

The Age of Universe...

and

deriving the Hubble constant

(Lecture 2)

Layout of the Course

This Week

Feb 5: Introduction / Overview / General Concepts

Feb 12: Age of Universe / Distance Ladder

Feb 19: Distance Ladder / Hubble Constant

Feb 26: Distant Measures / SNe science / Baryonic Content

Mar 4: Dark Matter Content of Universe / Cosmic Microwave Background

Mar 11: Cosmic Microwave Background

Mar 18: Cosmic Microwave Background / Large Scale Structure

Mar 25: Baryon Acoustic Oscillations / Dark Energy / Clusters

Apr 1: No Class

Apr 8: Clusters / Cosmic Shear

Apr 15: Dark Energy Missions / Review for Final Exam

May 13: Final Exam

Problem Set #1

Will make available on
BrightSpace by Monday
evening!

Due Sunday, February 25, 2023

Review Material from Last Week

Standard Cosmological Model

Founded on Two Principles:

- 1) Einstein's general theory of relativity...
- 2) general cosmological principle
(assume homogeneous and isotropic universe)

Second principle allows us to set up a metric -- the Friedmann-Robertson-Walker metric -- for measuring distances in space time:

$$ds^2 = c^2 dt^2 - R^2 \left\{ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right\}$$

Friedmann-Robertson-Walker metric

$R \propto$ size of universe
 R = “scale factor”

Other coordinates r, θ, ϕ
are comoving..

$k = 0, -1, +1$
flat, closed, open

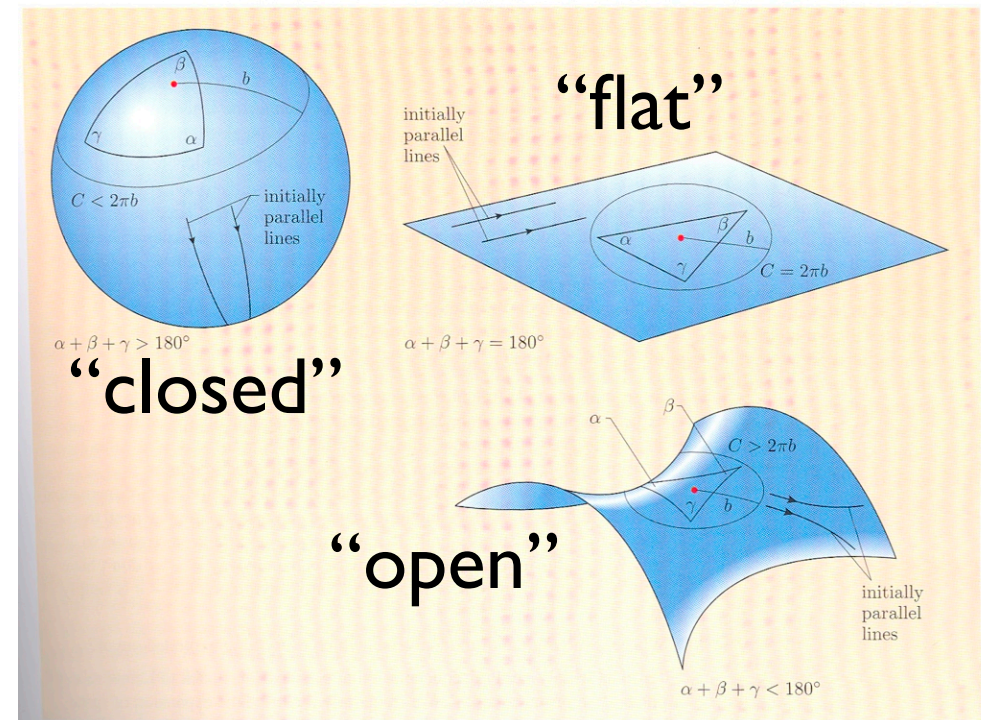
Options for the different geometries:

Different topologies:

$k = -1$ (open)

$k = 0$ (flat)

$k = +1$ (closed)



Take Einstein's field equation

Plug in the Friedmann-Robertson-Walker metric

This gives the following Friedmann equations....

$$\dot{R}^2 = \frac{8\pi G(\rho_m + \rho_r)R^2}{3} - kc^2 + \frac{\Lambda c^2 R^2}{3}$$

$$\ddot{R} = -4\pi G \left(\rho_m + \rho_r + \frac{3p}{c^2} \right) \frac{R}{3} + \frac{\Lambda c^2 R}{3}$$

An important number is the critical density:

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

$\rho_m > \rho_{crit} \Rightarrow$ Universe eventually recollapses

$\rho_m = \rho_{crit} \Rightarrow$ Universe expands forever

$\rho_m < \rho_{crit} \Rightarrow$ Universe expands forever

However for convenience, astronomers introduce a dimensionless quantity to describe these densities $\rho_m, \rho_r, \rho_\Lambda$ relative to the critical density:

$$\Omega_m = \frac{\rho_m}{\rho_{crit}}$$

$$\Omega_r = \frac{\rho_r}{\rho_{crit}}$$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{crit}}$$

