Sterrewacht Leiden

Origins & Evolution of the Universe

an introduction to cosmology — Fall 2018

Lecture 10: Cosmic Microwave Background Radiation (II) + Introduction to Structure Formation

Rychard Bouwens

Layout of the Course

Sep 24: Introduction and Friedmann Equations
Oct 1: Fluid and Acceleration Equations
Oct 8: Introductory GR, Space Time Metric, Proper Distance
Oct 15: Redshift, Horizons, Observable Distances
Oct 17: Problem Class #1
Oct 22: Observable Distances, Parameter Constraints
Oct 29: Thermal History, Early Universe
Nov 5: Early Universe, Inflation
Nov 12: Inflation, Lepton Era, Big Bang Nucleosythesis
Nov 14: Problem Class #2
Nov 19: Recombination, Cosmic Microwave Background Radiation
Nov 26: CMB Radiation (II), Introduction to Structure Formation
Dec 3: Introduction to Structure Formation (II)
Dec 5: Problem Class #3
Dec 10: Finishing Thoughts, Review
Dec 21: Final Exam

Problem set #3 was mailed to you last week

Due by Friday 13:30
December 5

Exam will take place 8-11 PM
on December 21

Have you registered?

Review Last Week

Radiative Era

The radiative era begins at the moment of the elimination of the electron - positron pairs at $T \sim 5 \times 10^9$ K or $t \sim 10$ s

The end of the radiative era occurs when the density of matter coincides with that of the relativistic particles, corresponding to a redshift:

$$1 + z_{eq} = (\rho_{0,c}\, \Omega_0)/(K_0\, \rho_{0,r}) \sim 4.3/K_0 \times 10^4\, \Omega_0\, h^2 \sim 3800 \text{ where } K_0 \sim 1.68 \text{ if } N_\nu = 3$$

$$T_{eq} = T_{0,r}\, (1+z_{eq}) = 10^5\, \Omega_0\, h^2 / K_0\ \mathrm{K}$$

neutrino density is $(7/8)(4/11)^{4/3} N_\nu = 0.68$ times the photon density

At these temperatures, the hydrogen and helium are fully ionized. As the temperature drops, the number of neutral atoms and He^+ atoms grows through the equilibrium reactions:

$$H^+ + e^- \rightleftharpoons H + \gamma \qquad He^{++} + e^- \rightleftharpoons He^+ + \gamma \qquad He^+ + e^- \rightleftharpoons He + \gamma$$

The number density of the individual components is determined through the Saha equation.

Saha Equation

Let us focus on Hydrogen $p^+ + e^- \rightleftharpoons H + \Upsilon$, the density of various particles is given by the Boltzmann distribution:

$$n_H / (n_p n_e) = g_H / (g_p + g_e)\ (m_H / (m_p\ m_e))^{3/2}\ (kT / 2\pi\hbar^2)^{-3/2} e^{[m_p + m_e - m_H]/kT}$$

$$= (m_e kT / 2\pi\hbar^2)^{-3/2} e^{Q/kT} \quad \text{where } Q = 13.6 \text{ eV}$$

If $x = n_p / (n_p + n_H) = n_p / n_{baryon}$ is the fractional ionization

Then,

$$(1-x)/x^2 = 3.84\eta\ (kT/m_e c^2)^{3/2}\ e^{Q/kT} \qquad \text{where } \eta = n_{baryon} / n_\Upsilon$$

If we define the momentum of recombination as the instant when $x=1/2$ assuming $\eta = 5.5 \times 10^{-10}$, the recombination temperature is

$kT = 0.323$ eV $= Q/42 << 13.6$ eV

This is the consequence of having a large number density of photons in the universe; something similar happens for deuterium formation

This corresponds to the temperature $T_{rec} \sim 3740$ K

$z_{rec} \sim 1370$,

$t_{rec} = 240{,}000$ years

Photon Decoupling / Saha Equation

Recombination is not instantaneous, but quite rapid:

$$(H/H_0)^2 = \Omega_{m,0} / a^3 = \Omega_{m,0}\ (1+z)^3$$

$x = 0.9$ at $z=1475$

$x = 0.1$ at $z=1255$

$\Delta t = 70{,}000$ years

Since the number density of free electrons drops rapidly during the epoch of recombination, the time of photon decoupling comes soon after the time of recombination.

The rate of photon scattering is $\Gamma(z) = n_e(z)\sigma_T\, c = x(z)(1+z)^3\, n_{baryon,0}\sigma_T\, c$

$$\Rightarrow \Gamma = 4.4 \times 10^{-21}\ s^{-1}\ x(z)(1+z)^3$$

At $z=0$, $H_0 = 2.5 \times 10^{-18}\ s^{-1}$ $\Rightarrow \Gamma(z) \ll H$

At $z=1500$, expansion rate $H(z) \ll \Gamma(z)$

so before recombination photons were well coupled to electrons

When recombination takes place, the universe is matter dominated, so

If $\Omega_{m,0} = 0.3$, $H(z) = 1.24 \times 10^{-18}\ s^{-1}\ (1+z)^{3/2}$

The redshift for photon decoupling is when the expansion rate equals the scattering rate $H = \Gamma$:

$\Rightarrow 1+z_{dec} = 43/x(z_{dec})^{2/3}$

$z_{dec} = 1130$

Recombination Epoch (z~1100)

Ionized Plasma → Neutral Gas

(z > 1100)

< 380,000 years

Temperature
> 3600 K

Hydrogen ionized

Photons Thomson-scattering off of the ionized hydrogen

(z < 1100)

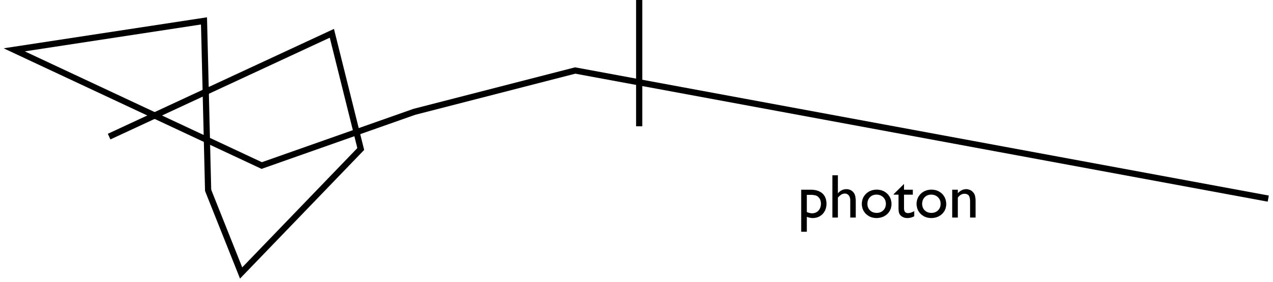
> 380,000 years

Temperature
< 3600 K

Hydrogen neutral

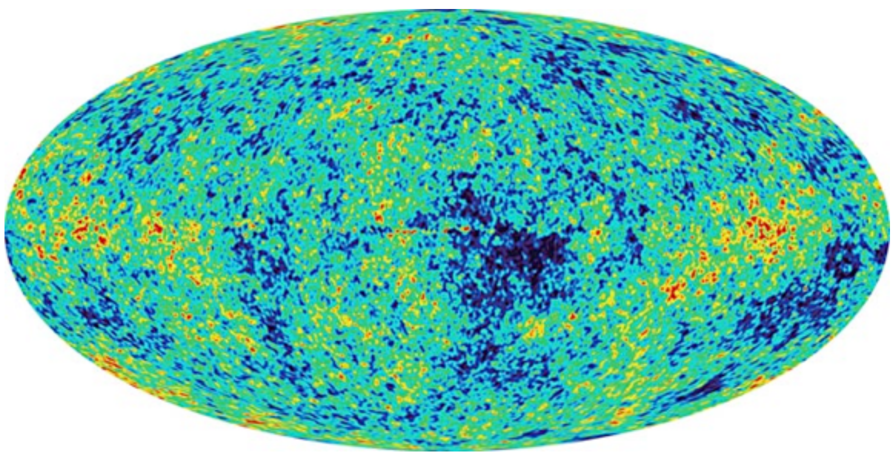
Almost no free electrons

Photons unbound from plasma

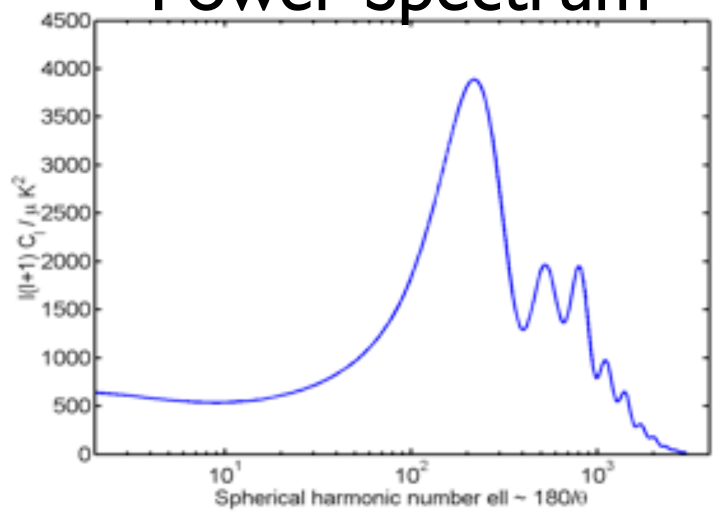


Power Spectra Derived from Fluctuations in CMB

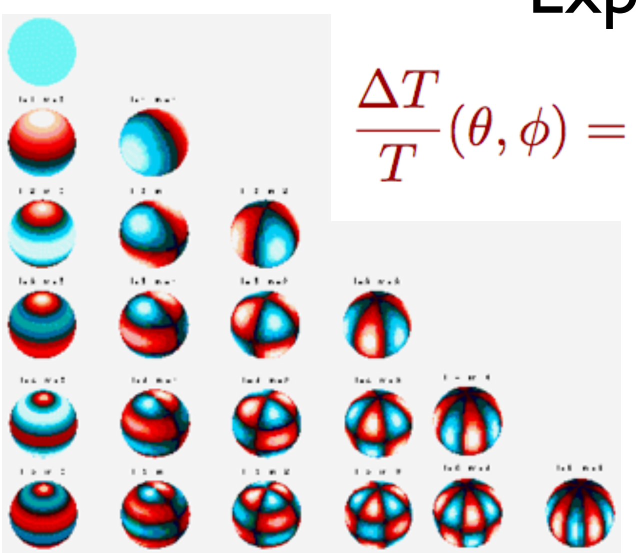
-- Use the spherical harmonic expansion to construct a power spectrum to describe anisotropies of the CMB on the sky



Power Spectrum



Expansion:



$$\frac{\Delta T}{T}(\theta,\phi) = \sum_{\ell=1}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m} Y_m^{\ell}(\theta,\phi)$$

