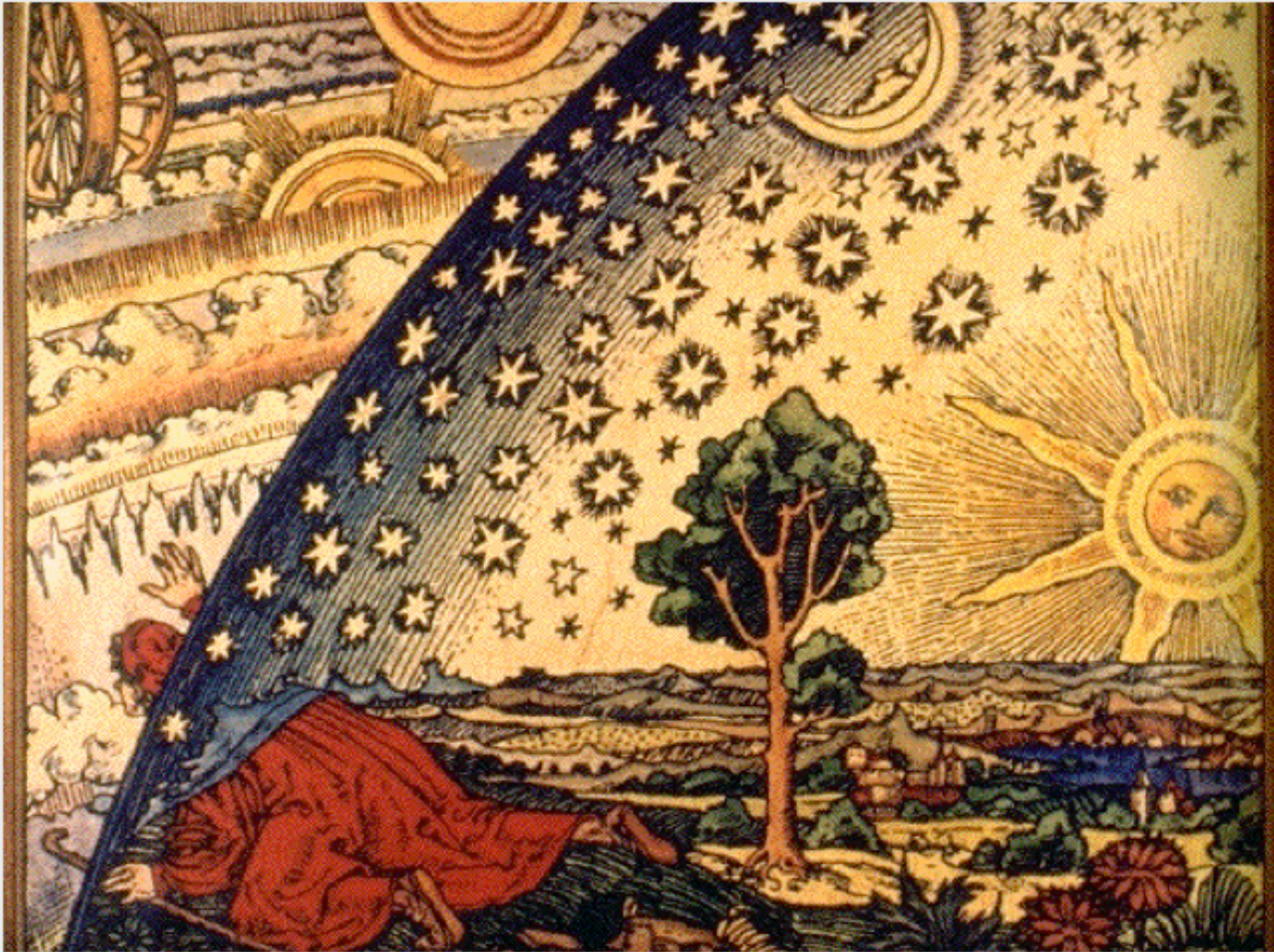


Standard Model of cosmology

[1] Parameter	[5] 2015F(CHM)	[6] 2015F(CHM) (Plik)
$100\theta_{\text{MC}}$	1.04094 ± 0.00048	1.04086 ± 0.00048
$\Omega_b h^2$	0.02225 ± 0.00023	0.02222 ± 0.00023
$\Omega_c h^2$	0.1194 ± 0.0022	0.1199 ± 0.0022
H_0	67.48 ± 0.98	67.26 ± 0.98
n_s	0.9682 ± 0.0062	0.9652 ± 0.0062
Ω_m	0.313 ± 0.013	0.316 ± 0.014
σ_8	0.829 ± 0.015	0.830 ± 0.015
τ	0.079 ± 0.019	0.078 ± 0.019
$10^9 A_s e^{-2\tau}$	1.875 ± 0.014	1.881 ± 0.014

The aim of this course is to explain these results.

Let's get started...



Basic principles

The Universe is enormous and we can observe only a single one...

We cannot carry out experiments to test our ideas: we use our knowledge of terrestrial (or solar system) physics to interpret our observations. This can lead to metaphysical considerations. Only in the past 60 years has cosmology been considered a “real” science.

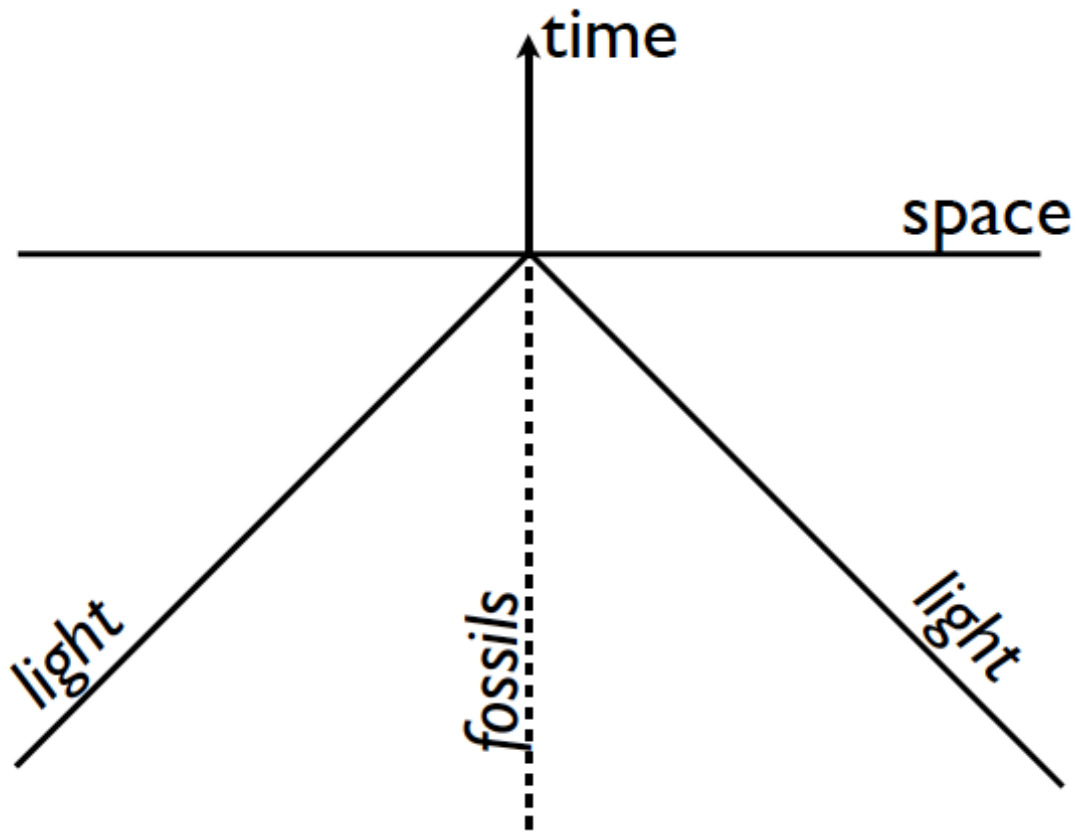
All we observe is the two-dimensional sky: the radial direction is a combination of distance and time into the past.

