

# What can we Learn from the Cosmic Microwave Background

# Layout of the Course

Feb 5: Introduction / Overview / General Concepts

Feb 12: Age of Universe / Distance Ladder / Hubble Constant

Feb 19: Distance Ladder / Hubble Constant / Distance Measures

Feb 26: Distance Measures / SNe science / Baryonic Content

Mar 4: Baryon Content / Dark Matter Content of Universe

Mar 11: Cosmic Microwave Background

Mar 18: Cosmic Microwave Background / Large Scale Structure

Mar 25: Baryon Acoustic Oscillations / Dark Energy / Clusters

Apr 1: No Class

Apr 8: Clusters / Cosmic Shear

Apr 15: Dark Energy Missions / Review for Final Exam

May 13: Final Exam

This Week



Will send around problem set 2  
later today

Will be due by March 27

**Review Material from Last Week**

# What is the matter composition of the universe?

## Baryonic Matter

$$\Omega_{\text{stars}} = 0.002$$

$$\Omega_{\text{cold gas, HI}} = 0.0003$$

$$\Omega_{\text{cold gas, molecular hydrogen}} = 0.0003$$

$$\Omega_{\text{ionized hydrogen}} = 0.02$$

$$\rightarrow \Omega_{\text{baryons}} = 0.04$$

## What is the evidence for dark matter?

Rotational Curves of Spiral Galaxies

Observations of Galaxy Cluster Collisions

Measurement of Masses for Galaxy Clusters

Peculiar Velocities of Galaxies in the Nearby Universe

One can determine  $\Omega_{\text{DM}}$  by measuring the ratio of the masses in baryons + DM in galaxy clusters

A) Measure mass in baryons by exploiting SZ effect.


B) Measure total mass in cluster using 3 different techniques:

1. Use velocity dispersion (motion of galaxies in cluster)
2. Measurement of gas profile (hydrostatic equilibrium)
3. Model gravitational lensing of background sources

$$\rightarrow \Omega_{\text{dark matter}} = 0.24$$

Measured M/L ratios increase towards largest scales indicating the increasing importance of dark matter on large scales:

Solar Neighborhood:  $M/L = 1$

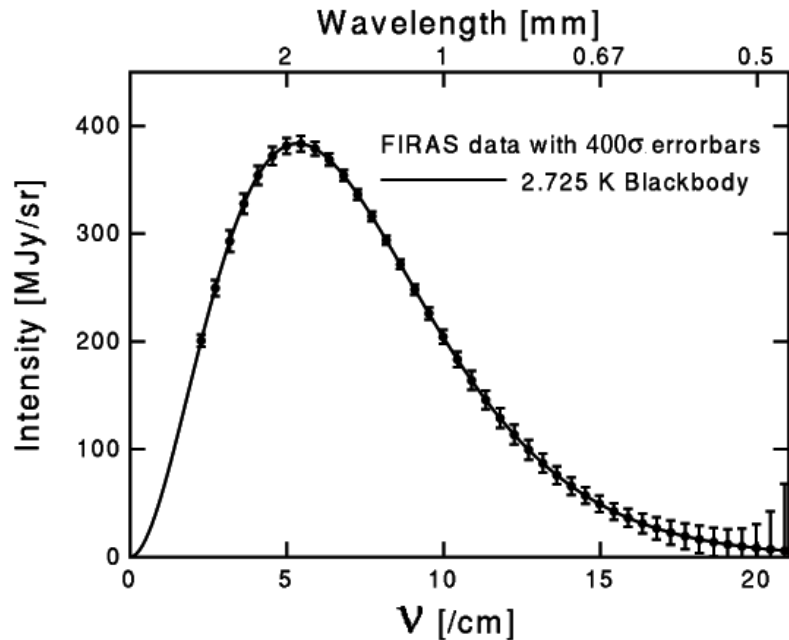

$$(M/L)_{\text{galaxy}} \sim 10\text{-}20 M_{\text{solar}}/L_{\text{solar}}$$

$$(M/L)_{\text{cluster}} \sim 100\text{-}200 M_{\text{solar}}/L_{\text{solar}}$$

Universe:  $M/L = \sim 200$

# Cosmic Microwave Background Radiation

Photons from the CMB have a spectral energy distribution which is almost a perfect black body.



Cosmic Microwave Background is Isotropic

Isotropic to one part in  $10^5$

$T = 2.728 \text{ K}$

==> New Material

What happened during the  
recombination epoch and  
how did it result in the  
cosmic microwave  
background?

# Recombination Epoch ( $z \sim 1100$ )

Ionized Plasma  $\longrightarrow$  Neutral Gas

( $z > 1100$ )  
< 380,000 years

Temperature  
> 3600 K

Hydrogen ionized

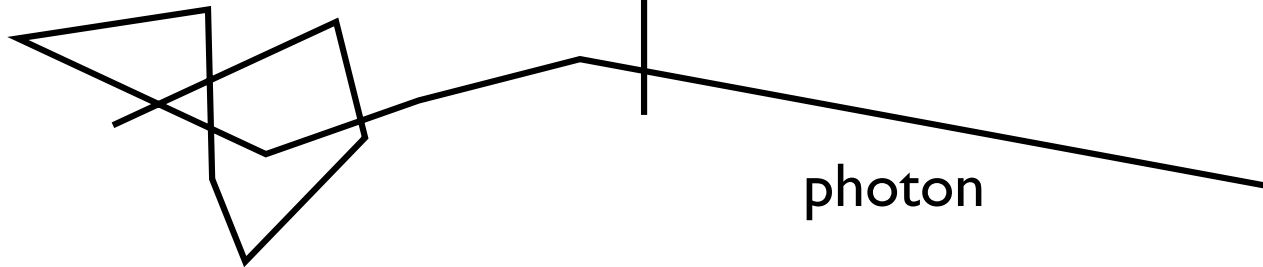
Photons Thomson-scattering  
off of the ionized hydrogen

( $z < 1100$ )  
> 380,000 years

Temperature  
< 3600 K

Hydrogen neutral

Almost no free electrons  
Photons unbound from  
plasma



# Recombination Epoch ( $z \sim 1100$ )

Ionized Plasma  $\longrightarrow$  Neutral Gas

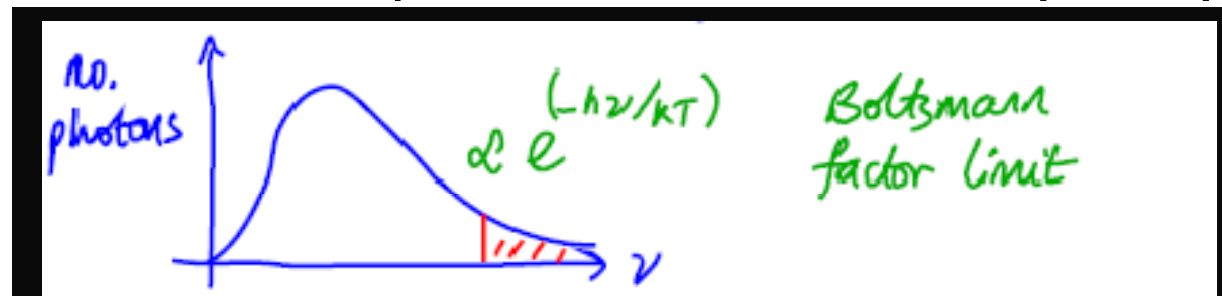
In more detail, transition occurs in three stages:

## 1. Recombination

- Temperature drops sufficiently that protons can recombine with electrons

## 2. Decoupling

- Stage where photons are no longer closely tied to baryons
- Occurs at a slightly later stage than the initial recombination, because the # of photons exceeds # of baryons by  $10^9$



Credit: Abdalla

- With so many more photons, the temperature of the background radiation can fall well below what would normally be necessary to ionize hydrogen. It is because of the high energy tail of distribution.

# Recombination Epoch ( $z \sim 1100$ )

Ionized Plasma  $\longrightarrow$  Neutral Gas

In more detail, transition occurs in three stages:

## 1. Recombination

- Temperature drops sufficiently that protons can recombine with electrons

## 2. Decoupling

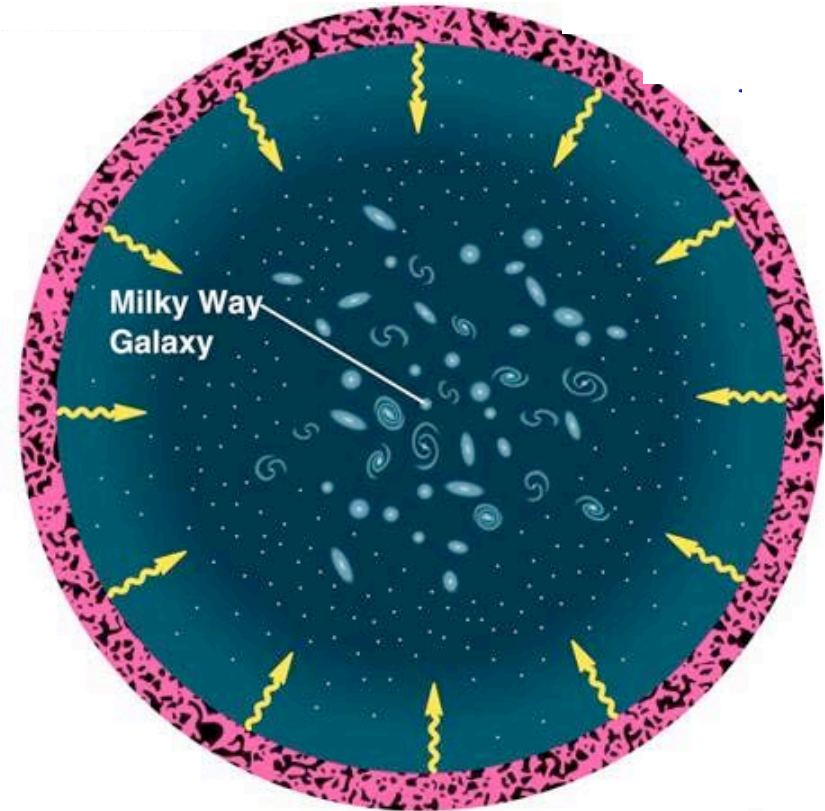
- Stage where photons are no longer closely tied to baryons
- Occurs at a slightly later stage than the initial recombination, because the # of photons exceeds # of baryons by  $10^9$

## 3. Last Scattering

- Last time a cosmic microwave photon scatters off of matter

# Origin of Microwave Background

- The last time cosmic microwave photons interacted with matter was at the last scattering surface
- Cosmic Microwave Photons we observe are a relic of the Big Bang
- We cannot observe the universe directly at any earlier time than the last scattering surface (~400,000 years after the Big Bang)

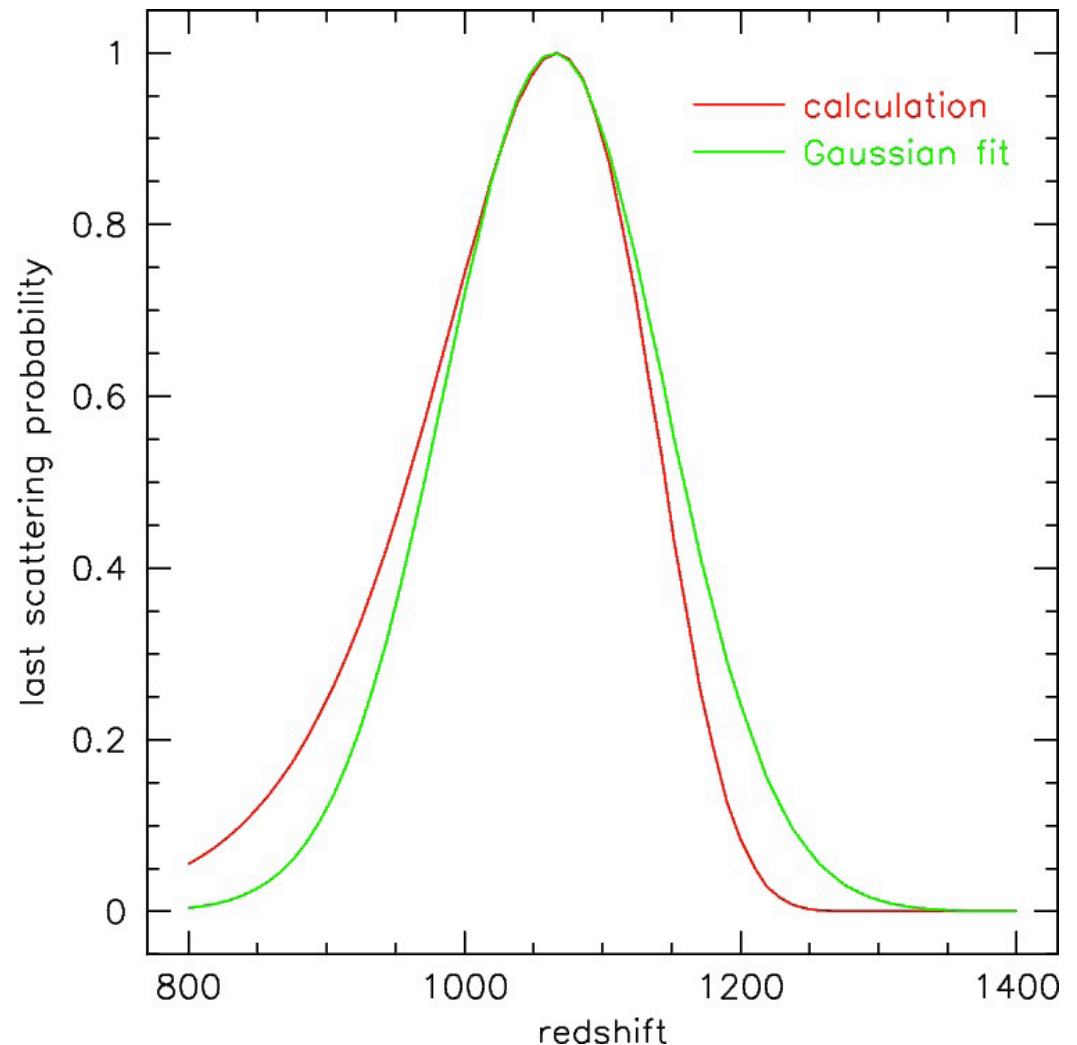


Credit: Pearson

# How extended is the surface of last scattering?

-- Distribution describes the probability that a photon from the cosmic microwave background was last scattered at a given redshift.

-- Can roughly be described by a normal distribution with mean  $z = 1080$  and standard deviation  $dz = 80$

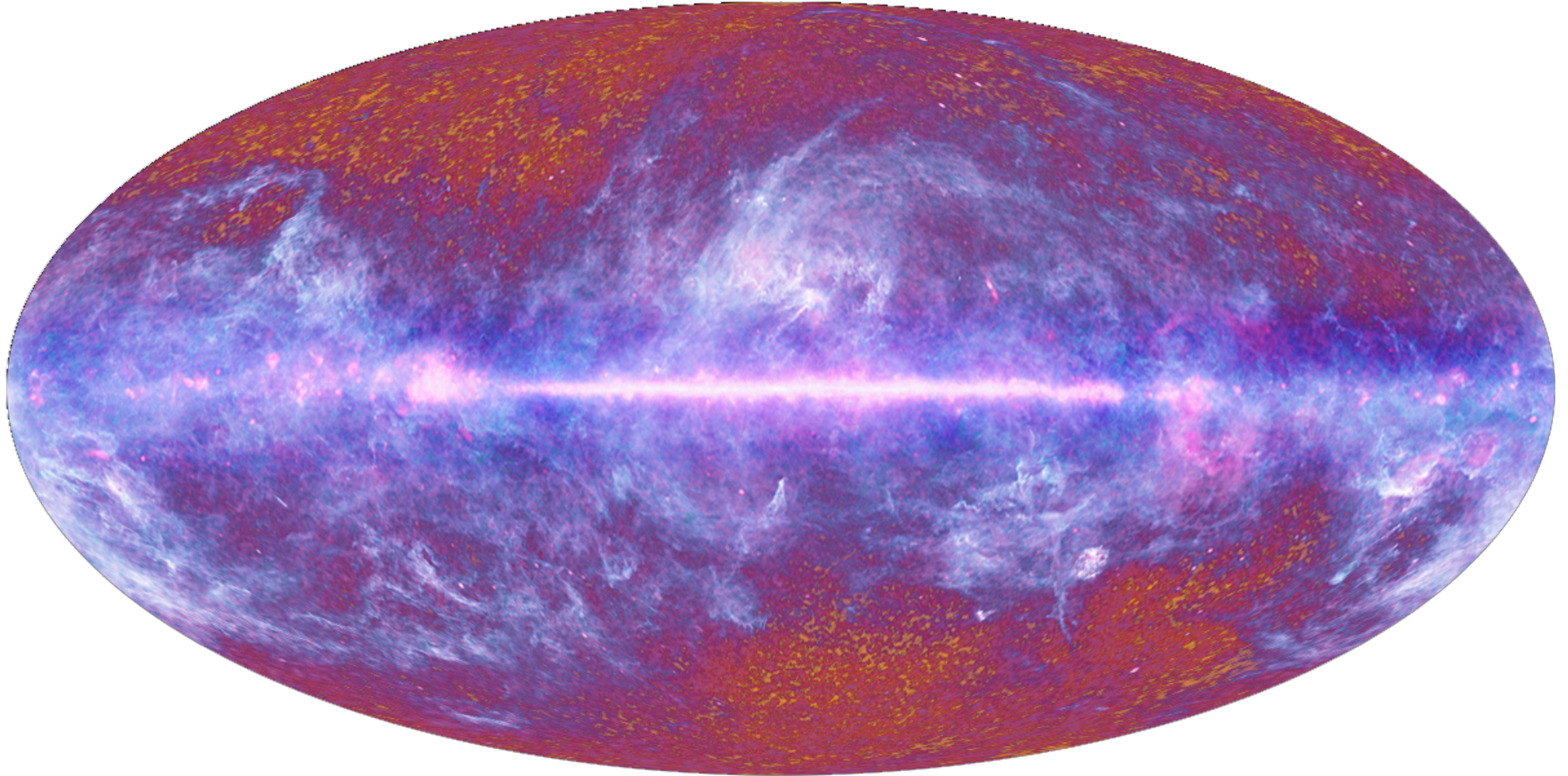


# Thermalization of the CMB:

- To have such a perfect blackbody shape, the rate of thermal and photon-scattering processes must be much faster than the rate of expansion of the universe. This happens at the redshifts  $z > 2 \times 10^6$  (2 months after big bang)
- This thermalization effectively removes any thermal and energy signatures from epochs before this point.
- Since the universe expands adiabatically, once a blackbody spectrum is set up, it is maintained.

What fundamentally do we  
observe?

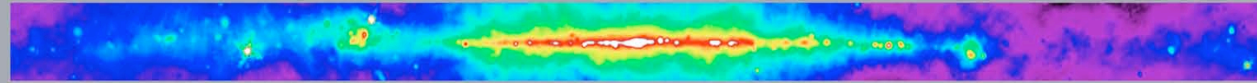
# View of the sky as seen by Planck



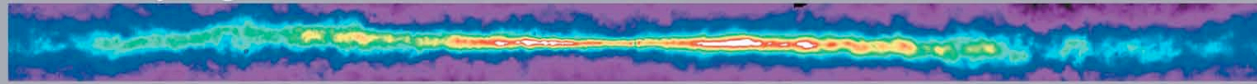
# Challenge of looking through the Milky Way

## Multiwavelength Milky Way

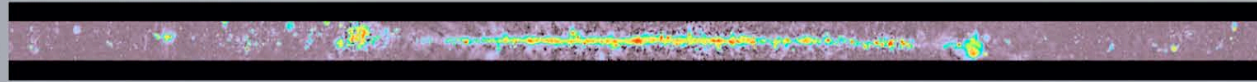
Radio Continuum 408 MHz Bonn, Jodrell Banks, & Parkes



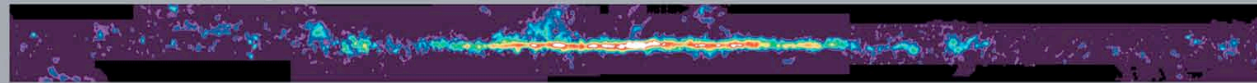
Atomic Hydrogen 21 cm Leiden-Dwingeloo, Maryland-Parkes



Radio Continuum 2.4-2.7 GHz Bonn & Parkes



Molecular Hydrogen 115 GHz Columbia-GISS



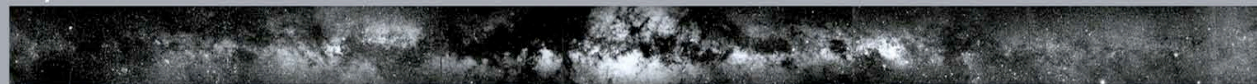
Infrared 12, 60, 100  $\mu\text{m}$  IRAS



Near Infrared 1.25, 2.2, 3.5  $\mu\text{m}$  COBE/DIRBE



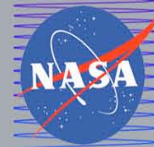
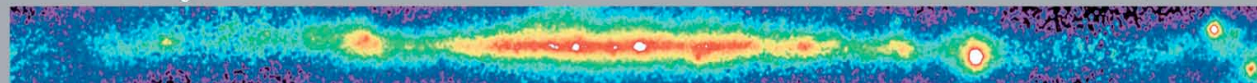
Optical Laustsen et al. Photomosaic



X-Ray 0.25, 0.75, 1.5 keV ROSAT/PSPC



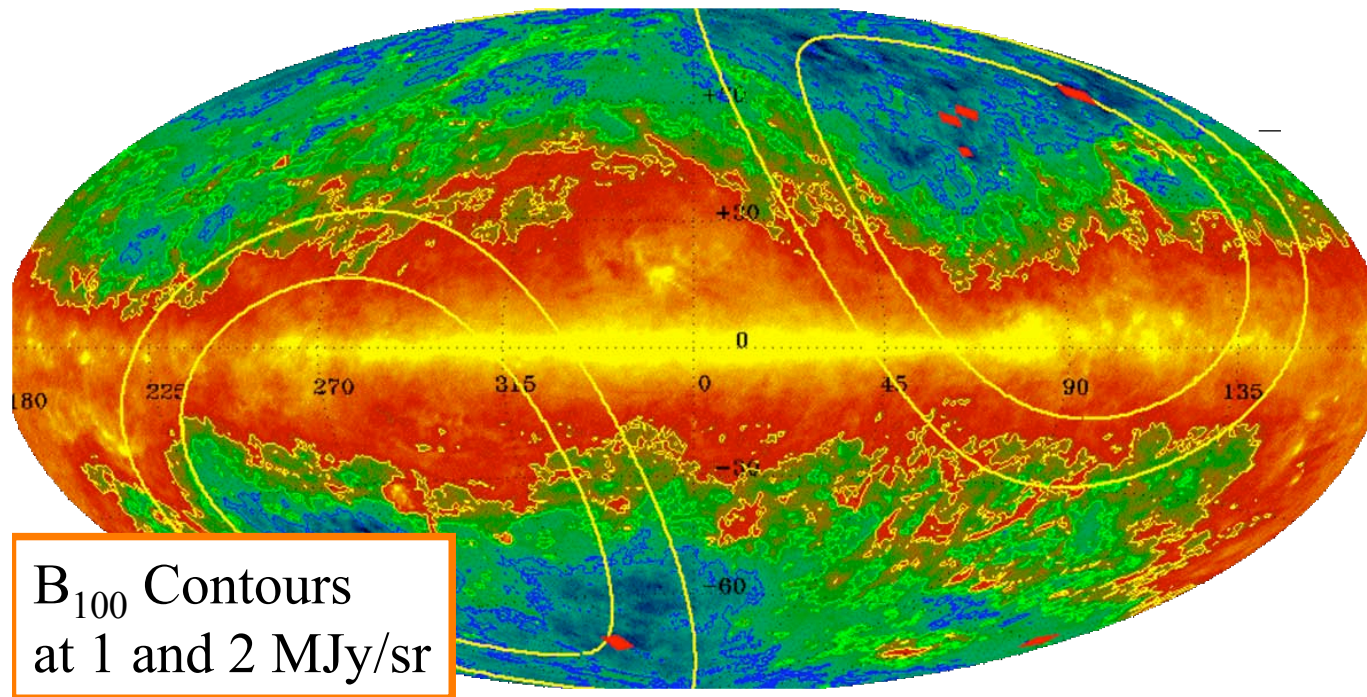
Gamma Ray >100 MeV CGRO/EGRET



# Substantial Foreground Light

## One Example is Infrared Cirrus:

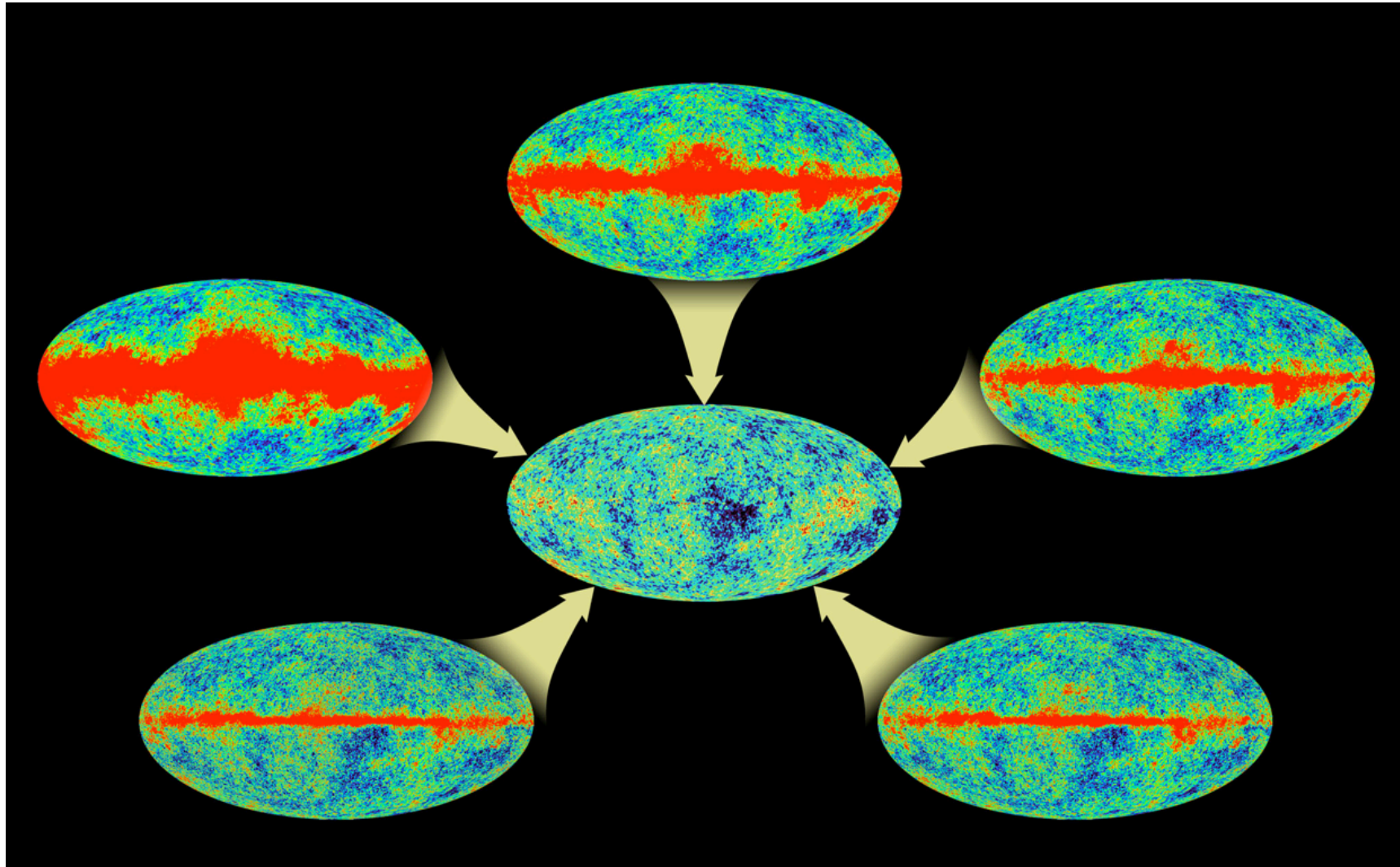
- Interstellar dust in our galaxy is heated by the interstellar radiation field.
- Emission depends on galaxy latitude and is significant longward of  $60\ \mu\text{m}$



Other Examples are Synchrotron (from supernovae remnants) and Free-free Emission (from ionized regions around hot stars)

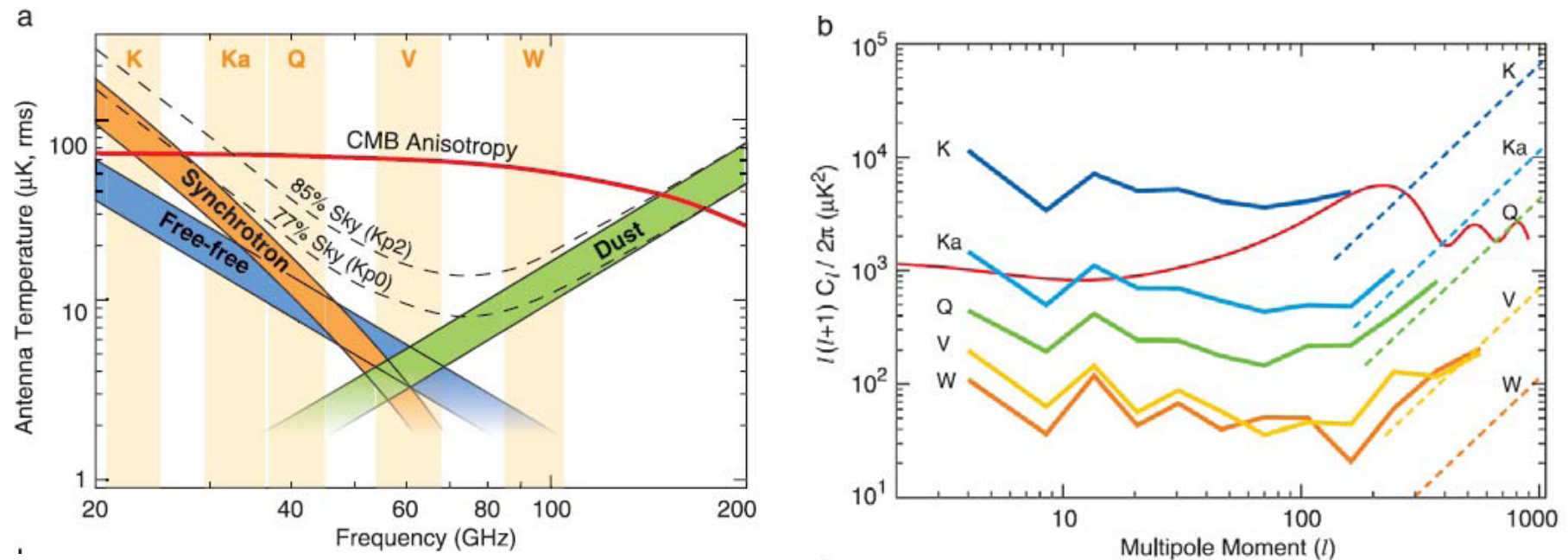
How can we distinguish  
microwave background light  
from foreground emission?