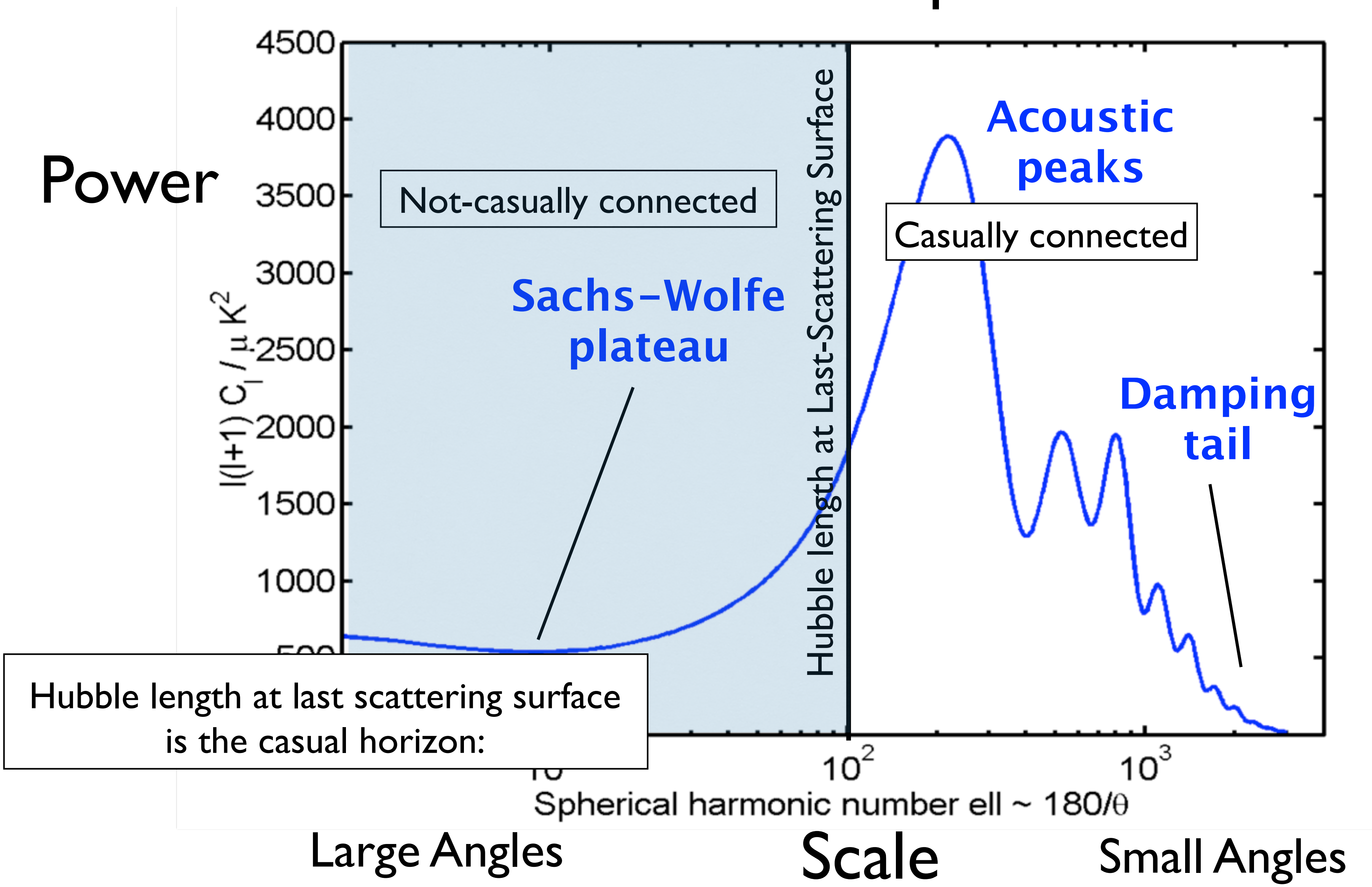
l = 180 / θ

After deriving the a_{lm} coefficients from the data, determine the statistical average

$$c_\ell = \langle |a_{\ell m}| \rangle^2$$

First question: how large can the angle become before the regions become casually disconnected?

Here is such a spectrum:



Sachs-Wolfe (1967)

Question: How do we explain the power spectrum of the anisotropies that are not casually connected, i.e., beyond the horizon?

These fluctuations are thought to be quantum fluctuations that are blown up in an initial inflationary phase of the universe

$\varepsilon_{DM} = \overline{\varepsilon_{DM}} + \delta\varepsilon_{DM}(r)$

variation in gravitational potential

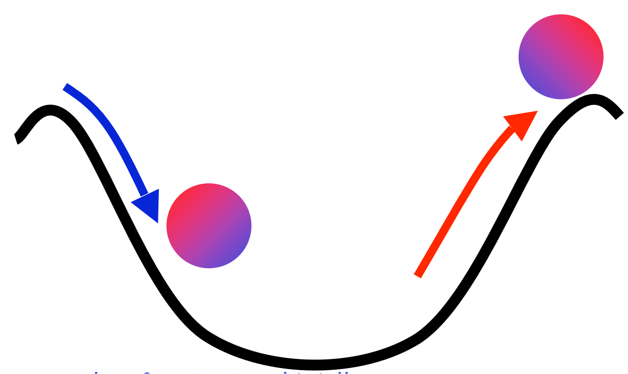
$\Rightarrow \nabla^2(\delta\varphi) = (4\pi G/c^2)\delta\varepsilon$

But how do these fluctuations translate into temperature fluctuations?

$\Delta\nu/\nu \sim \Delta T/T \sim \Phi/c^2$

Additional effect of time dilation while potential evolves (full GR):

$$\frac{\Delta T}{T} \sim \frac{1}{3}\frac{\Delta\Phi}{c^2}$$

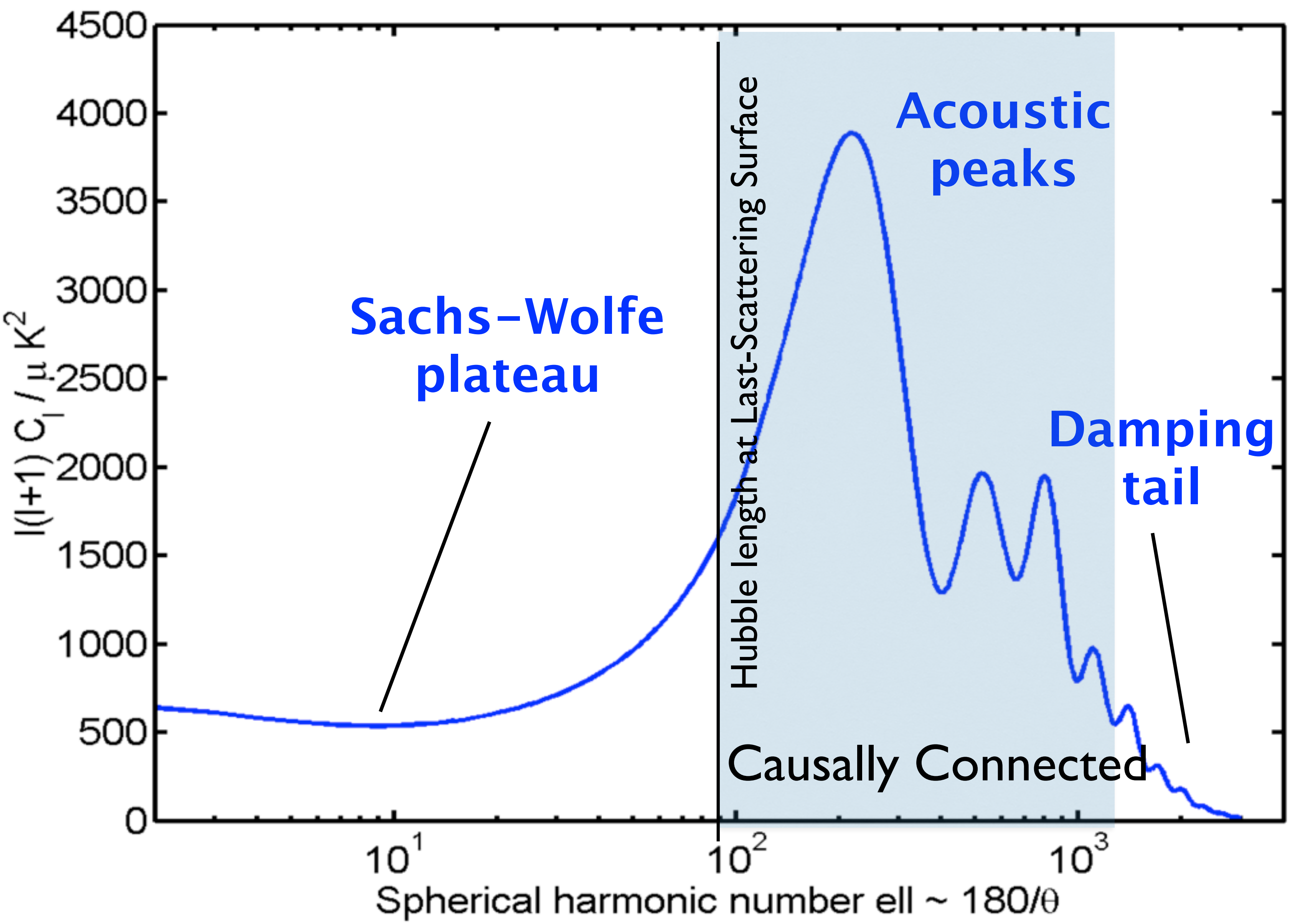


Photons climbs out of potential minimum, loses energy ↔ lower temperature

Photons falls out of potential maximum, gains energy ↔ higher temperature

What can we learn from the CMB?

Power



Credit: Porciano

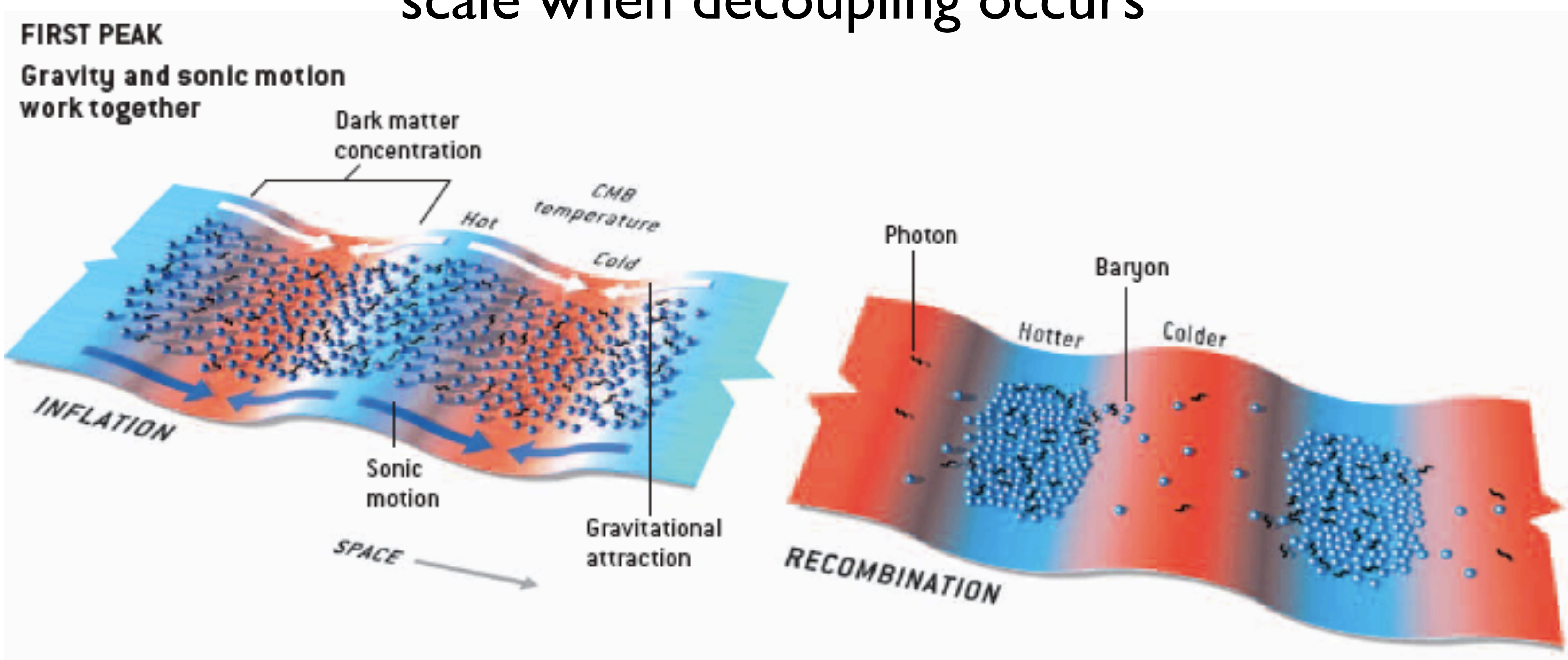
Large Angles | Scale | Small Angles

What can we learn from acoustic peaks

1st, 2nd, 3rd peaks correspond to compression, rarefaction, and compression modes

baryons fall onto over densities in the early universe, but are resisted by pressure in photon-baryon fluid and continues until decoupling…

for 1st peak, baryons are falling into over densities on a certain scale when decoupling occurs



[Image Credit: Hu & White 2004]

What do we learn from 1st peak?