$\Omega_m > 1 \Rightarrow$ Universe eventually recollapses

$\Omega_m = 1 \Rightarrow$ Universe expands forever

$\Omega_m < 1 \Rightarrow$ Universe expands forever

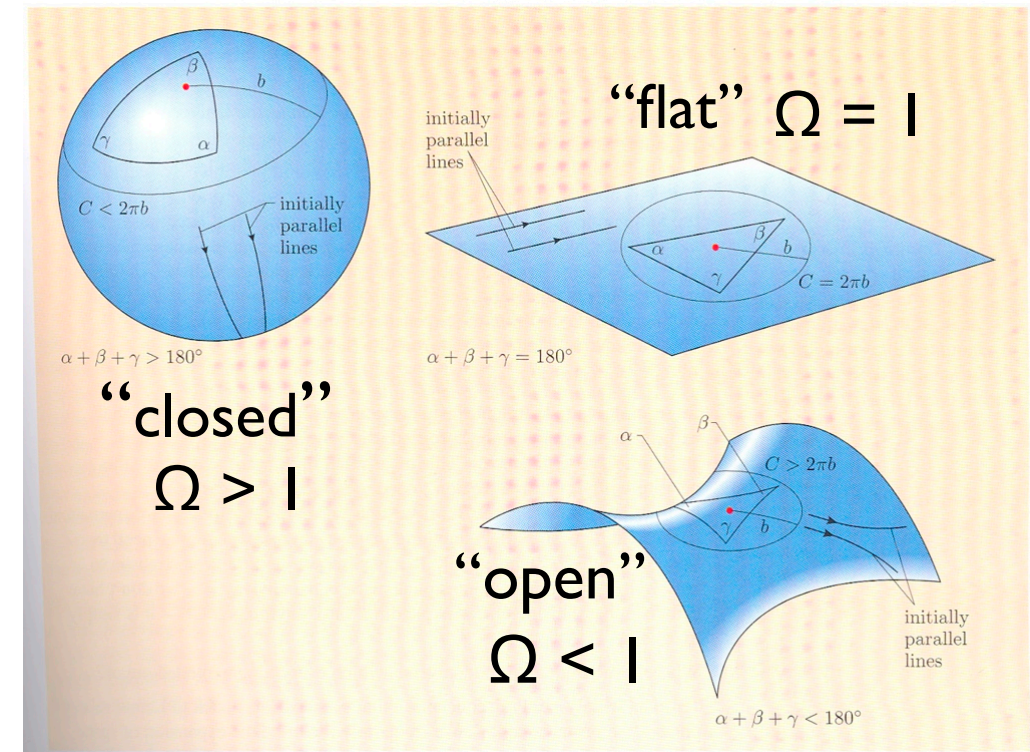
But for $\Omega_\Lambda \neq 0$, discussion is more complicated

$\Omega = \Omega_m + \Omega_r + \Omega_\Lambda$ determines geometry of universe

$\Omega > 1 \Rightarrow$ Universe is closed ($k = +1$)

$\Omega = 1 \Rightarrow$ Universe is flat ($k=0$)

$\Omega < 1 \Rightarrow$ Universe is open ($k=-1$)



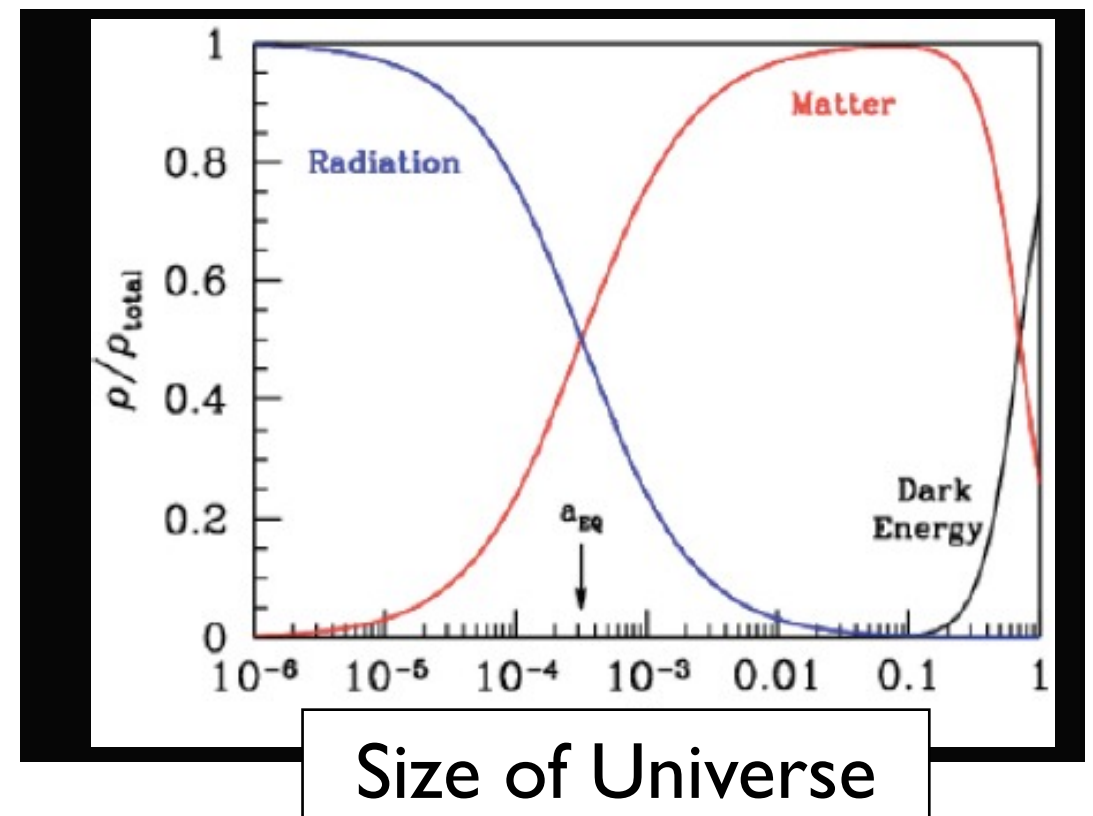
The expansion of the universe itself has an effect on the importance of the role or importance of each of the contents of the universe...