Fortunately all of these telescopes observe  
at multiple wavelengths



# Fortunately all of these telescopes observe at multiple wavelengths

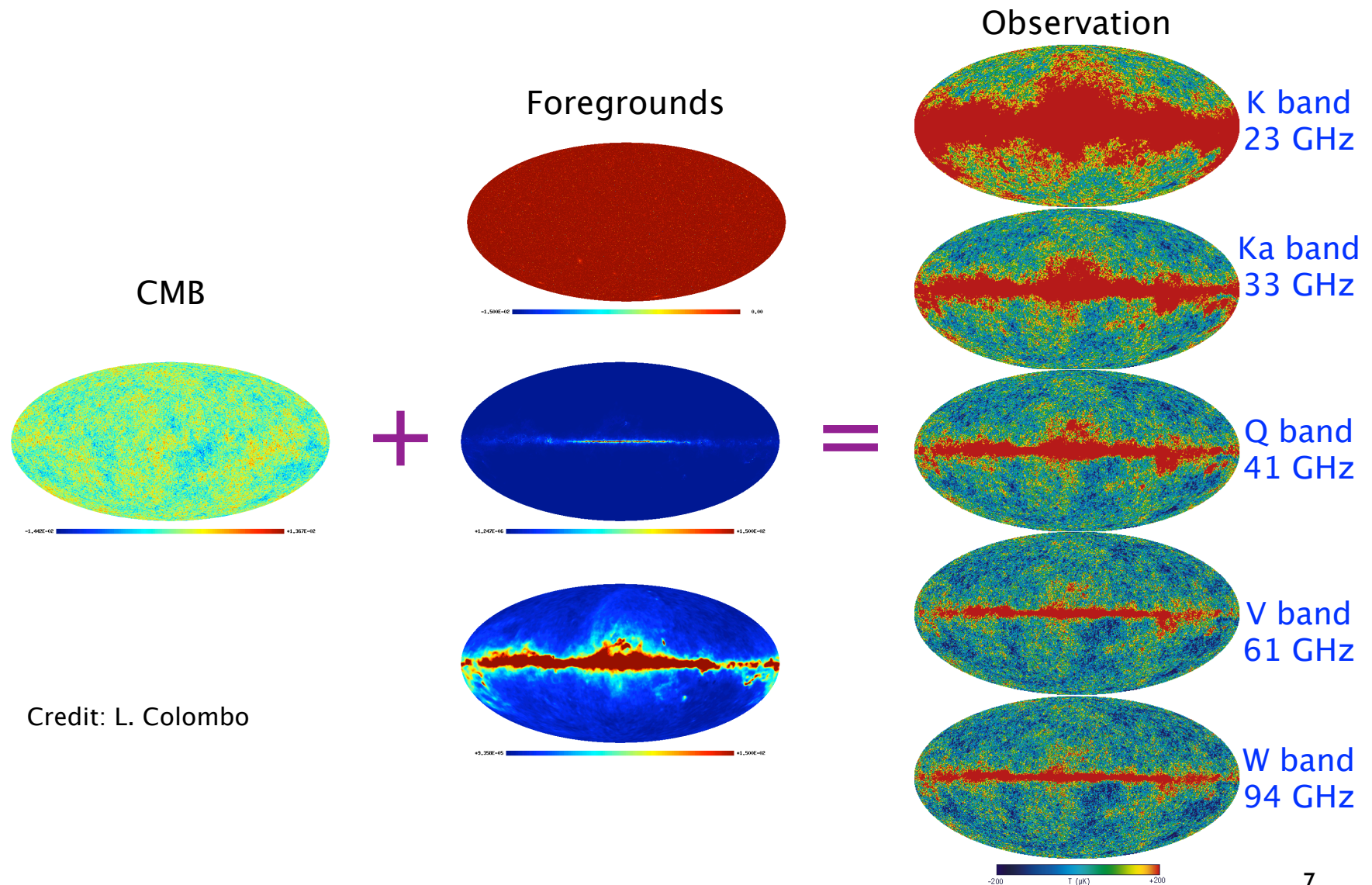
-- This foreground emission has a very different looking spectrum than the cosmic microwave background (each has unique multiwavelength signature)



CMB vs. foreground anisotropies (Bennett et al. 2003, WMAP 1st year)

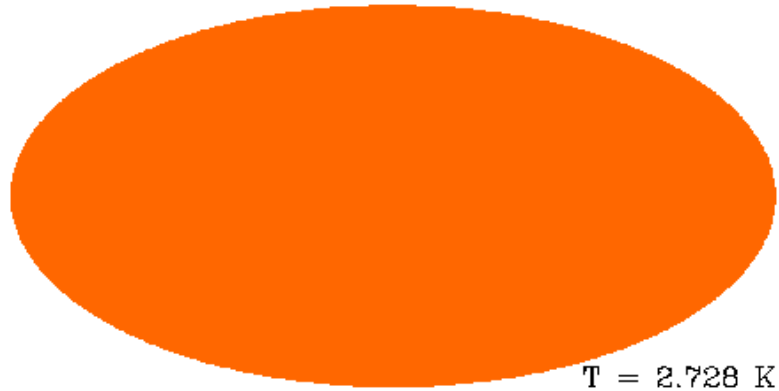
-- 5 Wavelength channels for telescopes like WMAP chosen at wavelengths where CMB is particularly prominent (9 channels for Planck)

Using unique multiwavelength signatures of the CMB and the foregrounds, find the right linear combination to match the multi-wavelength observations



Now that we've explained the  
observational procedure, let's  
look at the cosmic microwave  
radiation a little closer

# Examining the CMB at different contrast levels:

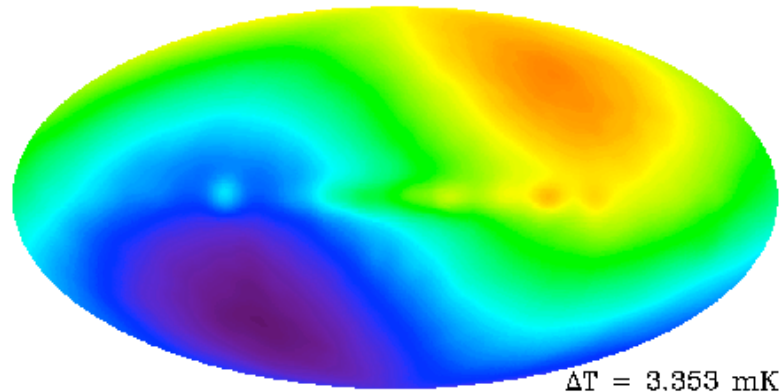


To first approximation, the cosmic microwave background is isotropic

# Examining the CMB at different contrast levels:

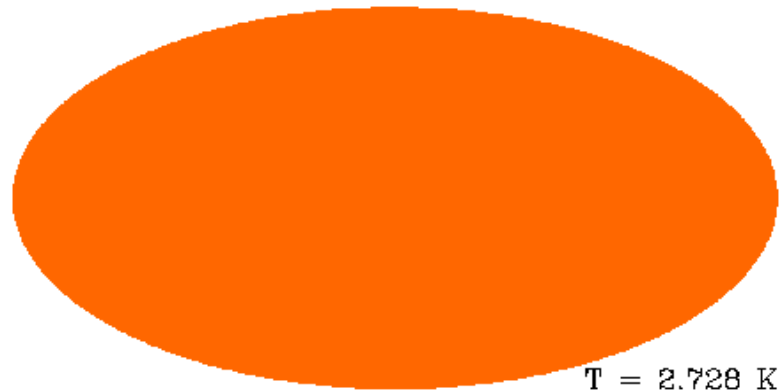


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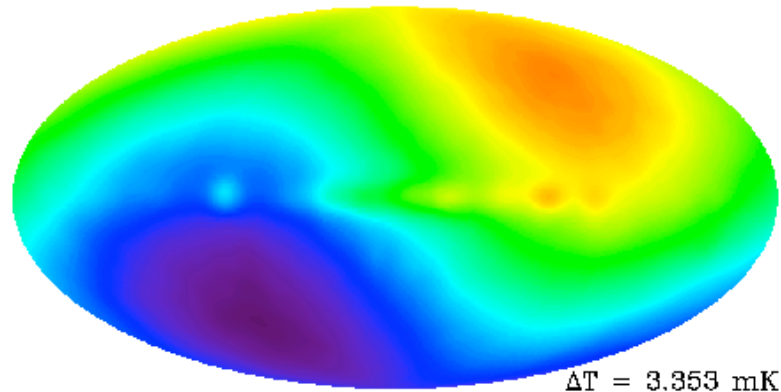


At the  $\sim 10^{-3}$  level, one finds a dipole -- that arises from the motion of the earth relative to the CMB frame

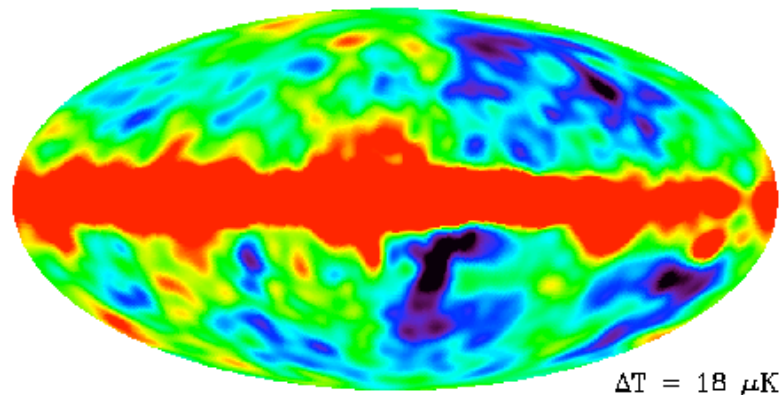
# Examining the CMB at different contrast levels:



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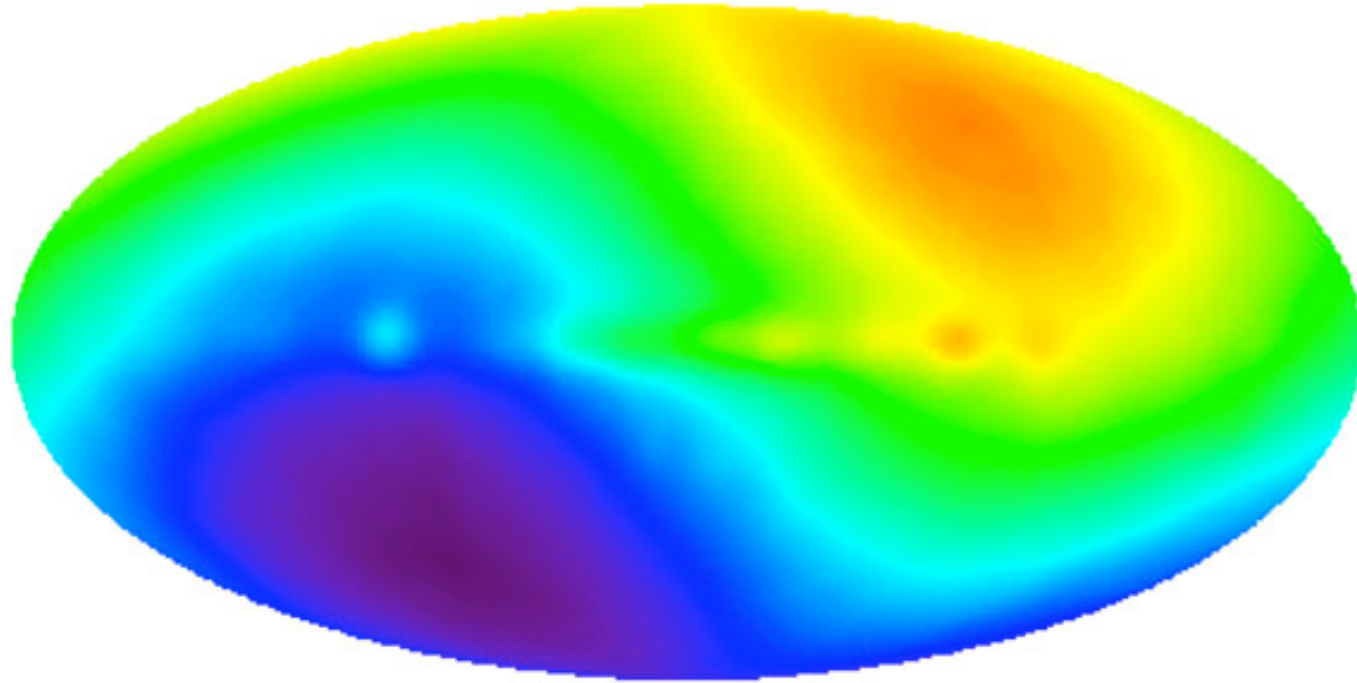


At the  $\sim 10^{-3}$  level, one finds a dipole -- that arises from the motion of the earth relative to the CMB frame



At the  $\sim 10^{-5}$  level (and subtracting the dipole), one observes anisotropies in the CMB.

# Dipole in the Cosmic Microwave Background



-- magnitude of the dipole is  $390 \pm 30$  km/s

-- if we correct for

satellite-earth  $\sim 8$  km s<sup>-1</sup>

earth-sun  $\sim 30$  km s<sup>-1</sup>

sun-galaxy  $\sim 220$  km s<sup>-1</sup>

galaxy-local group  $\sim 220$  km s<sup>-1</sup>

we find our local group moving towards Hydra at  $630 \pm 20$  km s<sup>-1</sup>

How do we analyze the  
cosmic microwave  
background?

# How to represent or model anisotropies in the CMB?

- Since the observed temperature of the CMB as a function of position on sky only differs by a small amount from the mean, represent the anisotropies as a temperature difference

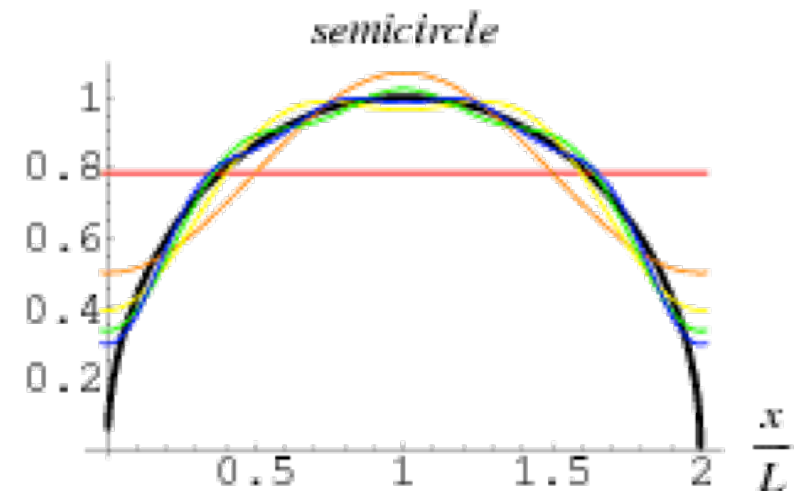
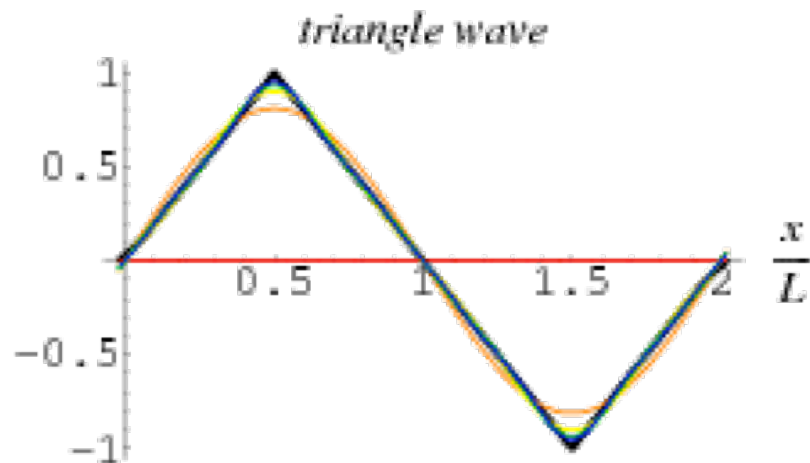
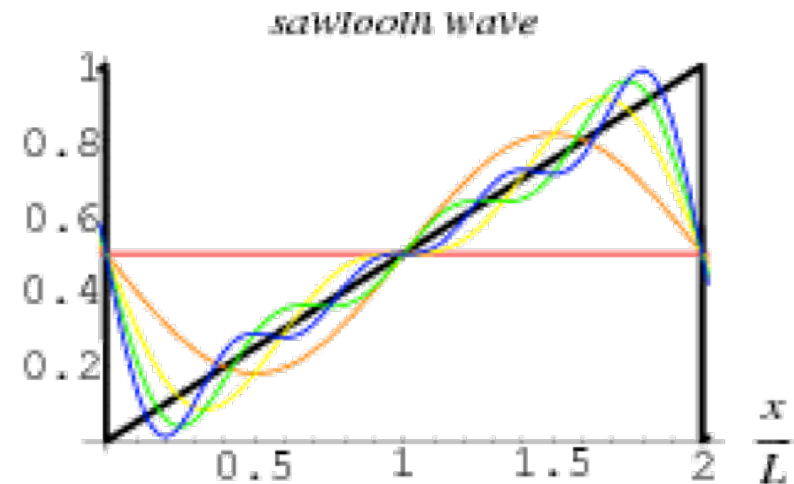
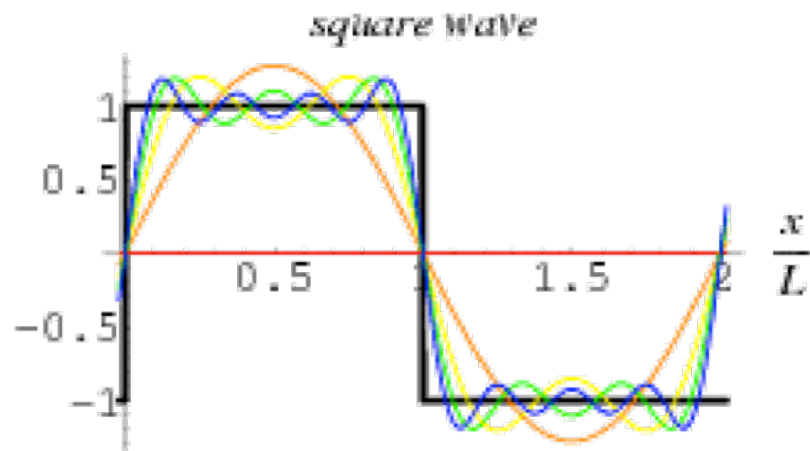
$$\frac{\Delta T}{T}(\theta, \phi) = \frac{T(\theta, \phi) - \bar{T}}{\bar{T}}$$

- Represent this temperature difference as a function of position using an equivalent Fourier series in spherical coordinates -- which are spherical harmonics

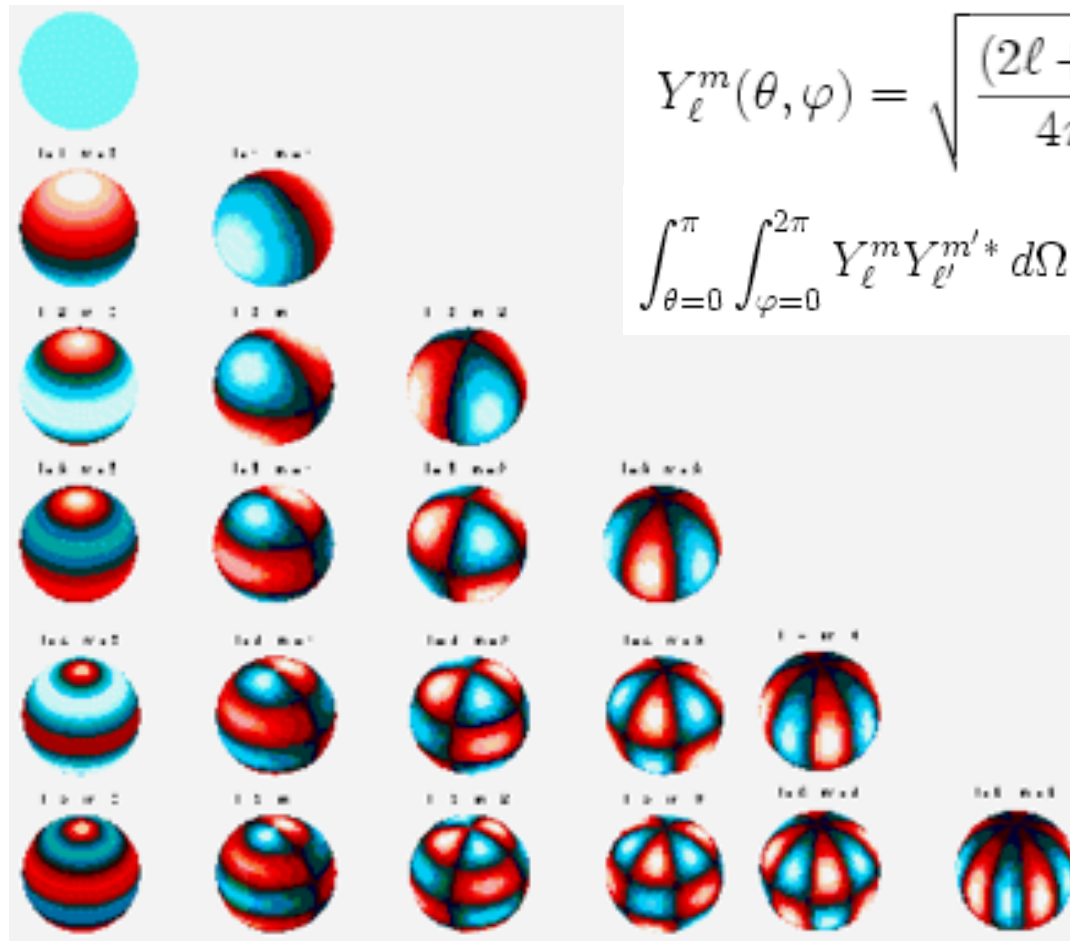
$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_m^{\ell}(\theta, \phi)$$

# Similar concept to Fourier Series

- Most of you are probably familiar with the fact that one can use a fourier series to represent an arbitrary one-dimensional function



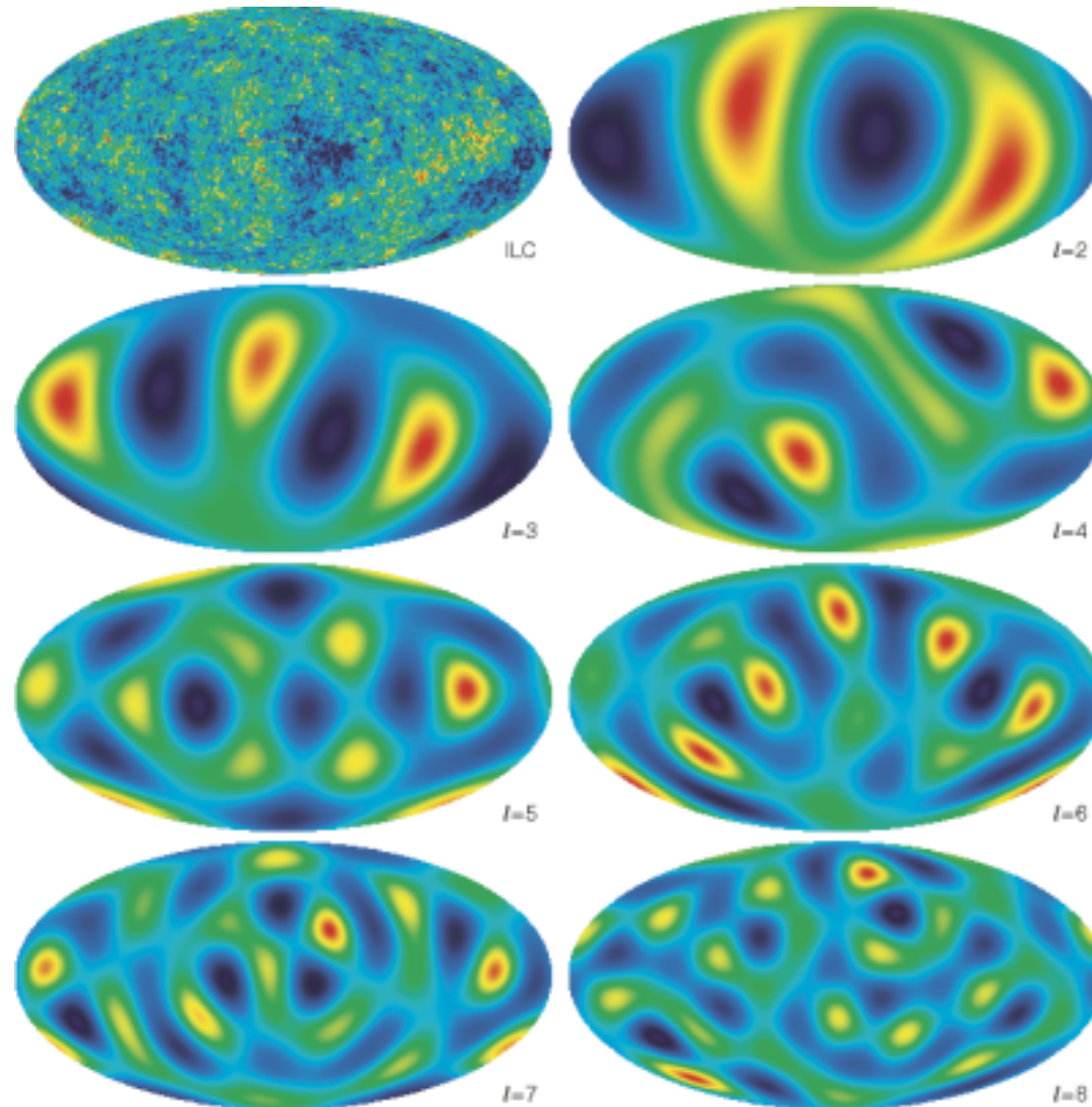
# Spherical harmonics (used to represent the anisotropies in the CMB)



$$Y_\ell^m(\theta, \varphi) = \sqrt{\frac{(2\ell + 1)(\ell - m)!}{4\pi(\ell + m)!}} \cdot e^{im\varphi} \cdot P_\ell^m(\cos \theta)$$

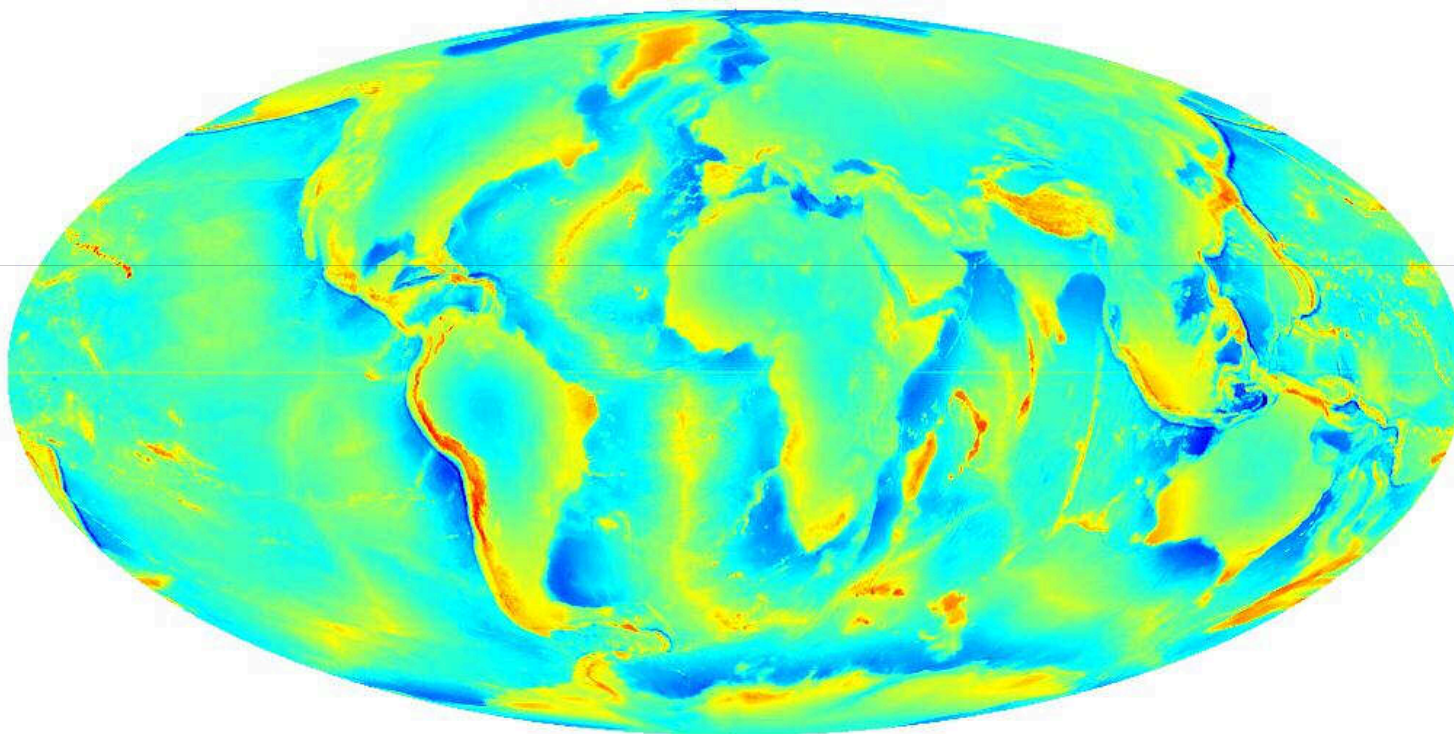
$$\int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} Y_\ell^m Y_{\ell'}^{m'*} d\Omega = \delta_{\ell\ell'} \delta_{mm'} \quad d\Omega = \sin \theta d\varphi d\theta$$

# Closer look at all the multipoles



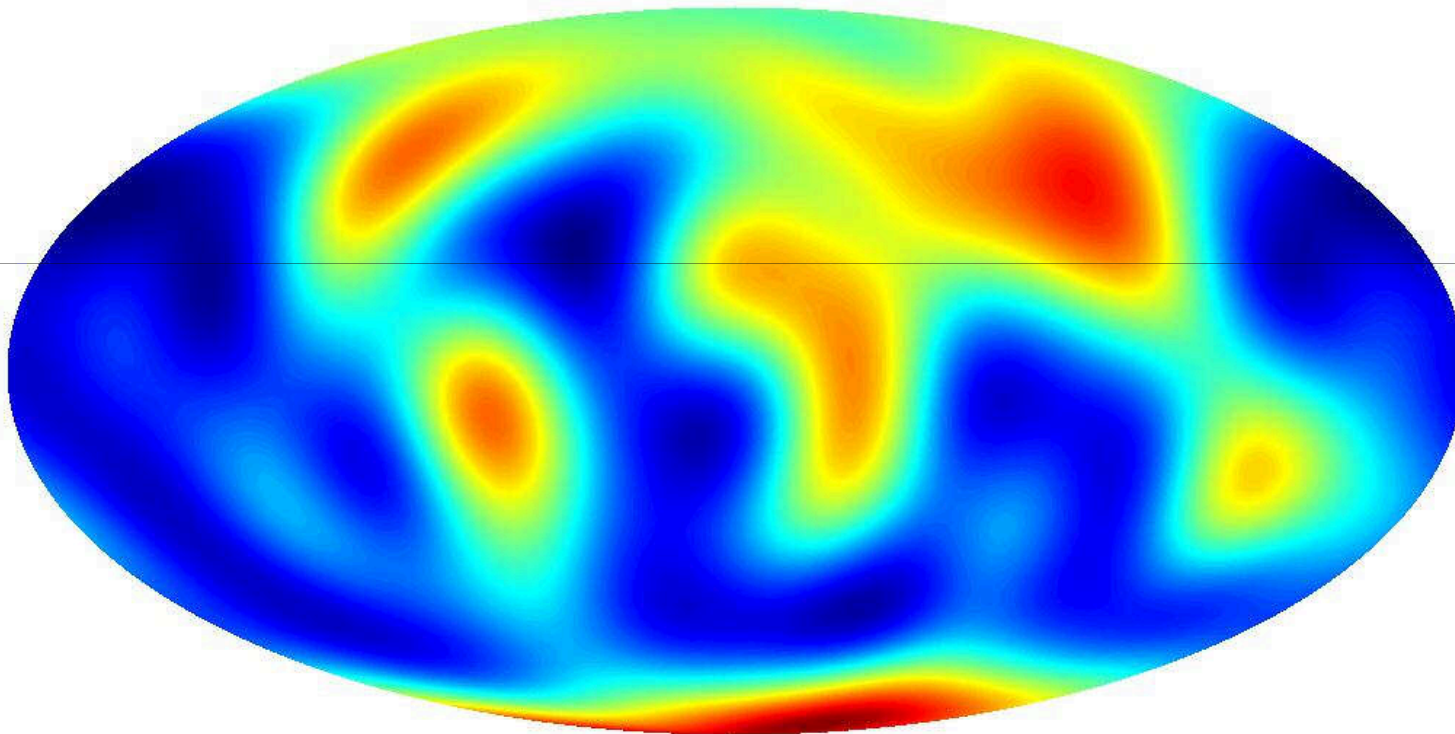
One can represent any spherical surface using an expansion in Legendre polynomials