(What can we learn from the angular scale at which is observed?)

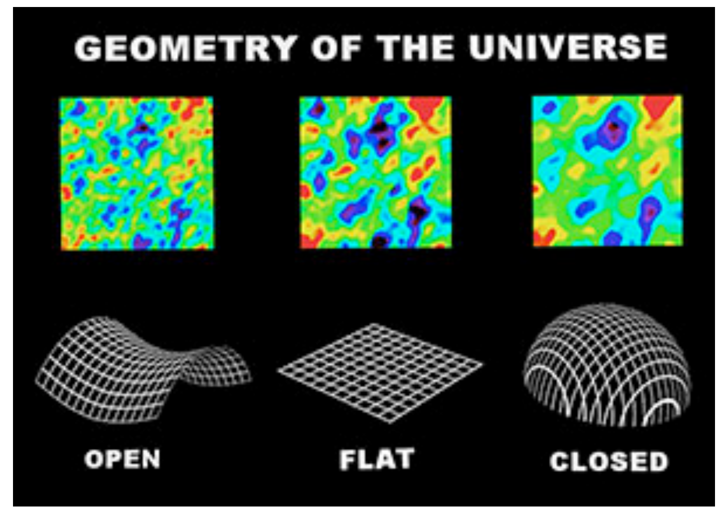
-- For this peak, baryonic matter would be falling onto these overdensities for the first time

-- Length scale spanned by peak is comoving length transversed by a sound wave to the point of last scattering:

$$L_S(t_R) = a(t_R) \int_0^{t_R} \frac{c_S\, dt}{a(t)}$$

where sound speed

$$c_S \approx \frac{c}{\sqrt{3}}$$



$$\theta = \frac{L_S(z)}{D_A(z)}$$

length scale traversed by matter (a standard rod)

angular diameter distance

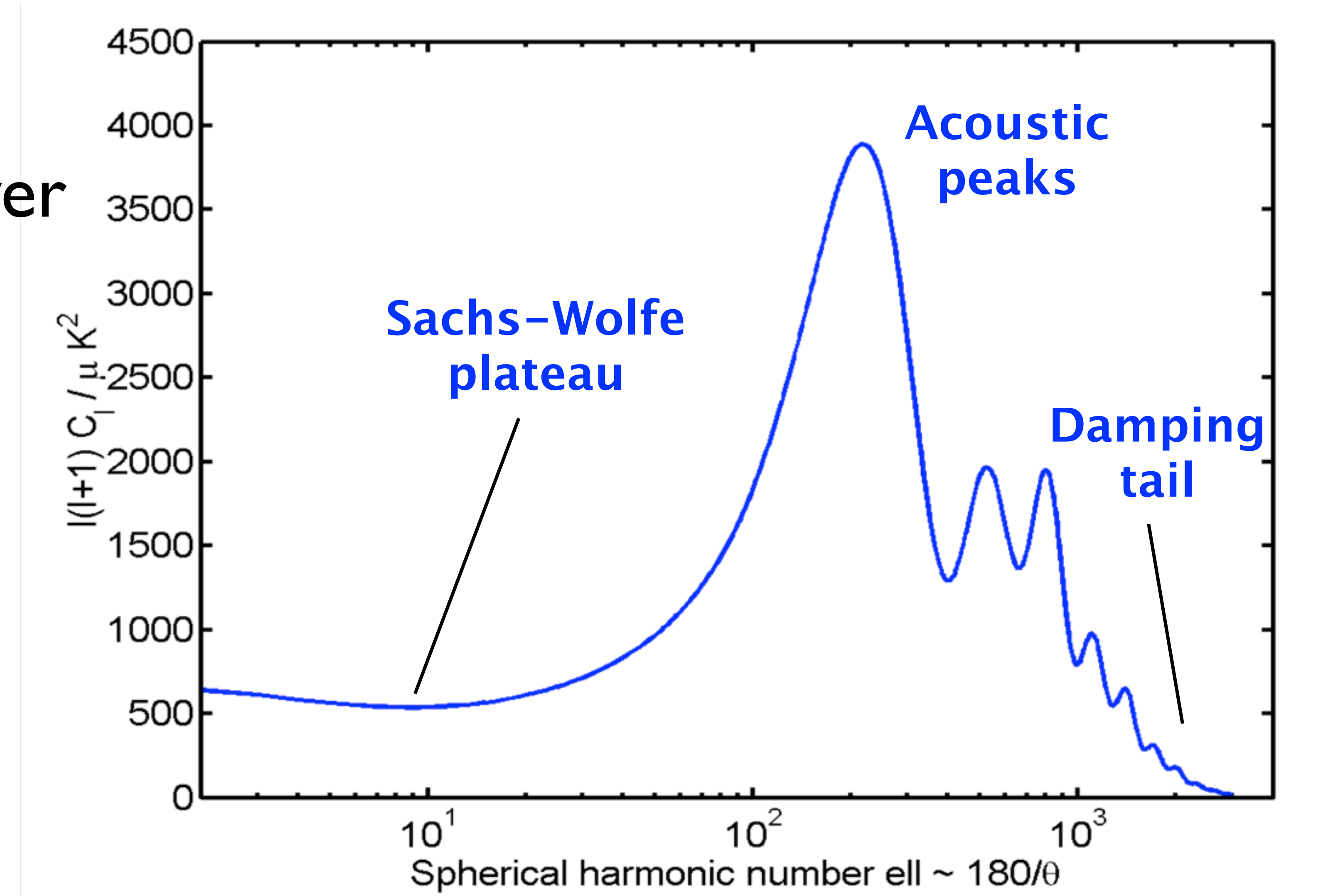
-- Can compute $L_S(z)$ and can measure θ (angle of 1st peak)

-- Can solve for D_A (z) and use to constrain geometry of universe

New Material for This Week

What about the damping tail?

Power

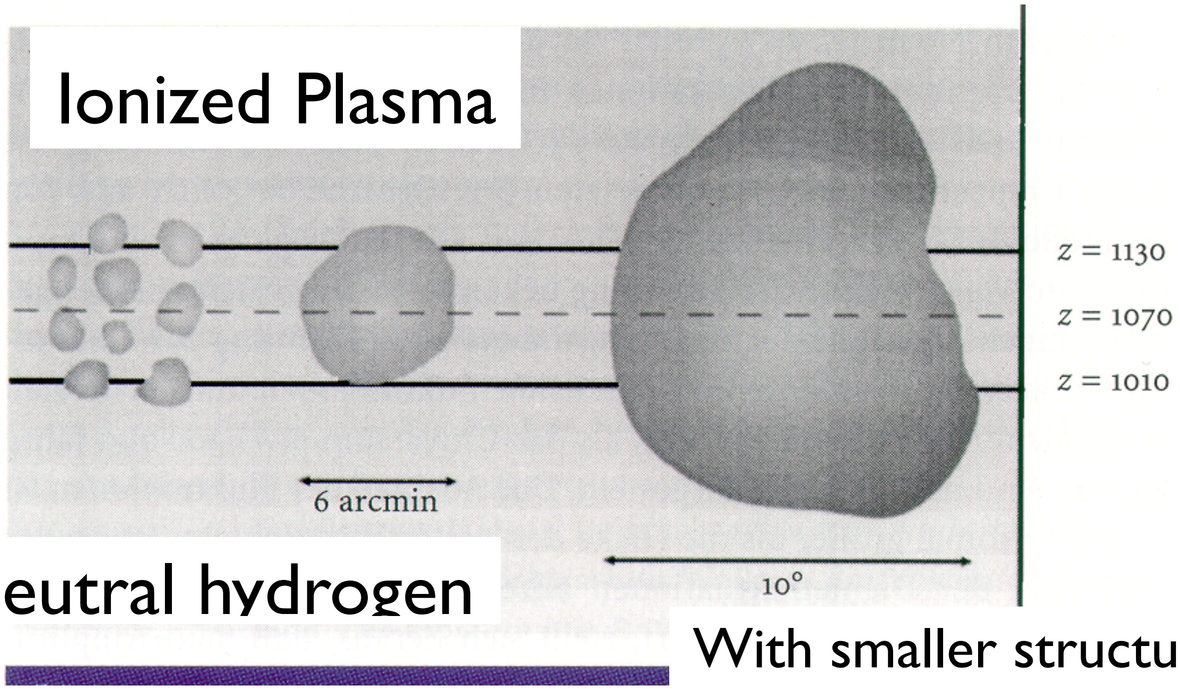


Large Angles Scale Small Angles

What about the damping tail?

-- Decoupling does not happen instantaneously (recall slide near the beginning of the presentation). This is not so important in viewing the last scattering surface for larger fluctuations. But for smaller fluctuations, the stuctures will overlap.

Note finite width of last scattering surface z = 1130 to 1010



Redshift

With smaller structures, projection effects will play a significant role in diluting signal

Silk Damping

Even before recombination, matter and radiation are not perfectly coupled: radiation leaks out of the perturbation, which leads to a dissipation of the perturbation.

This process occurs because photons bounce around (following a random walk) during recombination; for small scale fluctuations, the hot and cold photons can mix
=> on the scales corresponding to the distances photons can travel, the fluctuations are damped.

The dissipation scale $\lambda_D \sim 2c(\tau_{\gamma e}\, t)^{1/2}$ at time t

mean time before Thomson scattering: $\tau_{\gamma e} \propto n_e^{-1} \propto (1+z)^{-3}$

Before t_{eq}, $\lambda_D \propto (1+z)^{-5/2}$ since $t \propto (1+z)^{-2}$

After t_{eq}, $\lambda_D \propto (1+z)^{-9/4}$ since $t \propto (1+z)^{-1.5}$

The corresponding mass scale is $\rho(z)\lambda_D^3$, which gives $M_D \sim 10^{12}(\Omega_m h^2)^{-5/4} M_{sol}$ at recombination: ~cluster of galaxy scale

Without accounting for this Silk Damping the amplitude of an acoustic wave a mass scale < M_D would remain constant during radiation domination and decay as $\propto t^{-1/6}$ after t_{eq}; such structures are obliterated by photon diffusion.

(Material taught in the previous version of this course presented in a different manner here)

For $\theta < \theta_H$, the origin of the temperature fluctuations is complicated by the behavior of the photons and baryons.