A critical aspect is the establishment of a distance ladder that can be extended to cosmological scales. This work started in earnest with the work of Hubble, and continues to the present day with the Gaia satellite.

Supporting information may come from the ages of old stars, meteorites, etc.

Need for homogeneity

To be able to relate observations along different lines of sight we need to assume **homogeneity**: *without this assumption we cannot interpret observations.*

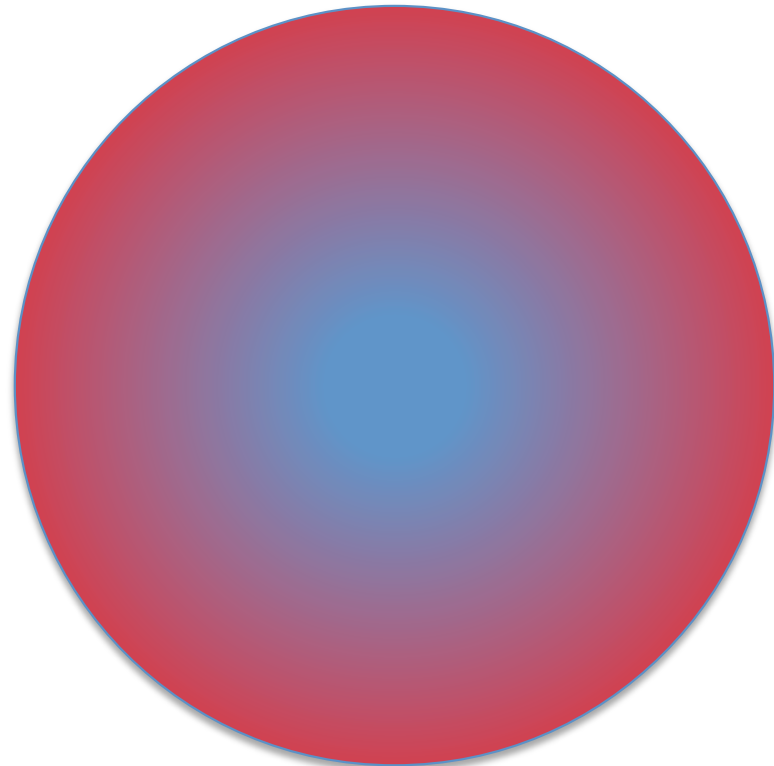
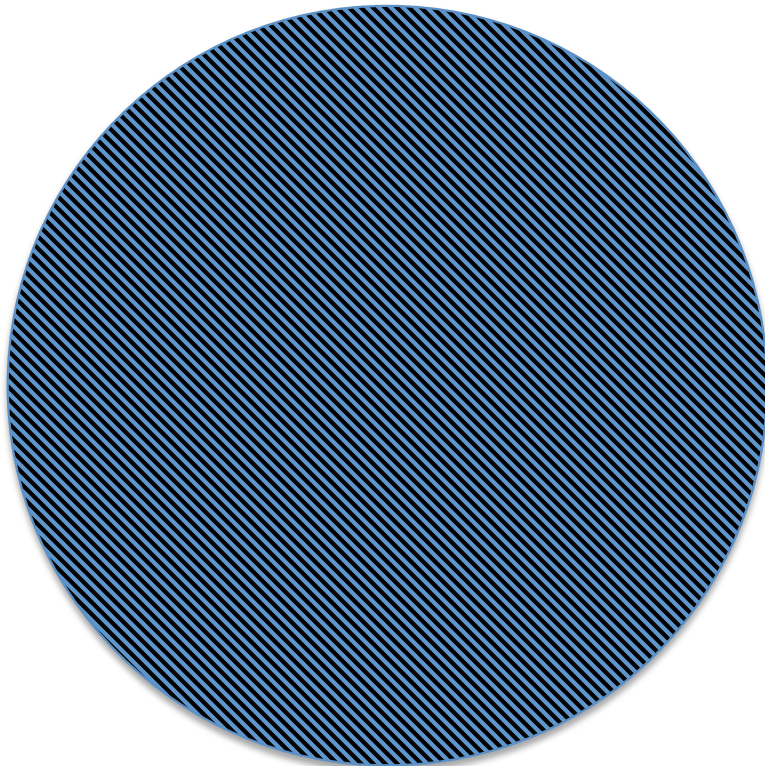


Homogeneity and Isotropy

Homogeneity: the Universe looks the same at each point.

Isotropy: the Universe looks the same in all directions.

Homogeneity does not imply isotropy, and isotropy does not imply homogeneity.



Copernican Principle

The Earth is not a special place in the Universe
(or: we are not privileged observers).

This is a major shift from the Ptolemaic system that formed the basis of medieval cosmology.

This implies that the Universe is isotropic about every point, which implies homogeneity as well.

Cosmological Principle

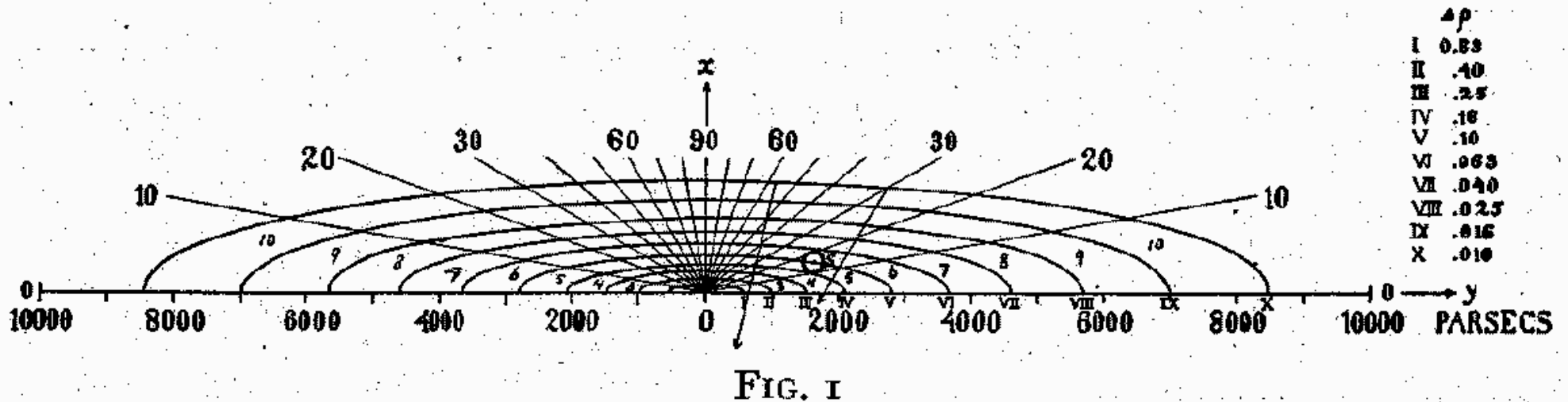
The **cosmological principle**:
the Universe is homogeneous *and* isotropic.

'Perfect' Cosmological Principle:

- "The universe is the same in all places, directions and times
- This motivated the 'Steady-State' universe (Bondi, Gold, Hoyle)

In fact it does not apply – the Universe evolves (as we shall see).

The Universe 100 years ago



Kapteyn (1922): used photographic star counts and estimated distances statistically based on parallaxes & proper motions of nearby stars.

He neglected interstellar absorption of starlight (assumes that stars were faint only because they far away, not because interstellar absorption blocks some of the light).

- Milky Way is a flattened disk about 15 kpc across and about 3 kpc thick
- The Sun is located slightly off-center.