Sum up to some high  $\ell$



Made by Matthias Bartelmann

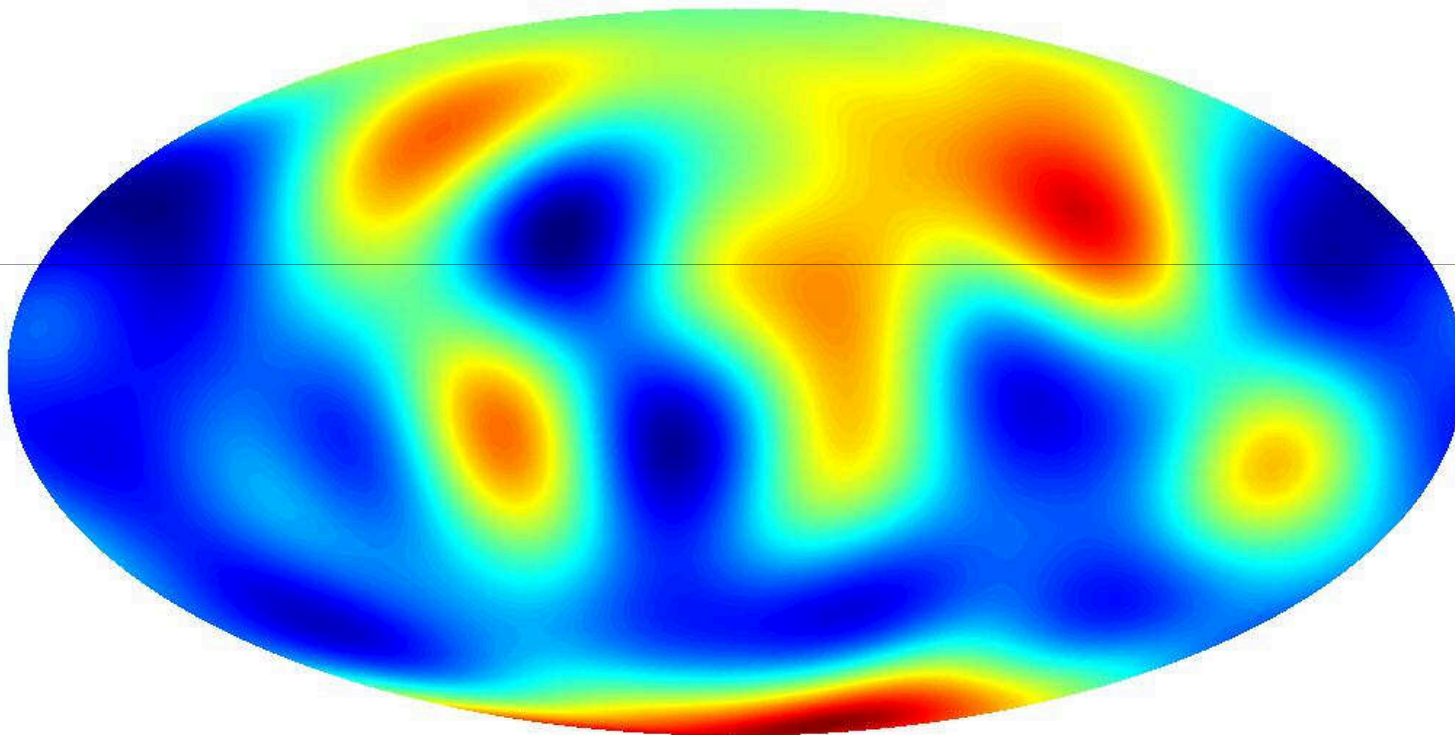
Sum  $\ell=1$  to 8



Made by Matthias Bartelmann

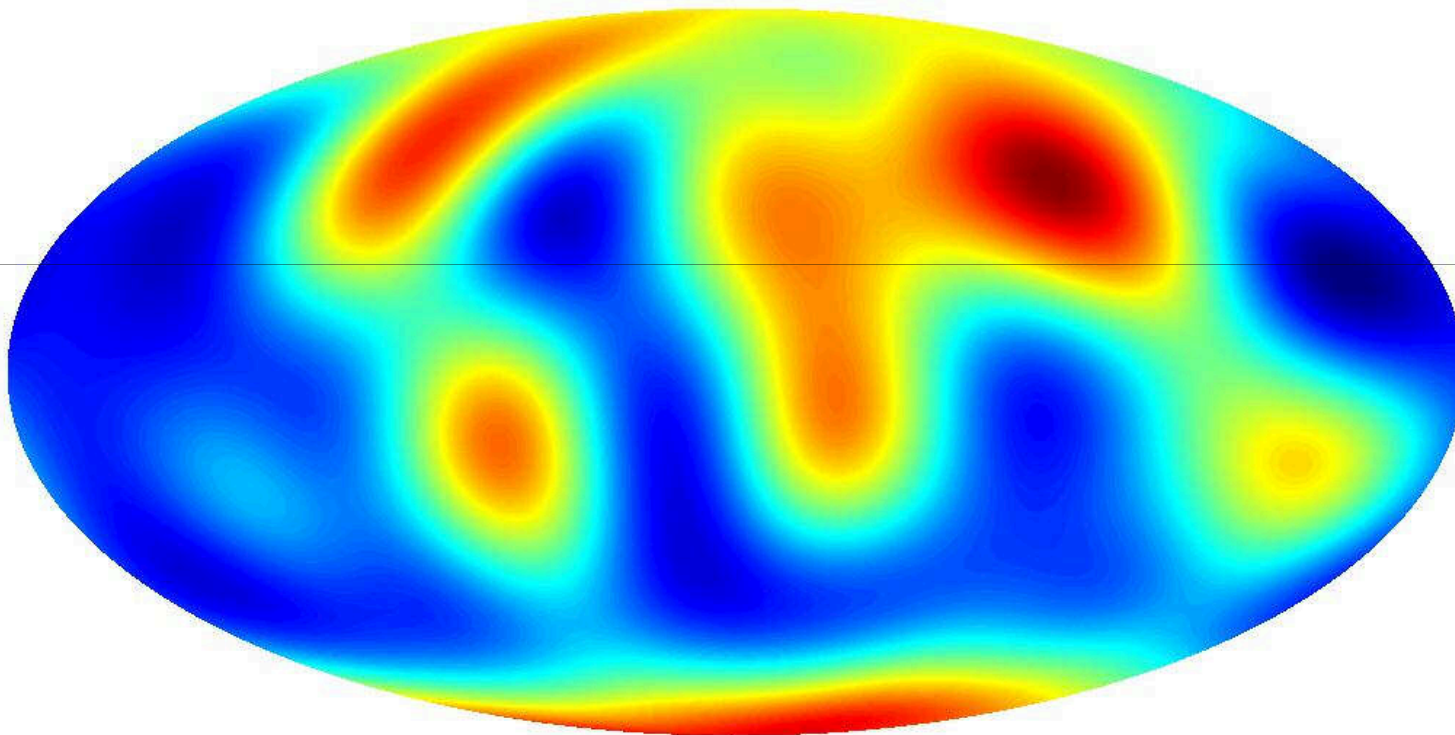
Credit: Bartelmann

Sum  $\ell=1$  to 7



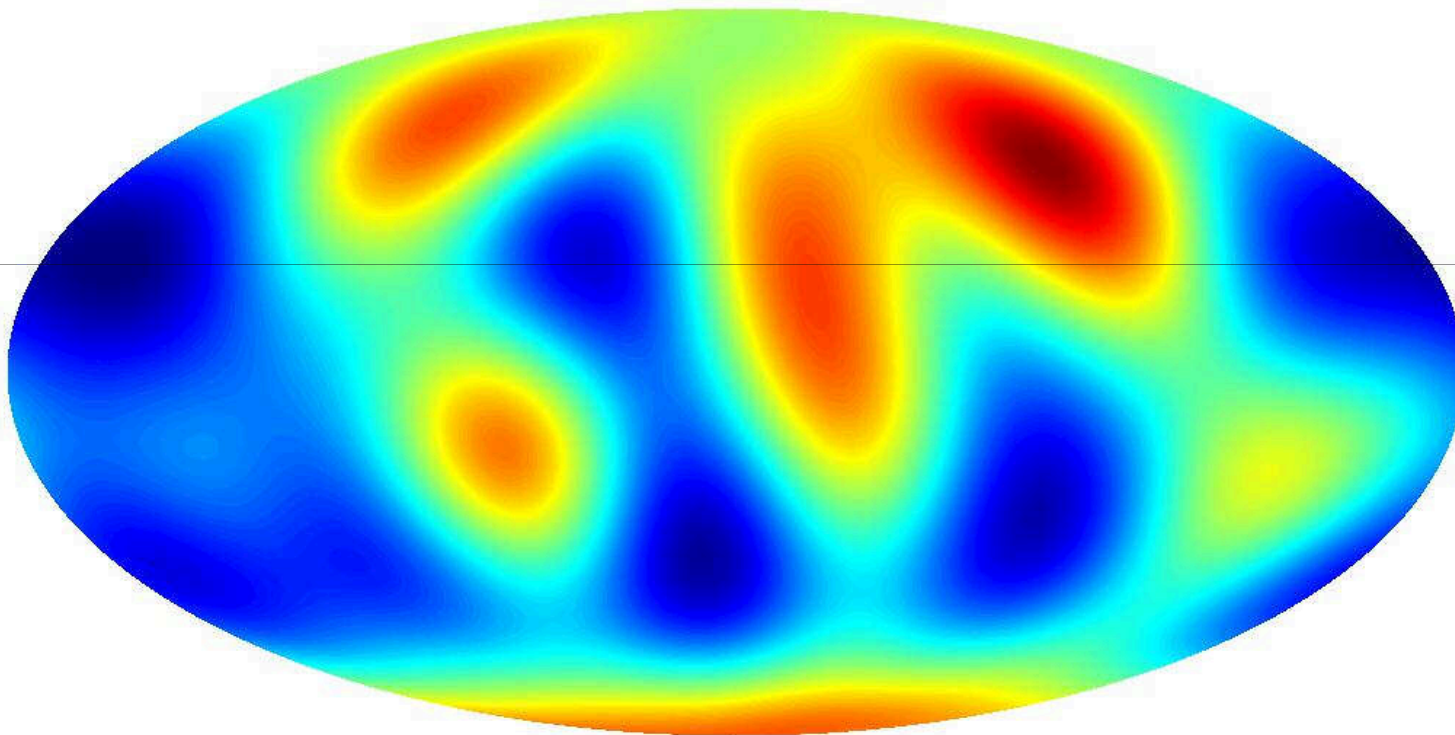
Made by Matthias Bartelmann

Sum  $\ell=1$  to 6



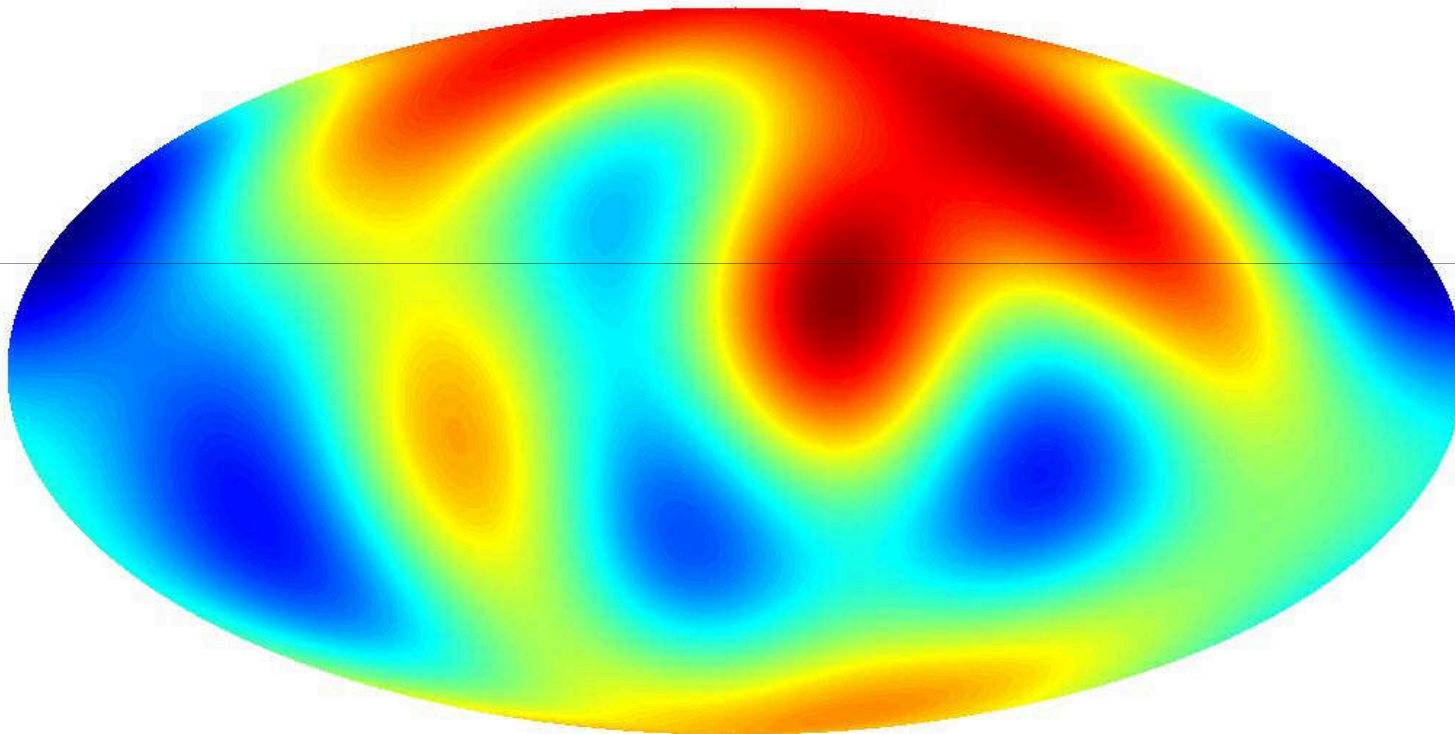
Made by Matthias Bartelmann

Sum  $\ell=1$  to 5



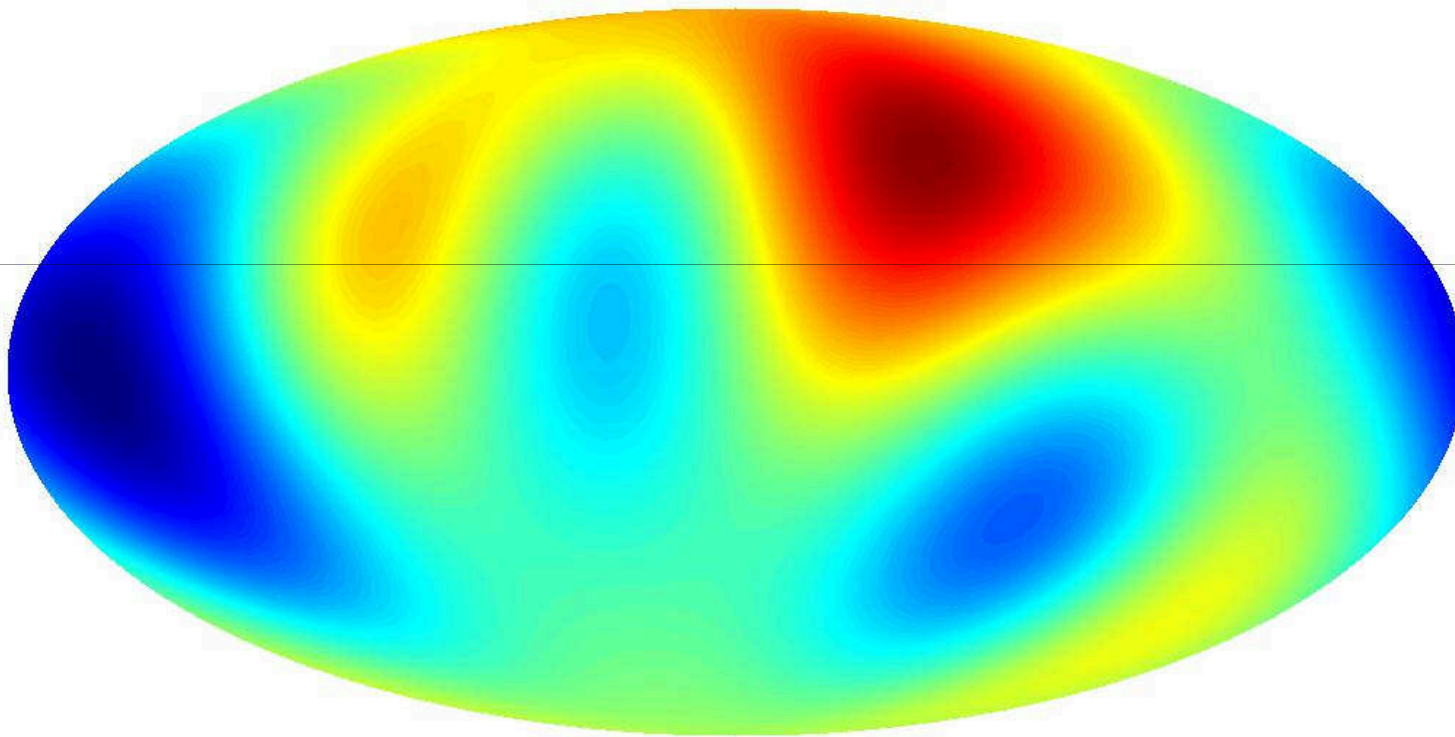
Made by Matthias Bartelmann

Sum  $\ell=1$  to 4



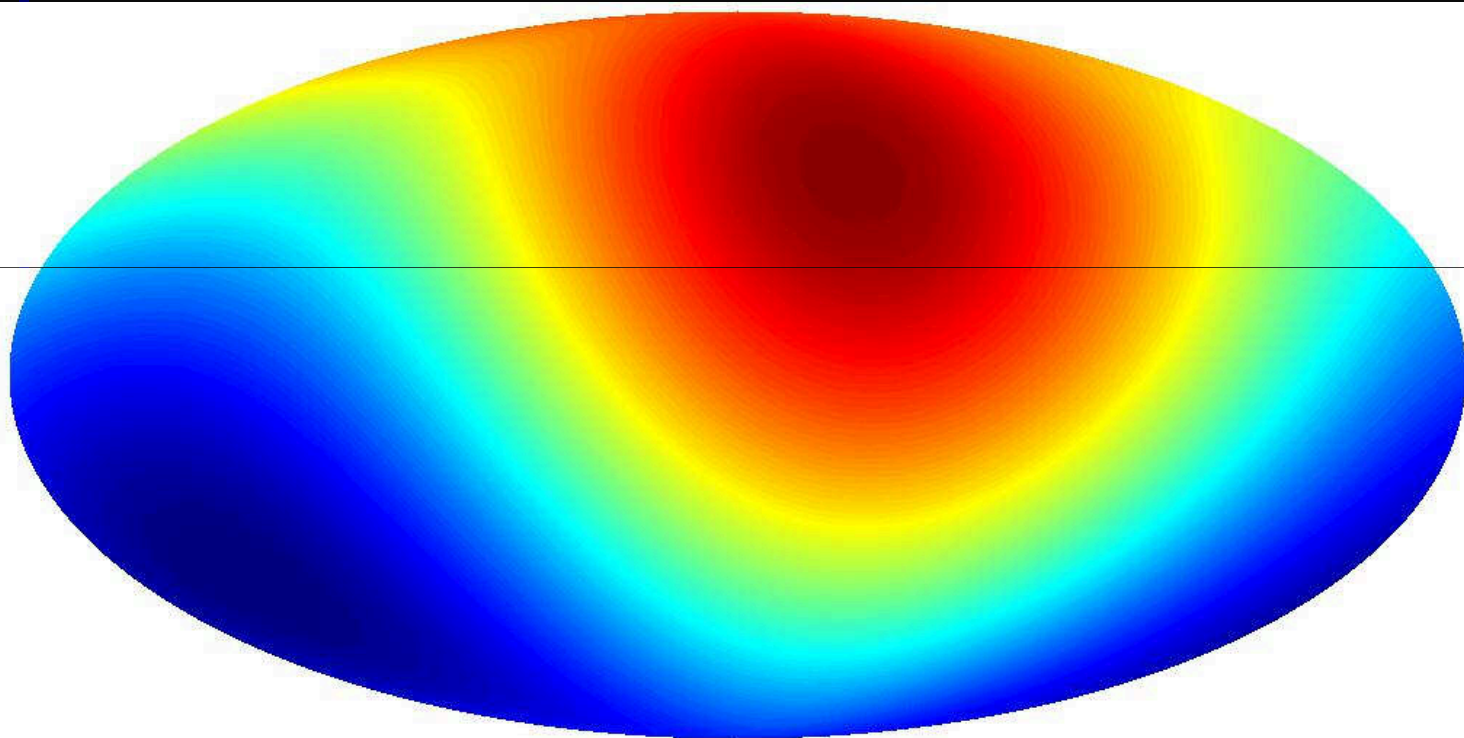
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$\ell=1$  plus  $\ell=2$  plus  $\ell=3$



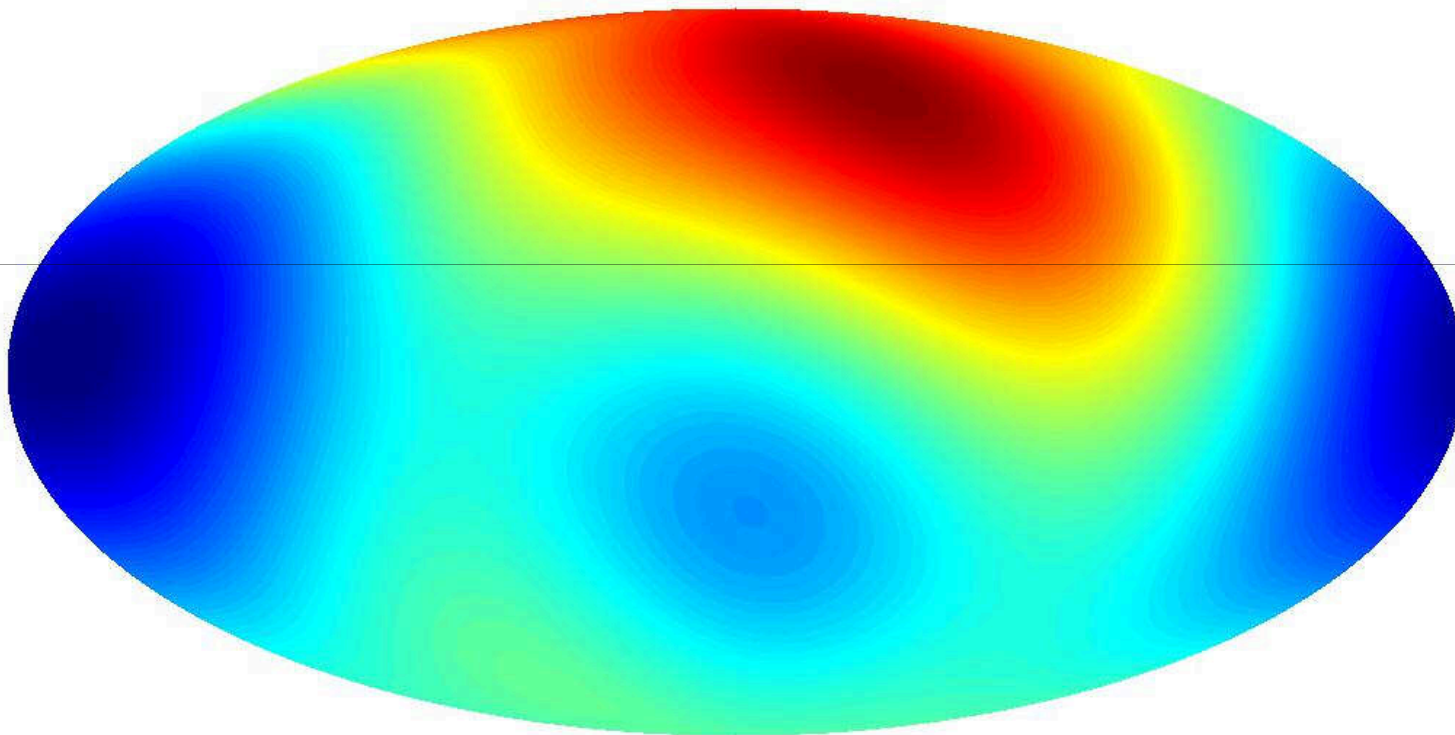
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Spherical harmonics:  
The spherical equivalent of sine waves  
 $\ell=1$



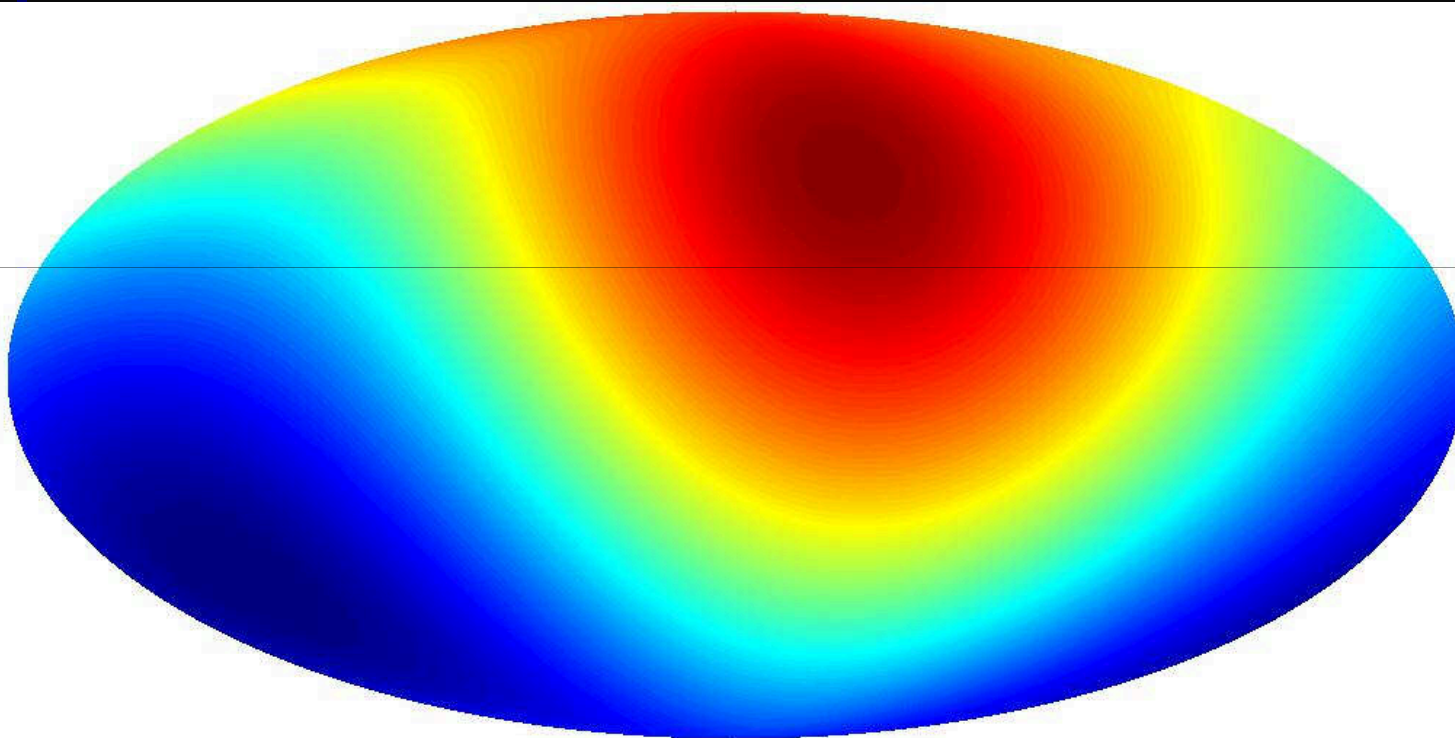
Made by Matthias Bartelmann

$\ell=1$  plus  $\ell=2$



Made by Matthias Bartelmann

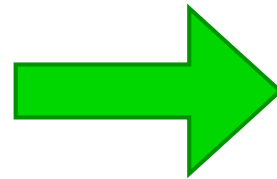
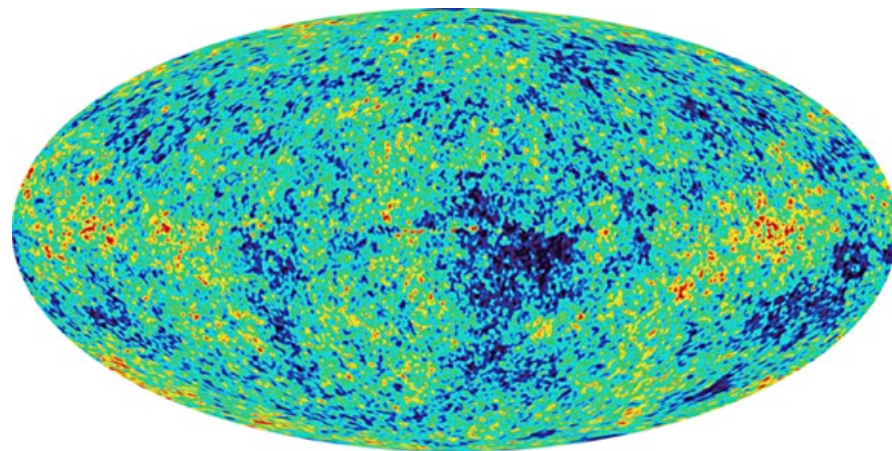
$\ell=1$



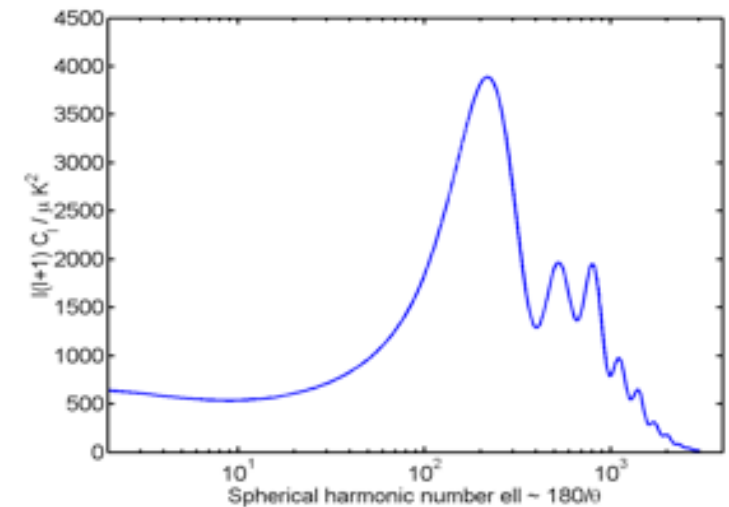
Made by Matthias Bartelmann

# Power Spectrum for CMB

- Use the spherical harmonic expansion to construct a power spectrum to describe anisotropies of the CMB on the sky



Power Spectrum



$$l = 180 / \theta$$

Expansion:

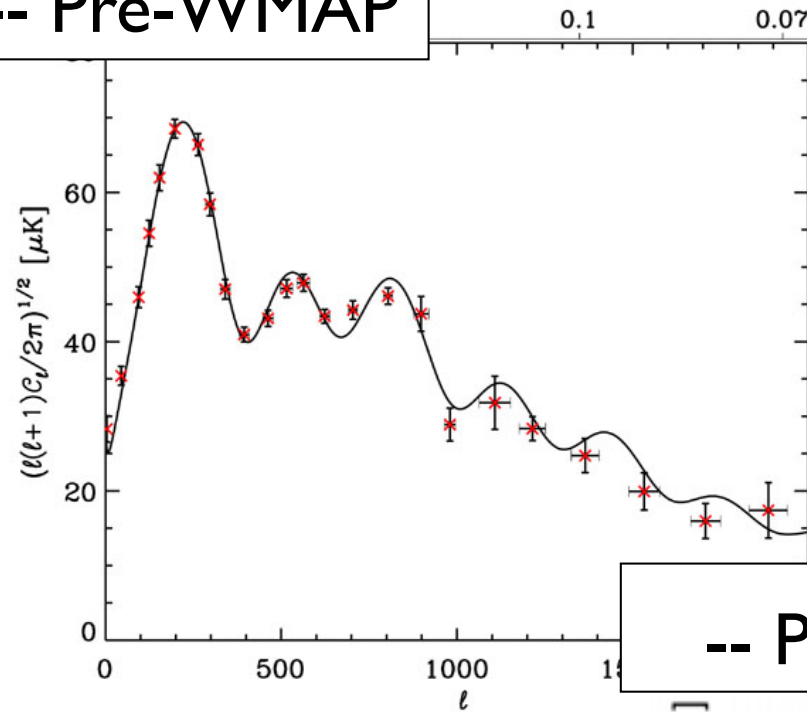
$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell}^m(\theta, \phi)$$

After deriving the  $a_{\ell m}$  coefficients from the data, determine the statistical average

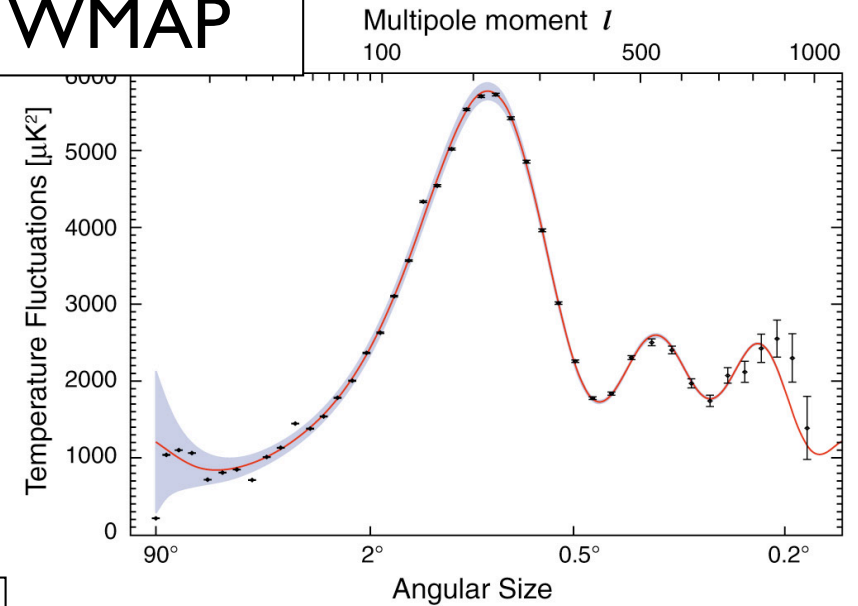
$$c_{\ell} = \langle |a_{\ell m}|^2 \rangle$$

# How do the new Planck results compare to earlier results!

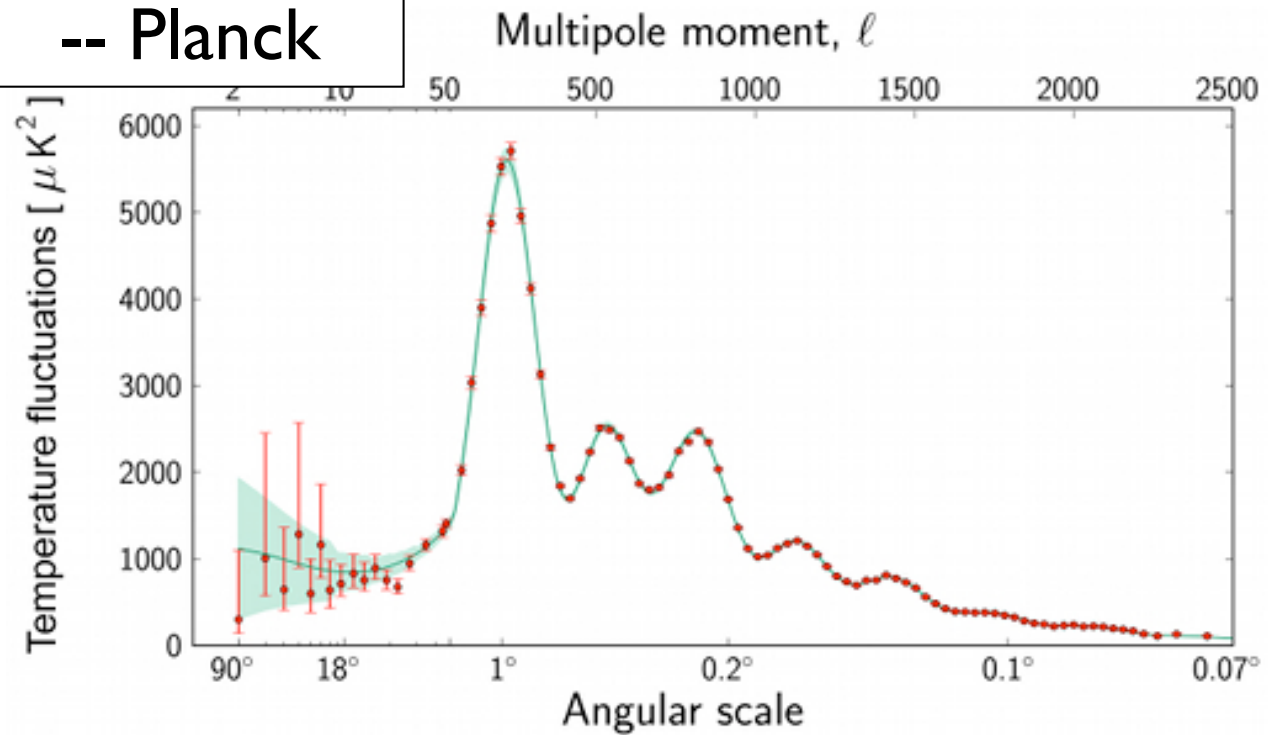
-- Pre-WMAP



-- WMAP



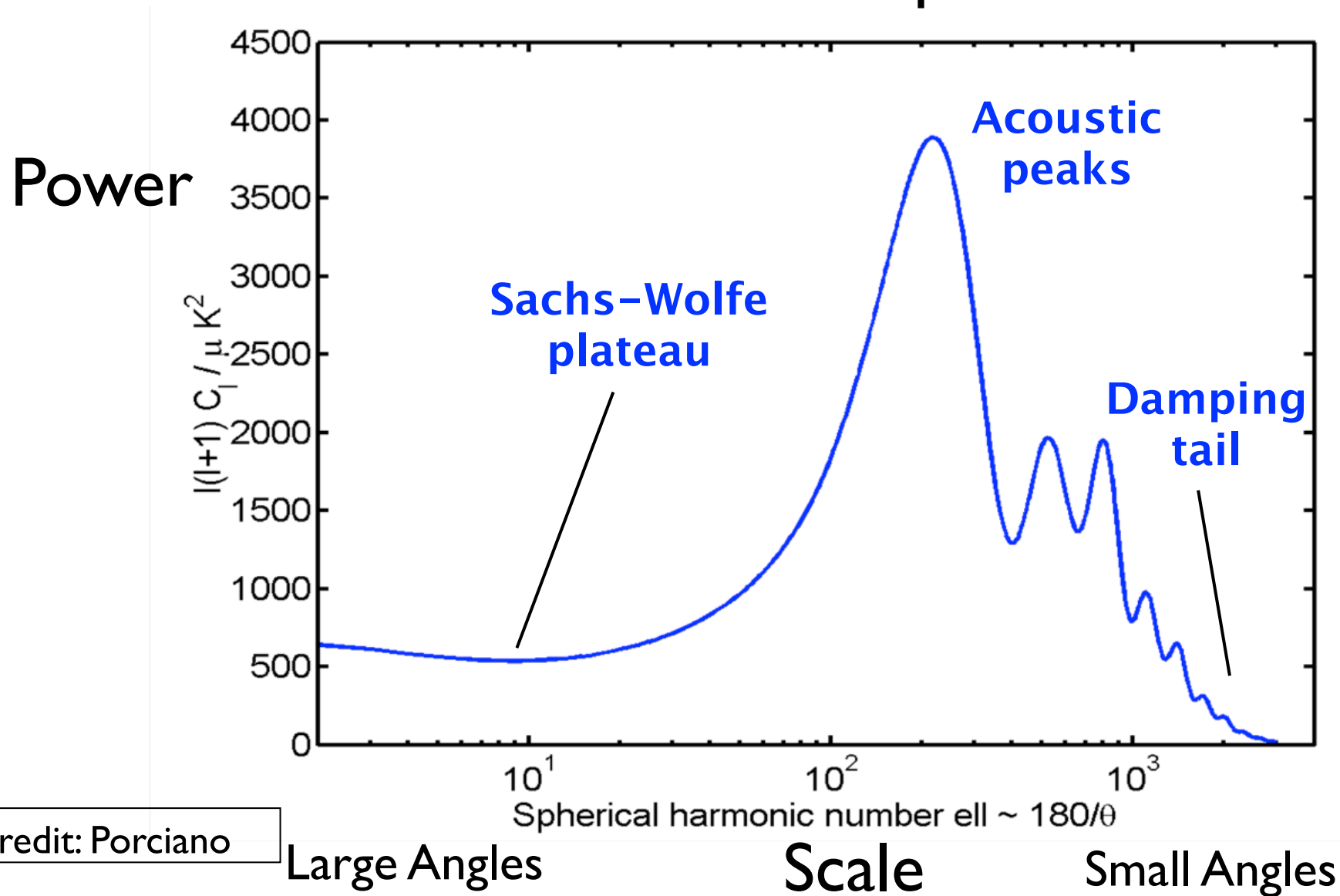
-- Planck



What can we learn from  
CMB power spectrum?

# What does the CMB power spectrum look like?

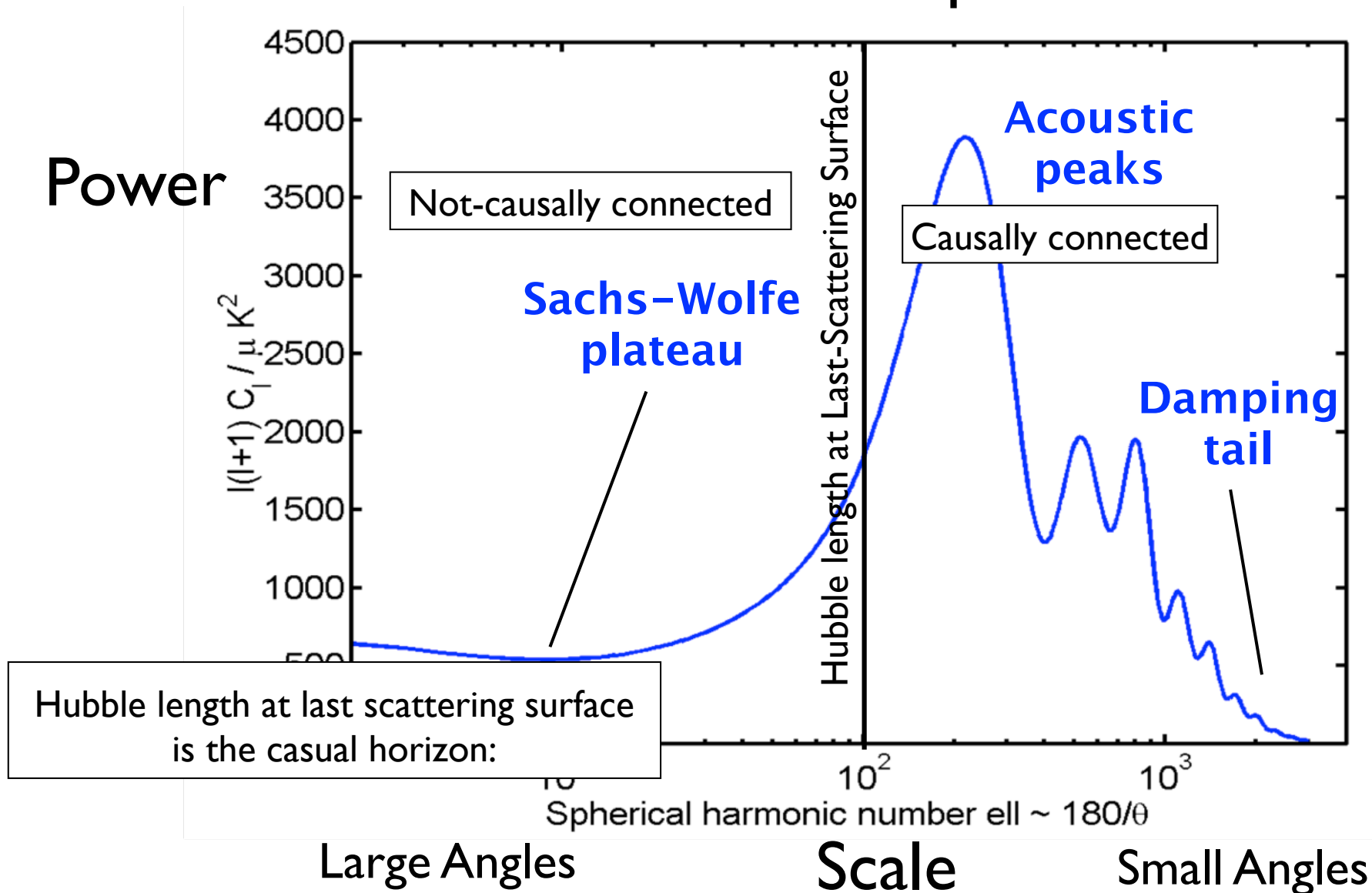
Here is such a spectrum:



First question: how large  
can the angle become  
before the regions become  
causally disconnected?