The energy density of the photon-baryon fluid is 30% of the DM. Its equation of state w is between 0 and 1/3. If it enters a potential well, the fluid is compressed by gravity but then the pressure rises until it is high enough to cause the fluid to expand outward.

If the baryon-photon fluid at maximum compression in well at decoupling, its density will be higher than average and as $T \propto \varepsilon^{1/4}$, the photons will be hotter than opposite holds for maximum expansion.
If the the plasma is in the process of expanding or contracting, the Doppler effect will blue or redshift the photons.

The resulting power spectrum can be computed given the initial conditions for primordial fluctuations / mix of ingredients.

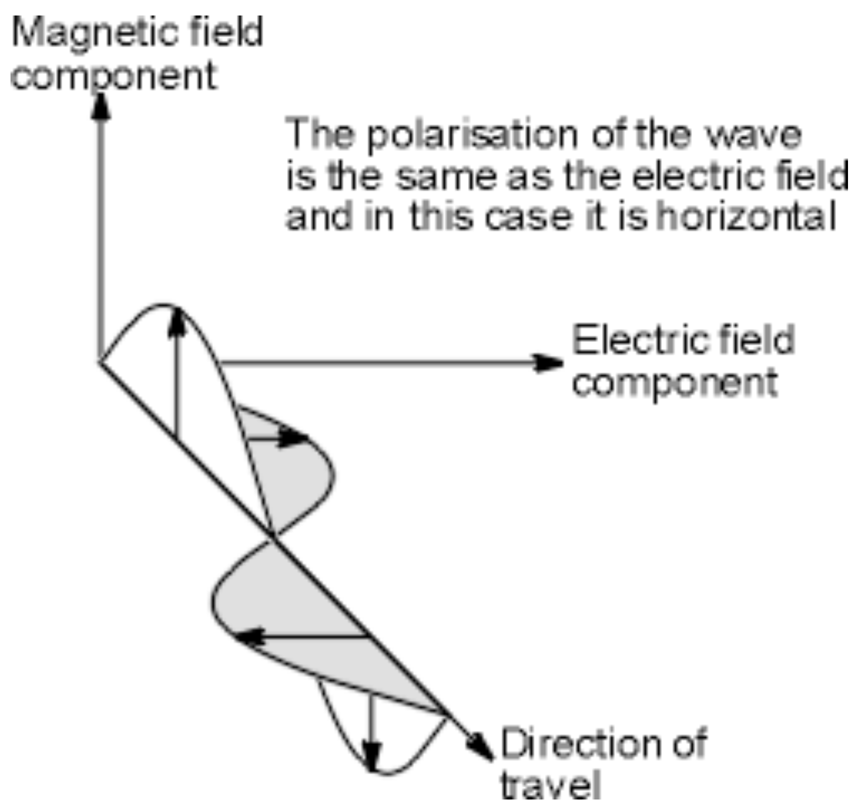
The location of the highest (first peak) corresponds to the potential well that just reached maximum compression which have size $c/H(z_{rec})$. Gives constraints on curvature Ω_k.

The amplitude depends on the sound speed of the plasma $c_s = (\omega_{plasma})^{1/2}c$
$\Rightarrow \Omega_{b,0}^{CMB} \sim 0.04$

How polarized is the cosmic microwave background overall?

most of the CMB light shows no net polarization

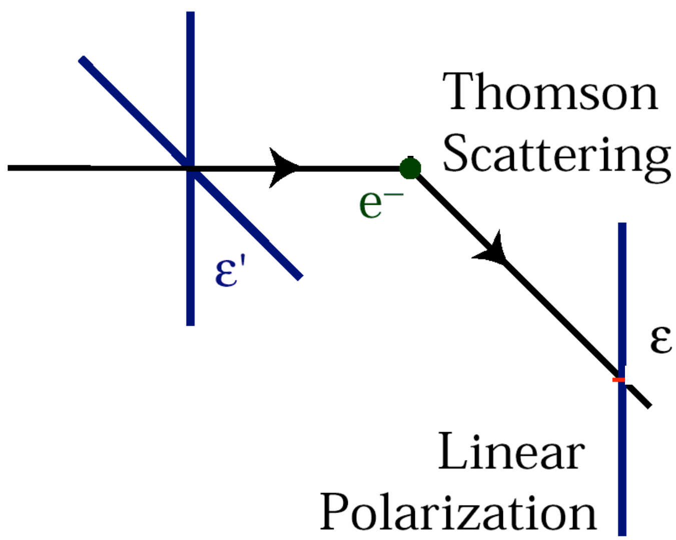
however there is a ~10% net polarization



Why are photons from the CMB polarized?

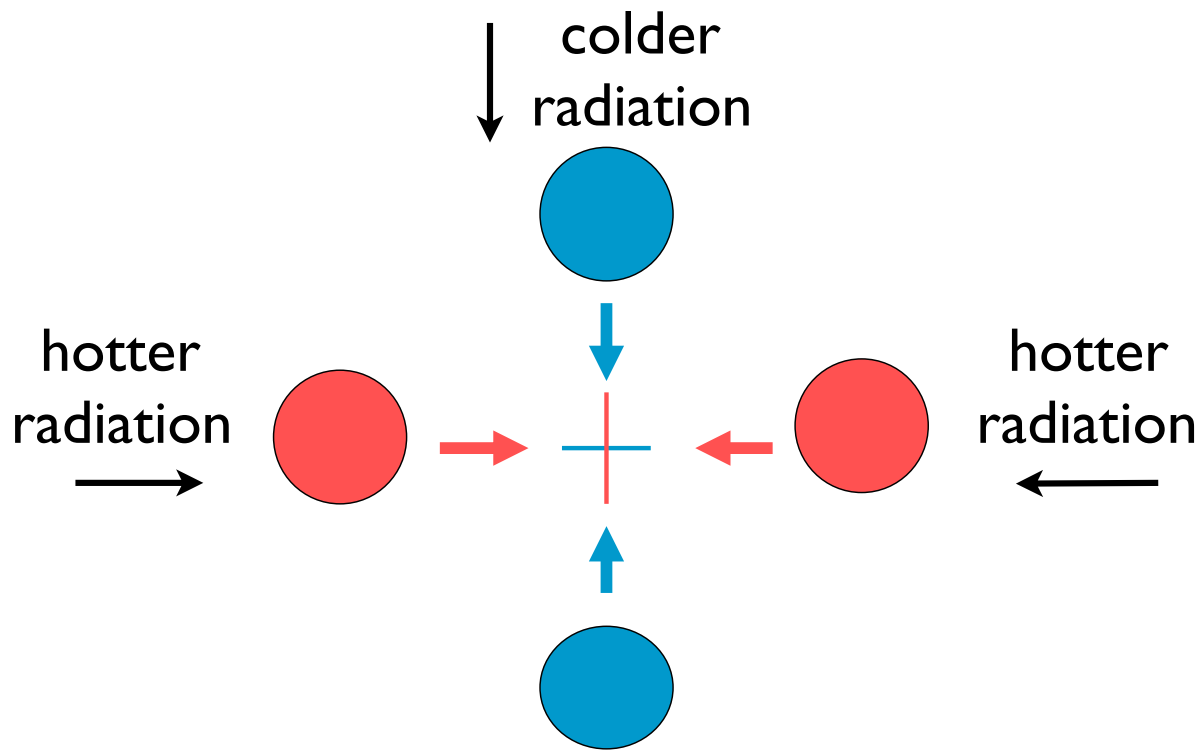
They are polarized from Thomson scattering

(valid in the limit that photon is much less than mass energy in the particle)



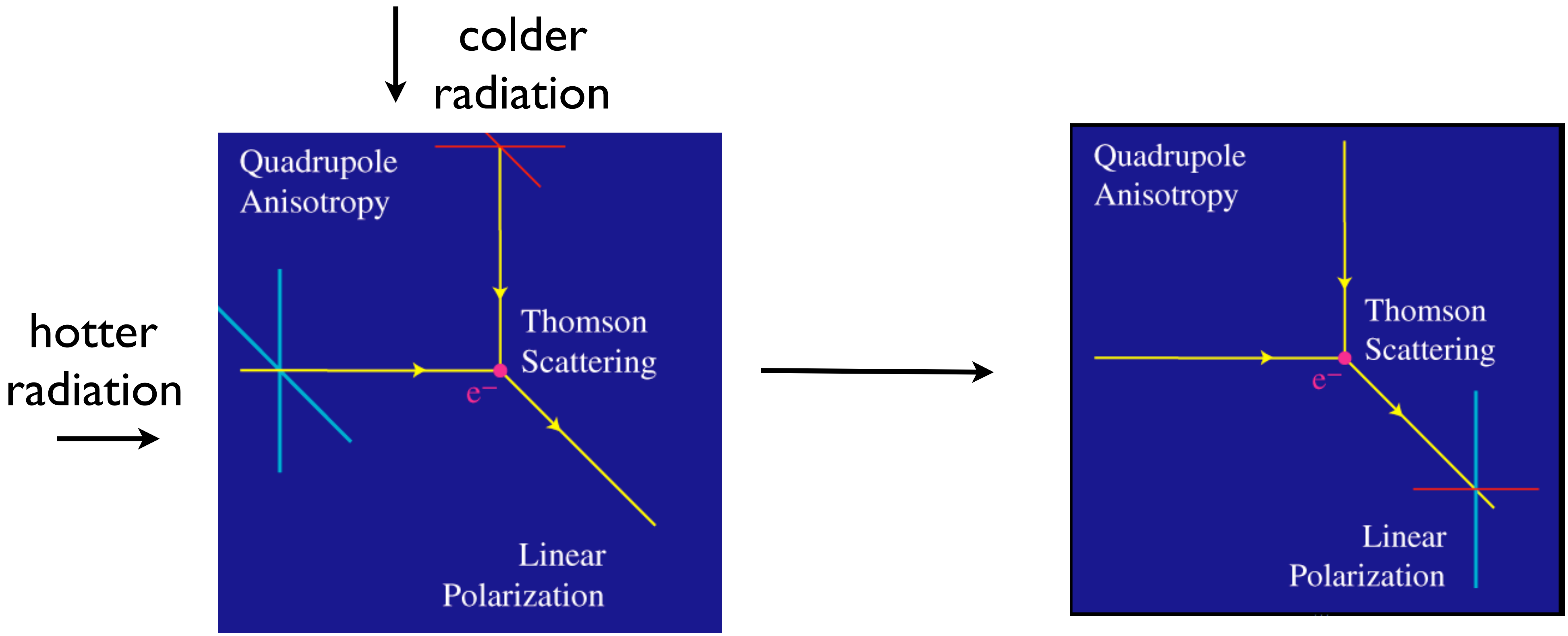
photons polarized perpendicular to incidence direction

How can this result in a polarized signal from the microwave background?



Net polarization only for a radiation field with a dipole from the CMB. Photons from highest temperature region dominate polarization signal

How can this result in a polarized signal from the microwave background?