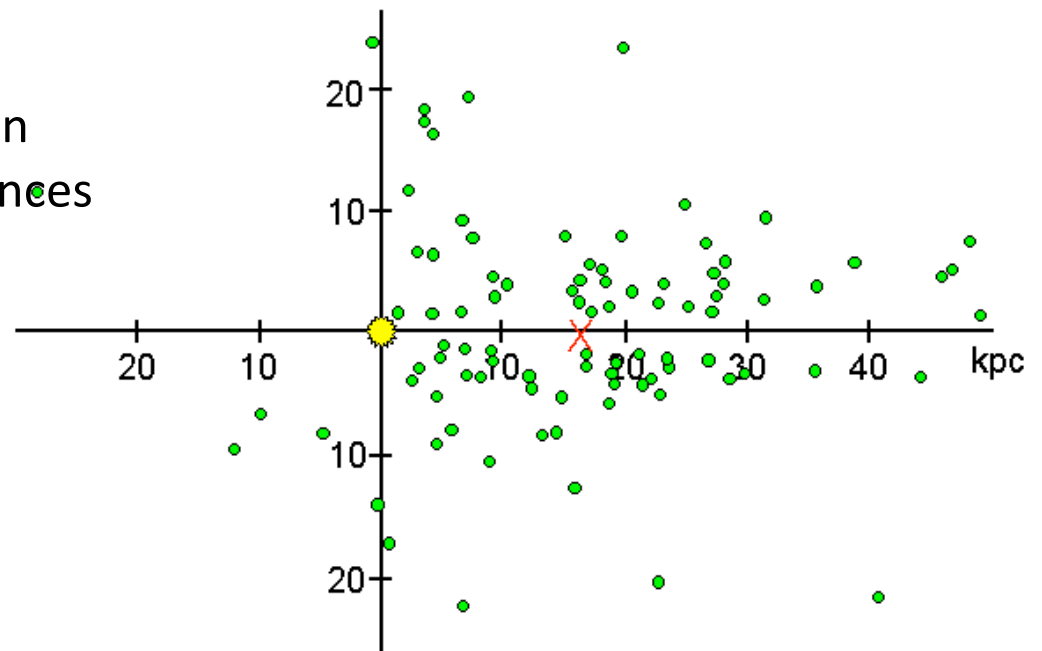
Shapley's Universe

Shapley's Results (1921):

Globular clusters form a subsystem centered on the Milky Way.
The Sun is 16 kpc from the MW center.
MW is a flattened disk about 100 kpc across

Right basic result, but too big:

Shapley ignored interstellar absorption
Caused him to overestimate the distances



Distances to other galaxies

At the same time there was an ongoing discussion about the nature of the “nebulae”: are they part of our Galaxy or “island Universes”.

Technology came to the rescue thanks to the construction of the 100-inch (2.5 m) Hooker telescope, which allowed Hubble to identify Cepheid variables in nearby galaxies.

He showed that the “nebulae” are not part of the Milky Way. By combining his distances with redshift measurements by Slipher and Humason he found that:

Velocity-Distance diagram

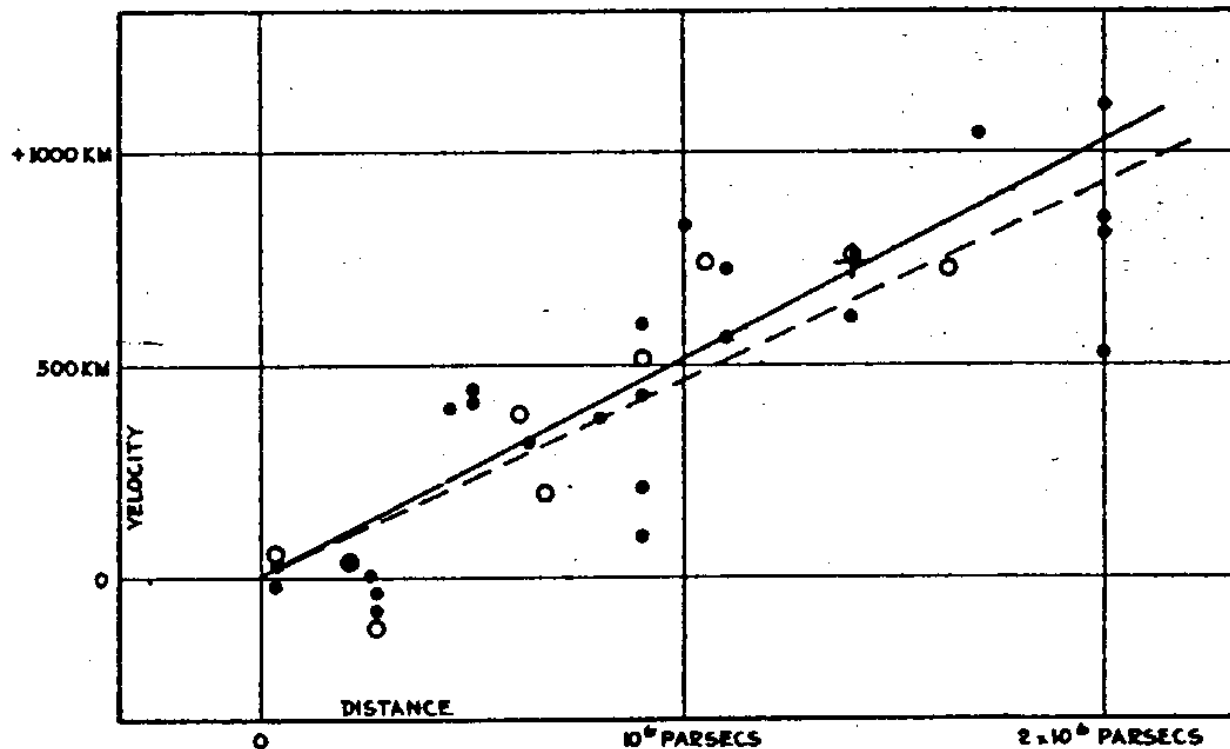


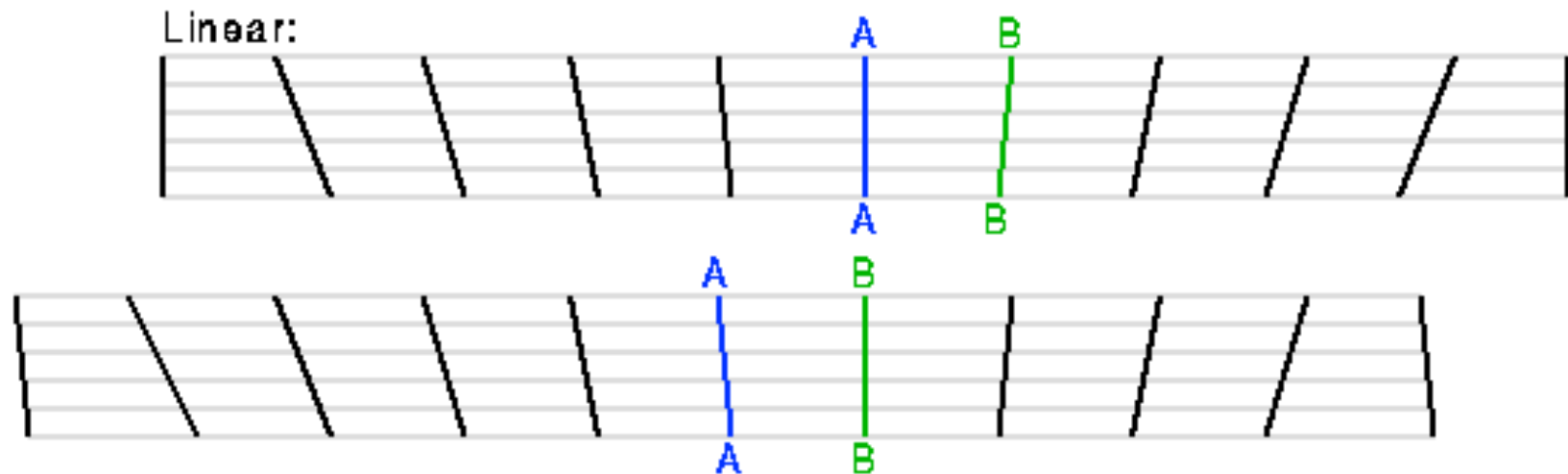
FIGURE 1

Hubble (1929): the recession velocity is proportional to the distance (after allowing for the Sun's motion): $v = H_0 r$.