# What does the CMB power spectrum look like?

Here is such a spectrum:



# Sachs-Wolfe Plateau

How do we explain the power spectrum of the anisotropies that are not causally connected, i.e., beyond the horizon?

These fluctuations are thought to be quantum fluctuations that are blown up in an initial inflationary phase of the universe

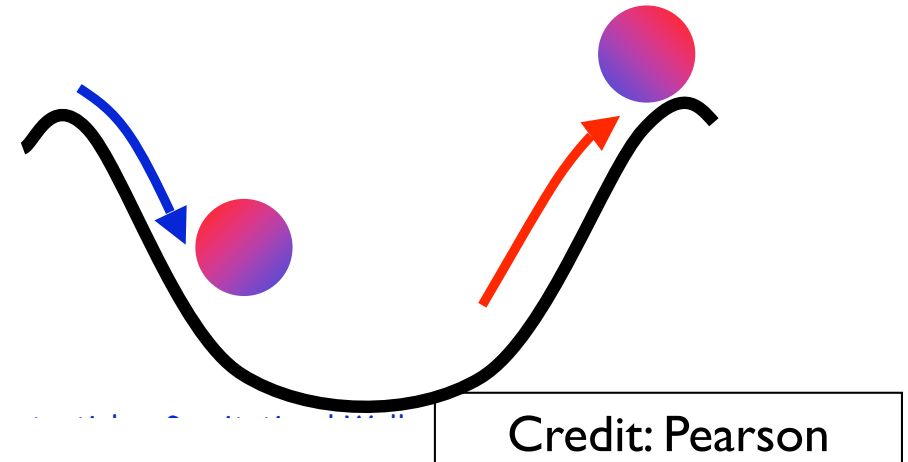
But how do these fluctuations translate into temperature fluctuations?

# Sachs-Wolfe Effect (1967)

$$\Delta\nu/\nu \sim \Delta T/T \sim \Phi/c^2$$

Additional effect of time dilation while potential evolves (full GR):

$$\frac{\Delta T}{T} \sim \frac{1}{3} \frac{\Delta\Phi}{c^2}$$



Photons climb out of potential minimum, lose energy  $\leftrightarrow$  lower temperature

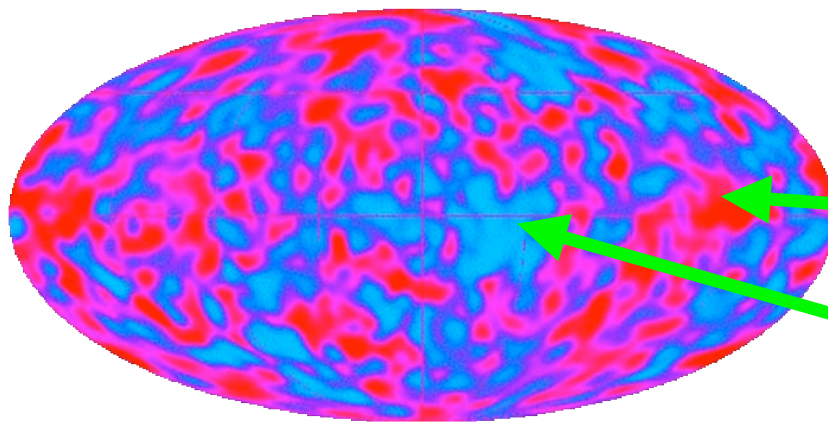
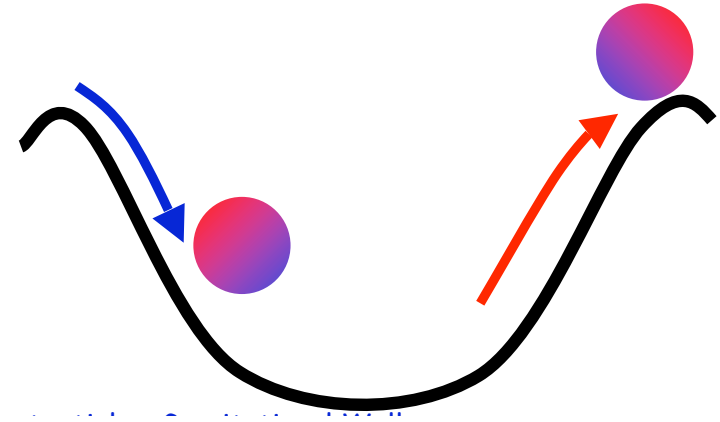
Photons fall out of potential maximum, gain energy  $\leftrightarrow$  higher temperature

# Sachs-Wolfe Effect (1967)

$$\Delta v/v \sim \Delta T/T \sim \Phi/c^2$$

Additional effect of time dilation while potential evolves (full GR):

$$\frac{\Delta T}{T} \sim \frac{1}{3} \frac{\Delta \Phi}{c^2}$$



red regions -- lower temperature (potential maxima)

blue regions -- higher temperature (potential minima)

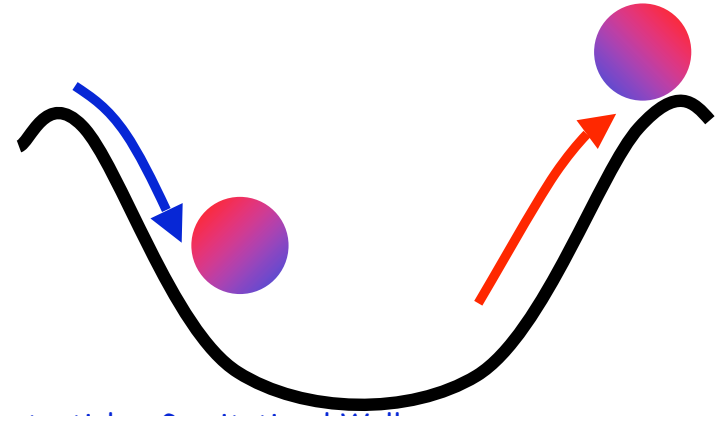
Credit: Pearson

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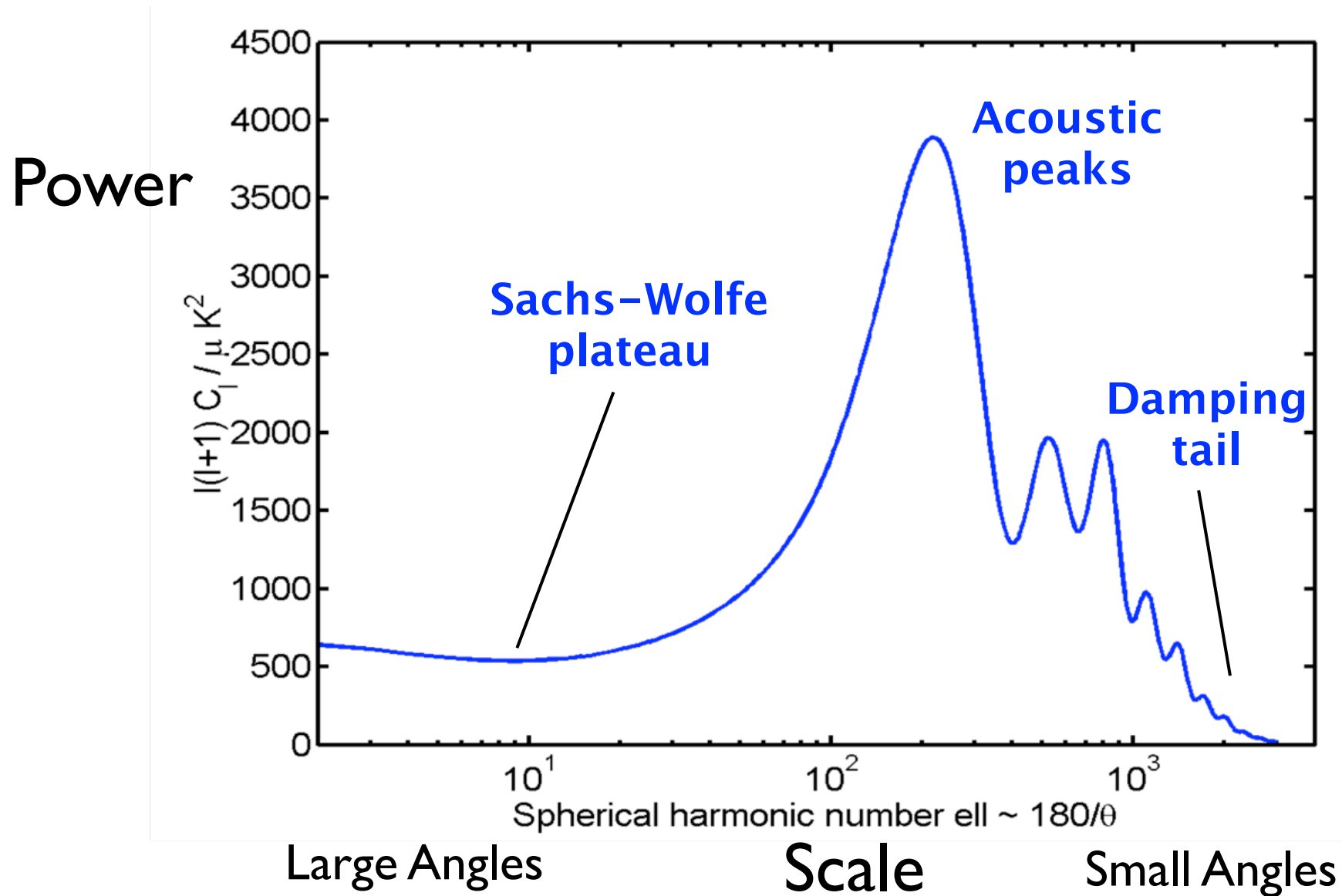
But what distribution of potential minima and maxima do we expect?

This comes from inflation

For a Harrison-Zeldovich power spectrum  $P(k) \propto k$  (expected from inflation), the CMB power spectrum is expected to be flat, i.e.,  $C_l \propto l/l(l+1)$

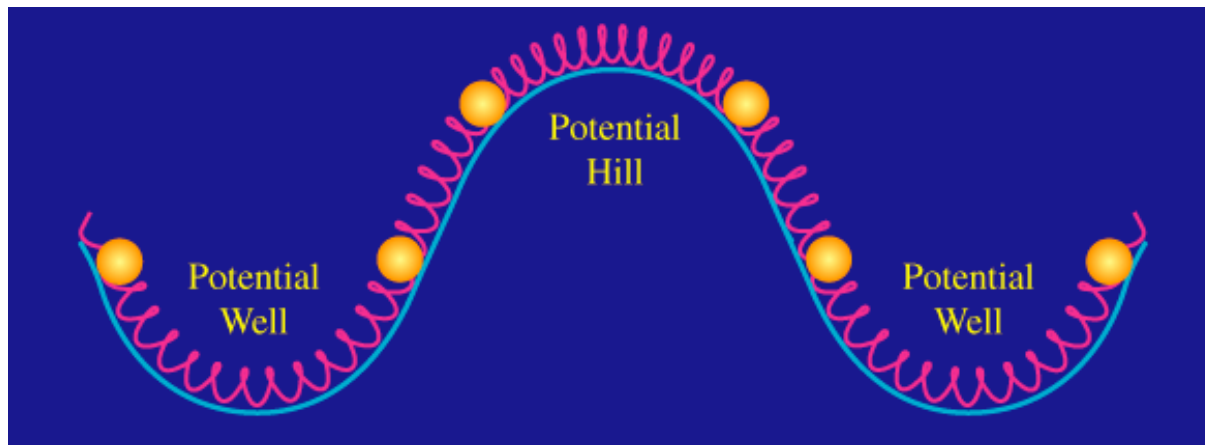
Second Topic: Now let's  
discuss the acoustic peaks in  
the CMB power spectrum

# What about the Acoustic Peaks?



# Acoustic Oscillations:

- Universe filled with slight dark matter overdensities on all scales
- Baryons will fall onto these overdensities due to the force of gravity heating the fluid up
- Large number of baryons falling onto overdensity causes an increase in pressure due to baryon-photon coupling -- which resists gravitational forces and causes it to expand (cooling the fluid down)
- An oscillation is set up and continues until decoupling

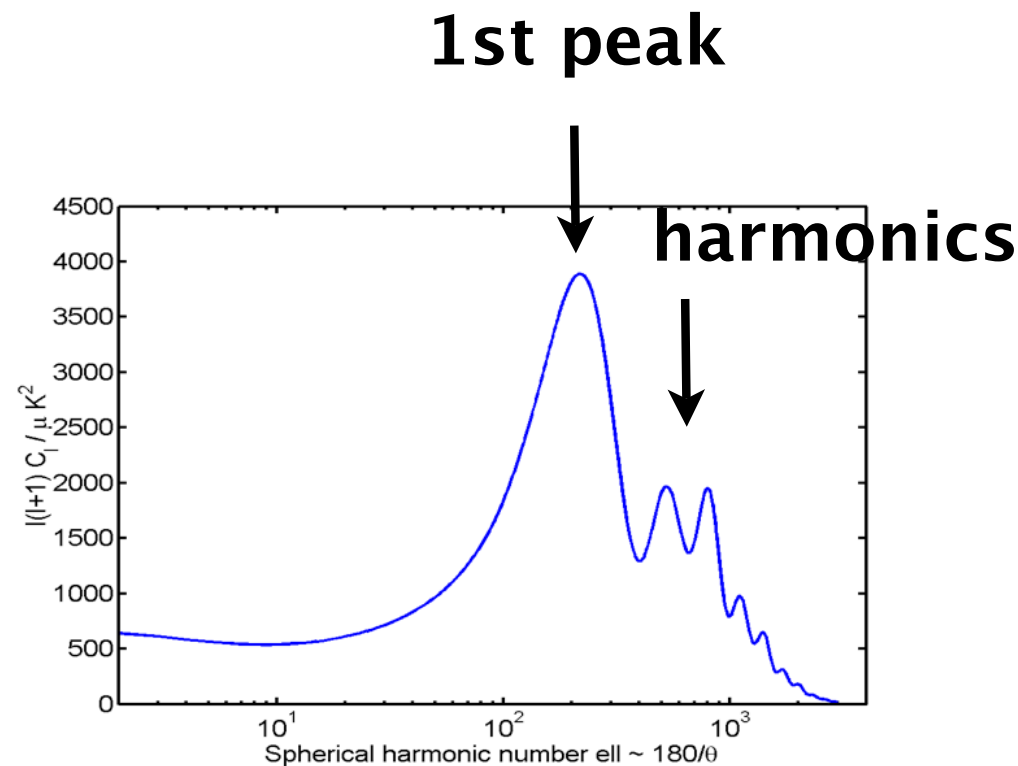


-- credit: Wayne Hu

# Acoustic Oscillations:

- First peak is a compression mode
- Second peak is a rarefaction mode
- Third peak is a compression mode

(Similar to harmonics on a musical instrument/string/pipe!)



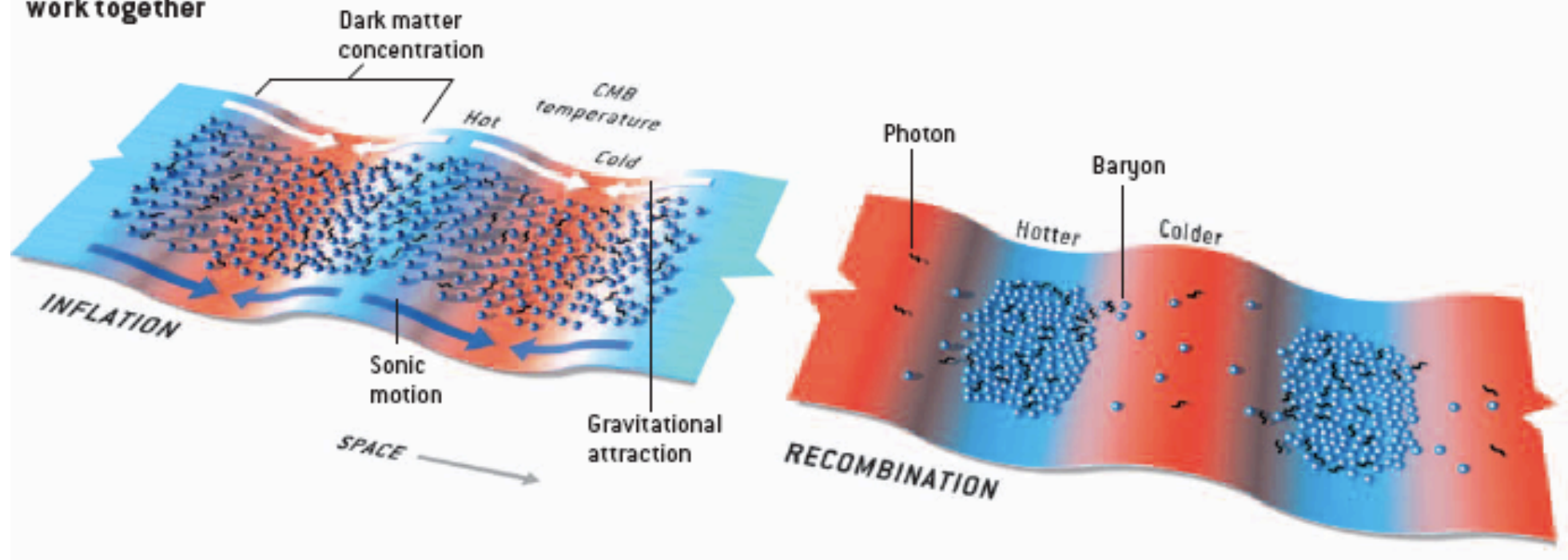
# First Peak: Illustration

INFLUENCE OF DARK MATTER modulates the acoustic signals in the CMB. After inflation, denser regions of dark matter that have the same scale as the fundamental wave (*represented as troughs in this potential-energy diagram*) pull in baryons and photons by gravitational attraction. (The troughs are shown in

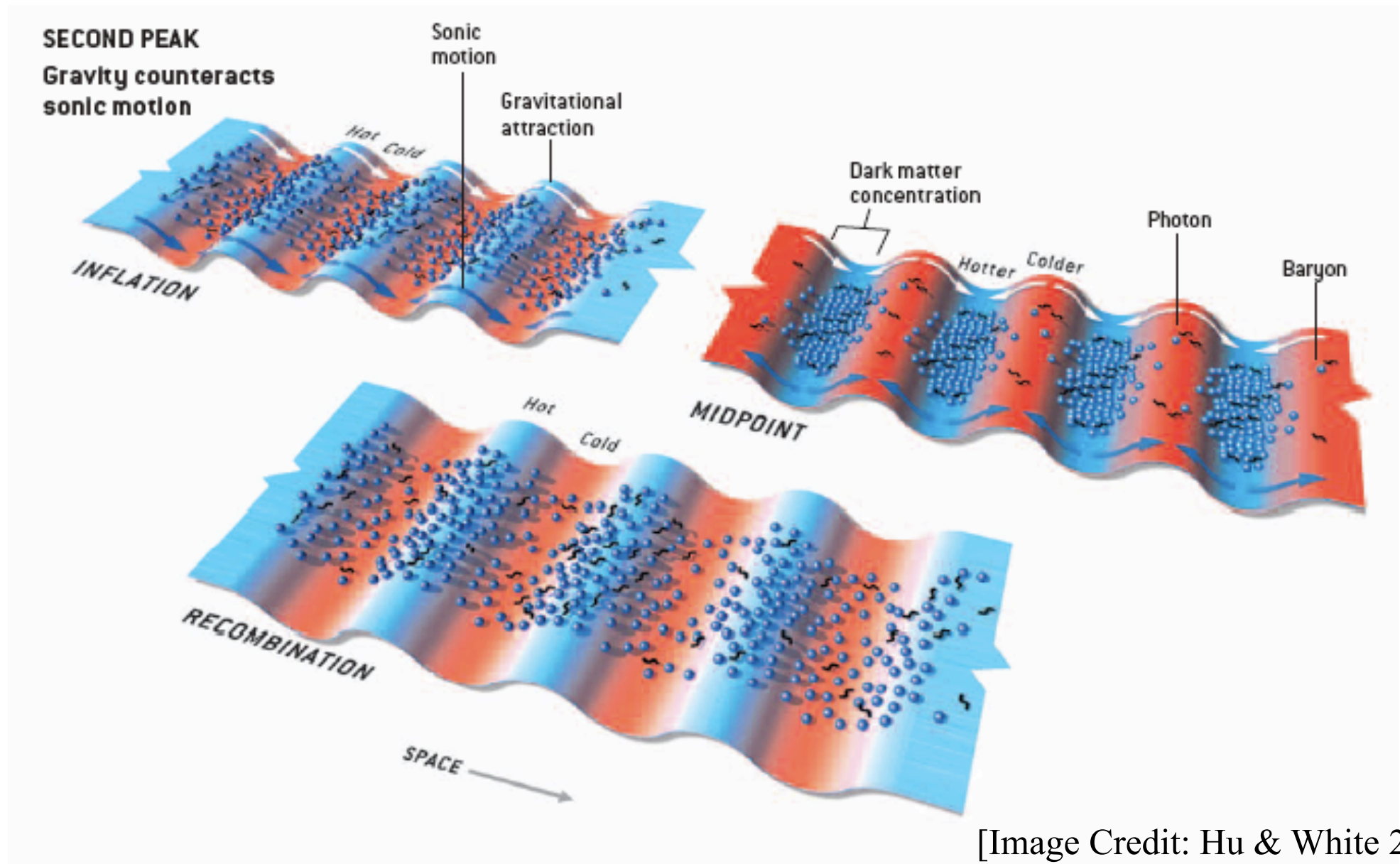
red because gravity also reduces the temperature of any escaping photons.) By the time of recombination, about 380,000 years later, gravity and sonic motion have worked together to raise the radiation temperature in the troughs (*blue*) and lower the temperature at the peaks (*red*).

## FIRST PEAK

Gravity and sonic motion work together



# Second Peak: Illustration

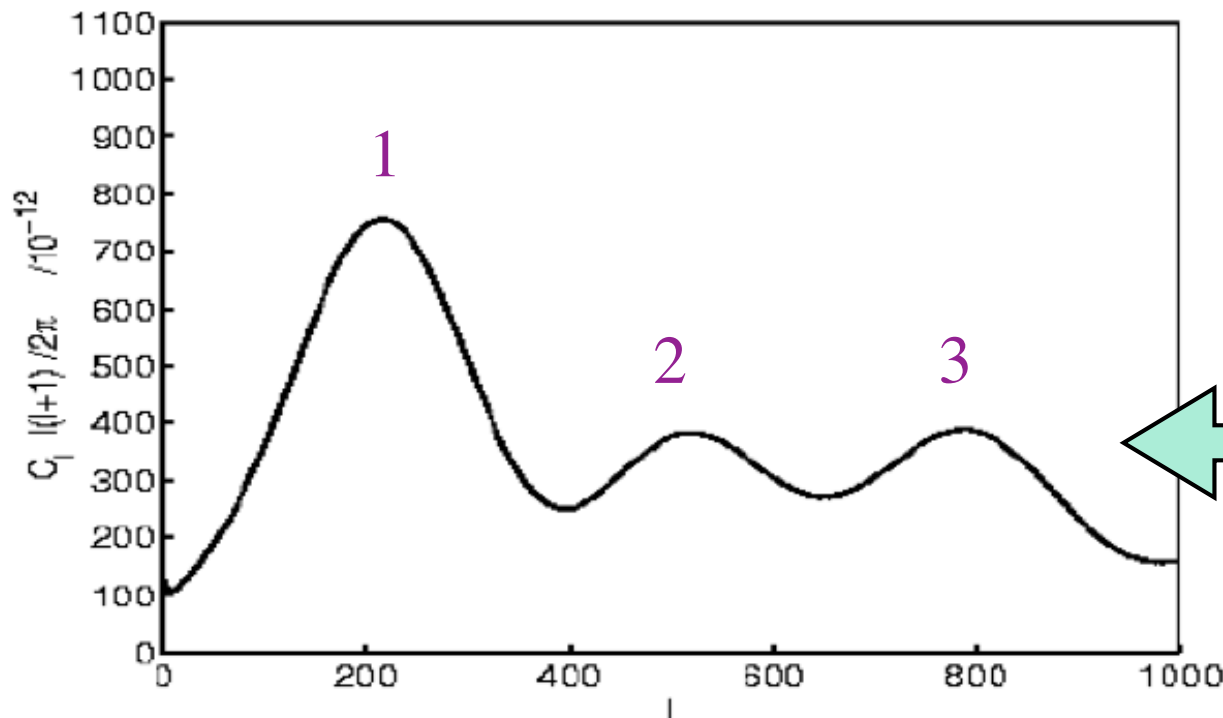


[Image Credit: Hu & White 2004]

# Acoustic Oscillations:

- First peak is a compression mode
- Second peak is a rarefaction mode
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(Similar to harmonics on a musical instrument/string/pipe!)



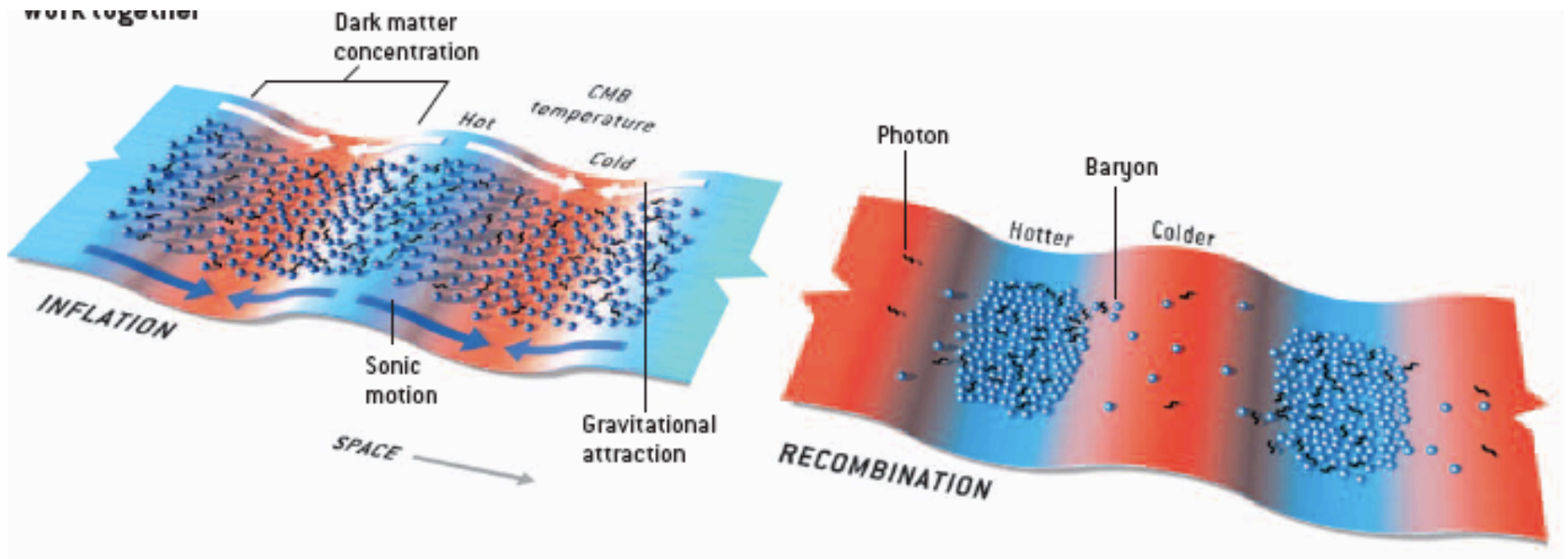
Peaks are spaced approximately equally in spherical harmonic number  $l$

What can we learn from the  
properties of these acoustic  
peaks?

# Let's examine acoustic peak #1

(What can we learn from the angular scale at which is observed?)  
(they give us standard rods to measure geometry of universe)

-- For this peak, baryonic matter would be falling onto this pattern of overdensities for the first time



# Let's examine acoustic peak #1

(What can we learn from the angular scale at which is observed?)  
(they give us standard rods to measure geometry of universe)

- For this peak, baryonic matter would be falling onto these overdensities for the first time
- Length scale spanned by peak is comoving length transversed by a sound wave to the point of last scattering:

$$L_S(t_R) = R(t_R) \int_0^{t_R} \frac{c_S dt}{R(t)} \quad \text{where} \quad \text{sound speed} \quad c_S \approx \frac{c}{\sqrt{3}}$$
$$\approx 110 \left( \frac{0.7}{h} \right) \left( \frac{0.3}{\Omega_M} \right)^{1/2} \text{ kpc}$$

- This length scale acts as a standard rod

# Let's examine acoustic peak #1

(What can we learn from the angular scale at which is observed?)

-- For this peak, baryonic matter would be falling onto these overdensities for the first time

-- Key Question: What is the angle of the peak on the sky?

$$\theta = \frac{L_S(z)}{D_A(z)}$$

length scale traversed by matter

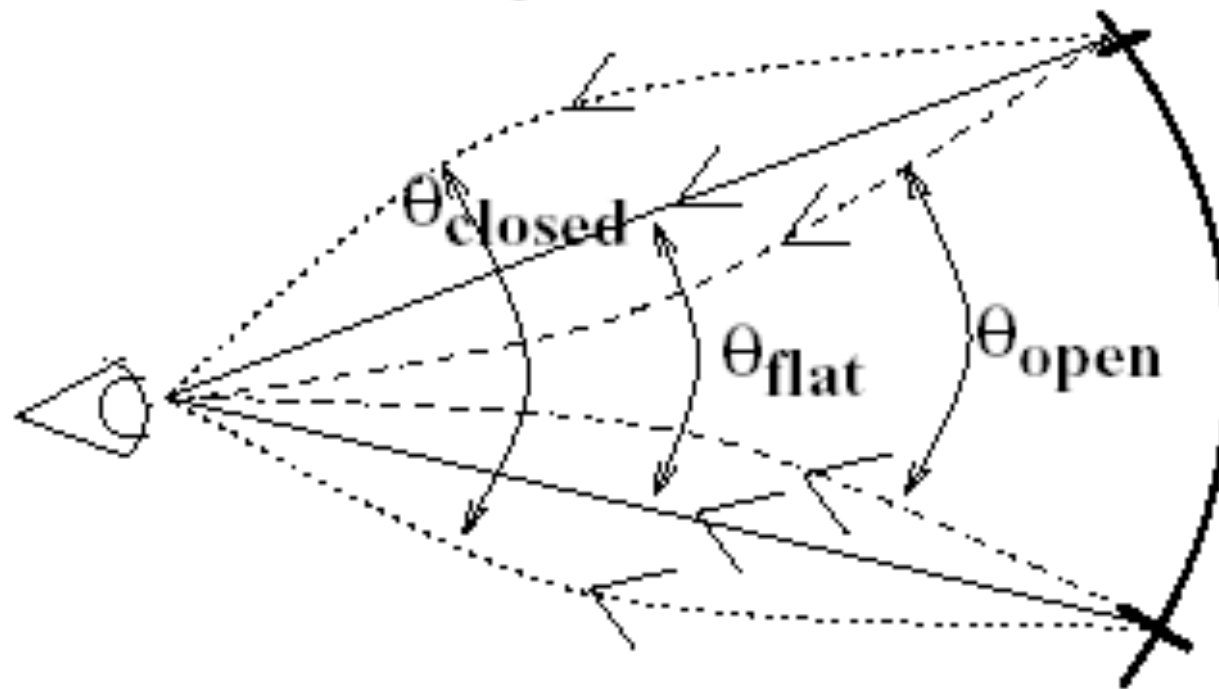
angular diameter distance

-- Can compute  $L_S(z)$  and can measure  $\theta$

-- Can solve for  $D_A(z)$  and use to constrain geometry of universe

# How does the angular diameter distance depend on the cosmological parameters?

## Geometry of the Universe

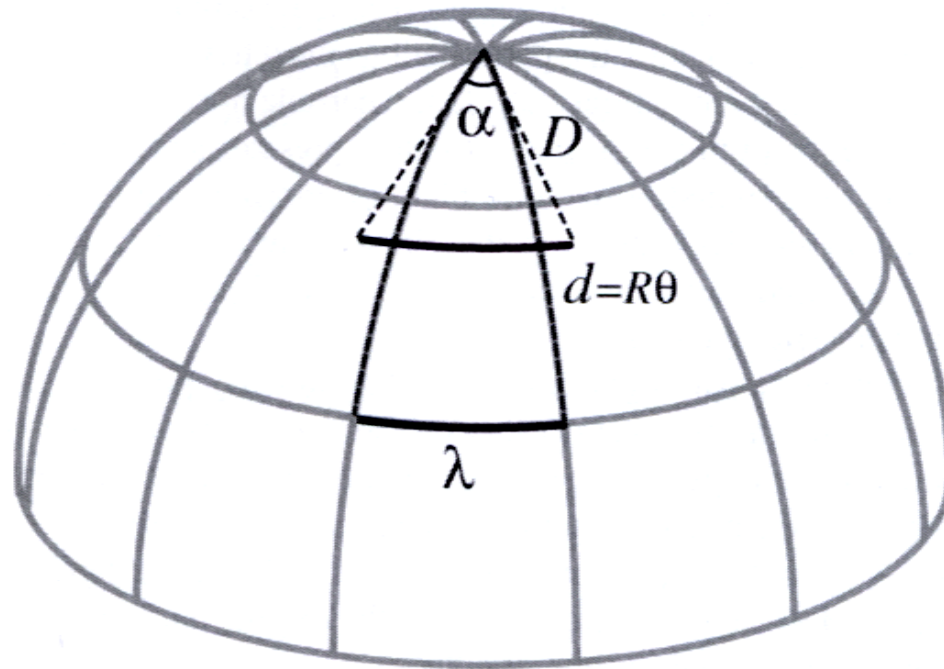


Fixed Distance  
Traversed by  
Baryons in First  
Acoustic Peak

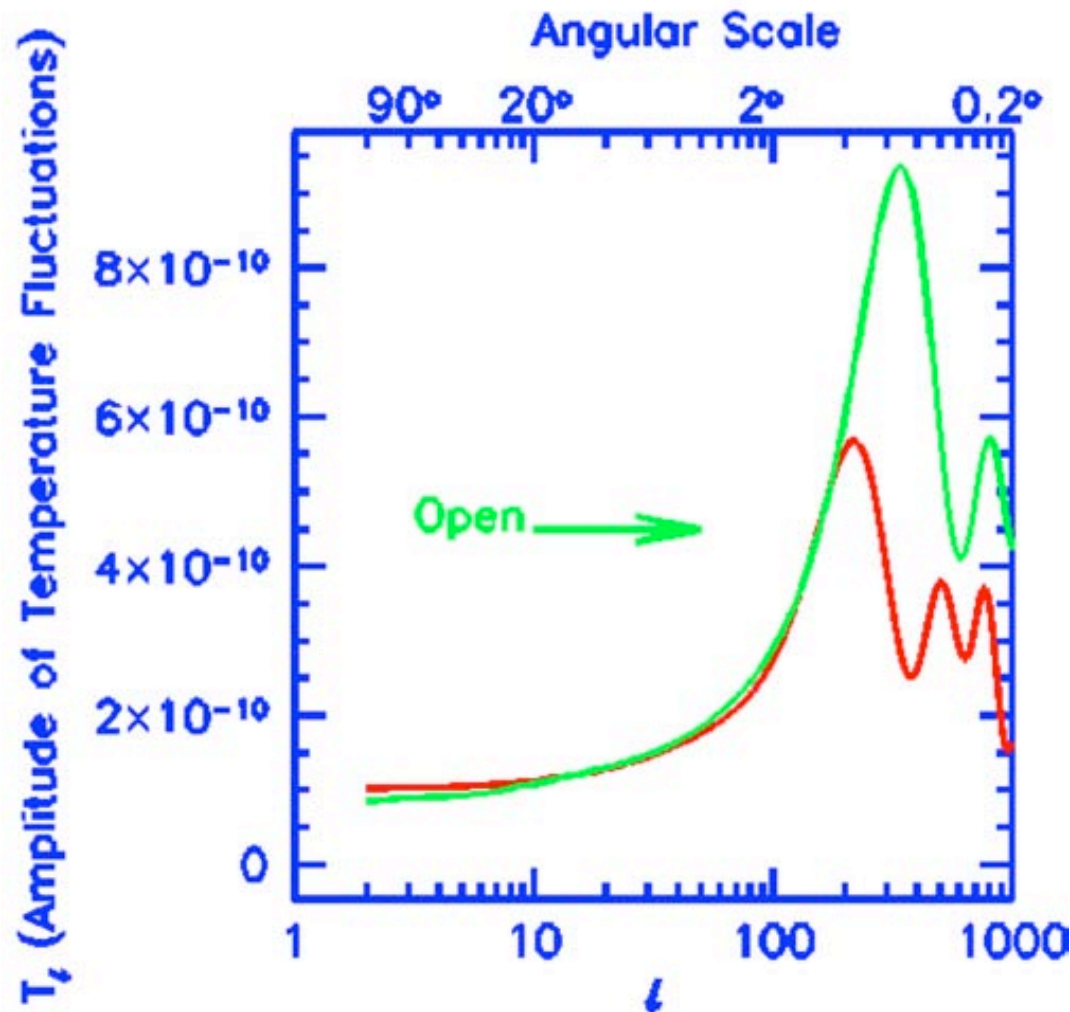
$$\theta_{\text{closed}} > \theta_{\text{flat}} > \theta_{\text{open}}$$

# How does the angular diameter distance depend on the cosmological parameters?

For example, in closed universe, objects subtend larger angle than they would in flat spacetime.