Net polarization only for a radiation field with a dipole from the CMB. Photons from highest temperature region dominate polarization signal

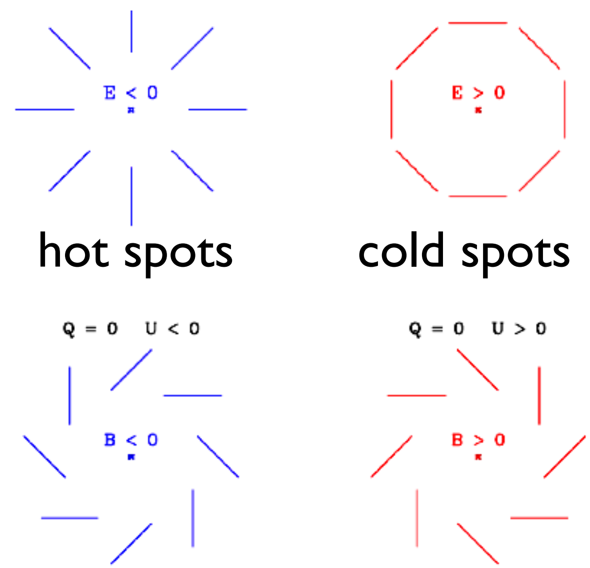
How CMB light can be broken down?

Measure Temperature and Polarization of Light

One tends to break down the polarization map into two modes (Helmholtz-Hodge theorem)

E-modes

B-modes



E-modes are curl free and can be written as the gradient of a potential

$$\nabla \times E = 0$$

B-modes have no divergence.

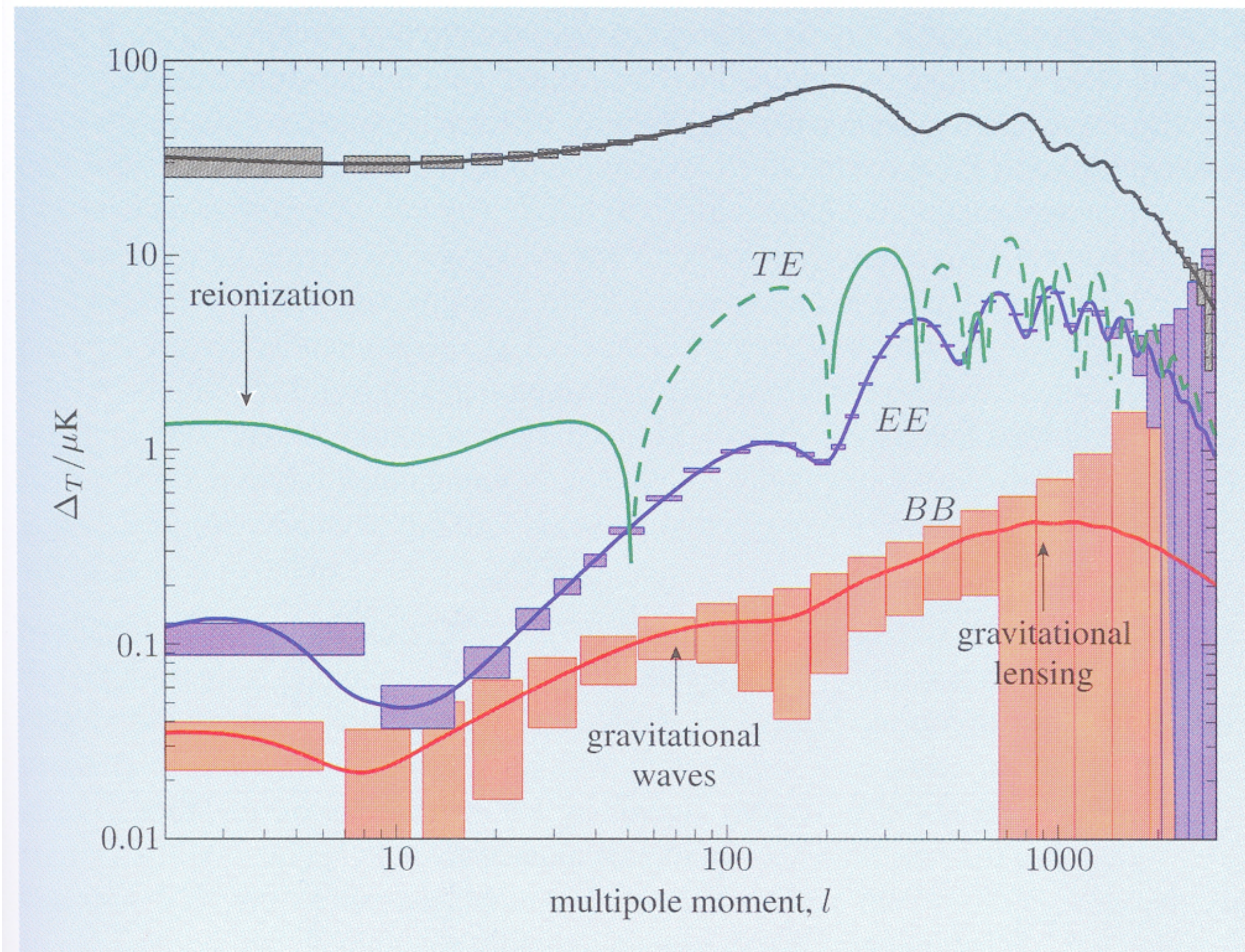
$$\nabla \cdot B = 0$$

The terms E and B modes simply reflect the general form of the polarization fields and are in analogy with similar fields in electromagnetism. However, they have no direct relation with electric or magnetic fields

E-modes have their origin in normal density perturbations such as make up the early universe

B-modes are only expected to arise from gravity waves in early universe (inflation) and from gravitational lensing (beween us and the last scattering surface)

But we can also look the TE, EE, and BB angular power spectra



Note that the EE, TE, and BB power spectra are not nearly as prominent as the TT power spectrum. This is because only 10% of the light from the CMB is polarized!

How are the TE, EE, and BB power spectra calculated?

Using an expansion of the polarization field in terms of spin-2 Y_{lm}'s. see Montroy+2005 (arXiv:0507514)

3.1. Power Spectrum Estimation

Both pipelines are polarized extensions of the Monte Carlo based MASTER method (Hivon et al. 2002) first used on B98 (Netterfield et al. 2002). These techniques rely on spherical harmonic transformations done on a partial map of the sky. For polarization data, the Q and U maps are expanded as a function of spin-2 spherical harmonics

$$(Q \pm iU)(\hat{n}) = \sum_{lm} (a^E_{lm} \pm i a^B_{lm})_{\pm 2} Y_{lm}(\hat{n}), \quad (1)$$

where a^E_{lm} and a^B_{lm} are the coefficients for E-mode and B-mode polarization respectively. These coefficients can be calculated in a manner similar to Legendre transformations,

$$a^E_{lm} = \frac{1}{2}\int d\Omega W(\hat{n})\Big[(Q+iU)(\hat{n})_{+2}Y_{lm}(\hat{n}) + (Q-iU)(\hat{n})_{-2}Y_{lm}(\hat{n})\Big], \quad (2)$$

$$a^B_{lm} = \frac{1}{2i}\int d\Omega W(\hat{n})\Big[(Q+iU)(\hat{n})_{+2}Y_{lm}(\hat{n}) - (Q-iU)(\hat{n})_{-2}Y_{lm}(\hat{n})\Big], \quad (3)$$

where $W(\hat{n})$ is an arbitrary weighting function and the integral extends only over the observed portion of the sky. From these transforms, we can build three observables:

$$C^{EE}_\ell = \frac{1}{2\ell+1}\sum_\ell |a^E_{lm}|^2, \quad (4)$$

$$C^{BB}_\ell = \frac{1}{2\ell+1}\sum_\ell |a^B_{lm}|^2, \quad (5)$$

$$C^{EB}_\ell = \frac{1}{2\ell+1}\sum_\ell a^E_{lm} a^{B*}_{lm}, \quad (6)$$

where $C^{\rm EE}_\ell$ the E-mode power spectrum, $C^{\rm BB}_\ell$ the B-mode power spectrum, and $C^{\rm EB}_\ell$ the cross-correlation between E-mode and B-mode polarization. $C^{\rm EB}_\ell$ is expected to be zero if parity is preserved in the early universe. Our estimates of the cross-correlations between temperature and polarization ($C^{\rm TE}_\ell$ and $C^{\rm TB}_\ell$) are discussed in Piacentini et al. (2005).

For spherical harmonic transforms done on the cut sky the measure of C_ℓ is biased; we describe them as pseudo-C_ℓ's ($\widetilde{C}_\ell$). For the polarization power spectra, the relationships between full-sky C_ℓ and $\widetilde{C}_\ell$ are expressed as

$$\widetilde{C}^{\rm EE}_\ell = \sum_{\ell'}\Big[{}_+K_{\ell\ell'}F^{EE}_{\ell'}B^2_{\ell'}C^{\rm EE}_{\ell'} + {}_-K_{\ell\ell'}F^{BB}_{\ell'}B^2_{\ell'}C^{\rm BB}_{\ell'}\Big] + \widetilde{N}^{\rm EE}_\ell, \quad (7)$$

$$\widetilde{C}^{\rm BB}_\ell = \sum_{\ell'}\Big[{}_+K_{\ell\ell'}F^{BB}_{\ell'}B^2_{\ell'}C^{\rm BB}_{\ell'} + {}_-K_{\ell\ell'}F^{EE}_{\ell'}B^2_{\ell'}C^{\rm EE}_{\ell'}\Big] + \widetilde{N}^{\rm BB}_\ell, \quad (8)$$

$$\widetilde{C}^{\rm EB}_\ell = \sum_{\ell'}\Big[{}_+K_{\ell\ell'} - {}_-K_{\ell\ell'}\Big]F^{EB}_{\ell'}B^2_{\ell'}C^{\rm EB}_{\ell'} + \widetilde{N}^{\rm EB}_\ell, \quad (9)$$

where $C^{\rm XY}_\ell$ represents the full-sky power spectrum, B_ℓ is the beam window function, F^{XY}_ℓ is the transfer function measured by signal-only Monte Carlo simulations, $\widetilde{N}^{\rm XY}_\ell$ is the noise bias measured by noise-only Monte Carlo simulations, ${}_+K_{\ell\ell'}$ is the primary coupling kernel and ${}_-K_{\ell\ell'}$ describes the geometric leakage between E-modes and B-modes (Chon et al. 2004). Both pipelines use roughly 500 Monte Carlo simulations of signal-only and noise-only data streams to estimate the signal transfer function and noise bias respectively. A similar number of signal+noise simulations can be used to estimate the uncertainty on the spectral estimate and check for bias in the pipeline.

Since we observe a small portion of the sky, we are not able to measure individual multipole moments. Instead, we parameterize the power spectrum as a piecewise continuous function

$$C^{\rm XY}_\ell = q^{XY}_b C^{(S){\rm XY}}_\ell, \quad (10)$$

where q^{XY}_b is the bandpower deviation over a range $(\Delta\ell)_b$ and $C^{(S){\rm XY}}_\ell$ is a shape parameter. Common choices for the shape parameter are those that keep $C^{(S)}_\ell$ constant over the band, those that keep $\ell(\ell+1)C^{(S)}_\ell/(2\pi)$ constant over the band (i.e. the flattened spectrum) or those that represent a theoretically motivated power spectrum (e.g. ΛCDM concordance model). The choice of parameterization depends in part on the nature of the expected signal and the noise in the maps.