Hubble obtained $H_0 = 464$ km/s/Mpc.

Implication of Hubble's discovery

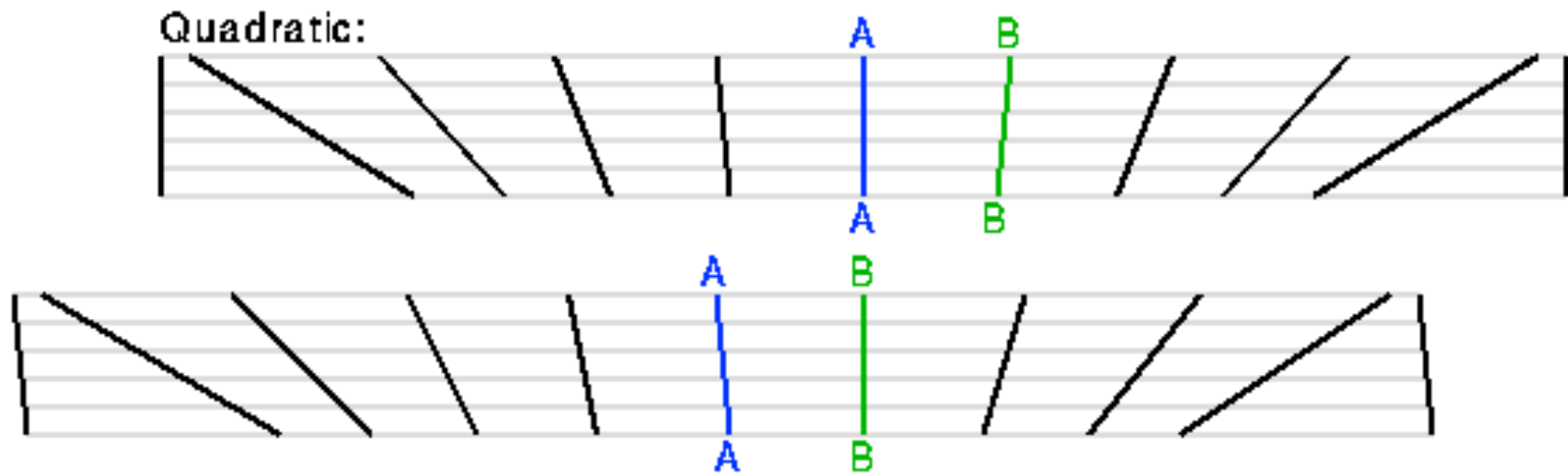
At first glance, this result may violate the Copernican Principle, but this is in fact not true!



The recession velocity is symmetric: if A sees B receding, then B sees that A is receding. The diagrams from the two different points of view are identical except for the names of the galaxies.

Implication of Hubble's discovery

The linear relation is the only one consistent with the Copernican Principle. For e.g. a quadratic relation different observers would see different relations.




Implication of Hubble's discovery

The Hubble law produces a **homologous** expansion: shapes of patterns in the Universe are not altered, but are merely scaled isotropically.

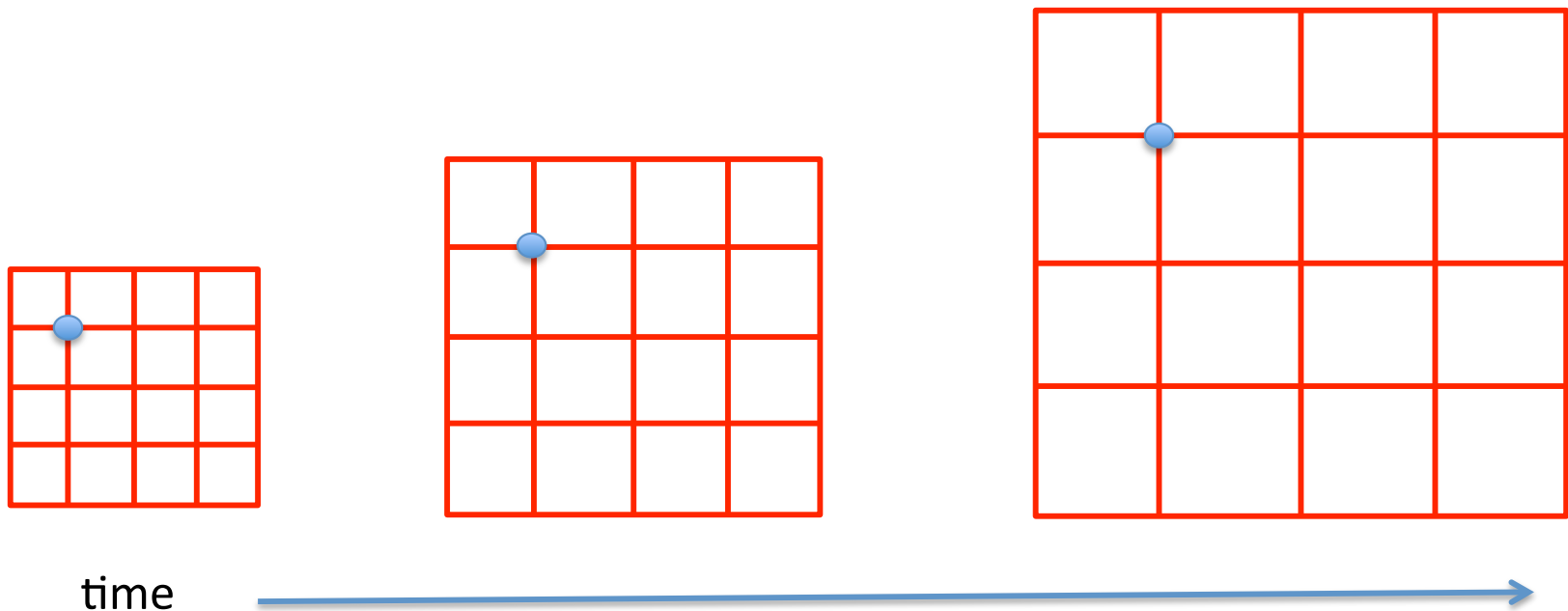
$$\mathbf{r}(t) = a(t)\mathbf{r}(t_0)$$

scale factor


$$\Rightarrow \mathbf{v}(t) = \frac{d\mathbf{r}(t)}{dt} = \frac{da(t)}{dt}\mathbf{r}(t_0)$$

Implication of Hubble's discovery

The Hubble law defines a special frame of reference at any point in the Universe. A **comoving** observer is at rest in this special frame of reference.



Age of the Universe

Note that the Hubble constant has dimension 1/time:

$$1/H_0 = (978 \text{ Gyr}) / (H_0 \text{ in km/s/Mpc})$$

If the expansion velocity is constant then all galaxies were in the same place $t=1/H_0$ years ago. With the current estimates of $H_0=71\pm3$ km/s this implies an age of 14 Gyr.

The expansion rate is not a constant, so the result is only an indication: typically the age of the Universe is $<1/H_0$.

Similarly c/H_0 is a natural scale for the Universe.

Age of the Universe

We can obtain independent constraints on the minimum age of the Universe by dating some of the constituents.