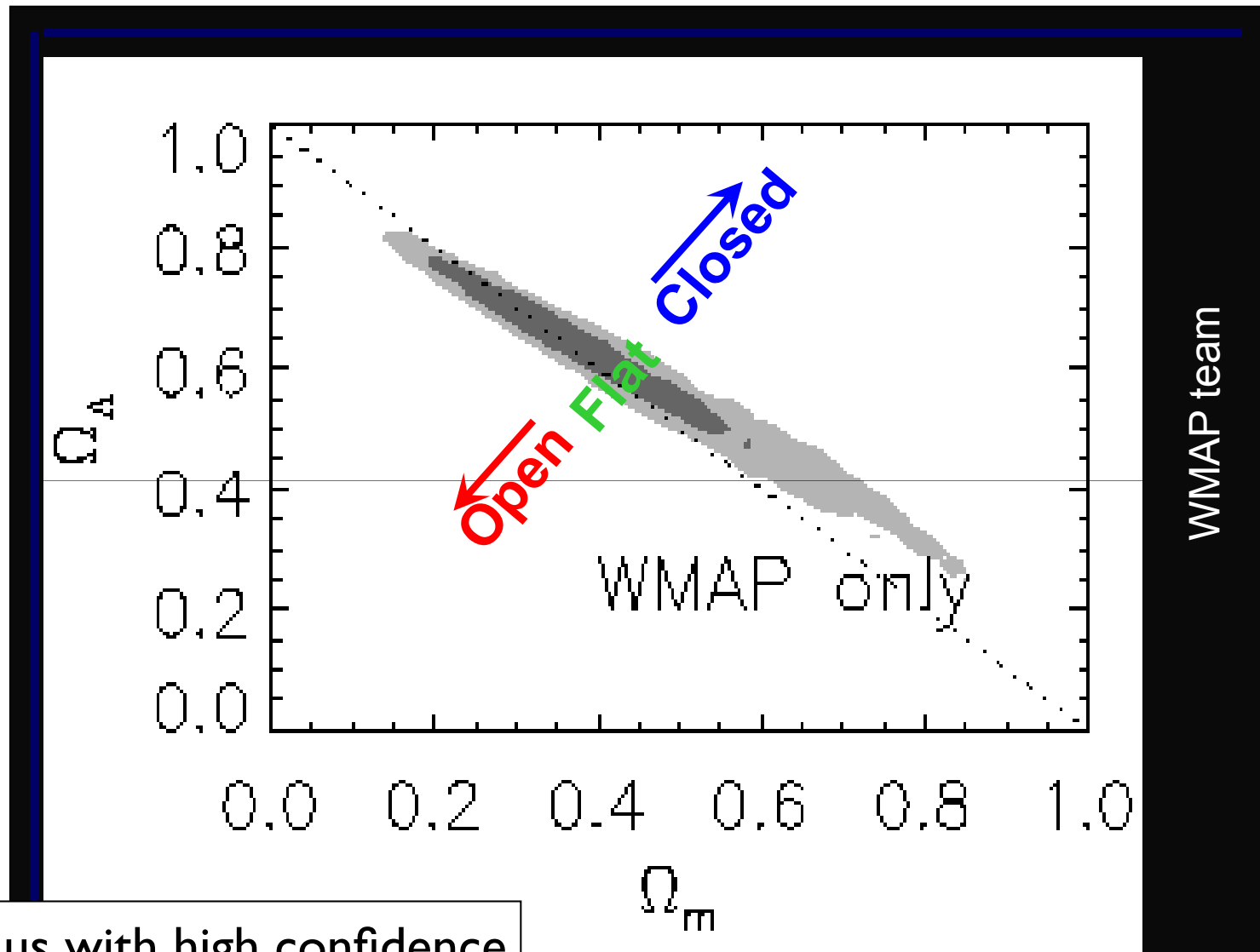


# How does the angular diameter distance depend on the cosmological parameters?



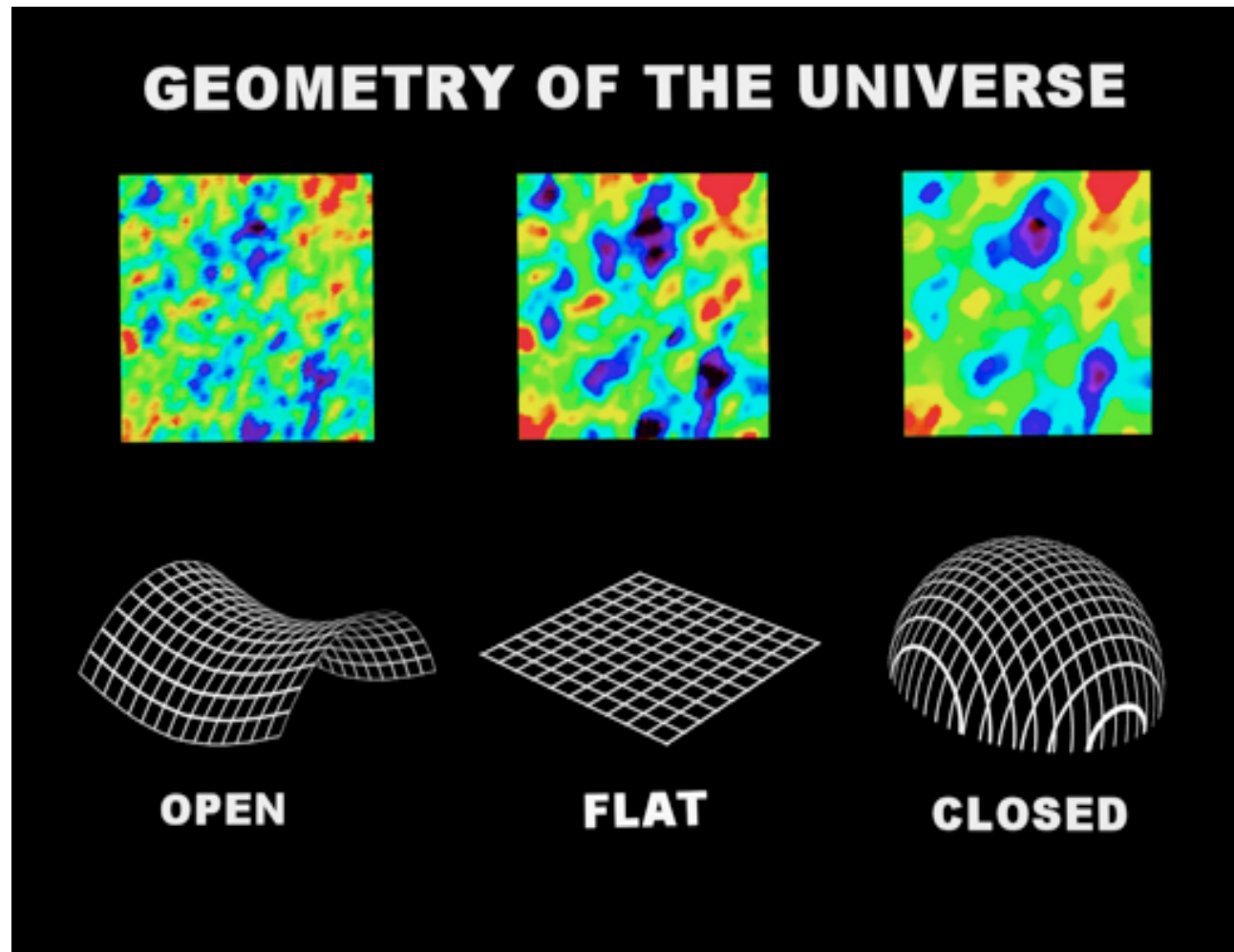
Here is how the peak would shift in an open universe (green) and a flat universe (red)

What does the position of the first acoustic peak teach about  $\Omega_M$  and  $\Omega_\Lambda$ ?



It shows us with high confidence that universe is likely flat

How might we expect the CMB anisotropies to look like on the sky for different geometries?

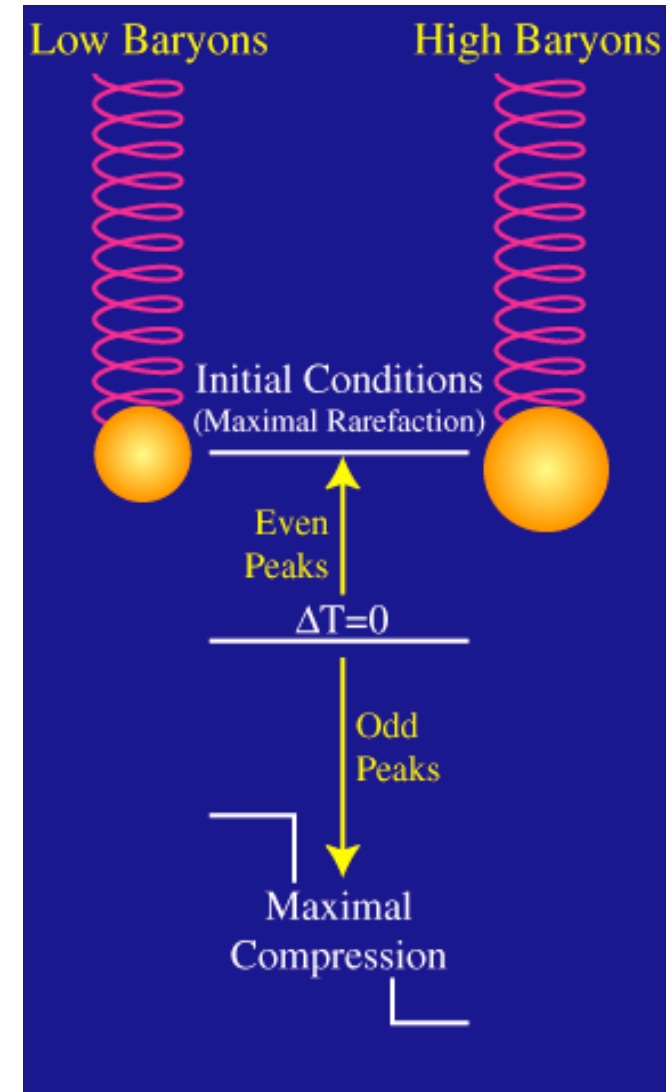


What can we learn from the  
other peaks?

# What can we learn from the other peaks?

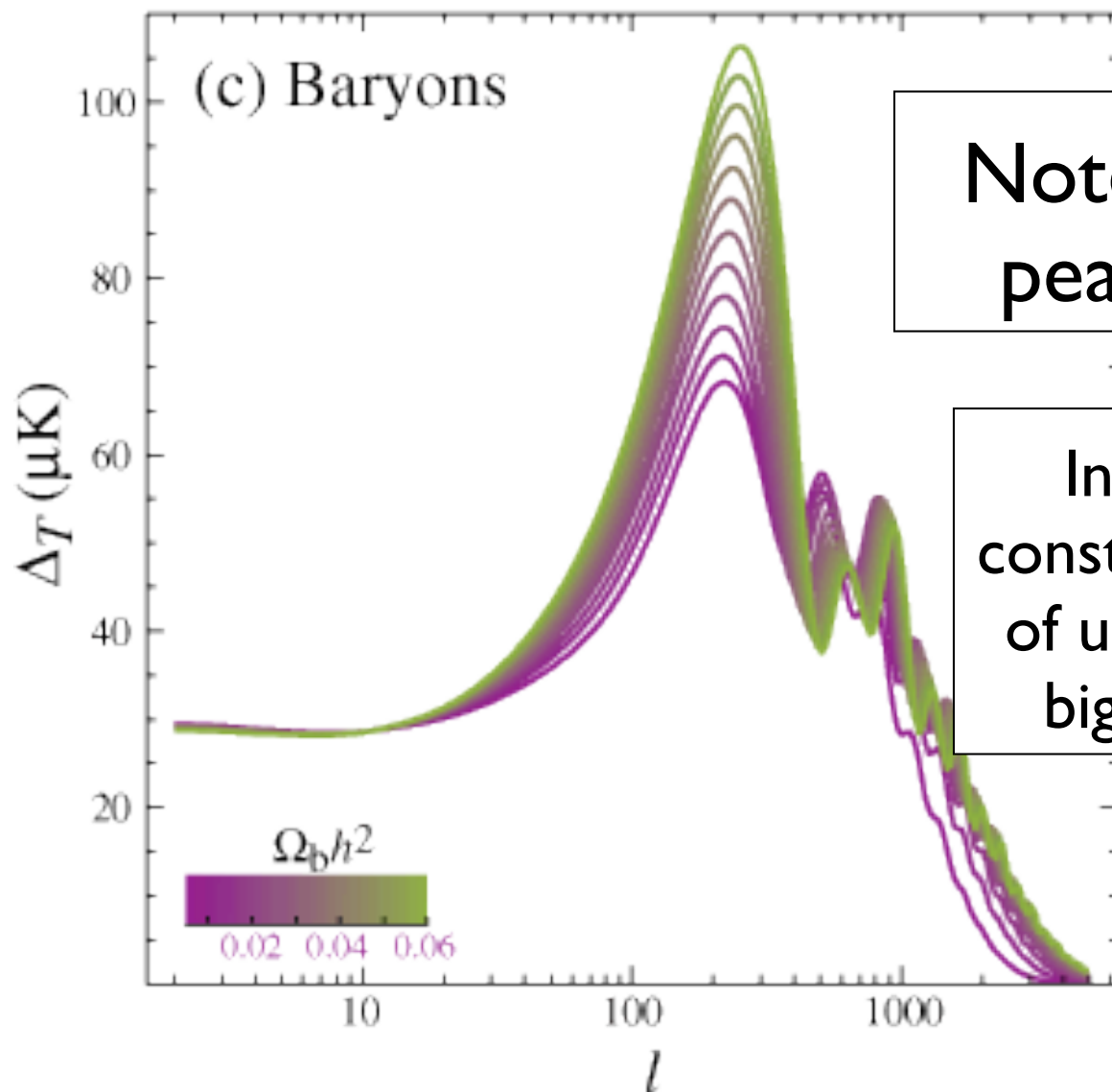
## Learn about baryon content

- The presence of more baryons increases the amplitude of the oscillations
- As a result, the fluid is compressed more before photon pressure can resist the compression
- This results in an asymmetry between the even and odd peaks



# What can we learn from the other peaks?

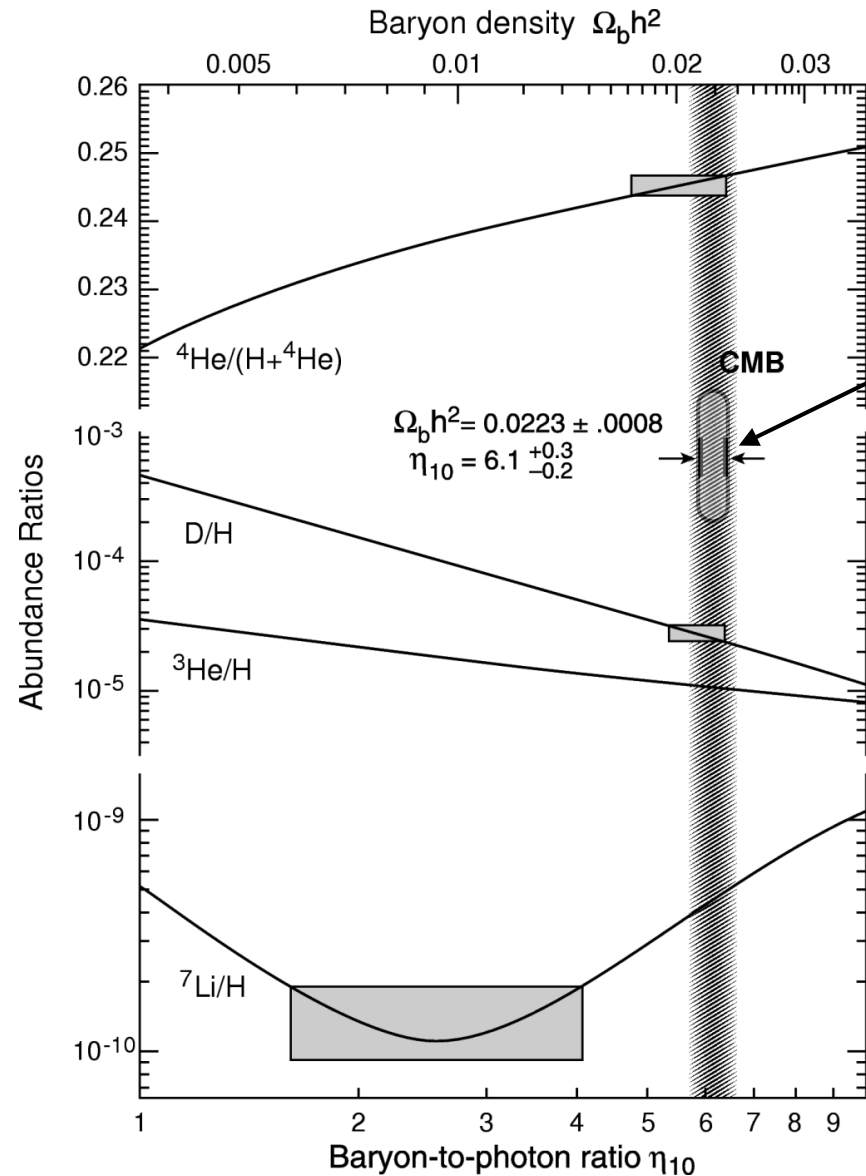
Learn about baryon content



Note how 1st and 3rd peaks are enhanced!

In fact, this provides best constraint on baryonic content of universe (even better than big-bang nucleosynthesis)

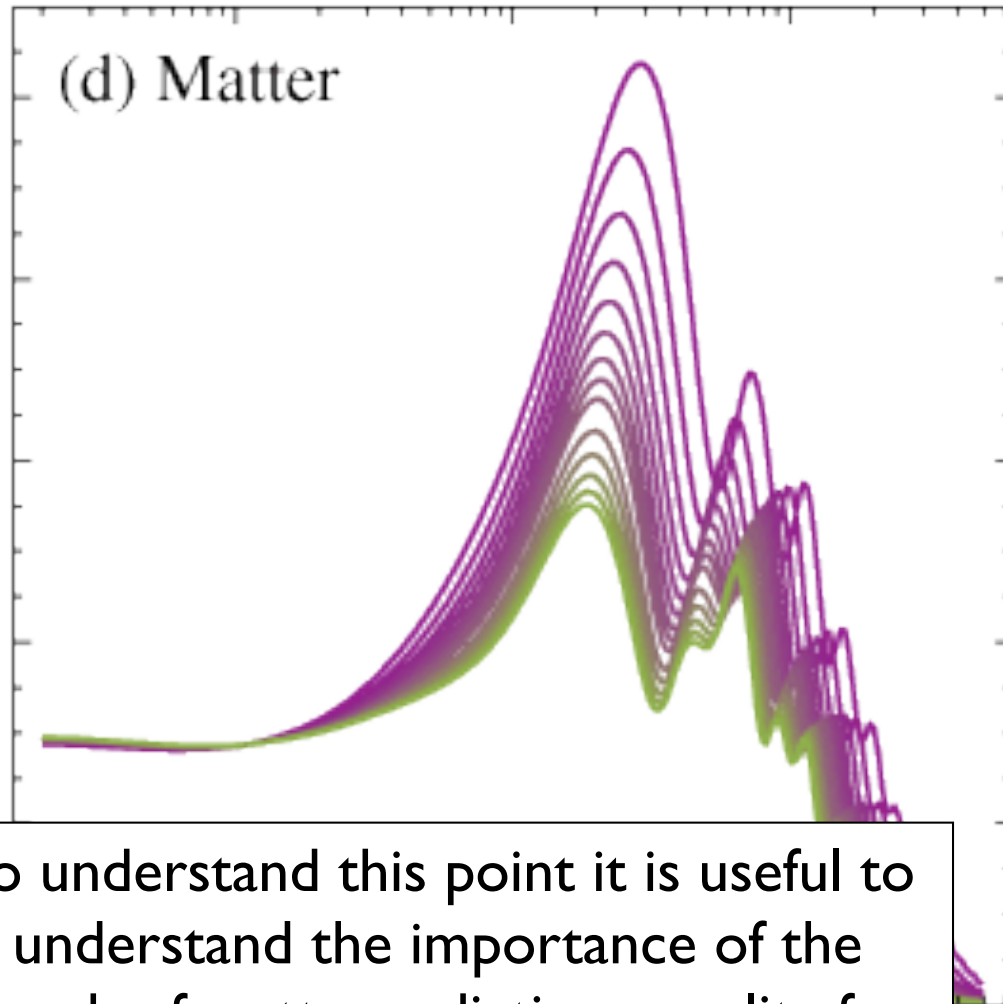
# How do constraints on $\Omega_{\text{baryon}}$ from the CMB compare with Big Bang Nucleosynthesis?



Constraints on  $\Omega_{\text{baryon}}$  from CMB in perfect agreement with Big Bang Nucleosynthesis

# What can we learn from the other peaks?

## Learn about dark matter content



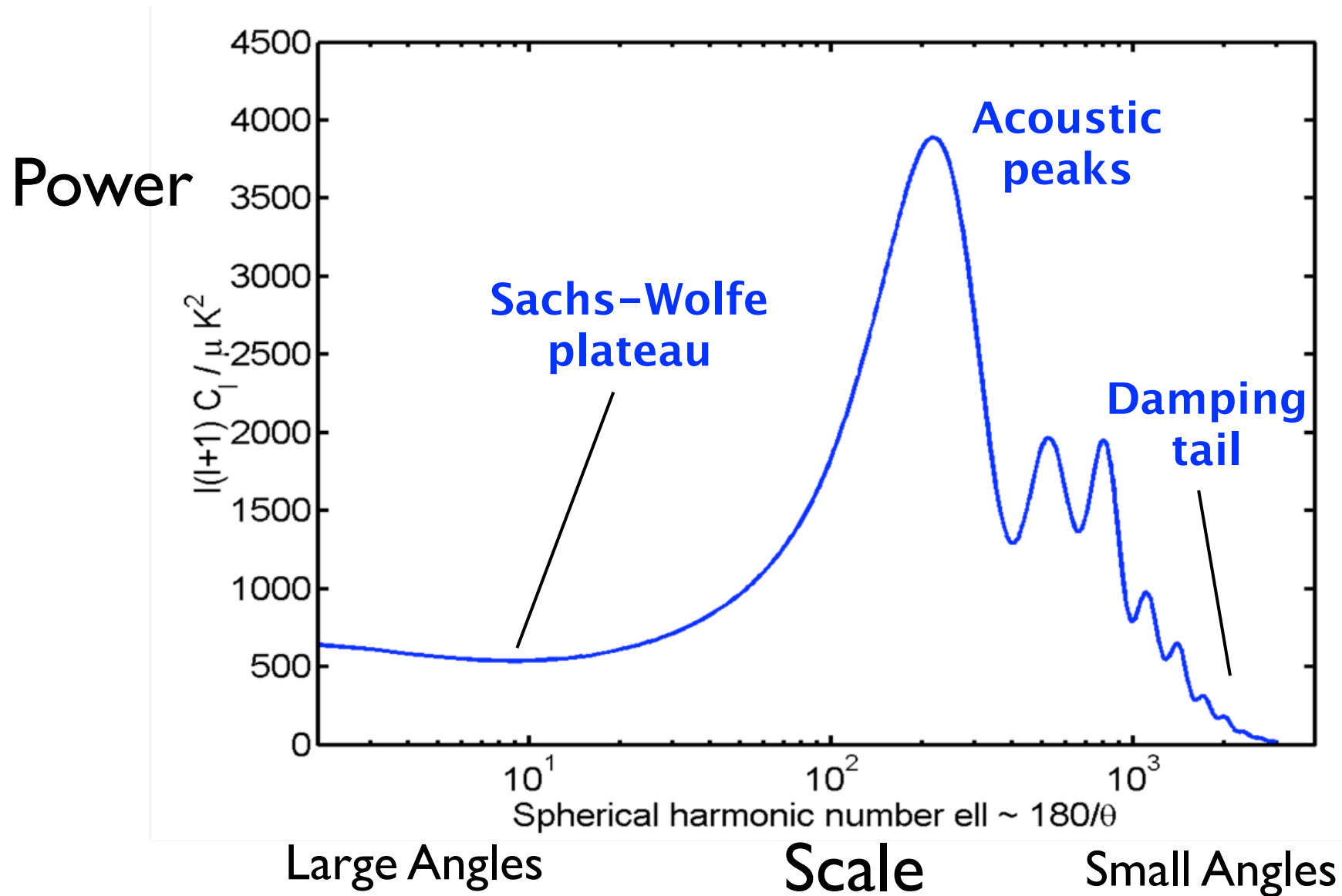
Note how 3rd peak is enhanced when dark matter density higher!

To ensure this peak is prominent, necessary to have a relatively high dark matter content earlier in universe. Otherwise, the universe will have a longer radiation dominated phase -- inhibiting the growth of fluctuations

To understand this point it is useful to understand the importance of the epoch of matter-radiation equality for the growth of fluctuations in the universe

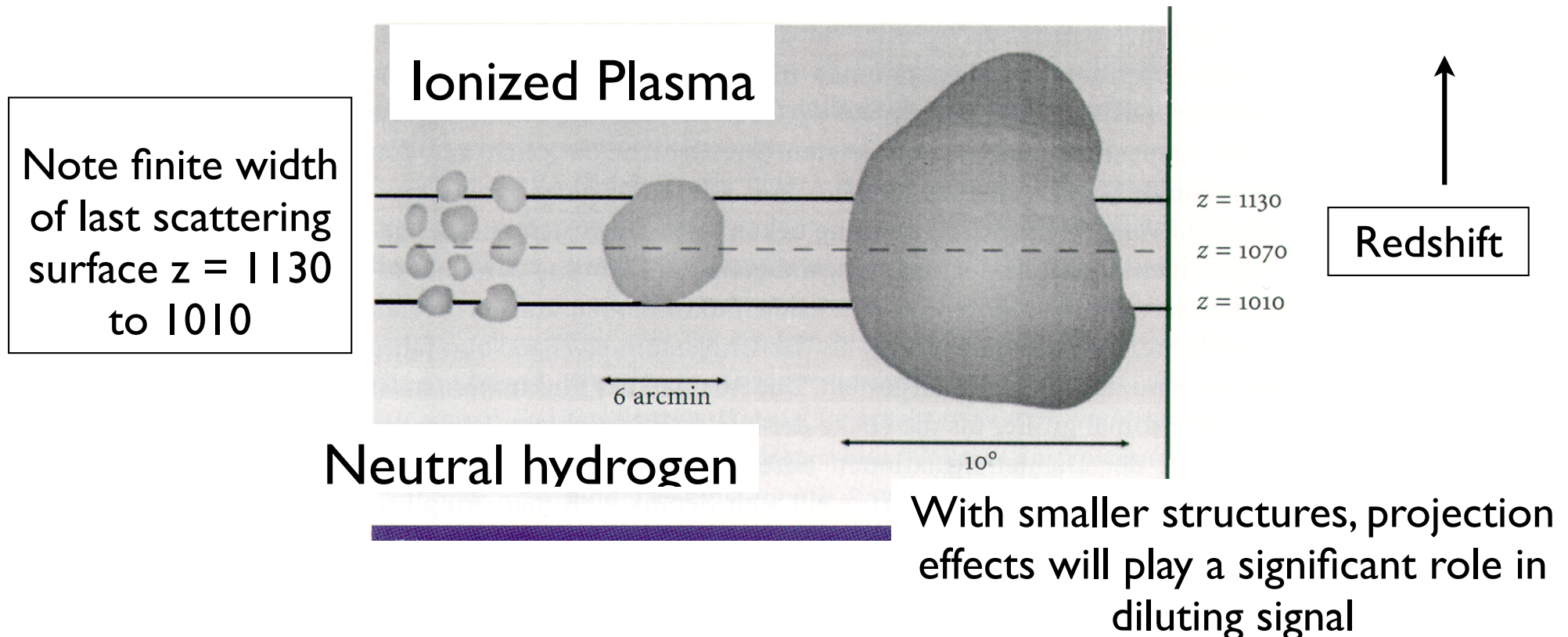
t:

# What about the damping tail?



# What about the damping tail?

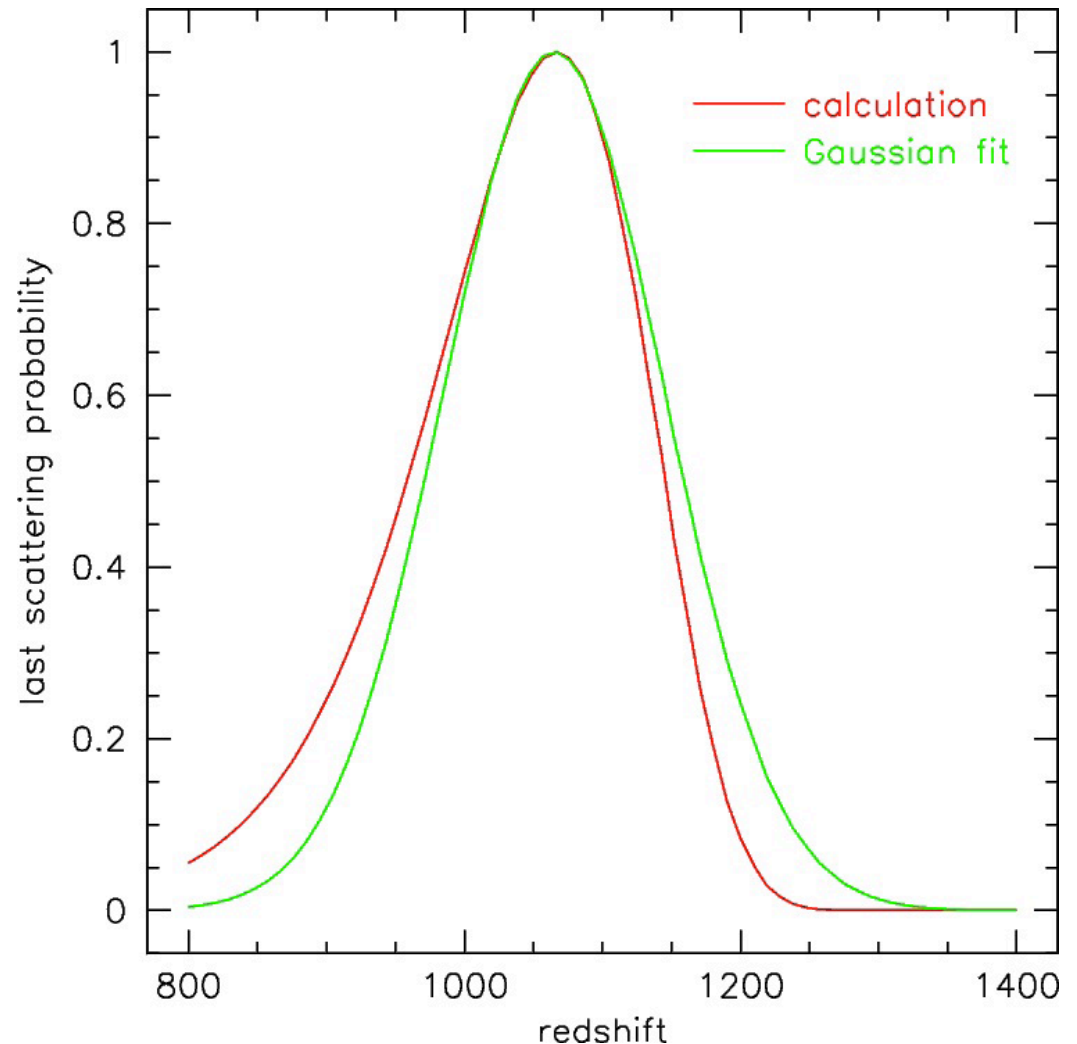
-- (I) **Radial Smearing:** Decoupling does not happen instantaneously. This is not so important in viewing the last scattering surface for larger fluctuations. But for smaller fluctuations, the structures will overlap.



# How extended is the surface of last scattering?

-- Distribution describes the probability that a photon from the cosmic microwave background was last scattered at a given redshift.

-- Can roughly be described by a normal distribution with mean  $z = 1080$  and standard deviation  $dz = 80$



# What about the damping tail?

-- **(2) Photon Diffusion / Silk Damping:** 2nd cause of the Damping tail results from photons in overdensities diffusing out of the overdensities via a random walk. This will wash out the overdensities in the baryonic material since the baryons are coupled to the photons before recombination.

Mean Free Path of Photons

$$l_{\text{mfp}} = \frac{1}{n_e \sigma_T},$$

Equation for Random Walk

$$\langle dl^2 \rangle = N l_{\text{mfp}}^2$$

Number of Scatterings

$$N = \frac{cdt}{l_{\text{mfp}}},$$

Length Scale of Photon Diffusion

$$\lambda_S^2 = \int_0^{t_{\text{dec}}} c l_{\text{mfp}} \frac{dt}{a^2}.$$

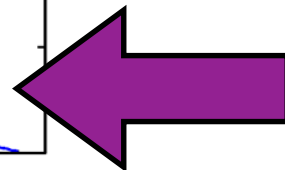
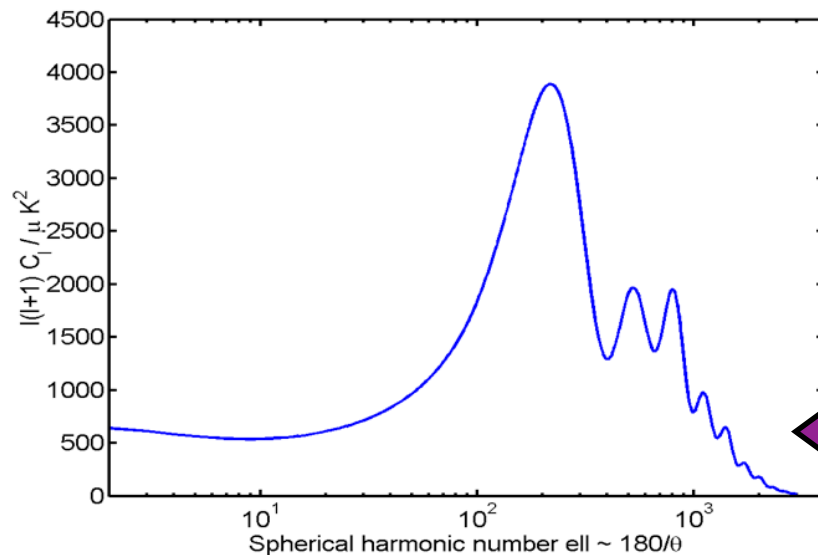
scale factor for universe



# What about the damping tail?

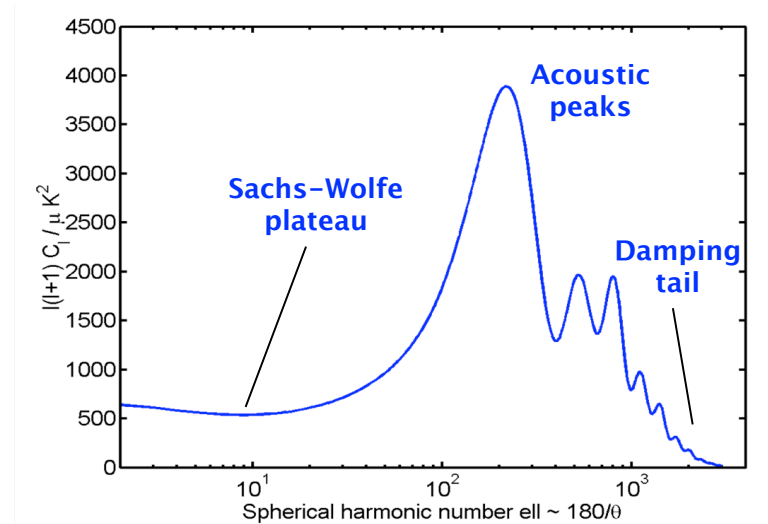
-- (1) **Radial Smearing:** Decoupling does not happen instantaneously, and so light from many smaller structures will overlap -- diluting the overall signal

-- (2) **Photon Diffusion / Silk Damping:** 2nd cause of the Damping tail results from photons in overdensities diffusing out of the overdensities via a random walk. This will wash out the overdensities in the baryonic material since the baryons are coupled to the photons before recombination.