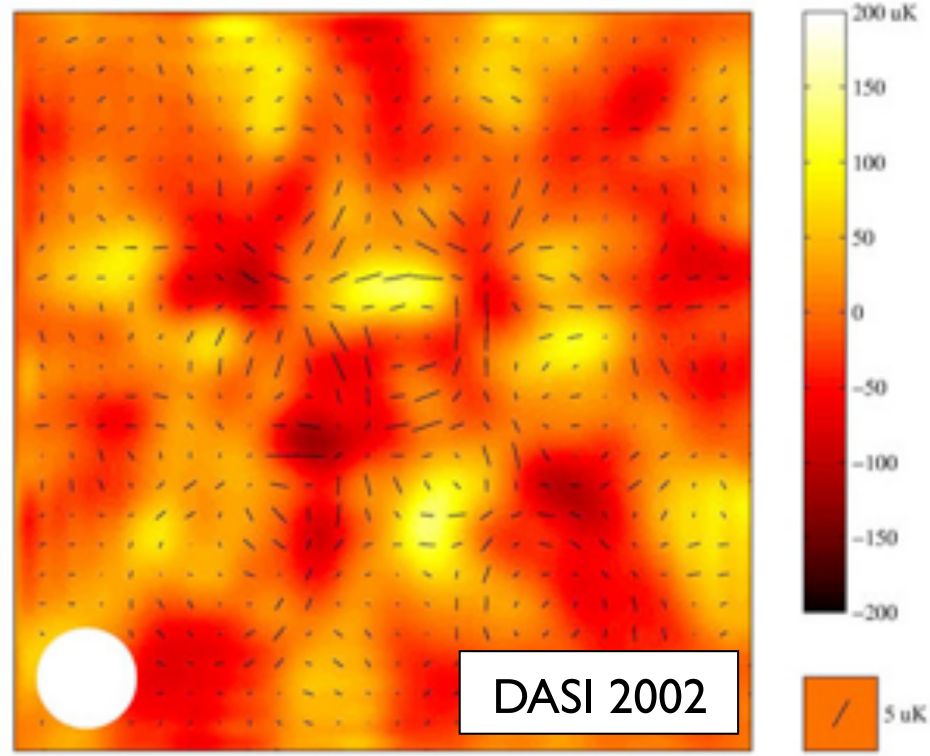
The output bandpower $\mathcal{C}^{XY}_b$ ($\mathcal{C}_\ell = \ell(\ell +$

First detection of polarization in CMB

-- DASI South Pole experiment (interferometer) first to detect E mode polarization (2002)

-- This was followed by WMAP reporting a measure of the C_{TE} power spectrum at low angular scales

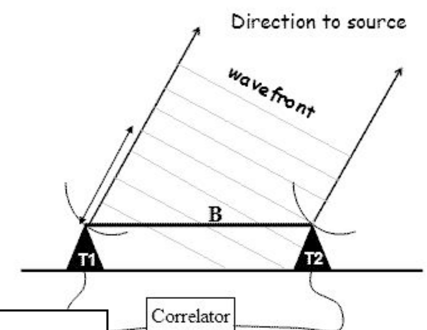
-- Measurements of the E-mode polarization also made with CARMAP, MAXIPOL, and QUAD



Map is 5 degrees square

DASI in South Pole

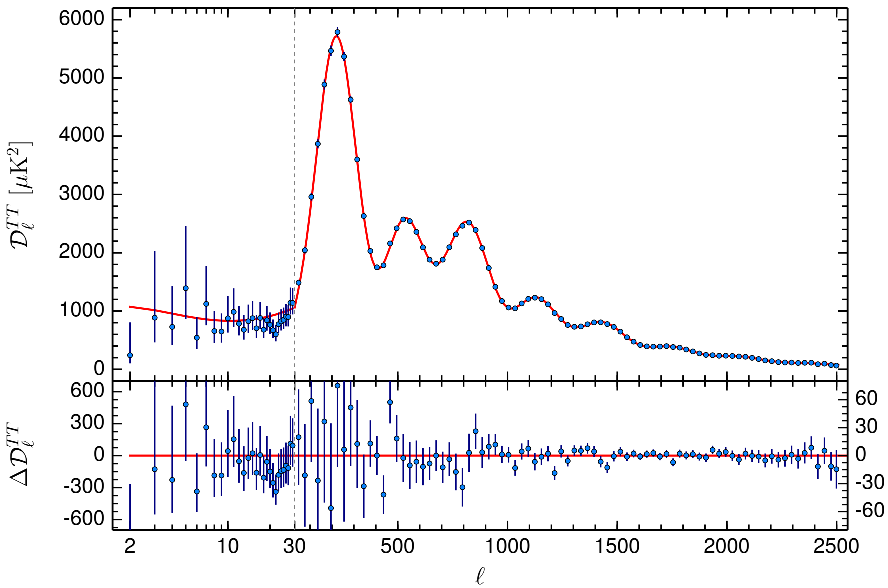
interferometer: collect coherent signals over certain angular scale on sky



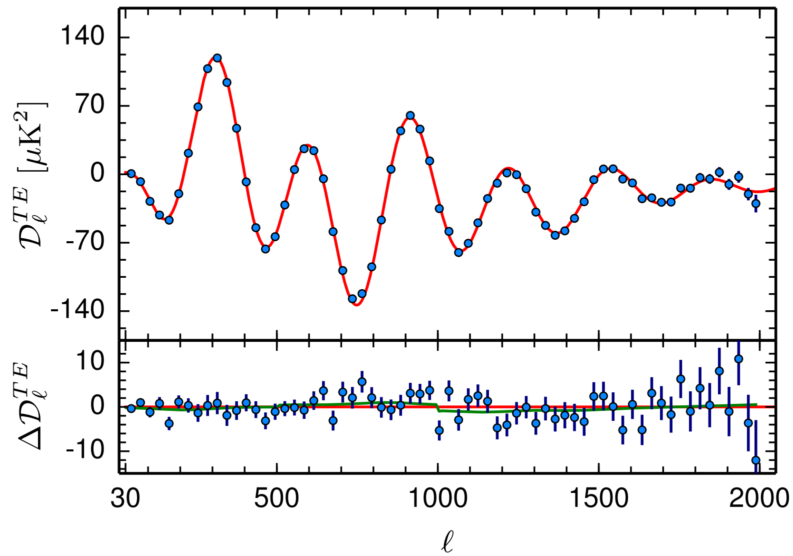
Credit: Basu

TT, TE, EE spectra derived from Planck

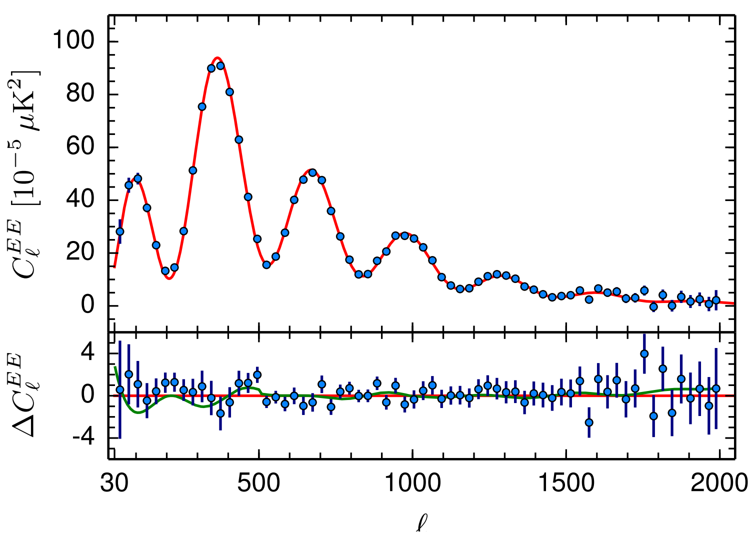
TT spectrum



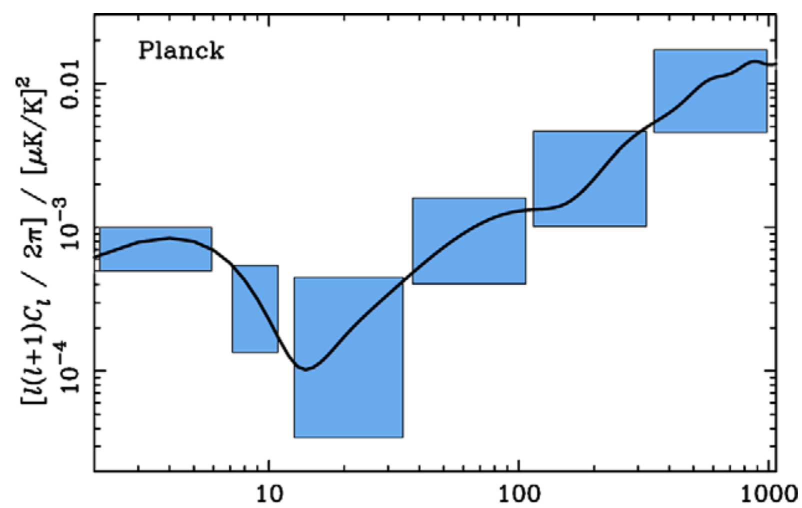
TE spectrum



EE spectrum



BB spectrum (no measurements quoted after first year day)

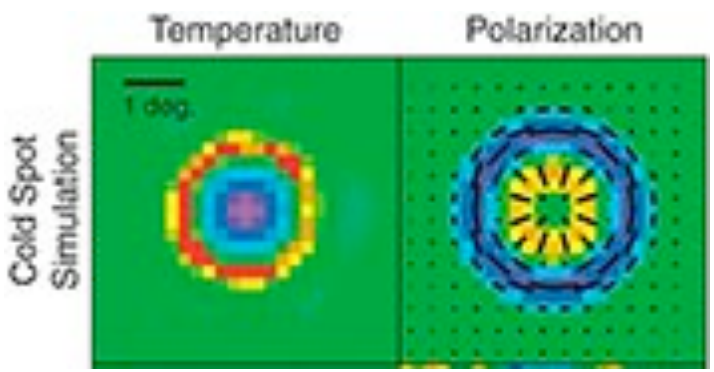


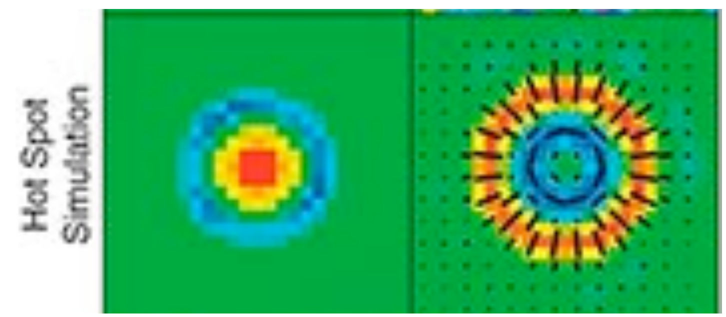
It is interesting that we can actually test whether our understanding of the polarization of CMB is correct

Around cold or hot spots, we expect a certain structure to the polarization signal

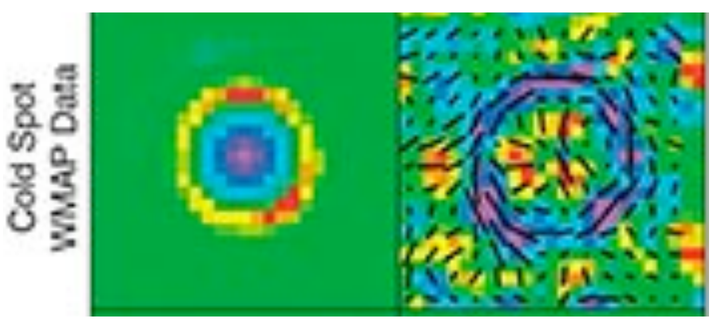
Can test this by looking at the polarization signal around hot or cold spots in the observations.