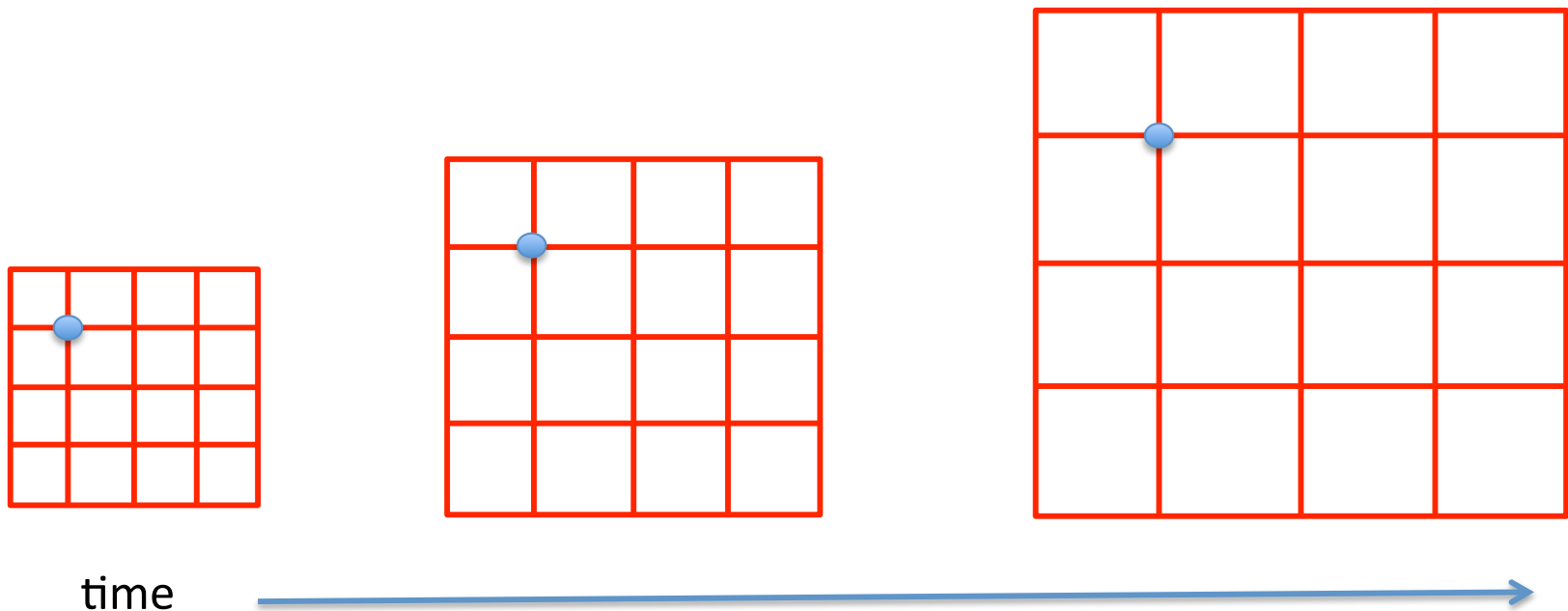
- Radioactive dating of the Solar system: > 4.6 billion years
- Cooling ages of old white dwarfs: best models give ages up to 11 Gyr
- Ages of star clusters: globular clusters have ages of 11 Gyr

Main Sequence stars: $L \propto M^4 \Rightarrow M/L \propto M^{-3} \propto L^{-3/4}$

Typical distance errors are 10% \Rightarrow 20% in L, or 15% in age.

Implication of Hubble's discovery

The Hubble law defines a special frame of reference at any point in the Universe. A **comoving** observer is at rest in this special frame of reference.

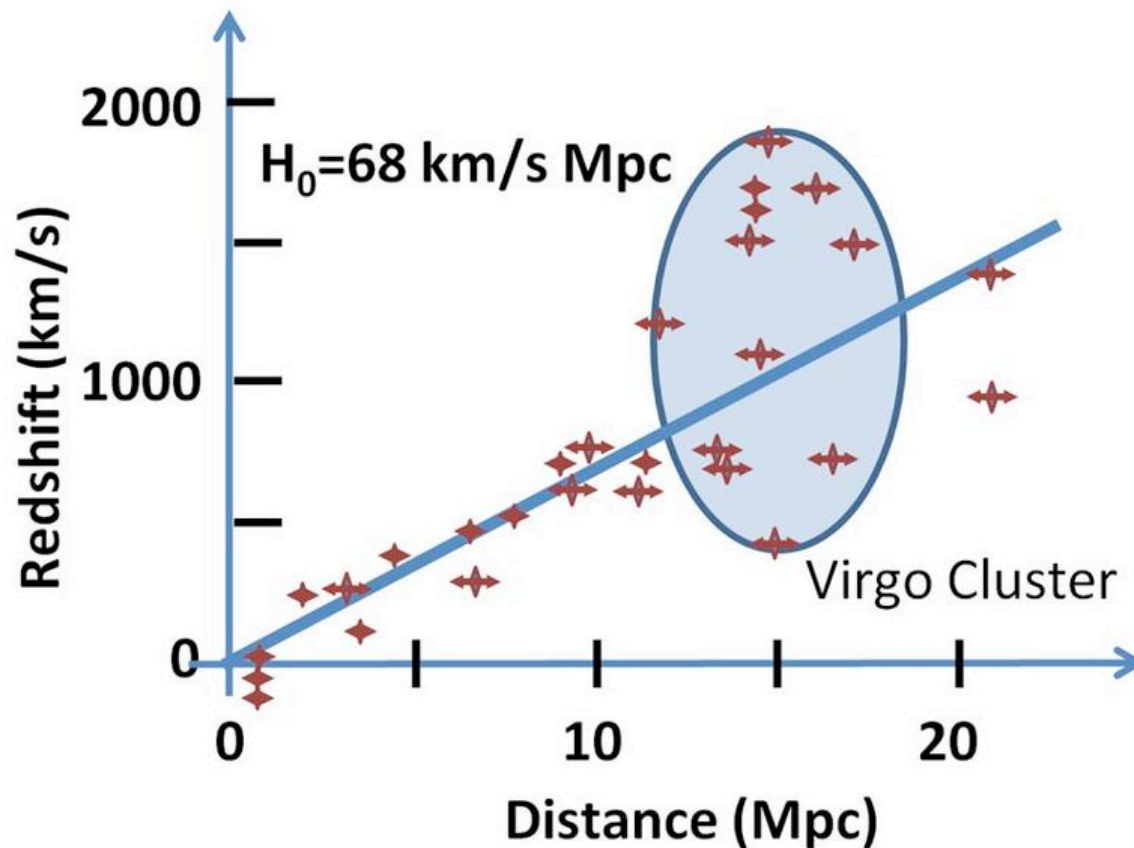


Reaching the Hubble flow

Our Solar System is not quite comoving: it moves with a **peculiar velocity** of 370 km/s relative to the observable Universe.

The Local Group of galaxies, which includes the Milky Way, appears to be moving at 600 km/sec relative to the observable Universe.