Power falls off

There are several other features in the CMB power spectrum to discuss

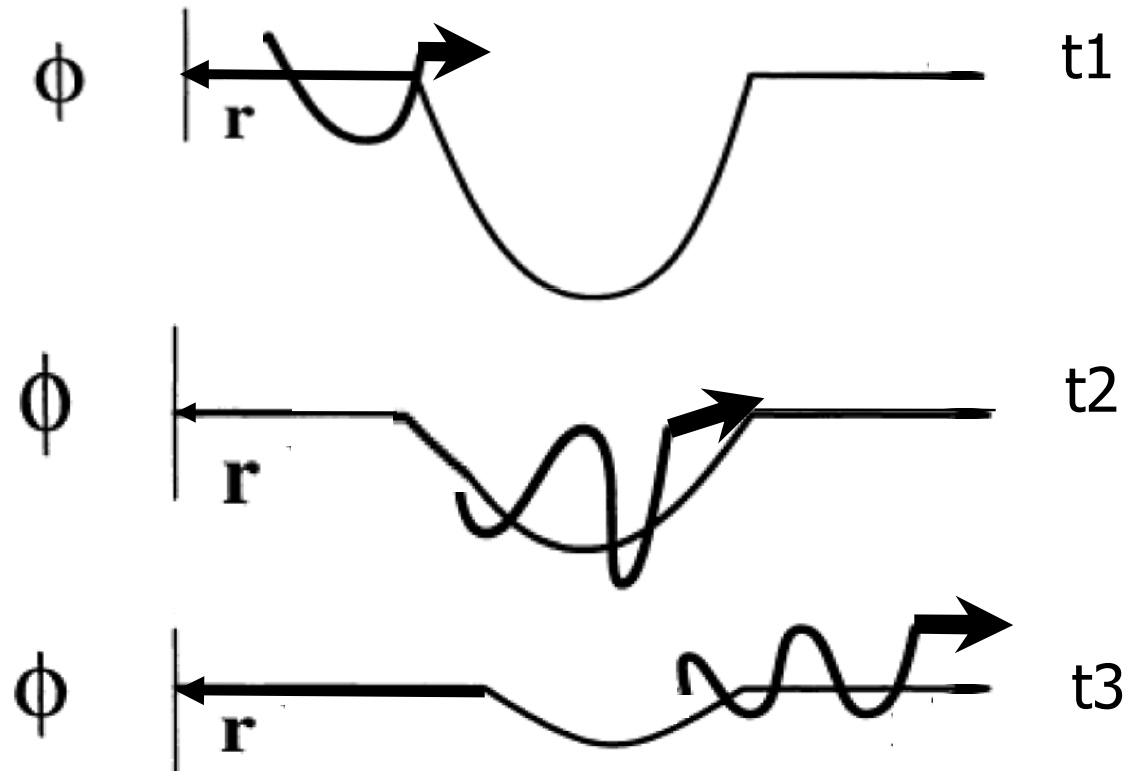


In addition to the temperature perturbations created by density fluctuations in the early universe (early Sachs Wolfe Effect), there is also the Integrated Sachs-Wolfe Effect

# Late Integrated Sachs Wolfe Effect

When CMB photons travel from the last scattering surface to us, they occasionally cross deep collapsed regions

When they traverse these regions, the photons gain energy when they fall into the potential and lose energy climbing out.



If the depth of the potential did not change as a function of cosmic time, you would expect the energy lost to be the same as the energy gained.

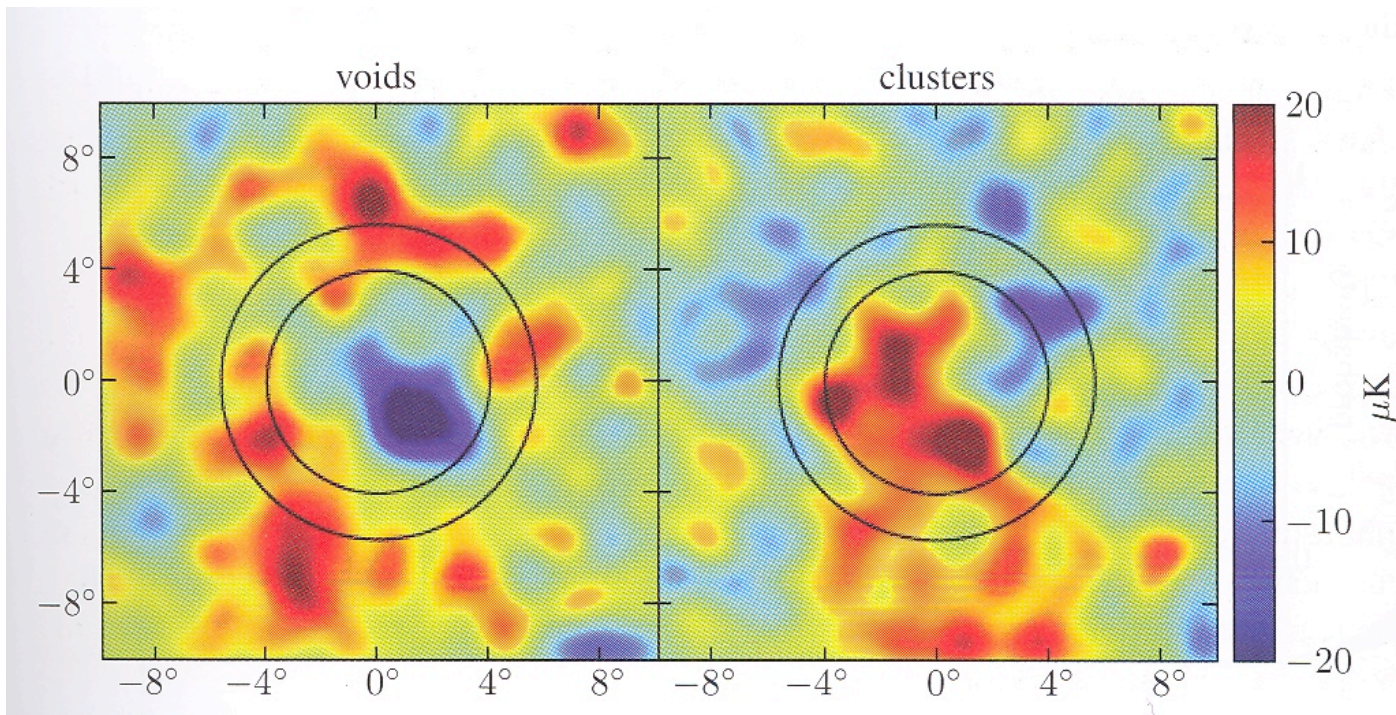
However, this is not the case -- if there is the universe has a substantial amount of dark energy

# Late Integrated Sachs Wolfe Effect

However, one can take advantage of the fact that one knows where the deep potential wells on the sky are from galaxy surveys.

By looking at spots in the CMB which traverse through overdensities (galaxy clusters), one expects to find hotter photons in general.

By looking at spots in the CMB which traverse through voids in the universe (underdensities), one expects the photons to be colder in general.



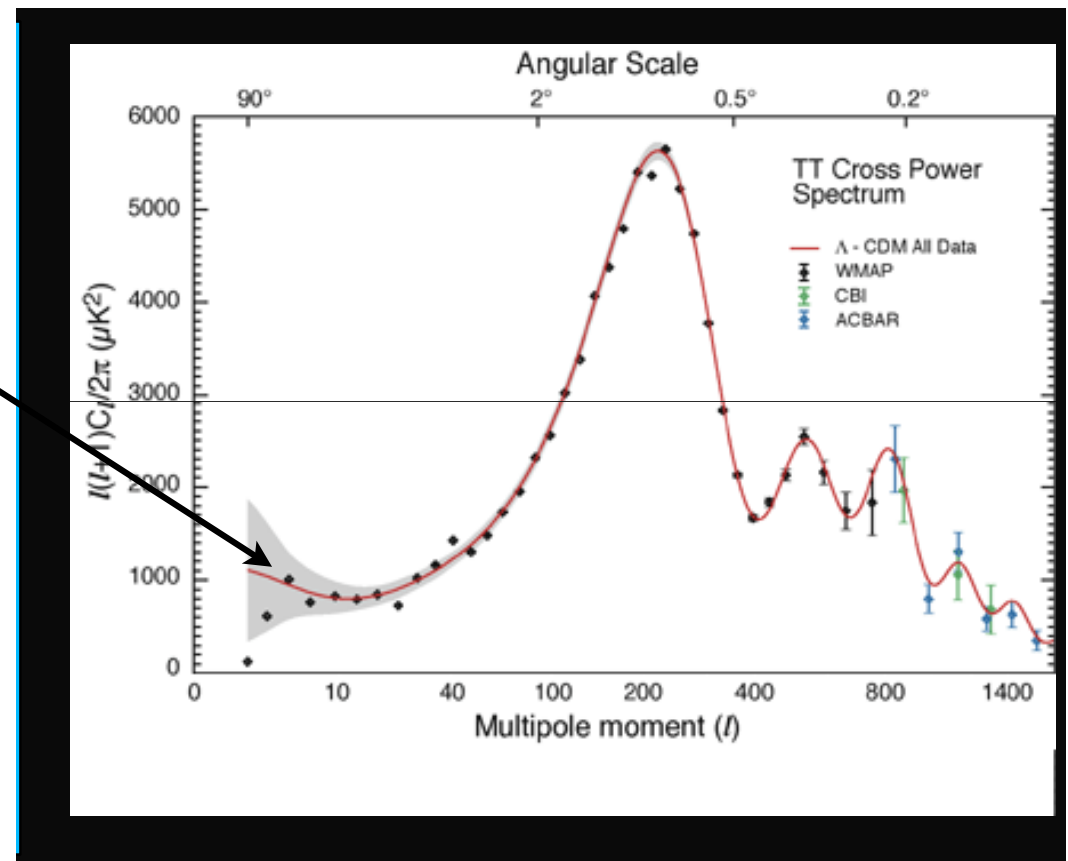
One can use this approach to constrain the amount of dark energy in the universe!

# Late Integrated Sachs Wolfe Effect

What is the effect of this on the power spectrum?

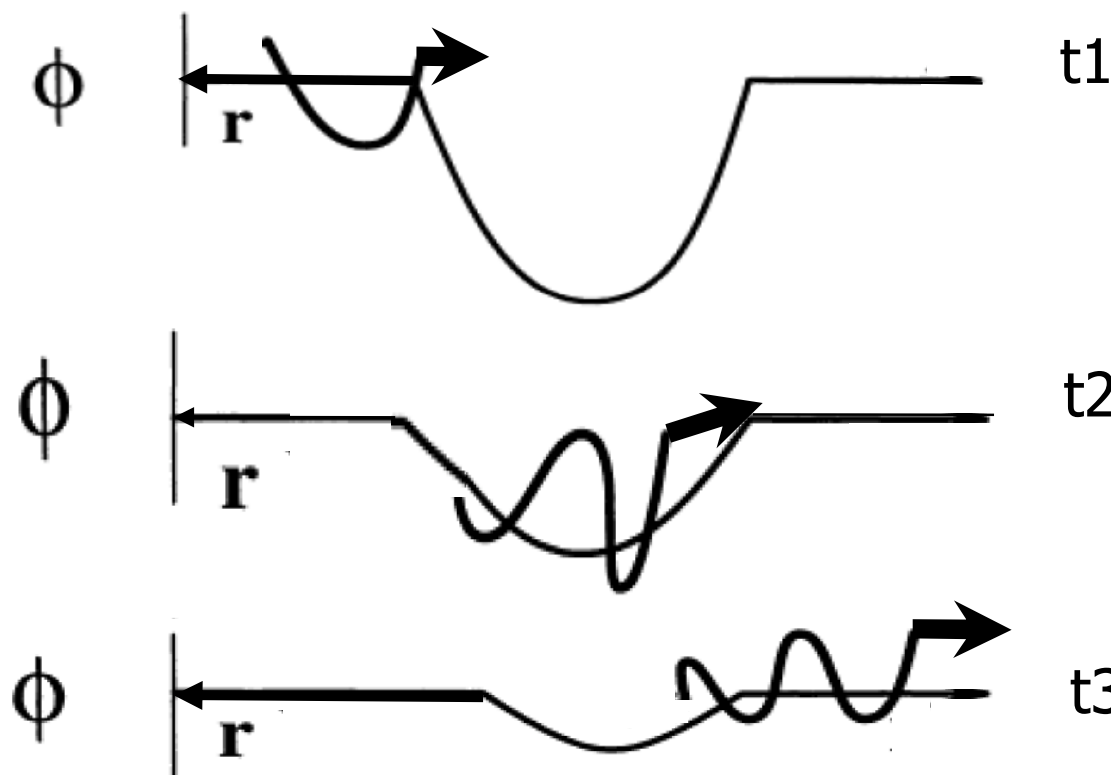
Results in slight tilt to  $C_l$  vs.  $l$  relationship

This is a difficult effect to observe in the general CMB power spectrum, since it results in a slight tilt at large scale (small  $l$  numbers) where spectrum is noisy due to cosmic variance



# There is also an early integrated Sachs Wolfe effect

This occurs due to the fact that the depth of potential wells is affected by radiation escapes from the potentials due to recombination



Before recombination, potential wells will include a contribution from radiation and be deeper

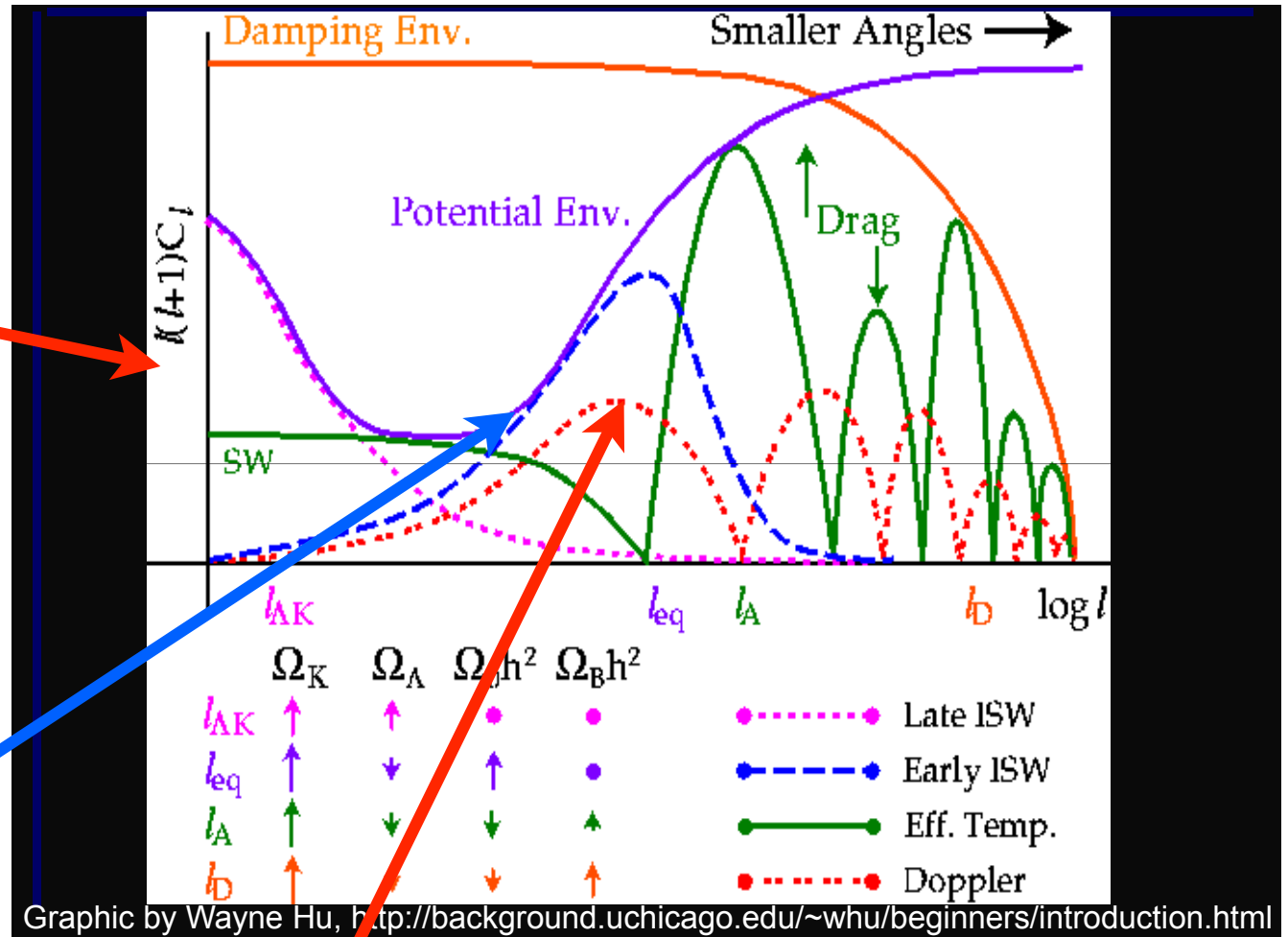
After recombination, potentials will lose this contribution from radiation and be shallower

# Illustration of the many effects impacting the CMB power spectrum

It's possible to break-down the power spectrum in a detailed manner

Also an early ISW effect due to the effect of photons climbing out of overdensities immediately after recombination

Doppler effect due to the expected motions of material in between modes at recombination epoch



Won't be tested on this figure

# One important limitation comes from cosmic variance

One thing that is important to remember is that actual density fluctuations in the real universe we see in the CMB are just one realization of a Gaussian-random process and may be different than the average perturbation size given an infinite number of universes

How well we can measure the fluctuation strength on a given physical (angular) scale, therefore, depends on how many fluctuation modes on this physical (angular) scale are available on the sky...

We can only see so many in the visible universe.

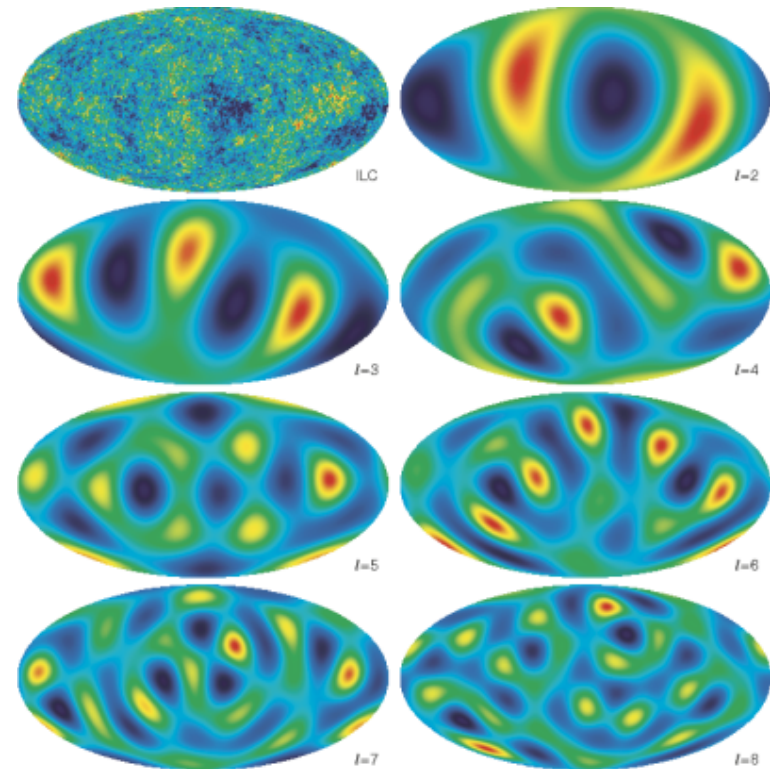
# How many fluctuation modes are available on a given scale?

It depends on the  $l$  number...

$$2l+1$$

from the  
Expansion:

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell}^m(\theta, \phi)$$



Each of the components  $a_{\ell m}$  are gaussian random variables.

and are used to determine  $c_{\ell} = \langle |a_{\ell m}|^2 \rangle$

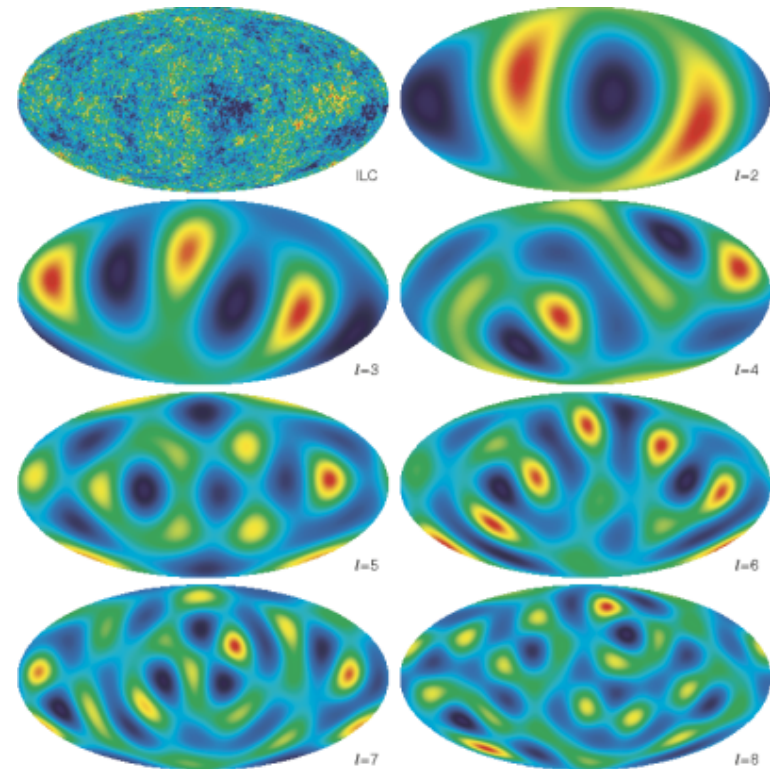
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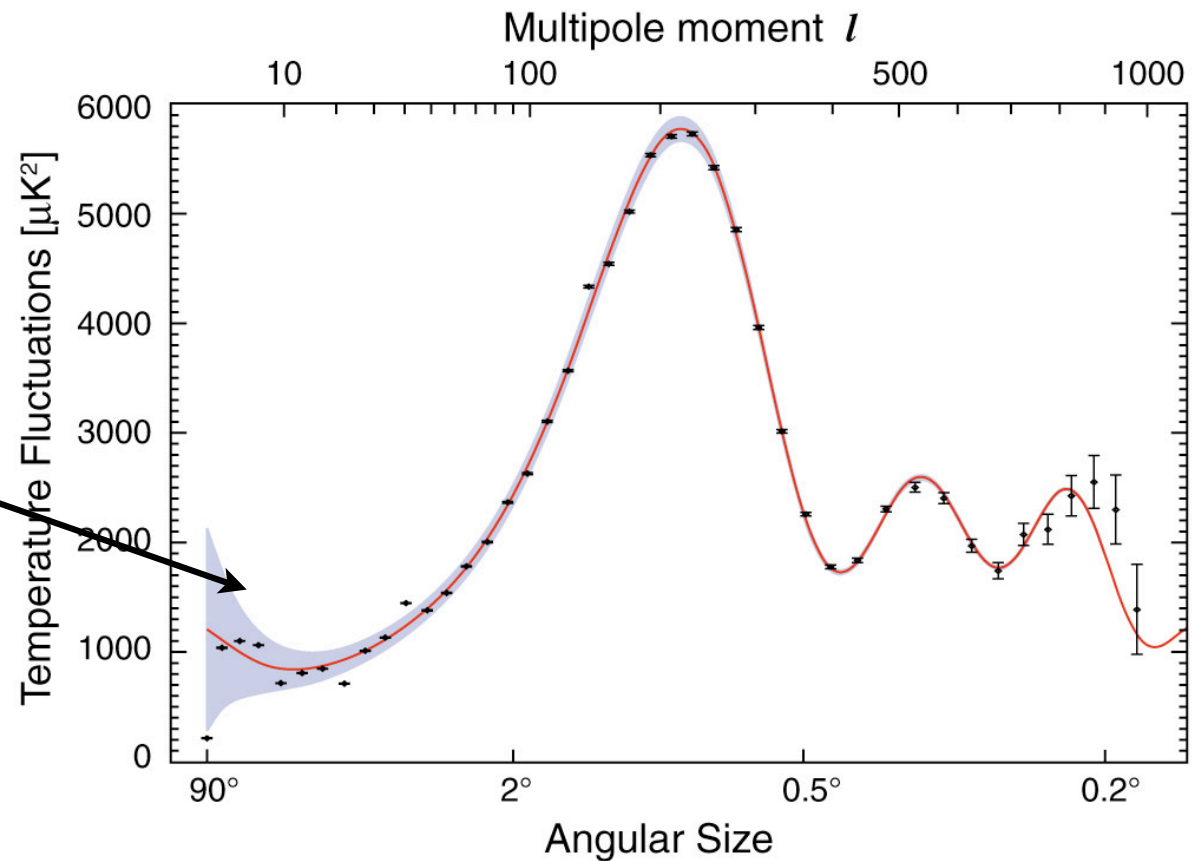


Clearly, we would expect the  $C_l$  to show less variance at smaller scales  
where more realizations (and  $l$  numbers) exist

# Concept of Cosmic Variance

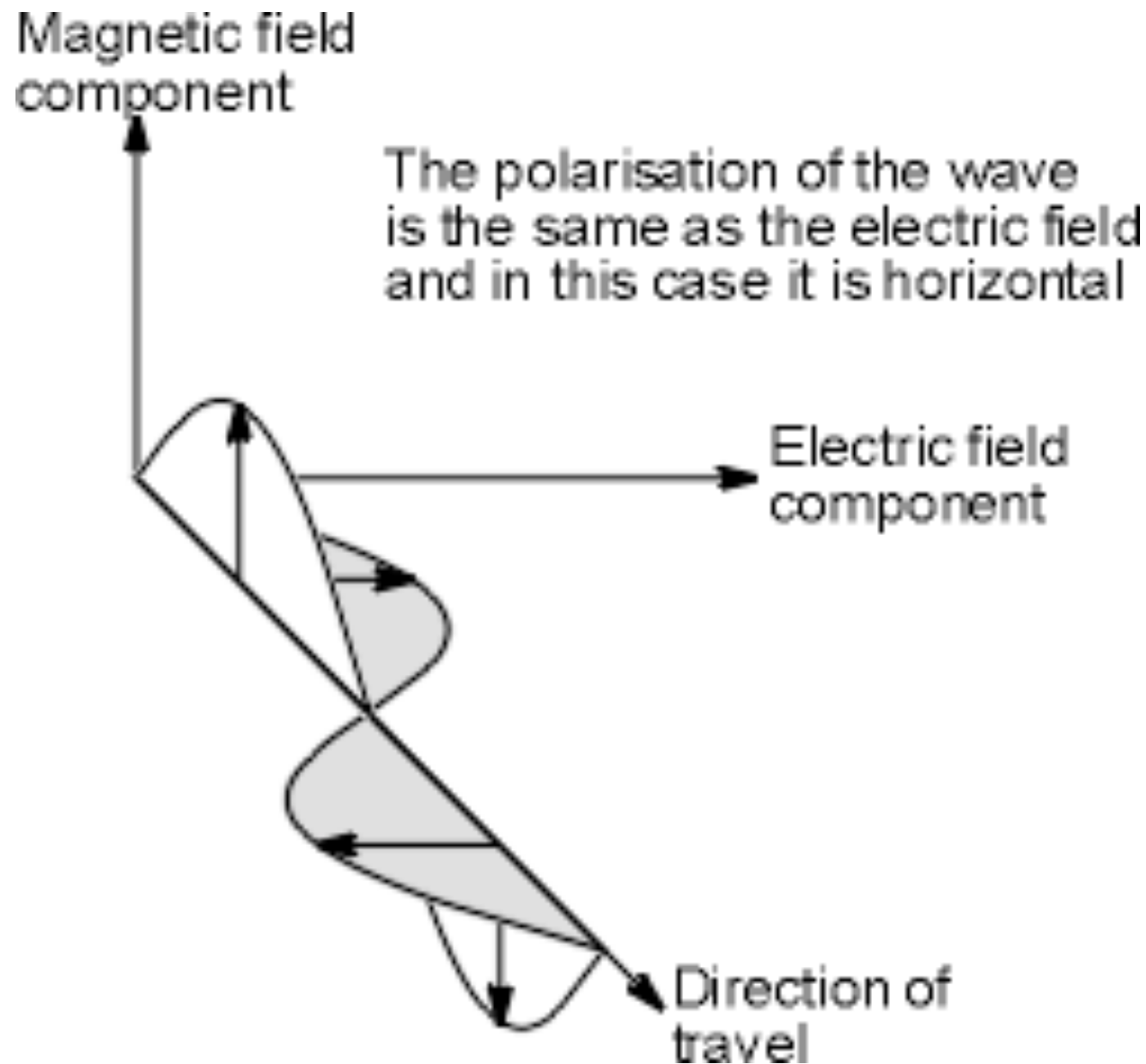
As a result, the intrinsic uncertainty on the CMB TT power spectrum became very large at low multipoles  $l$

Note how uncertainty blows up at small multipoles  $l$



# Polarisation Information in the Cosmic Microwave Background

# Polarization of Light



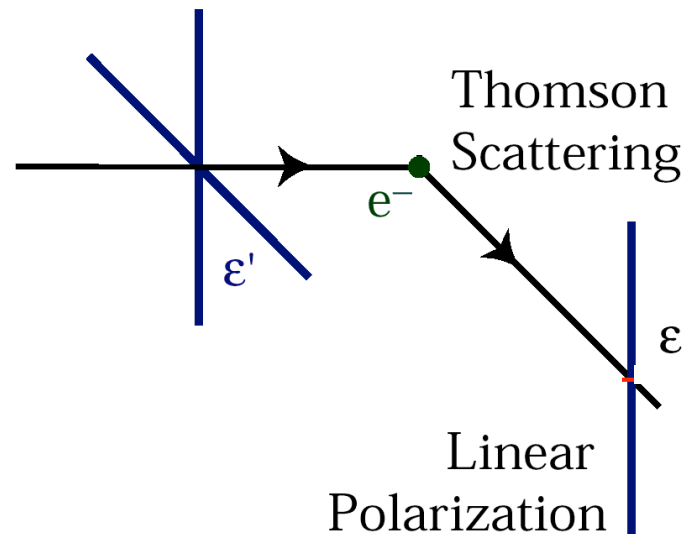
# How polarized is the cosmic microwave background overall?

most of the CMB light shows no net polarization

however there is a  $\sim 10\%$  net polarization

# Why are photons from the CMB polarized?

They are polarized from Thomson scattering  
(valid in the limit that photon is much less than  
mass energy in the particle)

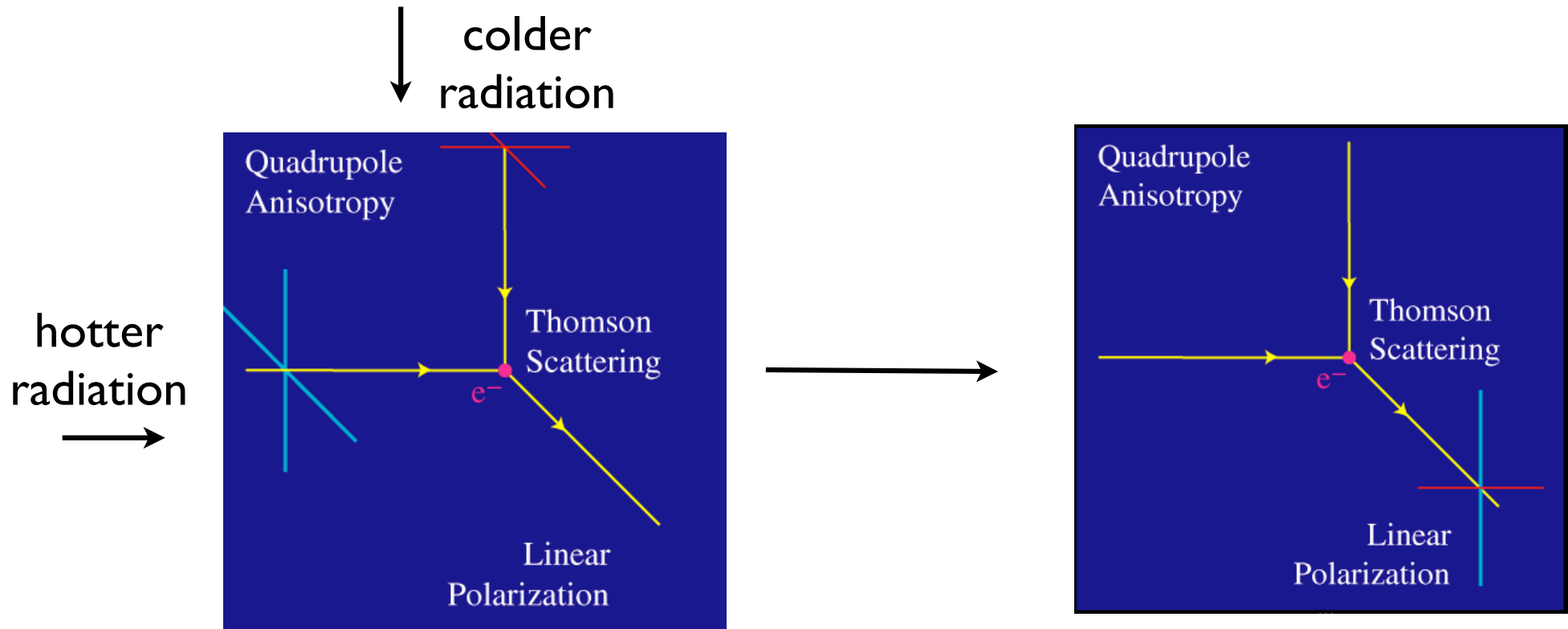


photons polarized  
perpendicular to incidence  
direction

How can this result in a polarized signal from the microwave background?

because of the relationship between the temperature structure of the CMB and polarization one gets from Thomson scattering

# How can this result in a polarized signal from the microwave background?

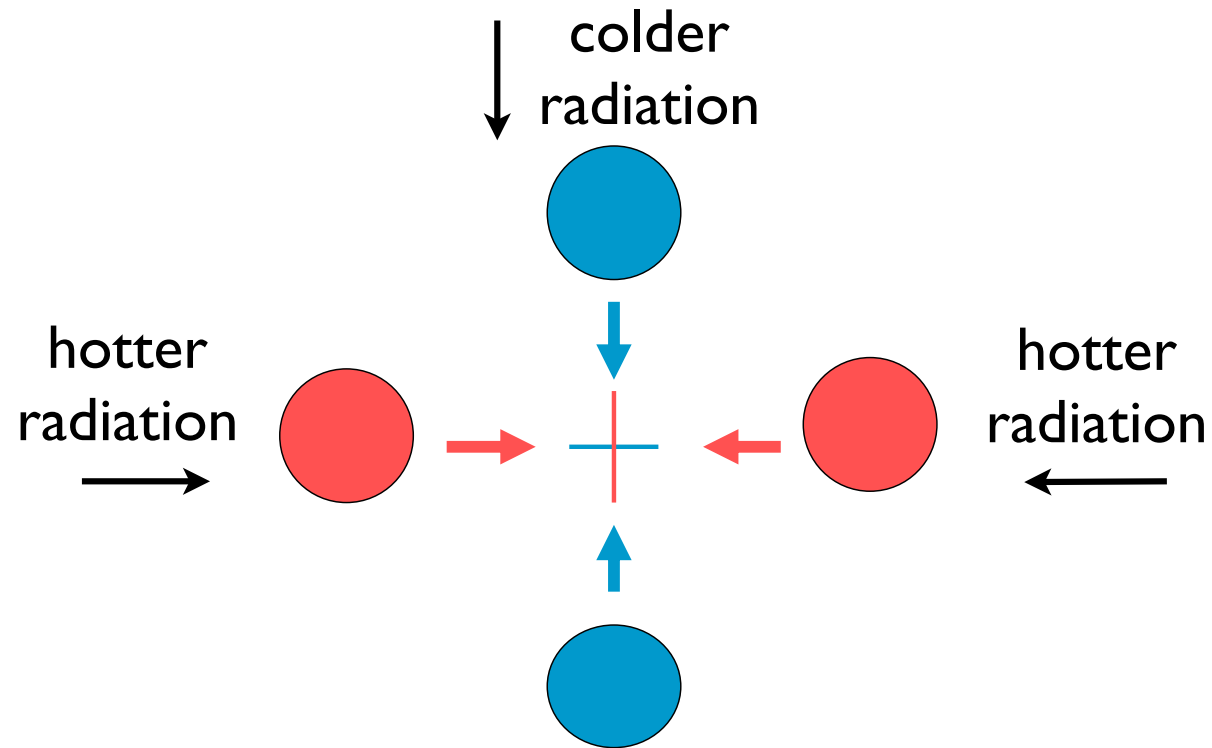


photons from hotter region will be observed with one polarization and those from colder region will be observed with another

No net polarization for an isotropic (or dipole) radiation field from the CMB. Only if the temperature structure has a quadrupole.