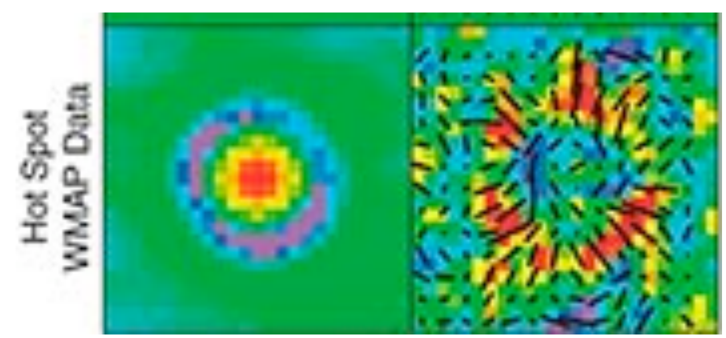
From theory



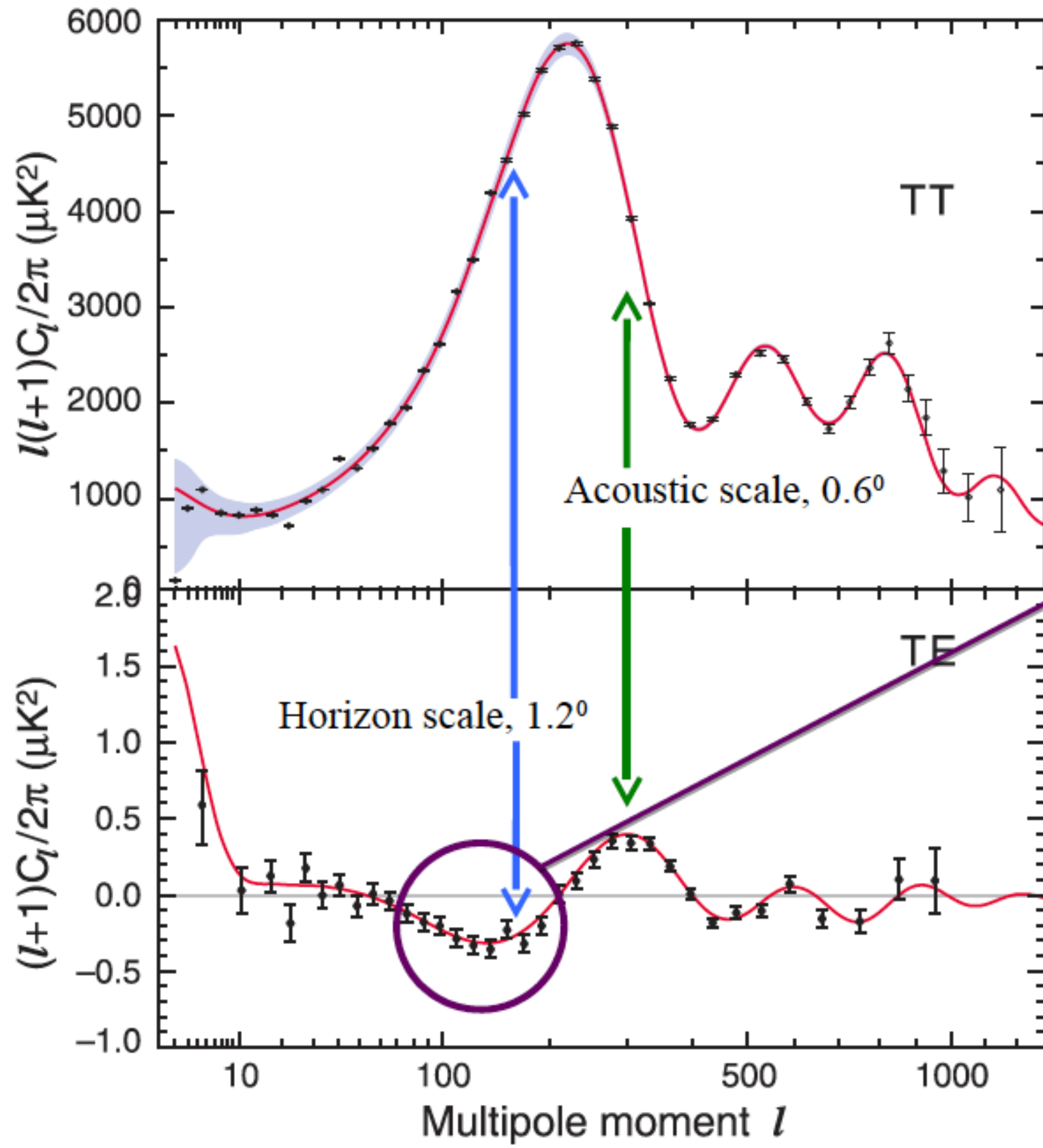


As observed by WMAP





Evidence for super horizon fluctuations



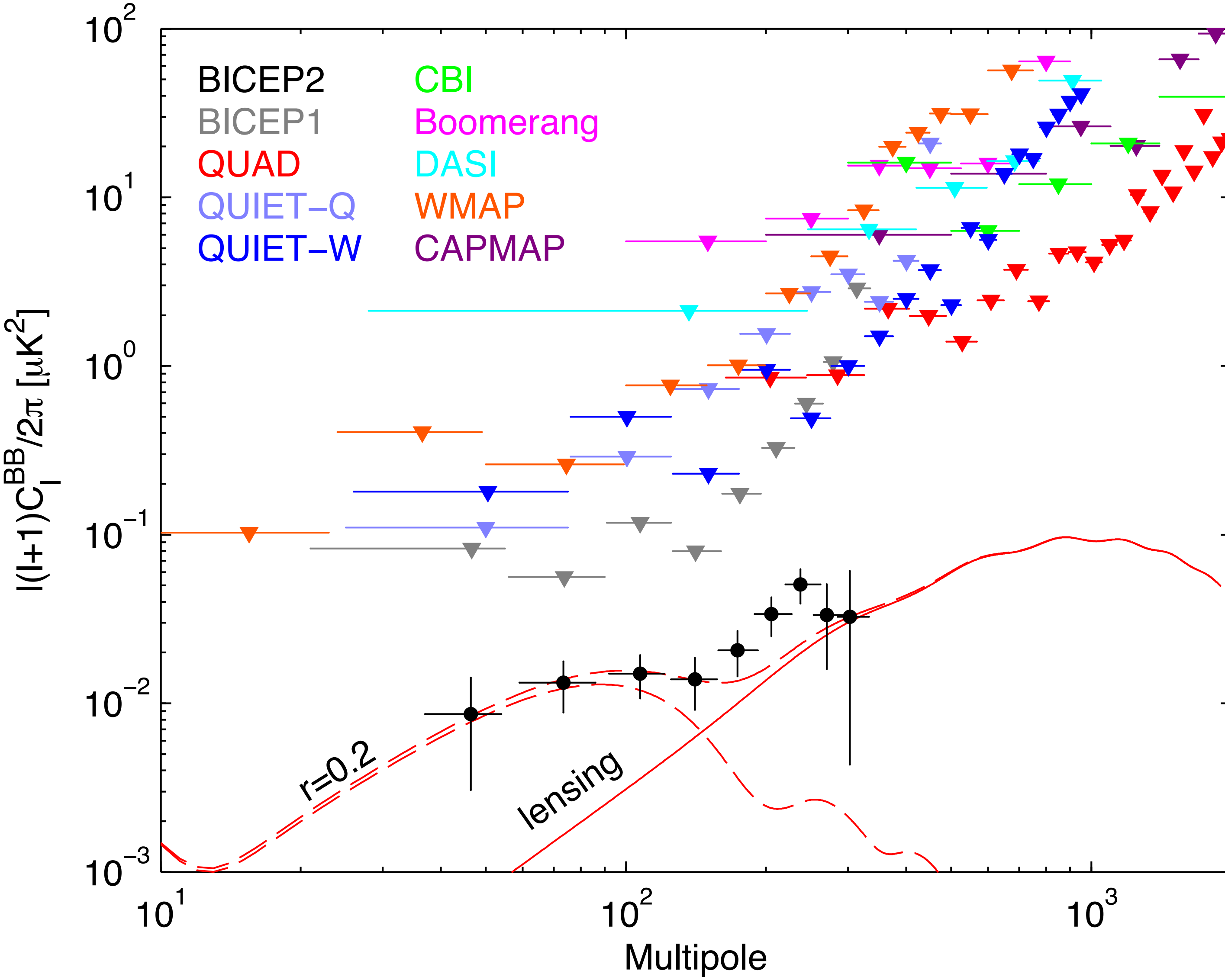
This TE anti-correlation is the best evidence for the existence of super horizon fluctuations, a key element of the standard model.

Spergel & Zaldarriaga (1997)

Peiris et al. (2003)

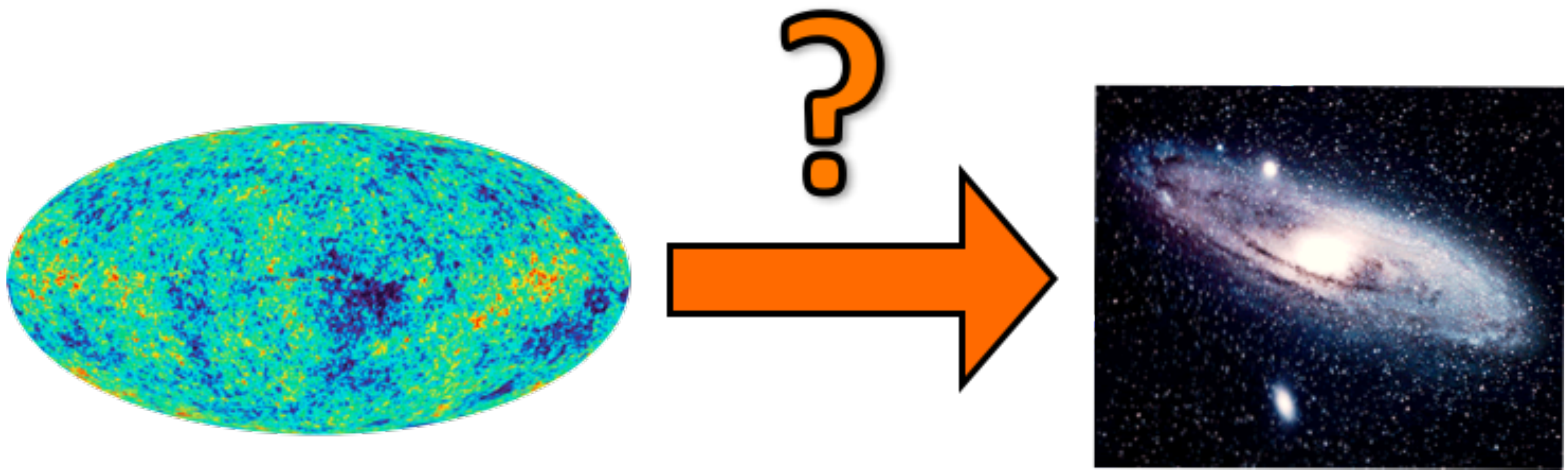
Bennett et al, 2013

Significant BB signal detected by BICEP II!