In particular, the density of these components of the universe scale as follows with respect to the size of the universe R :

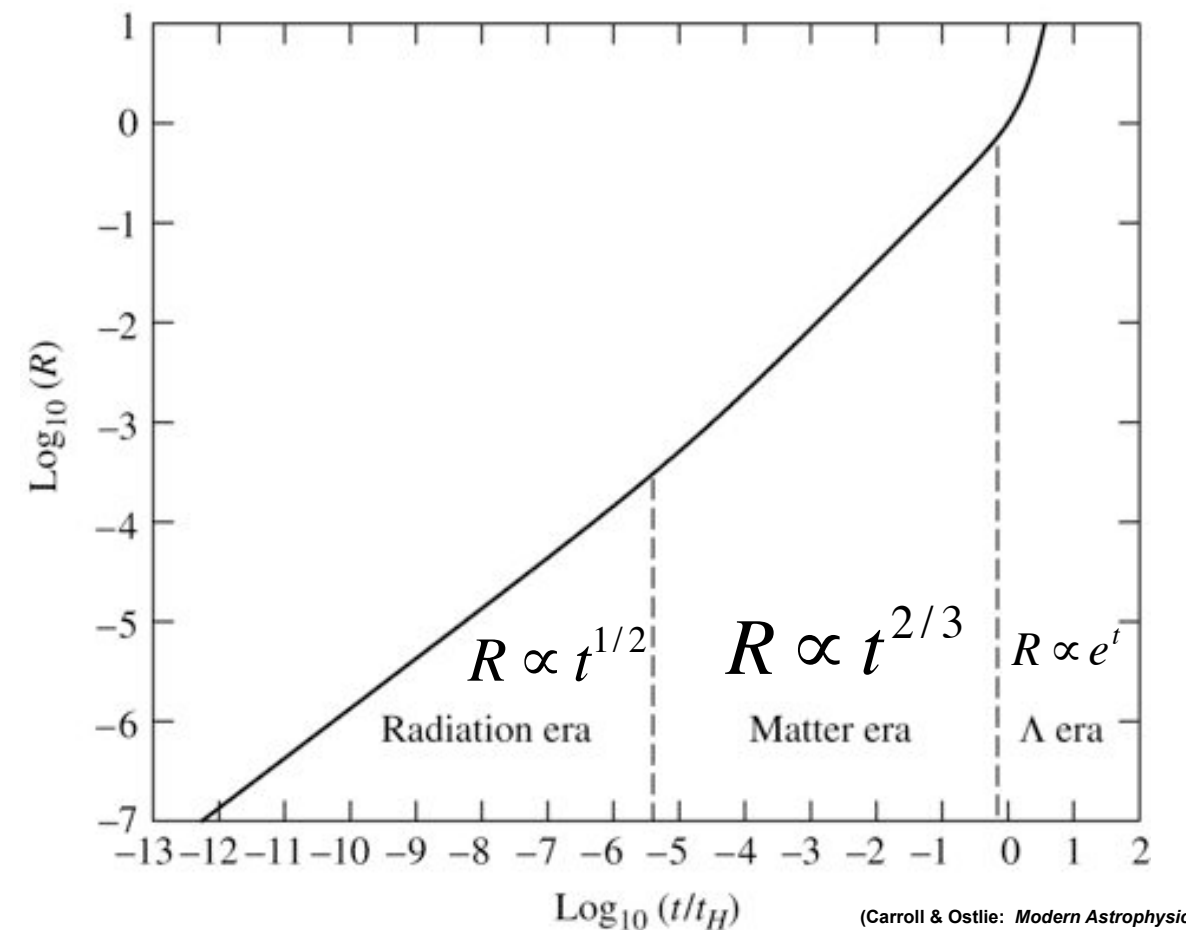
$$\rho_m \propto R^{-3}$$

$$\rho_r \propto R^{-4}$$

$$\Lambda = \text{const}$$



The expansion rate of the universe shows a different time dependence depending on which component of the universe dominates the energy density...



What are the key parameters we hope to determine in observational cosmology?

H_0
 Ω_M
 Ω_Λ
 Ω_b
 Ω_r
 n_s
 σ_8

Hubble Constant
 Matter Density in Matter
 Density of Dark Energy
 Matter Density in Baryons
 Energy Density in Radiation
 Slope of Primordial Power Spectrum
 RMS fluctuations of the mass density in spheres of $8h^{-1}$ Mpc

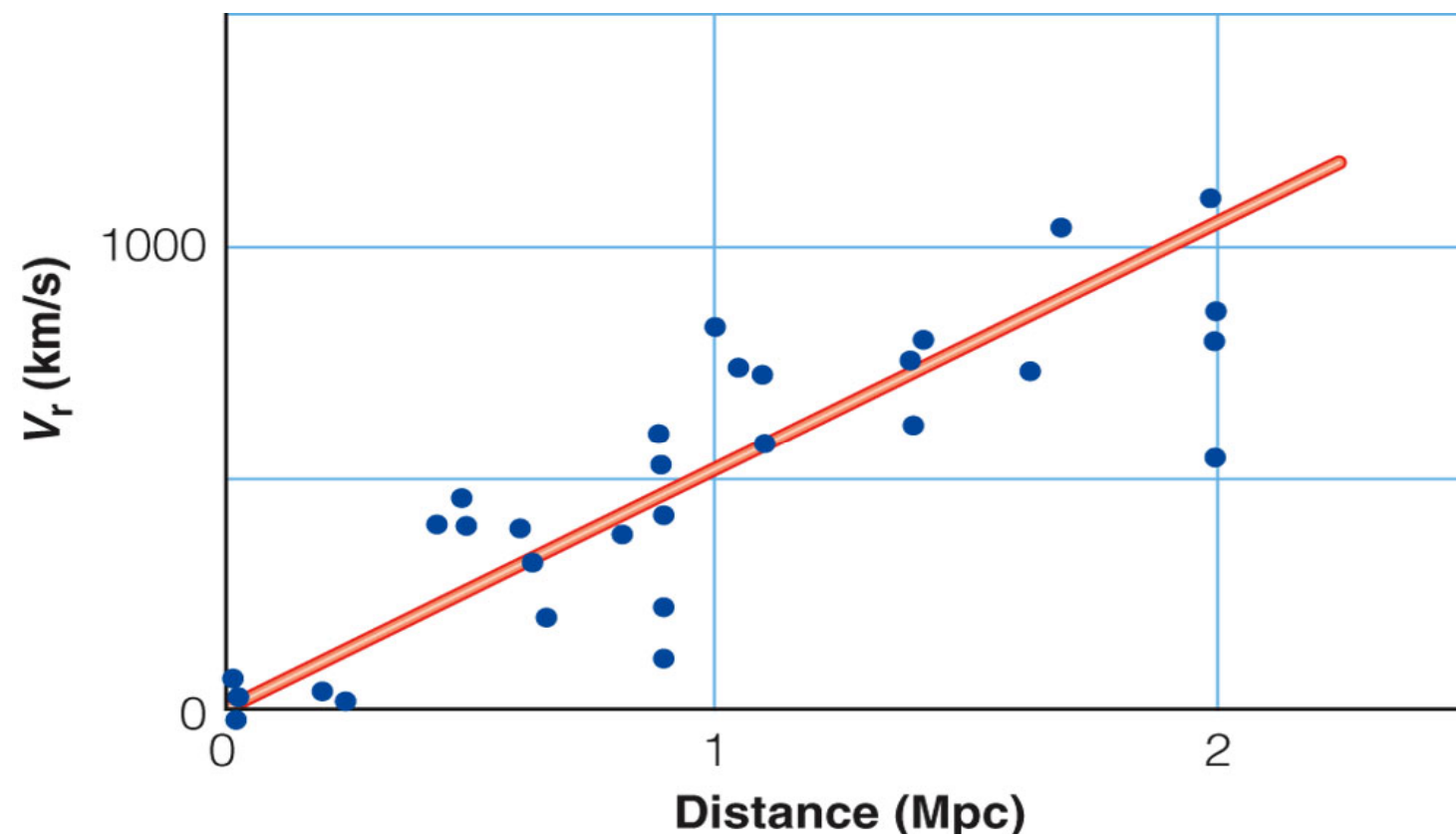
How do we determine what the Age of Universe is?