# How can this result in a polarized signal from the microwave background?

Same diagram but in plane of last scattering surface

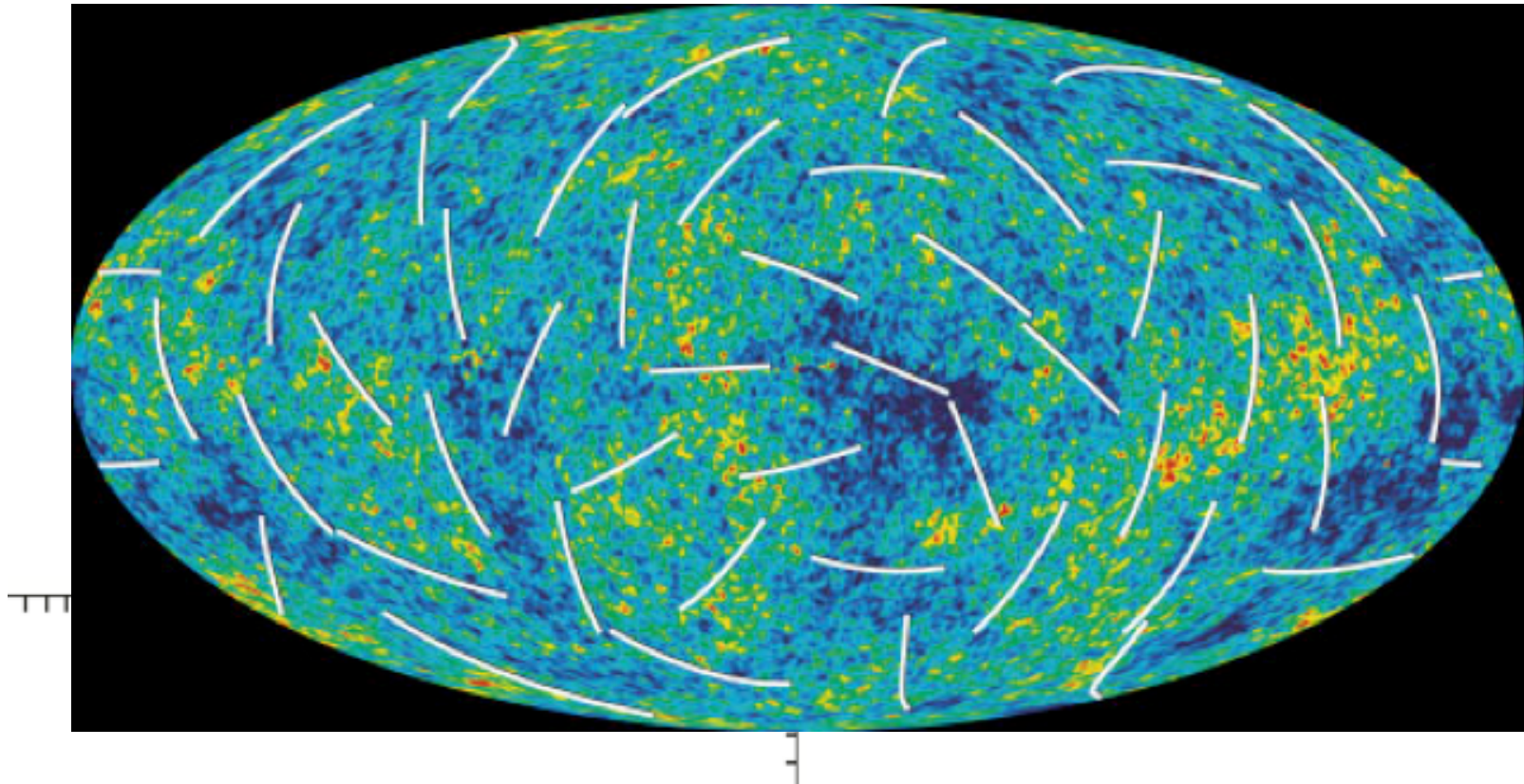


No net polarization for an isotropic (or dipole) radiation field from the CMB. Only if the radiation field has a quadrupole.

So as a result of this process, one finds a net polarization to the CMB radiation as a whole.

One can map out a polarization field for the  
entire CMB sky

e.g. with WMAP

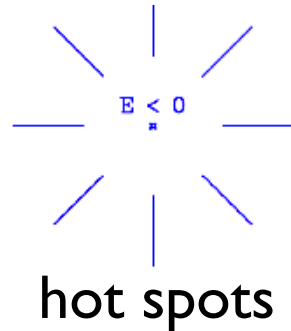


One tends to break down the polarization map into two modes (Helmholtz-Hodge theorem)

$$\begin{array}{ccc} & \mathbf{v} = \mathbf{E} + \mathbf{B} & \\ \nearrow & & \nwarrow \\ \mathbf{E} = \nabla\phi & & \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = 0 & & \end{array}$$

# One tends to break down the polarization map into two modes (Helmholtz-Hodge theorem)

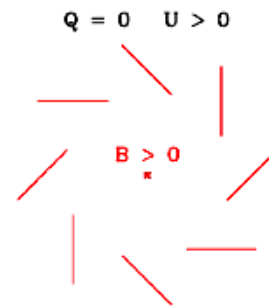
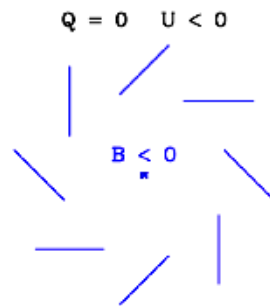
E-modes



E-modes are curl free and can be written as the gradient of a potential

$$\nabla \times \mathbf{E} = 0$$

B-modes



B-modes have no divergence.

$$\nabla \cdot \mathbf{B} = 0$$

The terms E and B modes simply reflect the general form of the polarization fields and are in analogy with similar fields in electromagnetism. However, they have no direct relation with electric or magnetic fields

# What is physical origin of E and B modes?

i.e., why look at them separately?

E-modes have their origin in normal density perturbations such as make up the early universe

B-modes are only expected to arise from gravity waves in early universe (inflation) and from gravitational lensing (between us and the last scattering surface)

Also have a temperature component to the CMB light which is entirely unpolarized, this is called the T mode (distinct from E and B modes)

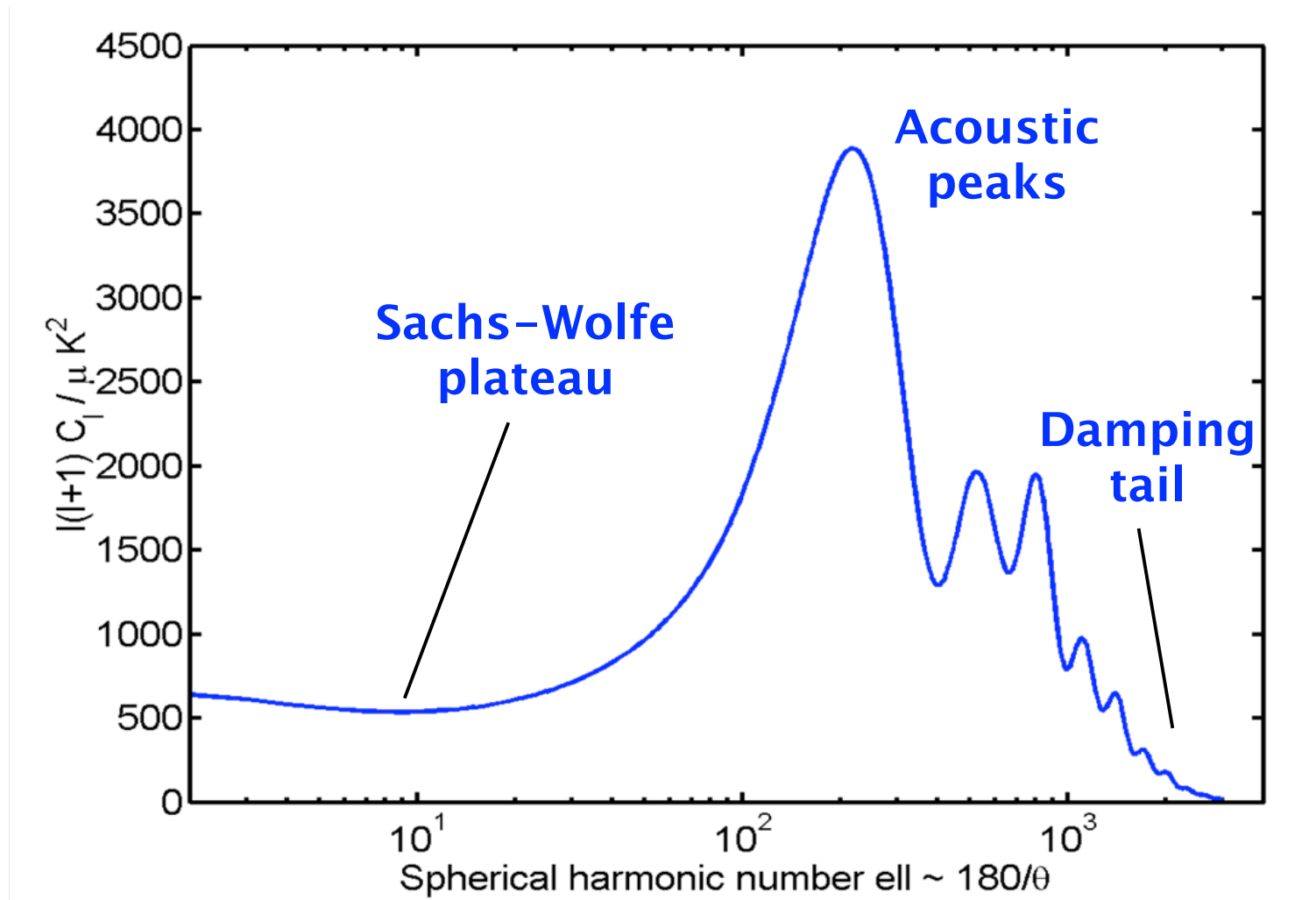
Why look at the polarized light separately?

teaches us new things..  
tests our assumptions...

What about these three components to  
temperature structures T, E, and B?

How does this relate to what we did before?

So, far what I have shown you the TT angular power spectrum...



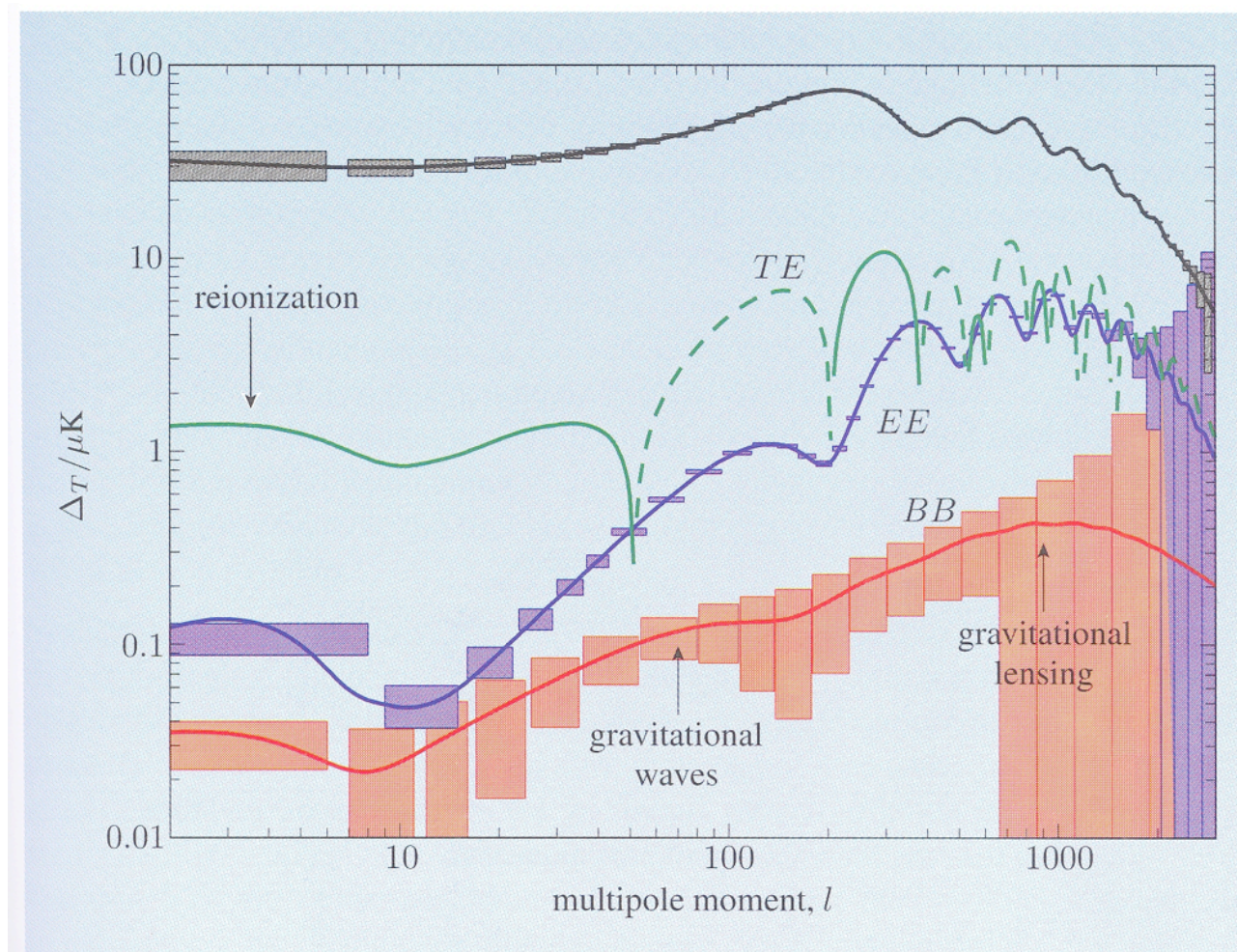
So, in addition to maps of the temperature  $T$  of the cosmic microwave background (all polarizations), we can look at maps of the temperature with an E-mode type polarization and B-mode type polarization

By cross-correlating the difference in temperature of the light from these different components T, E, and B, you can look at four different power spectra TT, TE, EE, BB...

Might imagine there could also TB and EB type power spectra...

However, since T and E modes have one type of symmetry and B modes have another, TB and EB always equal zero.

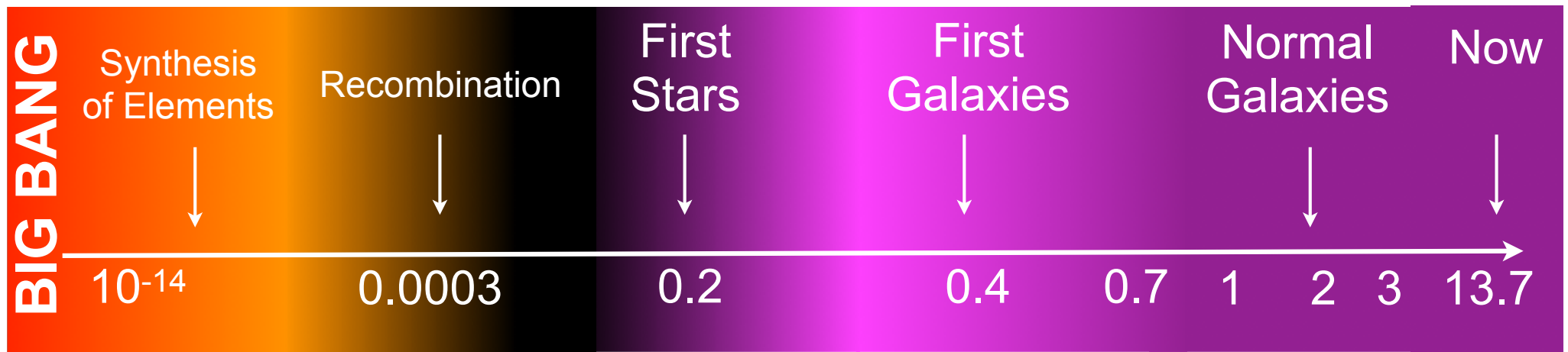
But we can also look the TE, EE, and BB angular power spectra



Note that the EE, TE, and BB power spectra are not nearly as prominent as the TT power spectrum. This is because only 10% of the light from the CMB is polarized!

# What new information do the TE, EE, and BB spectra provide?

Allows us to answer question how long did hydrogen in the universe in a neutral state, i.e., from 400,000 yrs after Big Bang to 1 Gyr

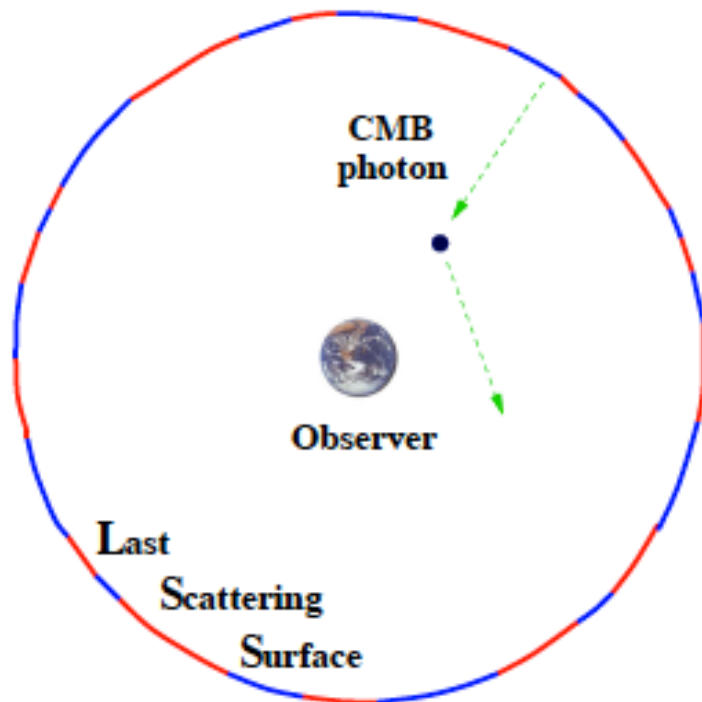


state of hydrogen

ionized → ← neutral → ← ionized

# What new information do the TE, EE, and BB spectra provide?

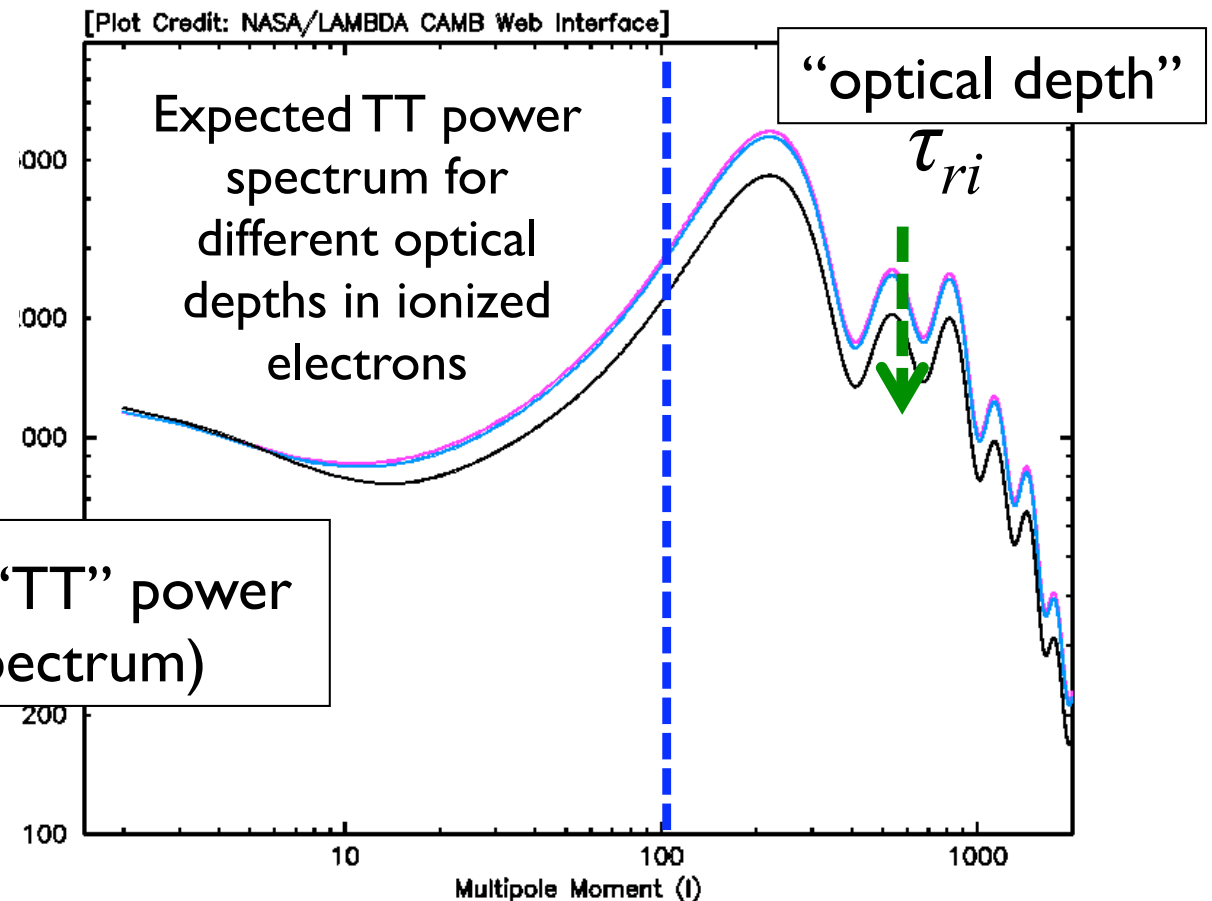
The microwave background helps us answer this question -- since photons from the microwave background scatter off of ionized electrons in the universe



Obviously, the longer the hydrogen remains in an ionized state, the more photons from the CMB we would expect to be scattered.

# What new information do the TE, EE, and BB spectra provide?

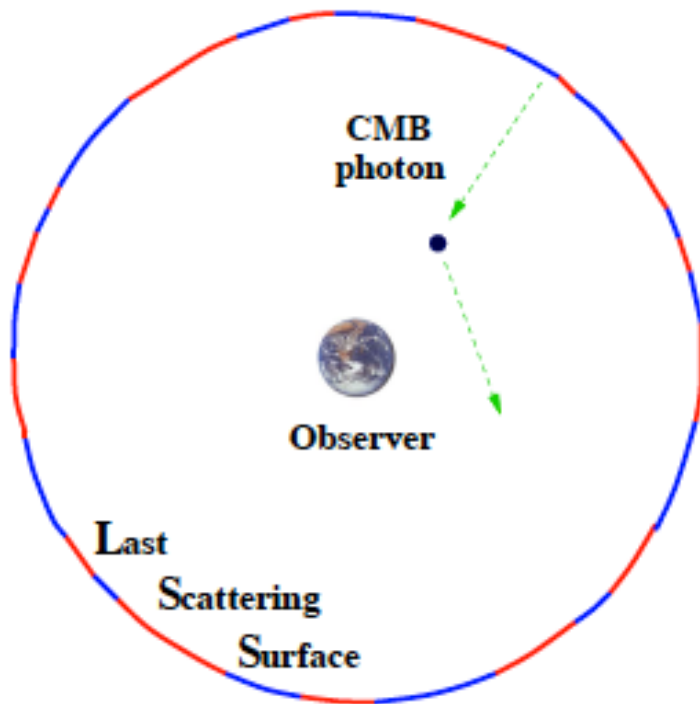
Information about reionization is present in the TT power spectrum, but it is degenerate with the underlying normalisation of the power spectrum.



Difficult to know to distinguish between scenarios where universe had less structure at early times and where the apparent structure washed out by Thomson scattering.

# What new information do the TE, EE, and BB spectra provide?

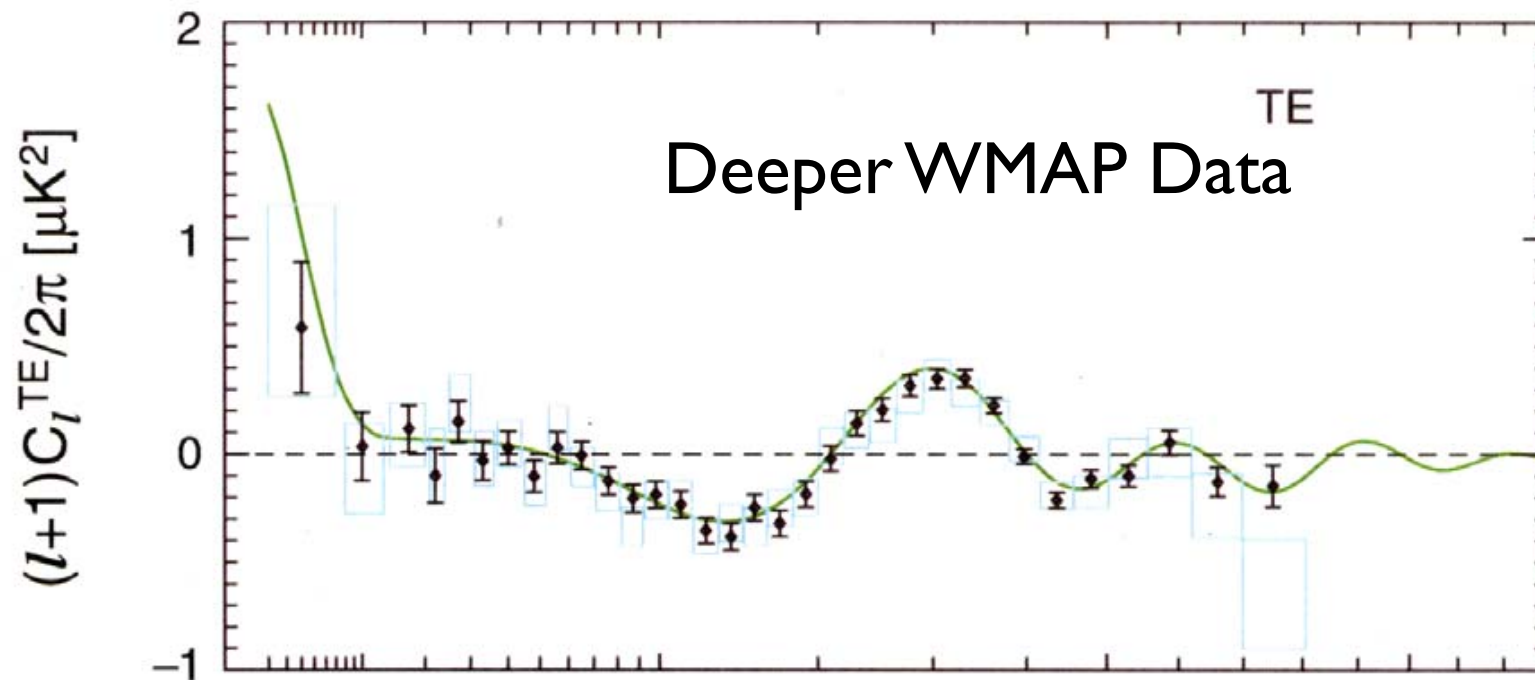
Since photons from the CMB are expected to have a certain polarization symmetry relative to the temperature structure of the CMB and this polarization would be mixed up if they are scattered by intervening matter, we can learn about the intervening ionized hydrogen



Measurements show that ~10% of CMB photons are so scattered

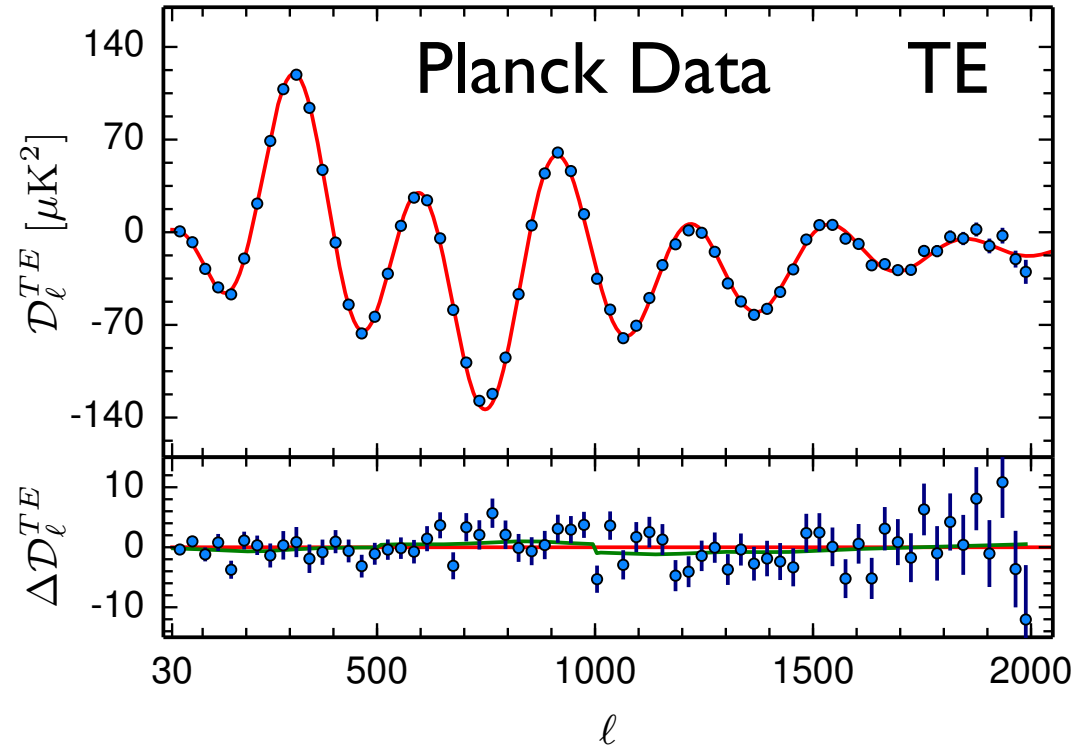
# What new information do the TE, EE, and BB spectra provide?

By looking at the polarization data, we can attempt to answer this question -- since the polarization of photons unscattered by ions in the intervening space will have different properties than those that are scattered.