BICEP2 results show a positive detection of BB modes. Attempted fit to gravity waves from inflation... Lensing contributes at small scales

Gravitational instability



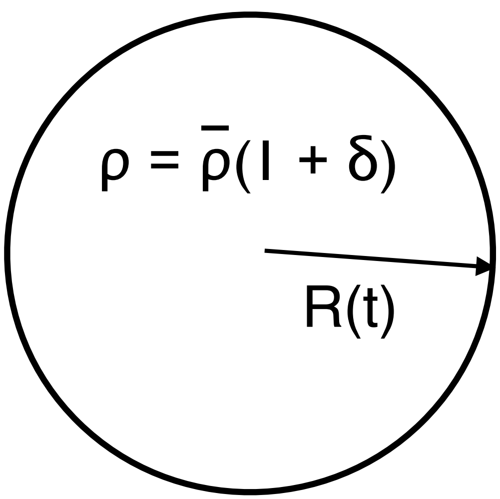
Growth of Structure

We start with tiny fluctuations in the background radiation temperature, which are related to density fluctuations. These grow into the very clustered universe we see today. We therefore need to study the density perturbations.

Gravitational Jeans Instability: Jeans showed that starting from a homogeneous and isotropic "mean" fluid, small fluctuations in the density $d\rho$ and velocity dv can evolve with time.

The simple criterion to decide whether a fluctuation will grow with time is that the typical length scale of a fluctuation should be larger than the Jeans length λ_J.

Consider a static, homogeneous matter-only Universe in which there is a spherical region that is overdense:



$$\delta(t) = (\rho - \bar{\rho})/\bar{\rho} << 1$$

$$d^2R/dt^2 = -G(\Delta M)/R^2 = -G((4/3)\pi R^3\bar{\rho}\delta)/R^2$$

$$d^2R/dt^2 / R = -(4\pi/3)G\bar{\rho}\delta$$

Hence, a mass excess $\delta > 0$ will cause the sphere to collapse

Conservation of mass gives $M = (4\pi/3)\rho[1+\delta(t)]R(t)^3$ = constant during collapse

Growth of Structure

Conversation of mass gives $M = (4\pi/3)\bar{\rho}[1+\delta(t)]R(t)^3$ = constant during collapse

$$R(t) = R_0\,[1+\delta]^{-1/3} \quad \text{where } R_0 = (3M/4\pi\rho)^{1/3}$$

If $\delta \ll 1$, then $R(t) = R_0[1-(1/3)\delta(t)]$ => $\quad d^2R/dt^2 = -(1/3)R_0 d^2\delta/dt^2$

mass conservation yields ==> $d^2R/dt^2 = -(1/3)R_0 d^2\delta/dt^2 \quad (\delta \ll 1)$

$d^2\delta/dt^2 = 4\pi G\bar{\rho}\delta$ which has solutions $\delta = A_1 e^{t/t_{dyn}} + A_2 e^{-t/t_{dyn}}$

where $t_{dyn} = 1/(4\pi G\bar{\rho})^{1/2}$ is the dynamical time for collapse

If the overdense sphere starts at rest, $d\delta/dt = 0$ at $t=0$ => $A_1 = A_2 = \delta(0)/2$. After a few dynamical times, only the growing mode matters → the fluctuations grow exponentially with time.

However, as the sphere collapses, pressure will build up. When a sphere is compressed by its own gravity, a pressure gradient will build up to counter the effects of gravity (e.g., in a star)

If the pressure gradient balances gravity, we have hydrostatic equilibrium.

The pressure gradient steepening takes time: any change in pressure travels with the speed of sound c_s; therefore the time to build up a pressure gradient is a sphere of radius R is pressure ~ R/c_s

$$c_s = c\,(dP/d\rho)^{1/2} = w^{1/2}\,c$$

Growth of Structure

For hydrostatic equilibrium to develop the gradient must build up before collapse:

$$R/c_s = t_{pressure} < t_{dyn} = (G\,\rho)^{-1/2} \;\; \Rightarrow R < c_s t_{dyn} = c_s/(G\bar{\rho})^{1/2} = \lambda_J$$

Jeans Length

A more accurate derivation yields $\lambda_J = c_s(\pi/G\rho)^{1/2} = 2\pi c_s\, t_{dyn}$

Consider a spatially flat Universe with mean density $\rho \;\; \Rightarrow 1/H = (3/8\pi G\bar{\rho})^{1/2} = (3/2)^{1/2}\, t_{dyn}$
$= 1.22\, t_{dyn}$

The Jeans length in an expanding universe will then be

$$\lambda_J = 2\pi c_s t_{dyn} = 2\pi(2/3)^{1/2} c_s/H$$

If we now focus on one component with equation of state w and $c_s = (w)^{1/2}\, c$

$$\lambda_J = 2\pi c_s t_{dyn} = 2\pi(2/3)^{1/2}\, w^{1/2}\, c/H$$

For a photon gas, $c_s = c/3^{1/2} \sim 0.58c \;\; \Rightarrow \lambda_J = 2\pi c_s t_{dyn} = 2\pi\; 2^{1/2}\, c/3H \sim 3c/H$

Density fluctuations in the radiative component will be pressure supported if they are smaller than 3 times the Hubble radius

Such fluctuations will oscillate. Only larger ones will collapse.

A universe containing only radiation will have density fluctuations $\lambda_J < 3c/H$ but they produce sound waves.

To get collapsed structures, we need a non-relativistic component with $w^{1/2} << 1$

Growth of Structure

Prior to decoupling, the baryons were coupled to the photons => no collapse possible

At $c/H(z_{rec}) \sim 0.2$ Mpc and $\varepsilon_\gamma = 1.4\ \varepsilon_{baryon}$

=> λ_J (before decoupling) = $3c/H(z_{rec}) = 0.6$ Mpc

baryon Jeans mass before decoupling = $7 \times 10^{18}\ M_{sun}$

30,000x coma cluster

After decoupling, we have two separate gases: for baryons, the sound speed drops to

c_s (baryon) = $(kT/mc^2)^{1/2}\ c$

c_s (baryon) = $(1.5 \times 10^{-5})c$ or 5 km/s

$kT = 0.26$ eV

$mc^2 = 1.22\ m_p\ c^2 = 1140$ MeV

$Y = 0.24$

Then, after decoupling, the Jeans length decreased by a factor of $c_s(\text{baryon})/c_s(\text{photon}) \sim 2.6 \times 10^{-5}$

M_J (after) $\sim 10^5\ M_{solar}$

much smaller than the mass of our galaxy,
~baryonic mass of the smallest dwarf galaxies

After decoupling, baryon density perturbations could start growing.

Growth of Structure

We can study the Jeans theory in a bit more detail, focusing first on the collisional fluids. The equations of motion are in the Newtonian approximation:

$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}) = 0 \qquad \text{continuity equation}$$

$$\partial\mathbf{v}/\partial t + (\mathbf{v}\cdot\nabla)\mathbf{v} + (1/\rho)P + \nabla\Phi = 0 \qquad \text{Euler equation}$$

$$\nabla^2\Phi - 4\pi G\rho = 0 \qquad \text{Poisson's Equation}$$

We will also neglect any dissipative terms arriving from viscosity or Thermal conductivity. Therefore, we have conservation of entropy per unit mass:

$$\partial S/\partial t + \mathbf{v}\cdot\nabla S = 0$$

A trivial solution is the following: $\rho = \rho_0$, $\mathbf{v} = 0$, $S = S_0$, $p = p_0$, $\nabla\Phi = 0$

Note that if $\rho = \rho_0 \neq 0$, then Φ must vary spatially => homogeneous distribution of ρ cannot be stationary, similar to what we saw when we derived the Friedmann equation

Although the derivation is formally incorrect, the results are qualitatively unchanged and the results can be "reinterpreted" to give correct results.

$\rho = \rho_0 + \delta\rho$, $\quad \mathbf{v} = \delta\mathbf{v}$, $\quad p = p_0 + \delta p$ $\quad S = S_0 + \delta S$ $\quad \Phi = \Phi_0 + \delta\Phi$

$$\partial\delta\rho/\partial t + \rho_0\nabla\cdot(\delta\mathbf{v}) = 0$$

$$\partial\delta\mathbf{v}/\partial t + (1/\rho_0)(dp/d\rho)_s\nabla\delta\rho + (1/\rho_0)(dp/ds)_\rho\nabla\delta s + \nabla\delta\phi = 0$$

$$\nabla^2\delta\phi - 4\pi G\delta\rho = 0 \qquad \partial\delta S/\partial t = 0$$

Modeling the Growth of Structure using Waves in Fluid

We will look for solution in the form of plane waves $\delta u_i = \delta_i\, e^{i\mathbf{k}\cdot\mathbf{r}}$ where $\delta u_i = \delta\rho, \delta v, \delta\phi, \delta s$

Given that the unperturbed solution do not depend on position, we can search for solutions:

$\delta_i(t) = \delta_{0,i}\, e^{i\omega t}$ amplitude D, V, Φ, Σ

Use that $c_s = (\partial P/\partial\rho)_s$ and $\delta_0 = D/\rho_0$

$$\Rightarrow \omega\, \delta_0 + \mathbf{k} \cdot \mathbf{V} = 0$$

$$\Rightarrow \omega\, \mathbf{V} + \mathbf{k}\, c_s^2\, \delta_0 + \mathbf{k}/\rho\, (dp/ds)_\rho\, \Sigma + \mathbf{k}\Phi = 0$$

$$\Rightarrow k^2\, \Phi + 4\pi G\rho_0\delta_0 = 0$$

$$\Rightarrow \omega\, \Sigma = 0$$

Let us consider solutions with $\omega \neq 0 \Rightarrow \Sigma = 0$: perturbations are adiabatic

Also, $\mathbf{k} \cdot \mathbf{V} \neq 0$, we can decompose into components parallel and perpendicular to $\mathbf{V}$

$\mathbf{k}$ perpendicular to $\mathbf{V} \Rightarrow \delta_0 = 0, \Phi = 0$, these vertical models do not imply a density perturbation.

$\mathbf{k}$ is parallel to $\mathbf{V}$

$$\Rightarrow \omega\, \delta_0 + kV = 0$$
$$\Rightarrow \omega V + kc_s^2\, \delta_0 + k\Phi = 0$$
$$\Rightarrow k^2\, \Phi + 4\pi G\rho_0\delta_0 = 0$$

$$\begin{pmatrix} \omega & k & 0 \\ kc_s^2 & \omega & k \\ 4\pi G\rho_0 & 0 & k^2 \end{pmatrix} \begin{pmatrix} \delta_0 \\ V \\ \Phi \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

This admits a non-zero solution for δ_0, V, Φ if and only if its determinant vanishes $\Rightarrow \omega$ and k must satisfy the dispersion relation

$$\Rightarrow \omega^2 - c_s^2\, k^2 + 4\pi G\rho_0 = 0$$

Modeling the Growth of Structure using Waves in Fluid

The solutions are of two types, depending on whether $\lambda = 2\pi/k$ larger or smaller than

$$\lambda_J = c_s\,(\pi/G\rho_0)^{1/2}$$

In the case that $\lambda < \lambda_J$, the value of ω is real and $\omega = \pm c_s\, k[1-(\lambda/\lambda_J)^2]^{1/2}$

These represent two sound waves in directions $\pm\mathbf{k}$ with a dispersion ω

If $\lambda > \lambda_J$, the frequency is imaginary: $\omega = \pm\, i\,(4\pi G\rho_0)^{1/2}\,[1-(\lambda_J/\lambda)^2]^{1/2}$

and the solution for the density is $d\rho/\rho_0 = \delta_0\, e^{i\mathbf{kr}}\, e^{\pm\omega t}$

The characteristic time scale for the evolution of the amplitude is

$$\tau = \omega^{-1} = 1/(4\pi G\rho_0)^{1/2}\,[1-(\lambda_J/\lambda)^2]^{-1/2}$$

for $\lambda >> \lambda_J$, this corresponds to the dynamical or free-fall time.