From the expansion rate of the universe

In 1929, Hubble showed that the velocities and distances are linearly correlated, and satisfy

$$v = H_0 d$$

where v is the recessional velocity (km/s) and d is the distance (Mpc). H_0 is a constant, “Hubble’s Constant” and has units of $\text{km s}^{-1} \text{ Mpc}^{-1}$.



In the simplest case that there is no acceleration in the expansion of the universe:

$$\text{Age of Universe} \sim 1/H_0$$

Now onto new material...
(With a repeat of the end of lecture I)

Age of Universe

(using solutions to Friedmann's equations)

$$\begin{aligned} t &= \int_0^t dt' \\ &= \int_0^{R_0} \frac{dR}{\dot{R}} \\ &= \int_0^{R_0} \frac{dR}{R(\dot{R}/R)} \\ &= \int_0^{R_0} \frac{dR}{RH(R)} \end{aligned}$$

R = size of universe

$H(R)$ = Hubble “constant” for
the universe with size R

In search of an equation for H(R):

One of the two Friedmann's equations:

$$\dot{R}^2 = \frac{8\pi G(\rho_m + \rho_r)R^2}{3} - kc^2 + \frac{\Lambda c^2 R^2}{3}$$

Divide by R^2

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G(\rho_m + \rho_r)}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{R^2}$$

Factor out H_0^2

$$H^2 = H_0^2 \left[\frac{8\pi G\rho_m}{3H_0^2} + \frac{8\pi G\rho_r}{3H_0^2} + \frac{\Lambda c^2}{3H_0^2} - \frac{kc^2}{R^2 H_0^2} \right]$$

Express $\rho_m = \rho_{m,0} (R_0/R)^3$,
 $\rho_r = \rho_{r,0} (R_0/R)^4$

$$H^2 = H_0^2 \left[\frac{8\pi G\rho_{m,0}}{3H_0^2} \frac{R_0^3}{R^3} + \frac{8\pi G\rho_{r,0}}{3H_0^2} \frac{R_0^4}{R^4} + \frac{\Lambda c^2}{3H_0^2} - \frac{kc^2}{R^2 H_0^2} \right]$$

In search of an equation for H(R):

$$H^2 = H_0^2 \left[\frac{8\pi G \rho_{m,0}}{3H_0^2} \frac{R_0^3}{R^3} + \frac{8\pi G \rho_{r,0}}{3H_0^2} \frac{R_0^4}{R^4} + \frac{\Lambda c^2}{3H_0^2} - \frac{kc^2}{R^2 H_0^2} \right]$$

Express using
 $\Omega_m, \Omega_r, \Omega_\Lambda,$
 Ω_k

$$\Omega_m = \frac{8\pi G \rho_m}{3H^2} \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H^2}$$

$$\Omega_r = \frac{8\pi G \rho_r}{3H^2} \quad \Omega_k = \frac{-kc^2}{H^2}$$

$$H(R)^2 = H_0^2 \left[\Omega_{m,0} (R/R_0)^{-3} + \Omega_{r,0} (R/R_0)^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0} (R/R_0)^{-2} \right]$$

$$H(R)^2 = H_0^2 E^2(R) \quad \text{where} \quad E^2(R) = [\Omega_{m,0} (R/R_0)^{-3} + \Omega_{r,0} (R/R_0)^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0} (R/R_0)^{-2}]$$

$$H(R) = H_0 E(R) \quad \text{where} \quad E(R) = [\Omega_{m,0} (R/R_0)^{-3} + \Omega_{r,0} (R/R_0)^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0} (R/R_0)^{-2}]^{1/2}$$

Age of Universe

(using solutions to Friedmann's equations)

$$\begin{aligned} t &= \int_0^t dt' \\ &= \int_0^{R_0} \frac{dR}{\dot{R}} \\ &= \int_0^{R_0} \frac{dR}{R(\dot{R}/R)} \\ &= \int_0^{R_0} \frac{dR}{RH(R)} \\ &= \frac{1}{H_0} \int_0^{R_0} \frac{dR}{RE(R)} \end{aligned}$$