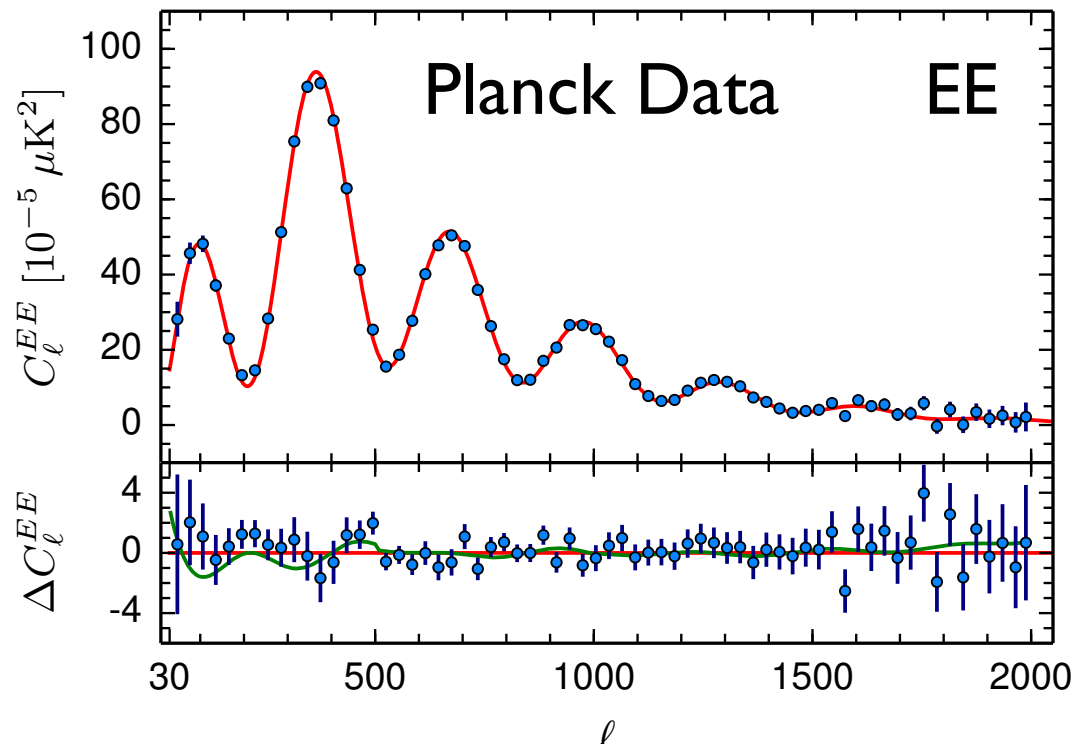
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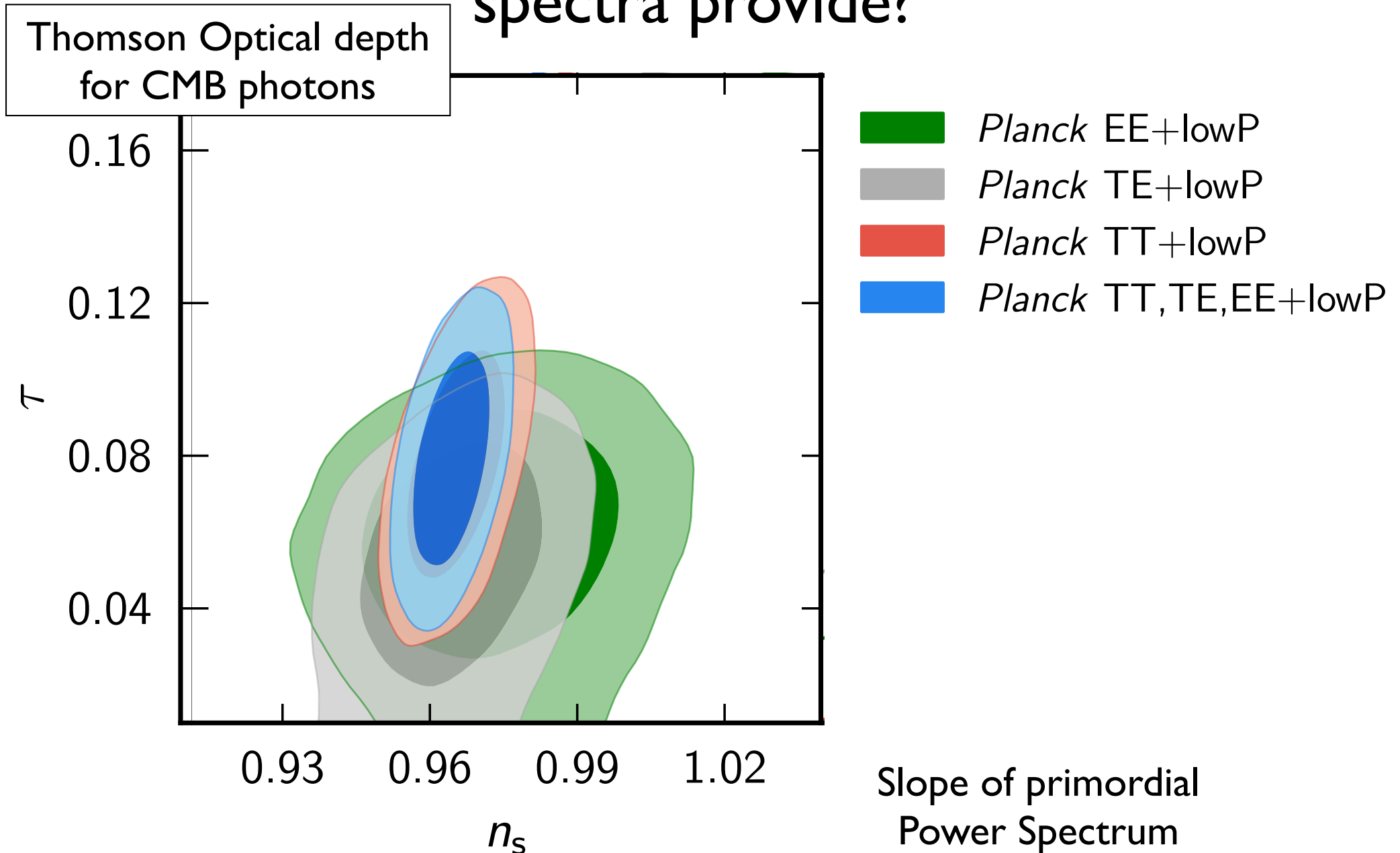


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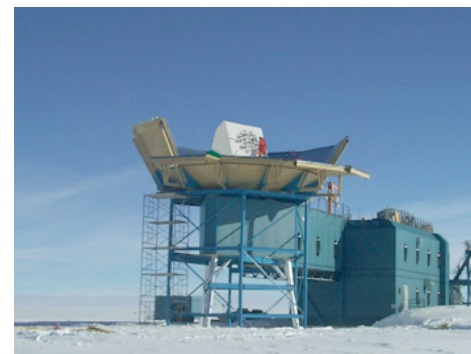
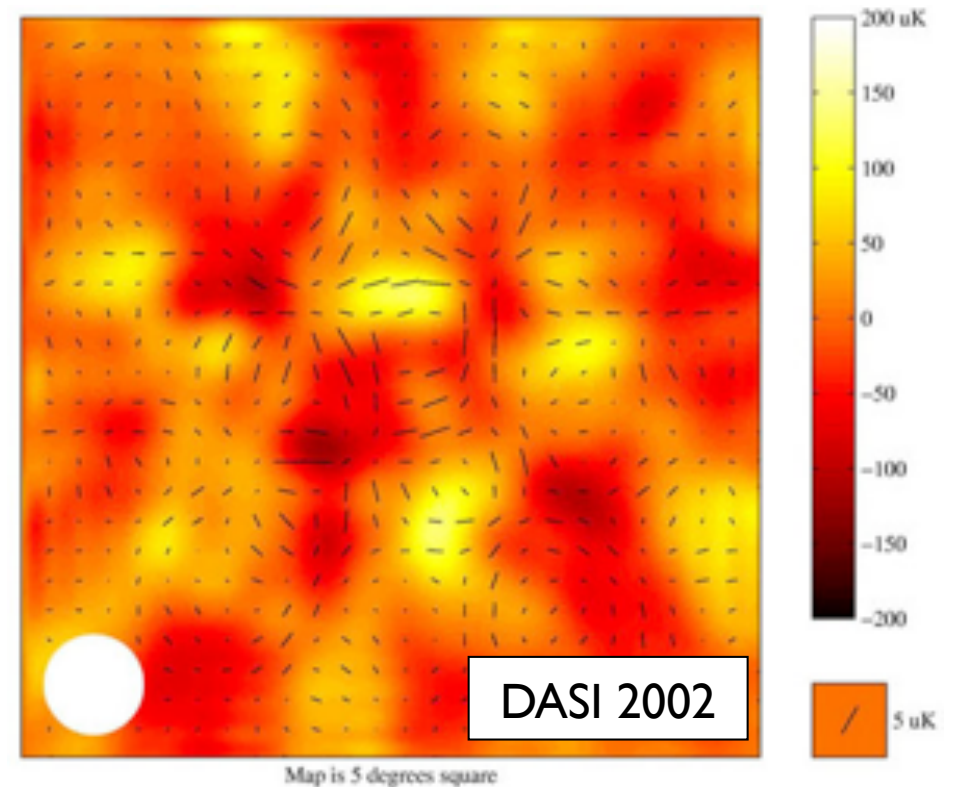


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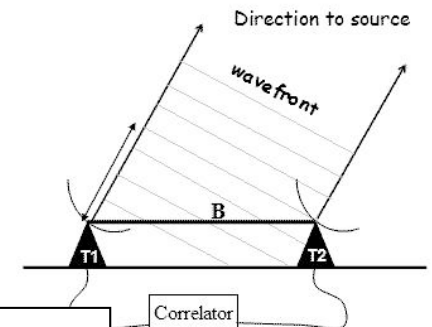
# First detection of polarization in CMB

- DASI South Pole experiment (interferometer) first to detect E mode polarization (2002)
- This was followed by WMAP reporting a measure of the  $C_{TE}$  power spectrum at low angular scales
- Measurements of the E-mode polarization also made with CARMAP, MAXIPOL, and QUAD



DASI in South Pole

interferometer: collect coherent signals over certain angular scale on sky



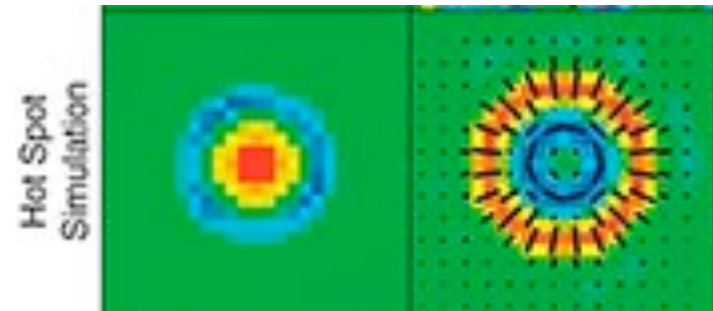
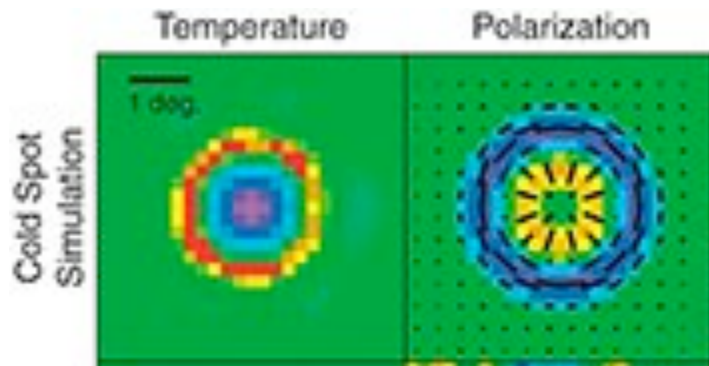
Credit: Basu

It is interesting that we can actually test whether our understanding of the polarization of CMB is correct

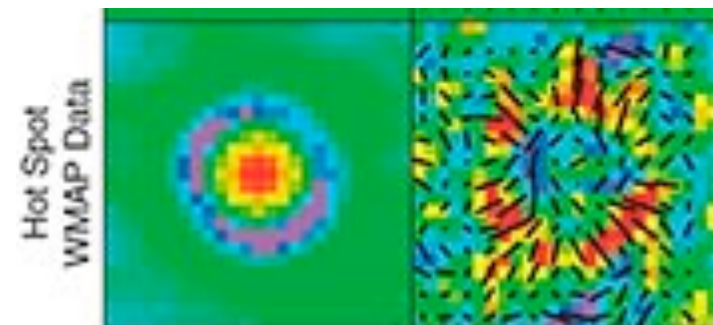
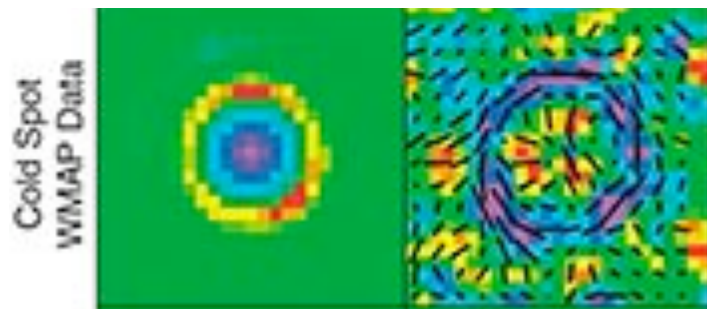
Around cold or hot spots, we expect a certain structure to the polarization signal

Can test this by looking at the polarization signal around hot or cold spots in the observations.

From theory

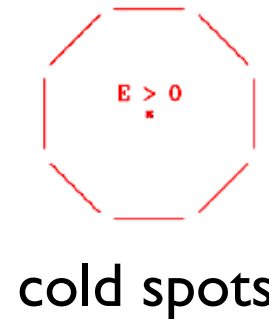
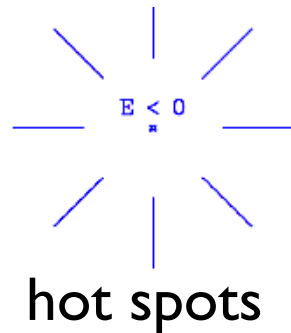


As observed by WMAP



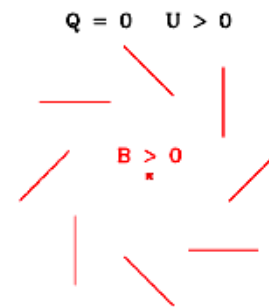
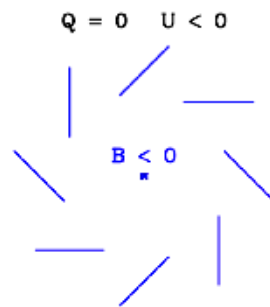
# One tends to break down the polarization map into two modes

E-modes



E-modes are curl free and can be written as the gradient of a potential

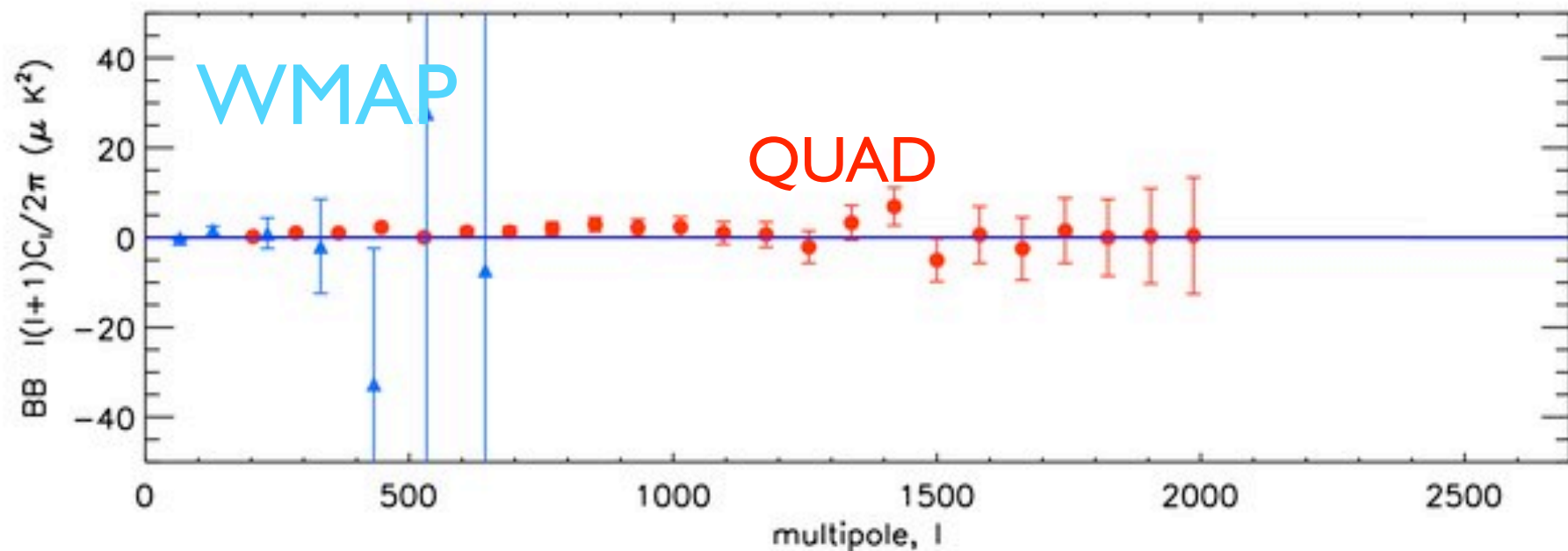
B-modes



B-modes are curl free and can be written as the gradient of a potential

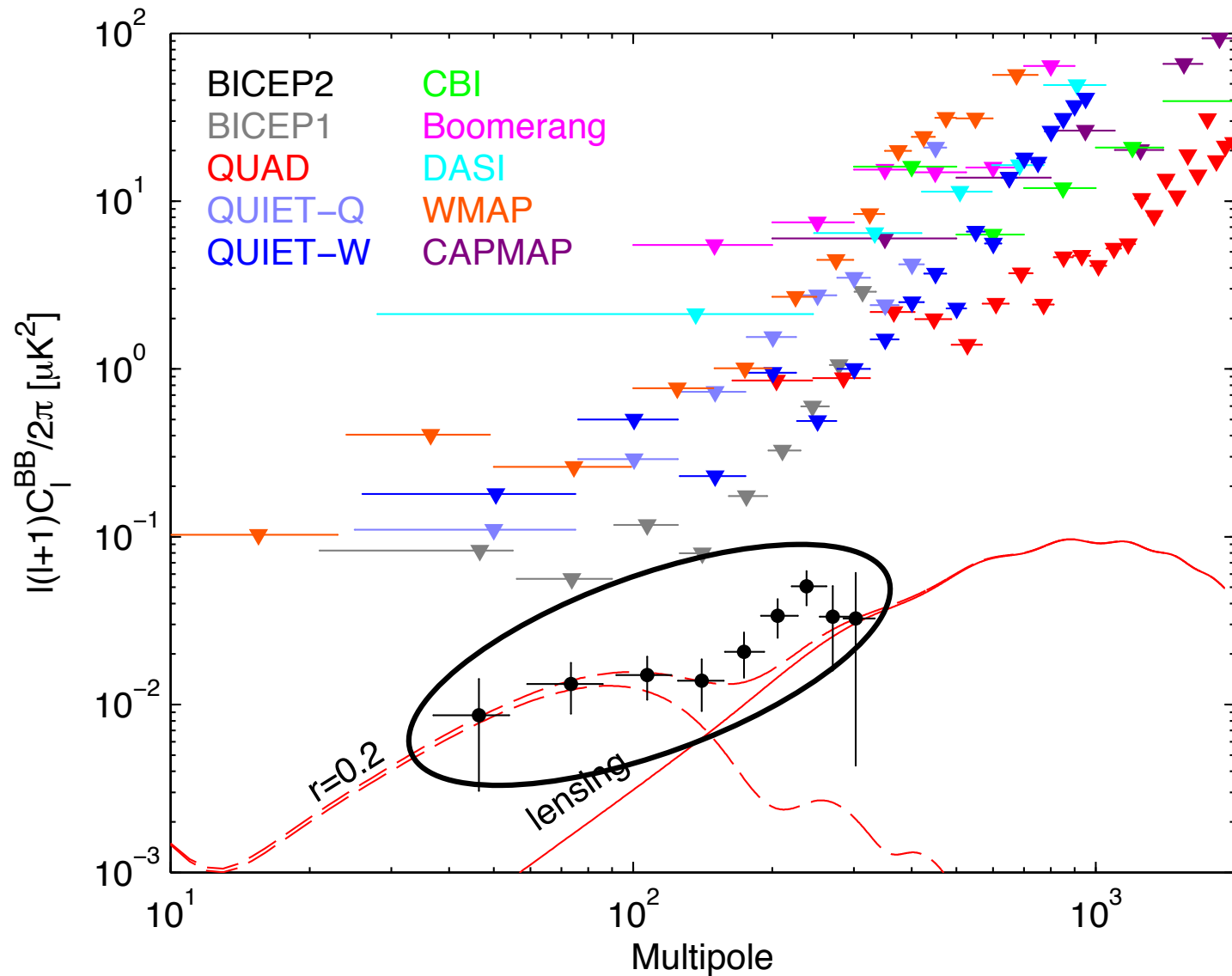
The terms E and B modes simply reflect the general form of the polarization fields and are in analogy with similar fields in electromagnetism. However, they have no direct relation with electric or magnetic fields

# No power in BB power spectrum detected as of 2013 -- goal of Planck!



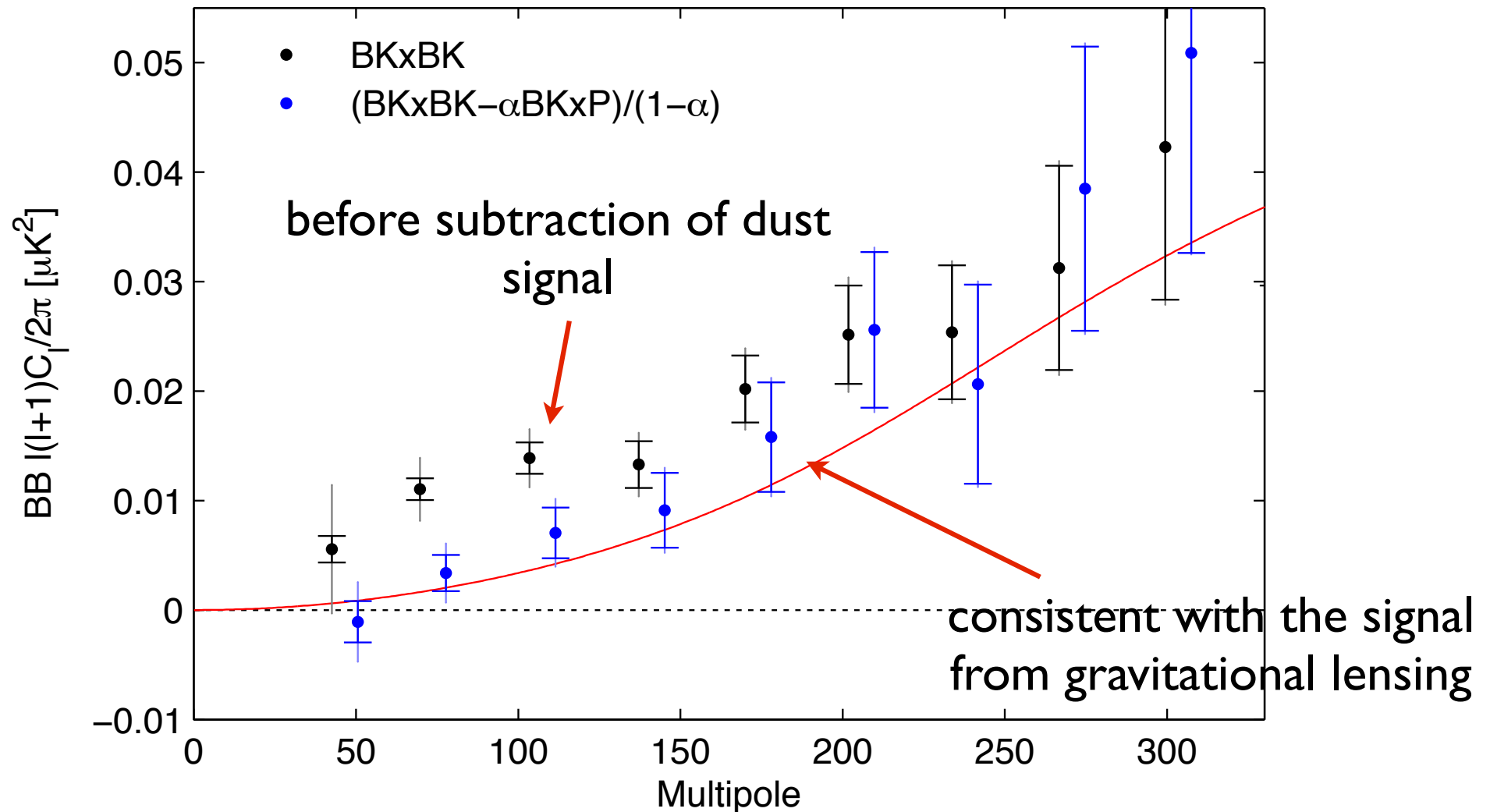
Was expected to the smoking gun test of inflation -- since the signal is expected to originate from gravity waves (from inflation) -- signal on smaller scales comes from gravitational lensing

# Significant BB signal detected by BICEP II!

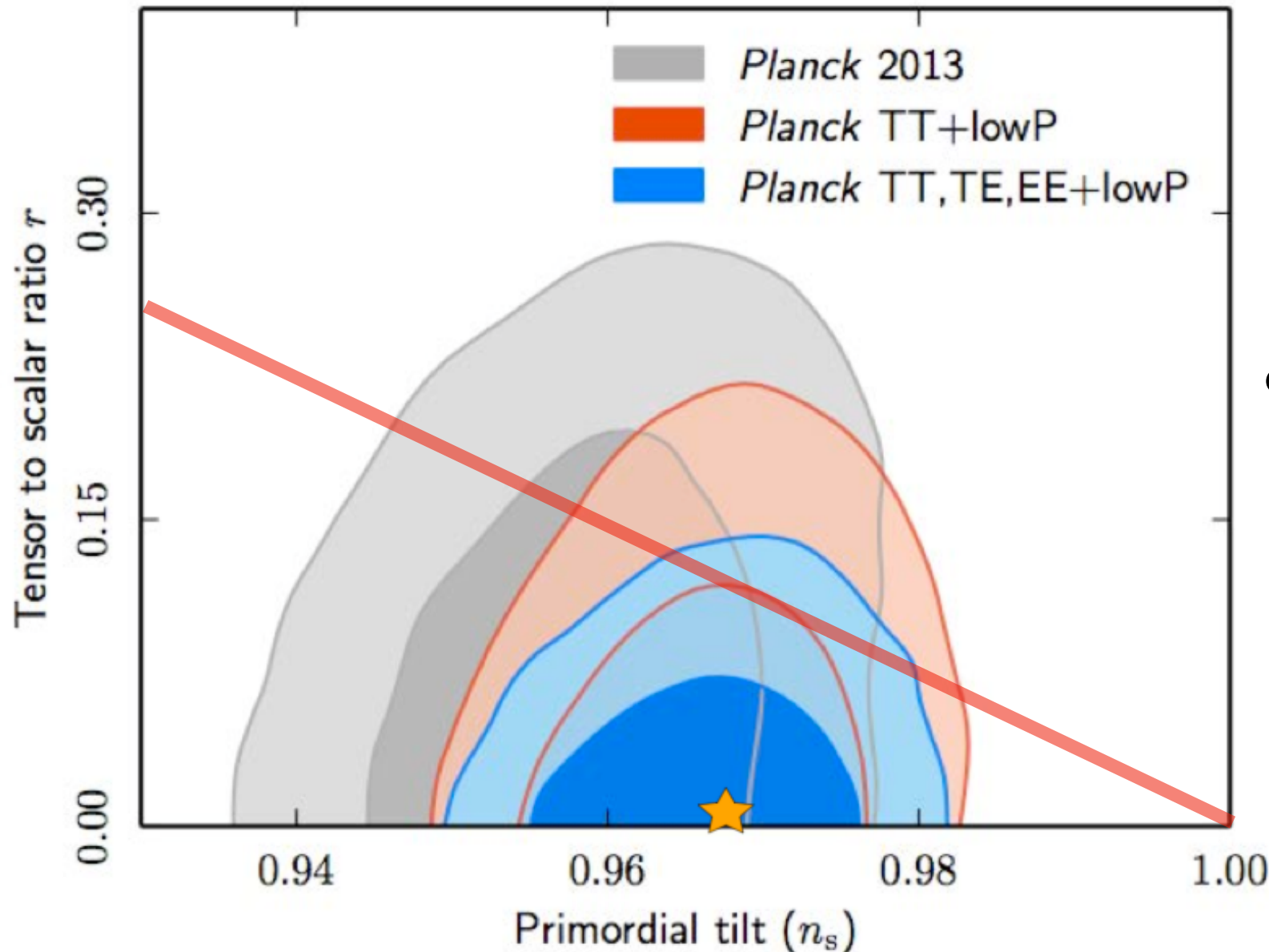


BICEP2 results show a positive detection of BB modes. Attempted fit to gravity waves from inflation... Lensing contributes at small scales

But current BB signal from BICEP II appears consistent with arising from dust in our own galaxy..



Current constraints on BB allow us to set constraints on  $r$ , the ratio of power tensor-to-scale modes.



For simplest inflation models, there is a relationship between the tilt of the primordial power spectrum And the tensor-to-scalar ratio  $r$ .

$$r = 8(1-n_s)$$

$$\Rightarrow r = 0.1-0.3$$

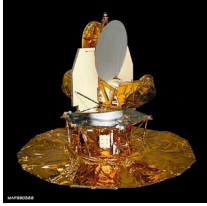
Both WMAP + Planck have provided us with  
an immense amount of information on the  
cosmological parameters

# Constraints on the cosmological parameters from WMAP observations (7-year)

WMAP launched June 2001



Credit: NASA



Note the same dual receivers as COBE. This design, added with the very stable conditions at the L2, minimizes the "1/f noise" in amplifiers and receivers.

Thus after 7 years, the data can still be added and noise lowered (of course, the improvement will be marginal).

WMAP Cosmological Parameters			
Model: $\Lambda$ CDM+SZ+lens			
Data: wmap7			
$10^2 \Omega_b h^2$	$2.258^{+0.057}_{-0.056}$	$1 - n_s$	$0.037 \pm 0.014$
$1 - n_s$	$0.0079 < 1 - n_s < 0.0642$ (95% CL)	$A_{\text{BAO}}(z = 0.35)$	$0.463^{+0.021}_{-0.020}$
$C_{220}$	$5763^{+38}_{-40}$	$d_A(z_{\text{eq}})$	$14281^{+158}_{-161}$ Mpc
$d_A(z_*)$	$14116^{+160}_{-163}$ Mpc	$\Delta_{\mathcal{R}}^2$	$(2.43 \pm 0.11) \times 10^{-9}$
$h$	$0.710 \pm 0.025$	$H_0$	$71.0 \pm 2.5$ km/s/Mpc
$k_{\text{eq}}$	$0.00974^{+0.00041}_{-0.00040}$	$\ell_{\text{eq}}$	$137.5 \pm 4.3$
$\ell_*$	$302.44 \pm 0.80$	$n_s$	$0.963 \pm 0.014$
$\Omega_b$	$0.0449 \pm 0.0028$	$\Omega_b h^2$	$0.02258^{+0.00037}_{-0.00036}$
$\Omega_c$	$0.222 \pm 0.026$	$\Omega_c h^2$	$0.1109 \pm 0.0056$
$\Omega_\Lambda$	$0.734 \pm 0.029$	$\Omega_m$	$0.266 \pm 0.029$
$\Omega_m h^2$	$0.1334^{+0.0056}_{-0.0055}$	$r_{\text{hor}}(z_{\text{dec}})$	$285.5 \pm 3.0$ Mpc
$r_s(z_d)$	$153.2 \pm 1.7$ Mpc	$r_s(z_d)/D_v(z = 0.2)$	$0.1922^{+0.0072}_{-0.0073}$
$r_s(z_d)/D_v(z = 0.35)$	$0.1153^{+0.0038}_{-0.0039}$	$r_s(z_*)$	$146.6^{+1.5}_{-1.6}$ Mpc
$R$	$1.719 \pm 0.019$	$\sigma_8$	$0.801 \pm 0.030$
$A_{\text{SZ}}$	$0.97^{+0.68}_{-0.97}$	$t_0$	$13.75 \pm 0.13$ Gyr
$\tau$	$0.088 \pm 0.015$	$\theta_*$	$0.010388 \pm 0.000027$
$\theta_*$	$0.5952 \pm 0.0016$ °	$t_*$	$379164^{+5187}_{-5243}$ yr
$z_{\text{dec}}$	$1088.2 \pm 1.2$	$z_d$	$1020.3 \pm 1.4$
$z_{\text{eq}}$	$3196^{+134}_{-133}$	$z_{\text{reion}}$	$10.5 \pm 1.2$
$z_*$	$1090.79^{+0.94}_{-0.92}$		

# Constraints on the cosmological parameters from Planck observations (final results)

2010-2014: The Planck satellite



Credit: ESA

Parameter	TT+lowE 68% limits	TE+lowE 68% limits	EE+lowE 68% limits	TT,TE,EE+lowE 68% limits	TT,TE,EE+lowE+lensing 68% limits	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$	$0.02212 \pm 0.00022$	$0.02249 \pm 0.00025$	$0.0240 \pm 0.0012$	$0.02236 \pm 0.00015$	$0.02237 \pm 0.00015$	$0.02242 \pm 0.00014$
$\Omega_c h^2$	$0.1206 \pm 0.0021$	$0.1177 \pm 0.0020$	$0.1158 \pm 0.0046$	$0.1202 \pm 0.0014$	$0.1200 \pm 0.0012$	$0.11933 \pm 0.00091$
$100\theta_{MC}$	$1.04077 \pm 0.00047$	$1.04139 \pm 0.00049$	$1.03999 \pm 0.00089$	$1.04090 \pm 0.00031$	$1.04092 \pm 0.00031$	$1.04101 \pm 0.00029$
$\tau$	$0.0522 \pm 0.0080$	$0.0496 \pm 0.0085$	$0.0527 \pm 0.0090$	$0.0544^{+0.0070}_{-0.0081}$	$0.0544 \pm 0.0073$	$0.0561 \pm 0.0071$
$\ln(10^{10} A_s)$	$3.040 \pm 0.016$	$3.018^{+0.020}_{-0.018}$	$3.052 \pm 0.022$	$3.045 \pm 0.016$	$3.044 \pm 0.014$	$3.047 \pm 0.014$
$n_s$	$0.9626 \pm 0.0057$	$0.967 \pm 0.011$	$0.980 \pm 0.015$	$0.9649 \pm 0.0044$	$0.9649 \pm 0.0042$	$0.9665 \pm 0.0038$
$H_0$ [km s <sup>-1</sup> Mpc <sup>-1</sup> ]	$66.88 \pm 0.92$	$68.44 \pm 0.91$	$69.9 \pm 2.7$	$67.27 \pm 0.60$	$67.36 \pm 0.54$	$67.66 \pm 0.42$
$\Omega_\Lambda$	$0.679 \pm 0.013$	$0.699 \pm 0.012$	$0.711^{+0.033}_{-0.026}$	$0.6834 \pm 0.0084$	$0.6847 \pm 0.0073$	$0.6889 \pm 0.0056$
$\Omega_m$	$0.321 \pm 0.013$	$0.301 \pm 0.012$	$0.289^{+0.026}_{-0.033}$	$0.3166 \pm 0.0084$	$0.3153 \pm 0.0073$	$0.3111 \pm 0.0056$
$\Omega_m h^2$	$0.1434 \pm 0.0020$	$0.1408 \pm 0.0019$	$0.1404^{+0.0034}_{-0.0039}$	$0.1432 \pm 0.0013$	$0.1430 \pm 0.0011$	$0.14240 \pm 0.00087$
$\Omega_m h^3$	$0.09589 \pm 0.00046$	$0.09635 \pm 0.00051$	$0.0981^{+0.0016}_{-0.0018}$	$0.09633 \pm 0.00029$	$0.09633 \pm 0.00030$	$0.09635 \pm 0.00030$
$\sigma_8$	$0.8118 \pm 0.0089$	$0.793 \pm 0.011$	$0.796 \pm 0.018$	$0.8120 \pm 0.0073$	$0.8111 \pm 0.0060$	$0.8102 \pm 0.0060$
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	$0.840 \pm 0.024$	$0.794 \pm 0.024$	$0.781^{+0.052}_{-0.060}$	$0.834 \pm 0.016$	$0.832 \pm 0.013$	$0.825 \pm 0.011$
$\sigma_8 \Omega_m^{0.25}$	$0.611 \pm 0.012$	$0.587 \pm 0.012$	$0.583 \pm 0.027$	$0.6090 \pm 0.0081$	$0.6078 \pm 0.0064$	$0.6051 \pm 0.0058$
$z_{re}$	$7.50 \pm 0.82$	$7.11^{+0.91}_{-0.75}$	$7.10^{+0.87}_{-0.73}$	$7.68 \pm 0.79$	$7.67 \pm 0.73$	$7.82 \pm 0.71$
$10^9 A_s$	$2.092 \pm 0.034$	$2.045 \pm 0.041$	$2.116 \pm 0.047$	$2.101^{+0.031}_{-0.034}$	$2.100 \pm 0.030$	$2.105 \pm 0.030$
$10^9 A_s e^{-2\tau}$	$1.884 \pm 0.014$	$1.851 \pm 0.018$	$1.904 \pm 0.024$	$1.884 \pm 0.012$	$1.883 \pm 0.011$	$1.881 \pm 0.010$
Age [Gyr]	$13.830 \pm 0.037$	$13.761 \pm 0.038$	$13.64^{+0.16}_{-0.14}$	$13.800 \pm 0.024$	$13.797 \pm 0.023$	$13.787 \pm 0.020$
$z_*$	$1090.30 \pm 0.41$	$1089.57 \pm 0.42$	$1087.8^{+1.6}_{-1.7}$	$1089.95 \pm 0.27$	$1089.92 \pm 0.25$	$1089.80 \pm 0.21$
$r_*$ [Mpc]	$144.46 \pm 0.48$	$144.95 \pm 0.48$	$144.29 \pm 0.64$	$144.39 \pm 0.30$	$144.43 \pm 0.26$	$144.57 \pm 0.22$
$100\theta_*$	$1.04097 \pm 0.00046$	$1.04156 \pm 0.00049$	$1.04001 \pm 0.00086$	$1.04109 \pm 0.00030$	$1.04110 \pm 0.00031$	$1.04119 \pm 0.00029$
$z_{drag}$	$1059.39 \pm 0.46$	$1060.03 \pm 0.54$	$1063.2 \pm 2.4$	$1059.93 \pm 0.30$	$1059.94 \pm 0.30$	$1060.01 \pm 0.29$
$r_{drag}$ [Mpc]	$147.21 \pm 0.48$	$147.59 \pm 0.49$	$146.46 \pm 0.70$	$147.05 \pm 0.30$	$147.09 \pm 0.26$	$147.21 \pm 0.23$
$k_D$ [Mpc <sup>-1</sup> ]	$0.14054 \pm 0.00052$	$0.14043 \pm 0.00057$	$0.1426 \pm 0.0012$	$0.14090 \pm 0.00032$	$0.14087 \pm 0.00030$	$0.14078 \pm 0.00028$
$z_{eq}$	$3411 \pm 48$	$3349 \pm 46$	$3340^{+81}_{-92}$	$3407 \pm 31$	$3402 \pm 26$	$3387 \pm 21$
$k_{eq}$ [Mpc <sup>-1</sup> ]	$0.01041 \pm 0.00014$	$0.01022 \pm 0.00014$	$0.01019^{+0.00025}_{-0.00028}$	$0.010398 \pm 0.000094$	$0.010384 \pm 0.000081$	$0.010339 \pm 0.000063$
$100\theta_{s,eq}$	$0.4483 \pm 0.0046$	$0.4547 \pm 0.0045$	$0.4562 \pm 0.0092$	$0.4490 \pm 0.0030$	$0.4494 \pm 0.0026$	$0.4509 \pm 0.0020$
$f_{2000}^{143}$	$31.2 \pm 3.0$			$29.5 \pm 2.7$	$29.6 \pm 2.8$	$29.4 \pm 2.7$
$f_{2000}^{143 \times 217}$	$33.6 \pm 2.0$			$32.2 \pm 1.9$	$32.3 \pm 1.9$	$32.1 \pm 1.9$
$f_{2000}^{217}$	$108.2 \pm 1.9$			$107.0 \pm 1.8$	$107.1 \pm 1.8$	$106.9 \pm 1.8$