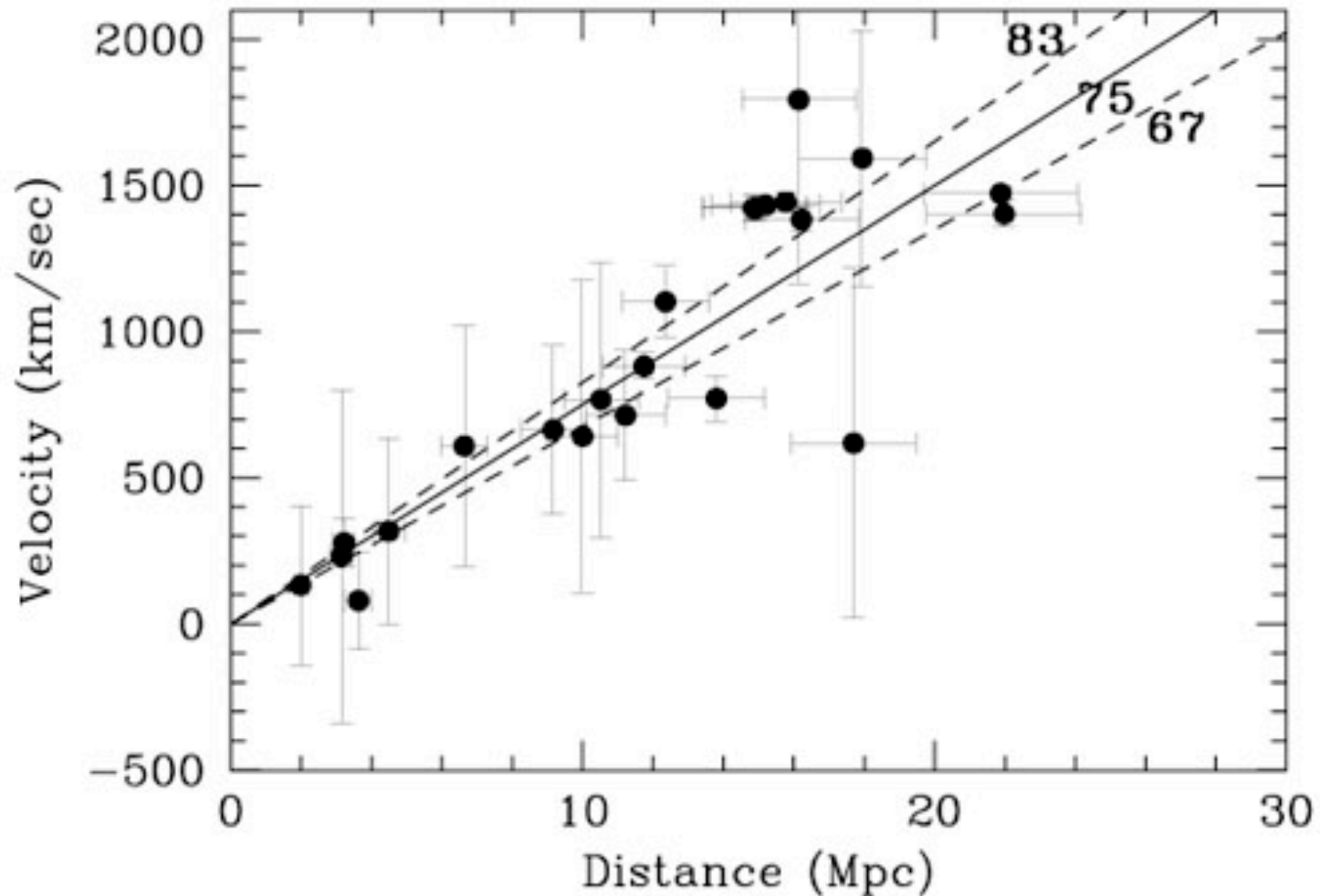
Reaching the Hubble flow



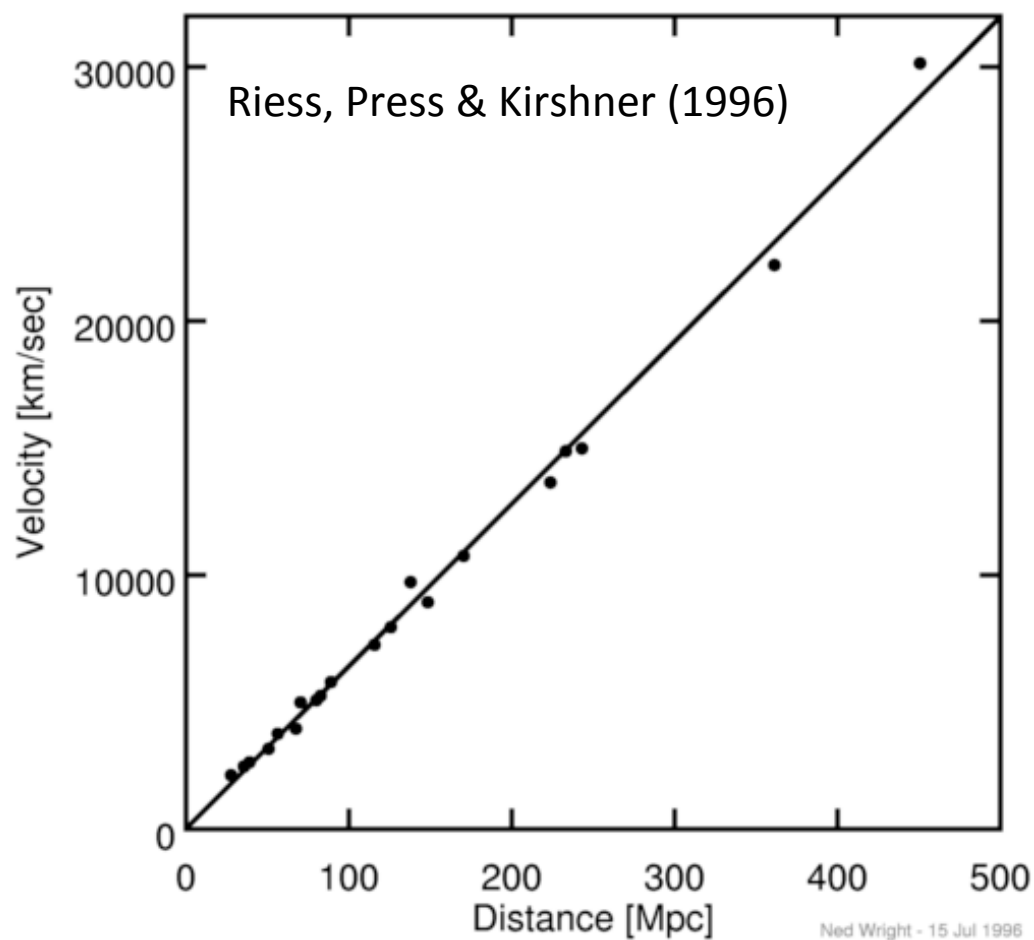
How far out do we need to measure to get a reliable measurement of H_0 if galaxies have **peculiar velocities** of 600 km/s?

Hubble key project

Hubble Diagram for Cepheids (flow-corrected)