\swarrow $H(R) = H_0 E(R)$

Age of Universe

(using solutions to Friedmann's equations)

$$H(R) = H_0(R/R_0)^{-3/2}$$

Consider Empty Universe:

$$\Omega_m = 0, \Omega_r = 0, \Omega_\Lambda = 0:$$

$$E(R) = [\Omega_{m,0}(R/R_0)^{-3} + \Omega_{r,0}(R/R_0)^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0}(R/R_0)^{-2}]^{1/2}$$

$$E(R) = [\Omega_k(R/R_0)^{-2}]^{1/2} \quad \Omega_k = 1 - \Omega_m - \Omega_r - \Omega_\Lambda = 1$$

$$E(R) = (R/R_0)^{-1}$$

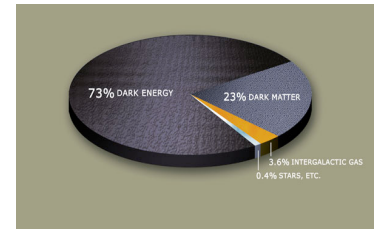
$$= \frac{1}{H_0} \int_0^{R_0} \frac{dR}{R E(R)} = (1/H_0)(R/R_0) \Big|_{R=0}^{R=R_0} = 1/H_0$$

Universe Expands at Same Rate for all of Cosmic Time!

How would things differ if we
include the other two main
components of the universe?

Age of Universe

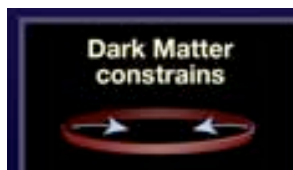
(using solutions to Friedmann's equations)



Matter Ω_m

(Baryons + Dark Matter)

“Opposes Expansion”



$$t \stackrel{?}{=} 1/H_0$$

$$t < 1/H_0$$

Dark Energy Ω_Λ

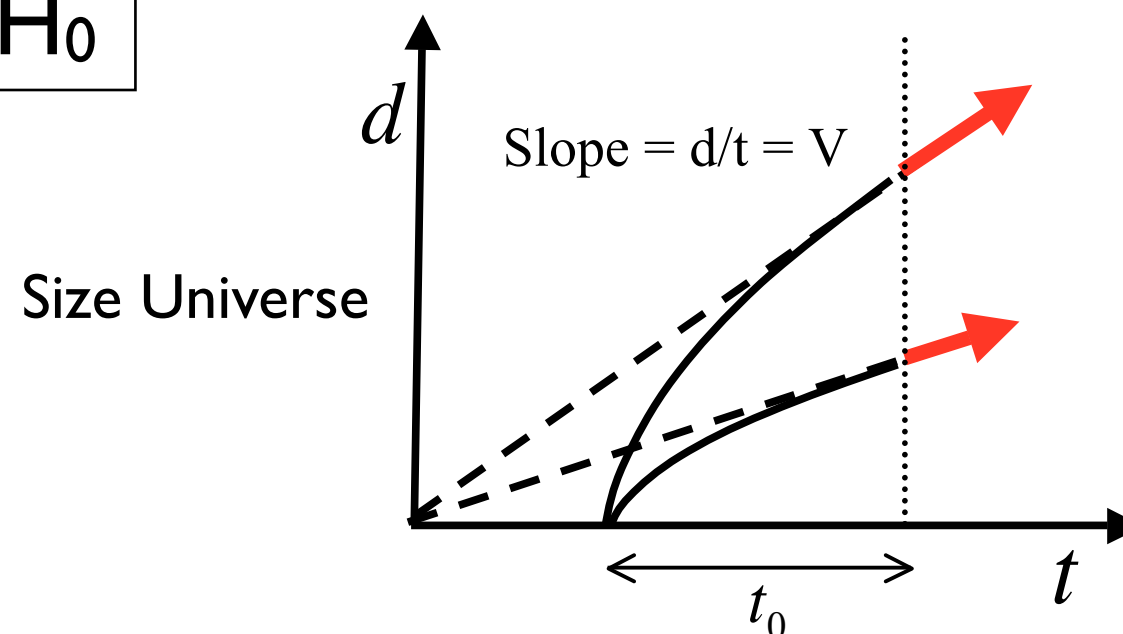
(Vacuum Energy)

“Speeds Expansion”



$$t \stackrel{?}{=} 1/H_0$$

$$t > 1/H_0$$



Age of Universe

(using solutions to Friedmann's equations)

Consider Flat Matter-Only Universe:

$$\Omega_m = 1, \Omega_r = 0, \Omega_\Lambda = 0:$$

$$E(R) = [\Omega_{m,0}(R/R_0)^{-3} + \Omega_{r,0}(R/R_0)^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0}(R/R_0)^{-2}]^{1/2}$$

$$E(R) = (R/R_0)^{-3/2}$$

$$t = \frac{1}{H_0} \int_0^{R_0} \frac{dR}{R E(R)} = \int_0^{R_0} \frac{R^{1/2} dR}{(R_0)^{3/2} H_0} = \frac{2(R/R_0)^{3/2}}{3H_0} = \frac{2}{3H_0}$$

Age of Universe

(using solutions to Friedmann's equations)

Consider Flat Radiation-Only Universe:

$$\Omega_m = 0, \Omega_r = 1, \Omega_\Lambda = 0:$$

$$E(R) = [\Omega_{m,0}(R/R_0)^{-3} + \Omega_{r,0}(R/R_0)^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0}(R/R_0)^{-2}]^{1/2}$$

$$E(R) = (R/R_0)^{-2}$$

$$t = \frac{1}{H_0} \int_0^{R_0} \frac{dR}{R E(R)} = 1/(2H_0)$$

Age of Universe

(using solutions to Friedmann's equations)

Consider Dark Energy-Only Universe:

$$\Omega_m = 0, \Omega_r = 0, \Omega_\Lambda = 1:$$

$$E(R) = [\Omega_{m,0}(R/R_0)^{-3} + \Omega_{r,0}(R/R_0)^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0}(R/R_0)^{-2}]^{1/2}$$

$$E(R) : 1$$

For the HW
problems...

Age of Universe

(using solutions to Friedmann's equations)

In empty universe (no change in expansion rate):

$$\text{Age of Universe} = 1/H_0$$

In matter-dominated universe (where gravity slows expansion):

$$\text{Age of Universe} < 1/H_0$$

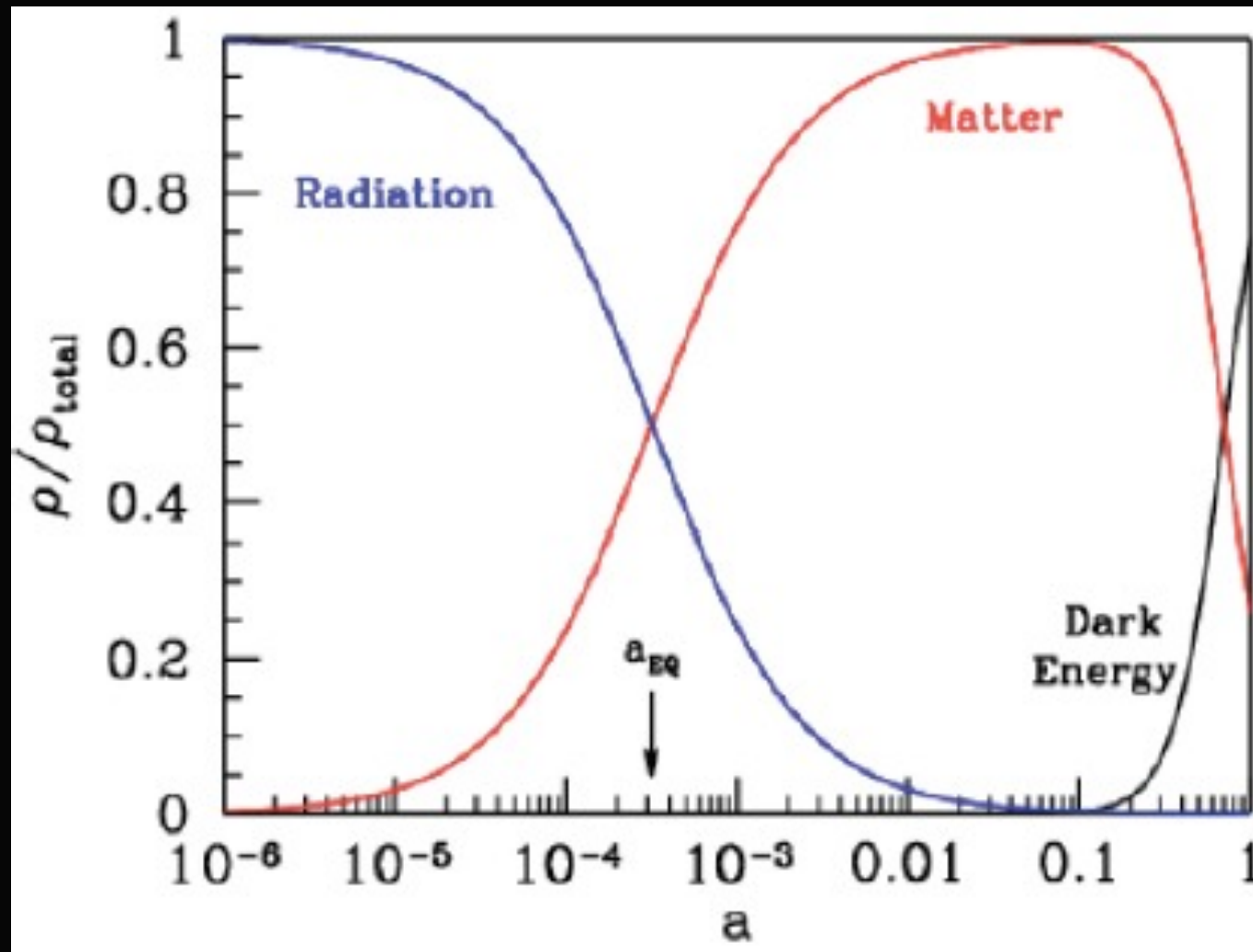
In dark energy dominated universe (where dark energy speeds expansion):

$$\text{Age of Universe} > 1/H_0$$

In all cases:

$$\text{Age of Universe} \sim 1/H_0$$

The real universe is more complicated than these simple cases...



As Consequently, we have both deceleration of the expansion of the universe (at early times) and an acceleration of the expansion of the universe...