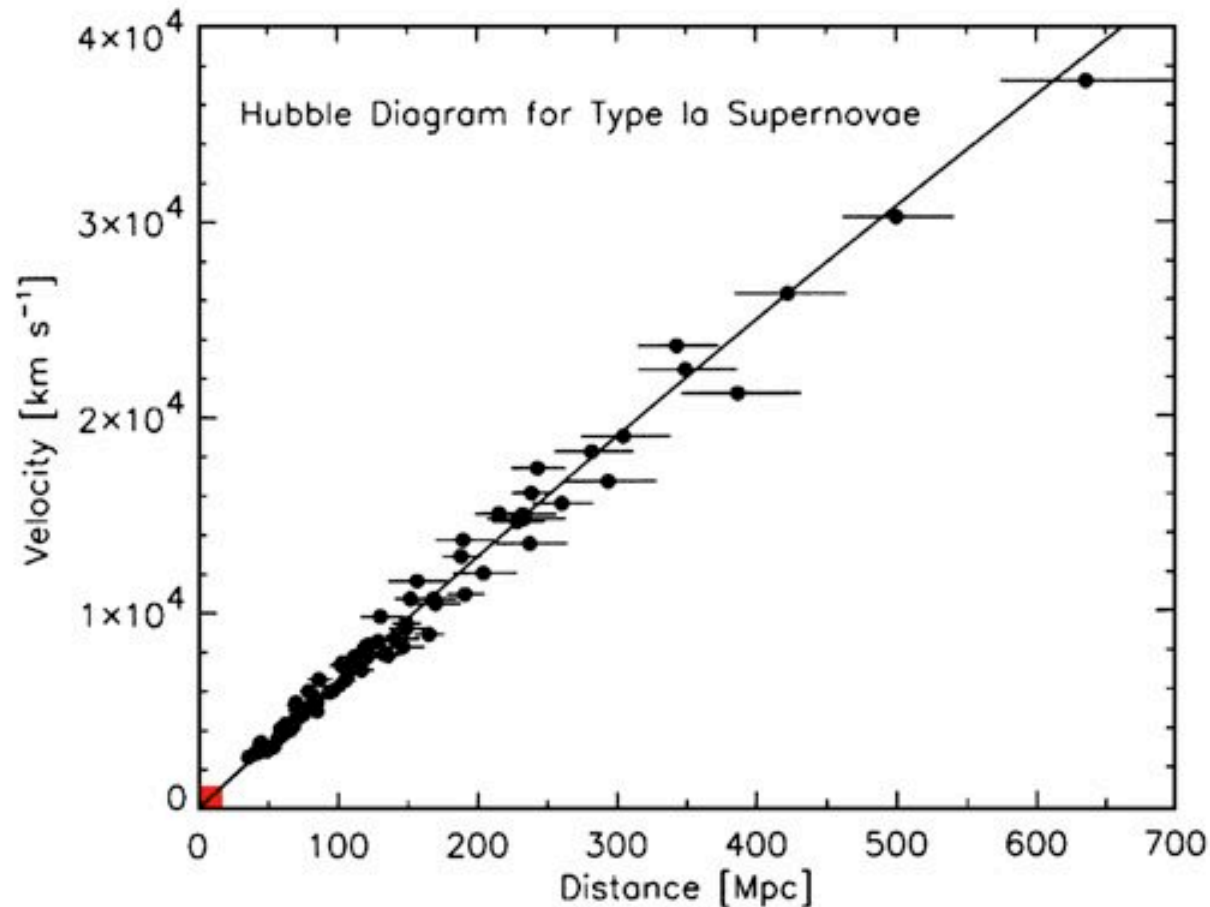


Linear relation on large scales



Riess et al. (1996) used type Ia SNe to extend the measurements beyond 30,000 km/sec and provide a dramatic confirmation of the Hubble law

Linear relation on large scales



Kirshner, Robert P. (2004) Proc. Natl. Acad. Sci. USA 101, 8-13

Type Ia SNe allow us to extend the measurements beyond 30,000 km/sec and provide a dramatic confirmation of the Hubble law

Evolving Universe!

The positive value for H_0 implies the Universe is expanding, which is a natural consequence of General Relativity.

The Universe was smaller in the past, and the physical conditions must have differed; observations of distant objects show a different epoch.

The Universe is evolving.

Deceleration

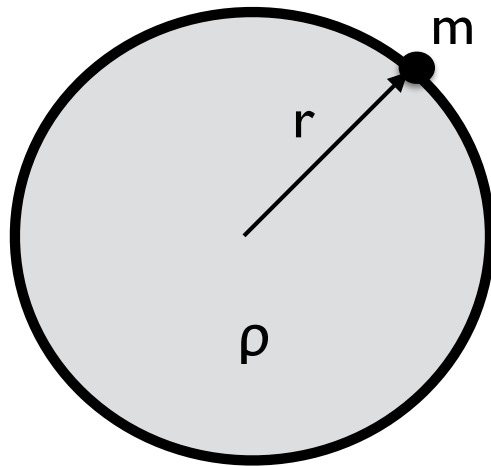
The expansion rate is expected to slow down due to the gravity of matter in the Universe. Therefore by measuring the rate of change we can “weigh” the Universe.

Therefore the Hubble constant is not constant, but varies with time: it is better to talk about the **Hubble parameter**.

To measure the deceleration, we need to extend the measurements to much larger distances, which became possible in the 1990s using type Ia supernovae.

The Expanding Universe

It is possible to derive the equations that describe an expanding homogeneous Universe using Newtonian gravity. Note that a correct physical interpretation does require General Relativity.



A particle feels no force from material at radii $>r$ and the mass inside r can be treated as if all concentrated in the central point.

$$\text{Potential energy of particle} = V = (-4\pi G/3)\rho m r^2$$

$$\text{Kinetic energy of particle} = T = (1/2)m(dr/dt)^2$$

$$U = T+V = \text{constant} = (1/2)m(dr/dt)^2 - (4\pi G/3)\rho m r^2$$

Homogeneity means applies for all particles, so we can change coordinates.

$$\begin{array}{ccc} & \mathbf{r} = a \mathbf{x} & \\ \text{physical coordinates} \nearrow & \uparrow & \nwarrow \text{co-moving coordinates} \\ & \text{scale factor} & \end{array}$$

$$U = (1/2)m x^2 (da/dt)^2 - (4\pi G/3)\rho m (ax)^2$$

The Expanding Universe

$$U = (1/2)mx^2(da/dt)^2 - (4\pi G/3)\rho m(ax)^2$$

Now multiply the above by $2/(ma^2x^2)$

$$2U/ma^2x^2 = (da/dt)^2 / a^2 - (8\pi G/3)\rho$$

Rearrange:

$$(da/dt)^2 / a^2 = (8\pi G/3)\rho + 2U/ma^2x^2$$

$$(da/dt)^2 / a^2 = (8\pi G/3)\rho - kc^2/a^2$$

← $k = -2U/mc^2x^2$
(first of the Friedman equations)

$$H^2 = ((da/dt) / a)^2$$

Consider three different cases for k : $k < 0$, $k = 0$, and $k > 0$

Suppose $k = 0$:

$$(da/dt)^2 / a^2 = (8\pi G/3)\rho$$

$$\rho = \rho_0 / a^3$$

$$(da/dt)^2 / a^2 = (8\pi G/3)\rho_0 / a^3$$

$$da/dt = (8\pi G\rho_0 / 3a)^{1/2}$$

So as $a \rightarrow \infty$, $da/dt \rightarrow 0$

Expands forever, asymptotically slowing expansion

The Expanding Universe

Suppose $k < 0$:

$$(da/dt)^2 / a^2 = (8\pi G/3)\rho - kc^2/a^2$$

small

As $a \rightarrow \infty$,

$$(da/dt)^2 = |k|c^2$$

Expands forever, asymptotically slowing expansion

Suppose $k > 0$:

$$(da/dt)^2/a^2 = (8\pi G/3)\rho - kc^2/a^2$$
$$(8\pi G/3)\rho_0/a^3 - kc^2/a^2$$

As $a \rightarrow \infty$, first term grow slower than second

So eventually expansion will stop and $da/dt = 0$, so

$$(8\pi G/3)\rho_0/a^3 - kc^2/a^2 = 0$$
$$a = (3kc^2/8\pi G\rho)^{1/2}$$

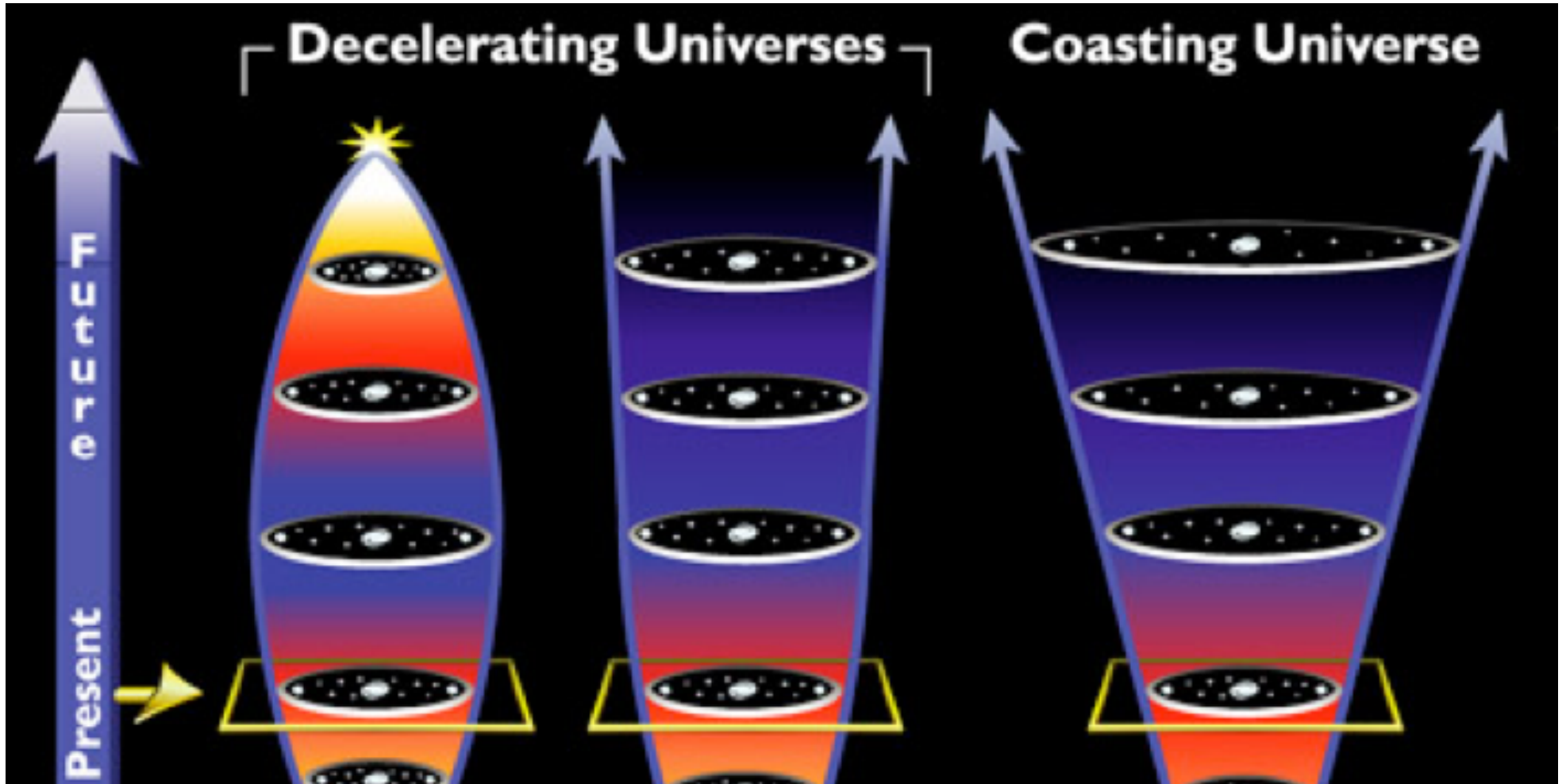
Let's look at how the evolution of the scale factor proceeds