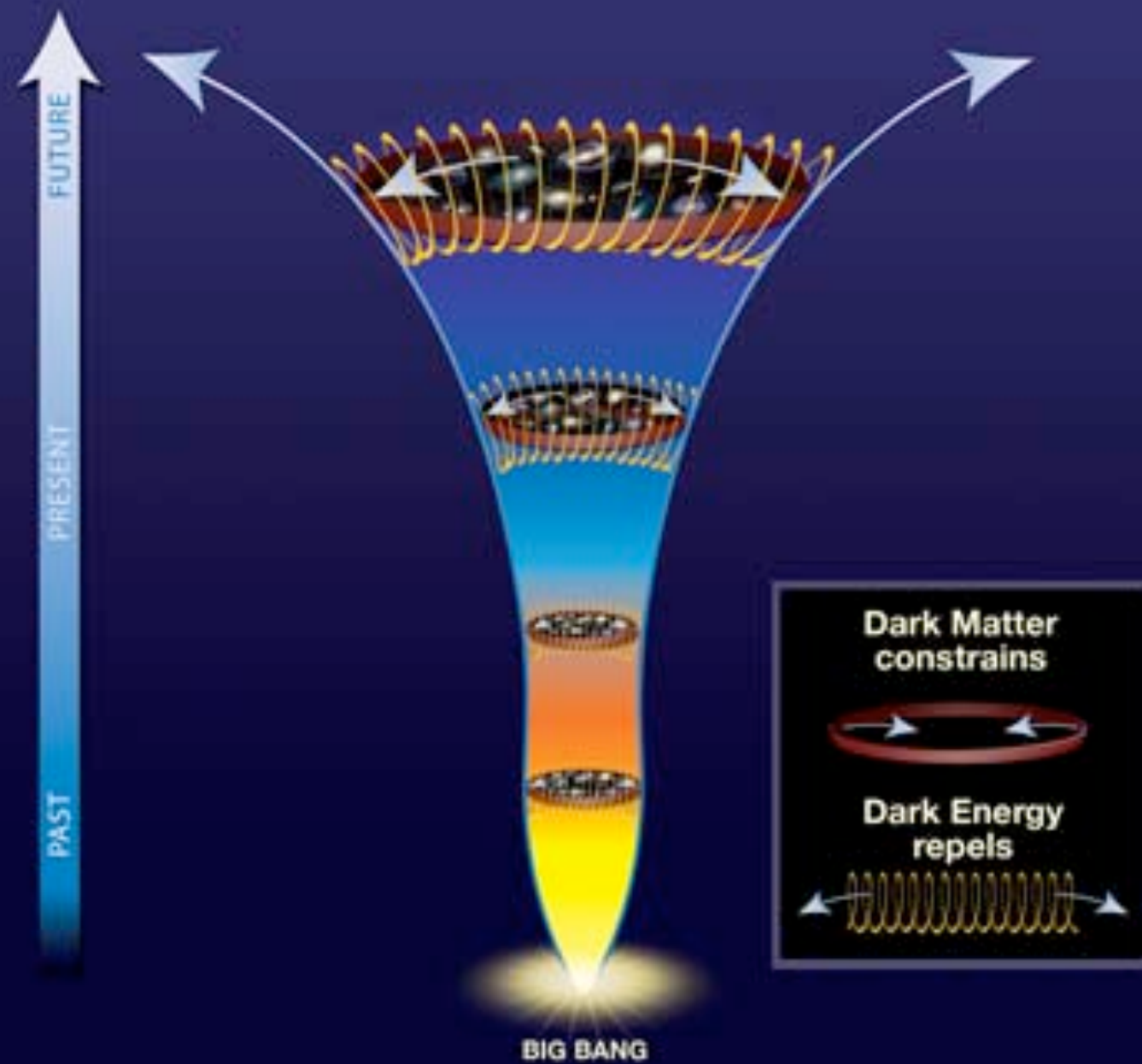
Therefore, age of universe $\sim 1/H_0$

be slowed due to self gravity from matter + radiation

We would then expect the expansion to accelerate due to the dark energy

Cosmic tug of war

The force of dark energy surpasses that of dark matter as time progresses.



Age of Universe

(From Direct Observational
Evidence)

Age of the Universe (From Radioactive Decay)

Age of the Universe (From Radioactive Decay)

-- We have a rough understanding of how the heavy elements (heavier than Iron) are created...

Created through r-process events (likely in supernovae)

Created through s-process events (likely in AGB stars)

-- We also have predictions for the abundances we would expect for the different elements and isotopes from these processes

-- Of course, some of these elements or isotopes will be unstable and suffer radioactive decay. By measuring the abundance of these elements or isotopes at some later time t and compare their relative abundance, we can measure how much time has passed.