$$da/dt = (8\pi G\rho_0 / 3a - kc^2)^{1/2}$$

$$d^2a/dt^2 = (1/2(da/dt))(-8\pi G\rho_0 / 3a^2)(da/dt) = -4\pi G\rho_0 / 3a^2 < 0$$

Expansion stops... turns around such that the universe recollapses...

The fate of the Universe



What sets the value of k ?

The Expanding Universe

Suppose $k = 0$:

$$da/dt = (8\pi G\rho_0 / 3a)^{1/2}$$

$$da a^{1/2} = (8\pi G\rho_0 / 3)^{1/2} dt$$

$$\int da a^{1/2} = \int (8\pi G\rho_0 / 3)^{1/2} dt$$

$$(2/3)a^{3/2} = (8\pi G\rho_0 / 3)^{1/2} t$$

$$a = t^{2/3} (6\pi G\rho_0)^{1/2}$$

If we take $t_0 = (1/6\pi G\rho_0)^{1/2}$, then

$$a = (t/t_0)^{2/3}$$

As such, the scale factor a equals 1 at the present day t_0

Note that $H = (da/dt)/a = (2/3)(t/t_0)^{-1/3} / (t/t_0)^{2/3} = 2/3 (1/t_0)$

$$t_0 = 2/(3H_0)$$

Note that the $k = 0$ is between the $k < 0$ and $k > 0$ cases and hence represents the critical density.

$$H^2 = (da/dt)^2/a^2 = (8\pi G/3)\rho_{\text{crit}}$$

$$\rho_{\text{crit}} = 3H^2 / (8\pi G)$$

$$\rho_{\text{crit},0} = 3H_0^2 / (8\pi G)$$

The Expanding Universe

With these definitions, we can rewrite Friedmann's equation as follows:

$$H^2 = (8\pi G/3)\rho - kc^2/a^2$$

$$H^2 = H_0^2(8\pi G/3H_0^2)\rho - k(H_0^2c^2)/(a^2H_0^2)$$

$$H^2 = H_0^2(\rho/\rho_{\text{crit}} - kc^2/a^2H_0^2)$$

Define $\Omega(a) = \rho/\rho_{\text{crit}}$ $\Omega_k = -kc^2/H_0^2$

Finally, we can write this as follows:

$$H^2 = H_0^2(\Omega(a) + \Omega_k/a^2)$$

Note that ρ scales as $1/a^3$ for a universe filled with matter, $\Omega(a)$ can be rewritten as Ω_0/a^3

$$H(a)^2 = H_0^2(\Omega_0 / a^3 + \Omega_k/a^2)$$

Note that we will be looking in detail at how ρ scales with scale factor in the next lecture for different components of the universe.

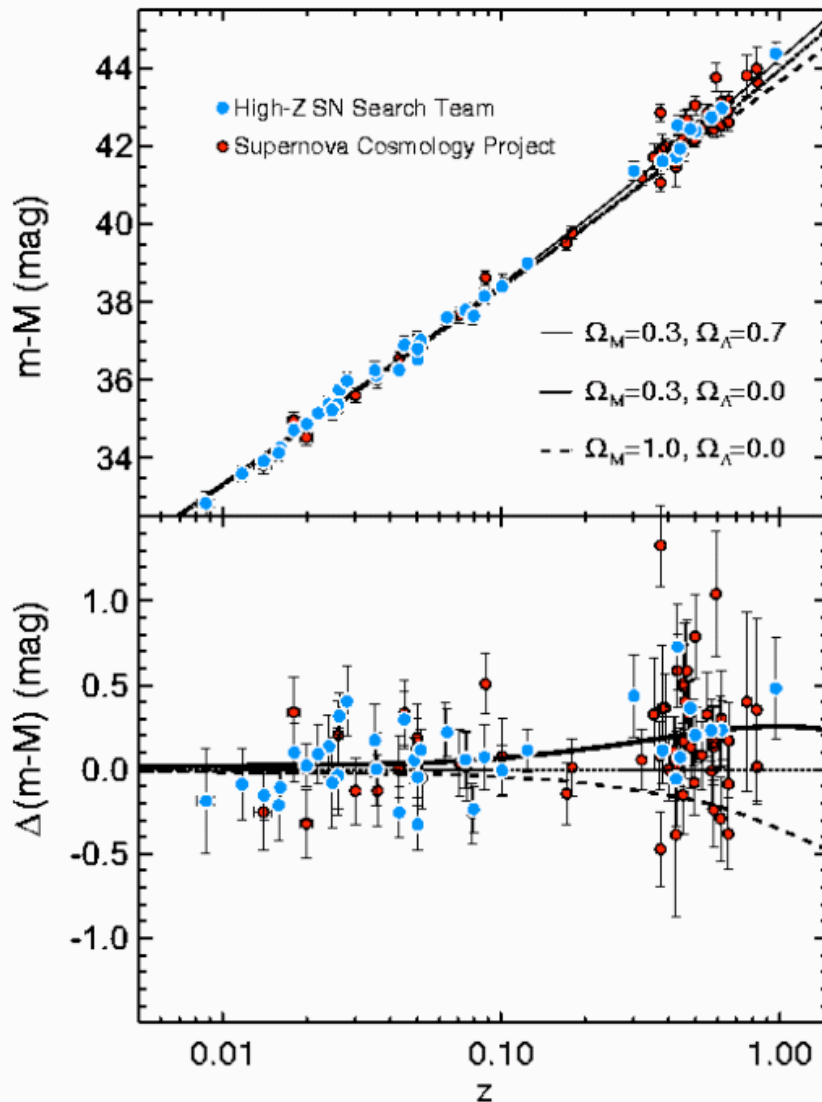
Deceleration

The expansion rate is expected to slow down due to the gravity of matter in the Universe. Therefore by measuring the rate of change we can “weigh” the Universe.

Therefore the Hubble constant is not constant, but varies with time: it is better to talk about the **Hubble parameter**.

To measure the deceleration, we need to extend the measurements to much larger distances, which became possible in the 1990s using type Ia supernovae.

Very distant supernovae



The discovery in 1998 was that the distances to the supernovae were larger than expected:

Instead of decelerating, the expansion is actually accelerating!

Explaining this result is the biggest question in cosmology to date.