$$N_{238} = N_{238,0} \times e^{-\lambda t}$$

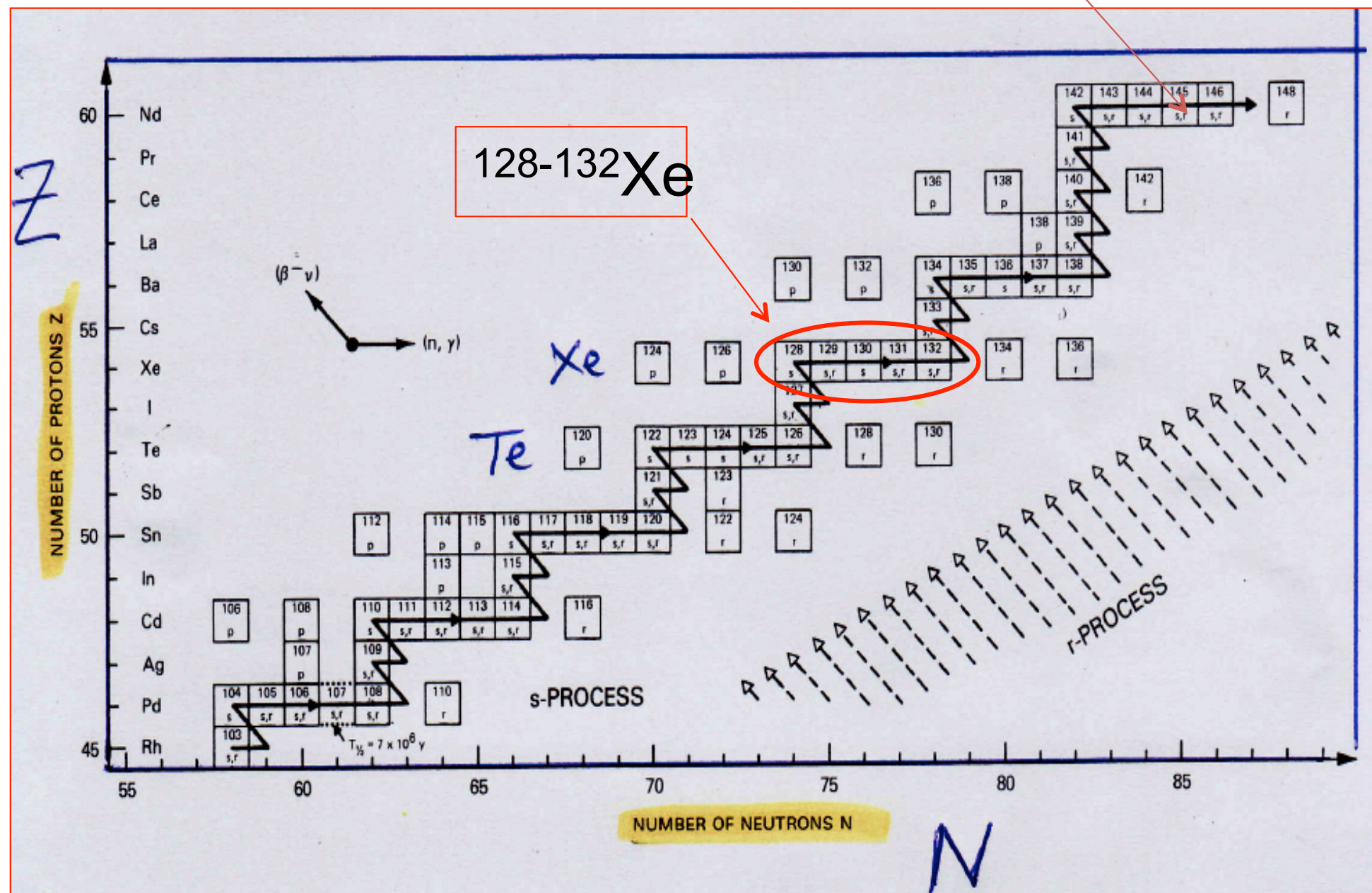
Abundance
when observed

Initial
Abundance

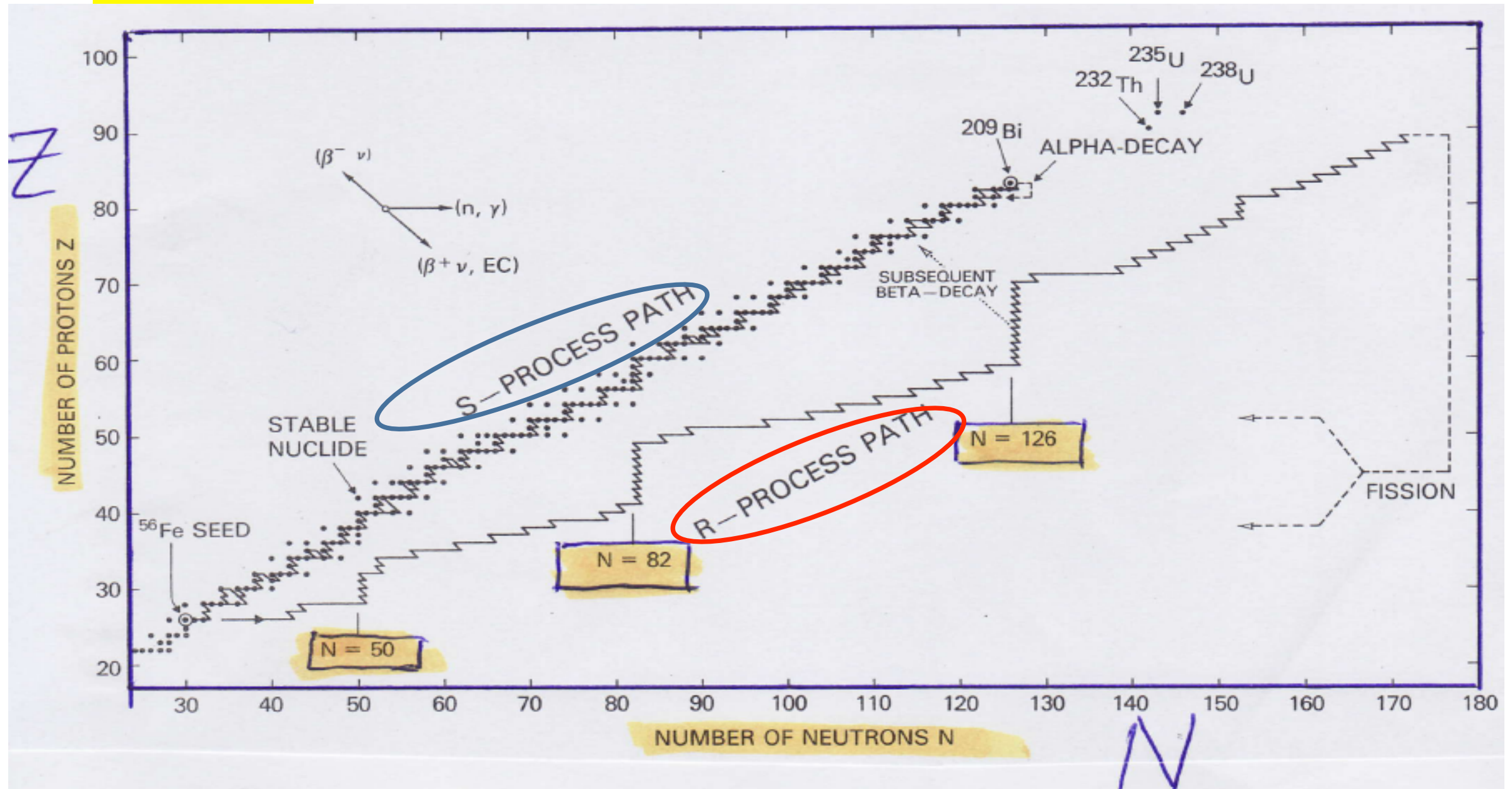
Fraction surviving

(n, γ) reactions increases the mass number (A) . However, if the isotope is unstable , then subsequent process depends on the neutron flux and the life time (τ)

(a) $\tau_{n\gamma} \gg \tau_{\beta}$ Standard s-process (beta decay wins the game). The produced nuclei are closed to the valley of beta stability (see



(b) $\tau_{n\gamma} \ll \tau_{\beta}$ (neutron capture wins the game, one deals with the r-process)



Beta decay lifetimes: $(10^{-3} - 10^{-4})$ s.

For $\tau_{n\gamma} = 10^{-4}$ s : $N_n = 3 \times 10^{20}$ neutron/cm³

Age of the Universe (From Radioactive Decay)

Age date universe



$^{232}\text{Th} \rightarrow ^{208}\text{Pb}$ (Thorium Series)
Half-life = 14 Gyr

$^{238}\text{U} \rightarrow ^{206}\text{Pb}$ (Radium series)
Half-life = 4.5 Gyr

$^{235}\text{U} \rightarrow ^{207}\text{Pb}$ (Actinium series)
Half-life = 0.7 Gyr

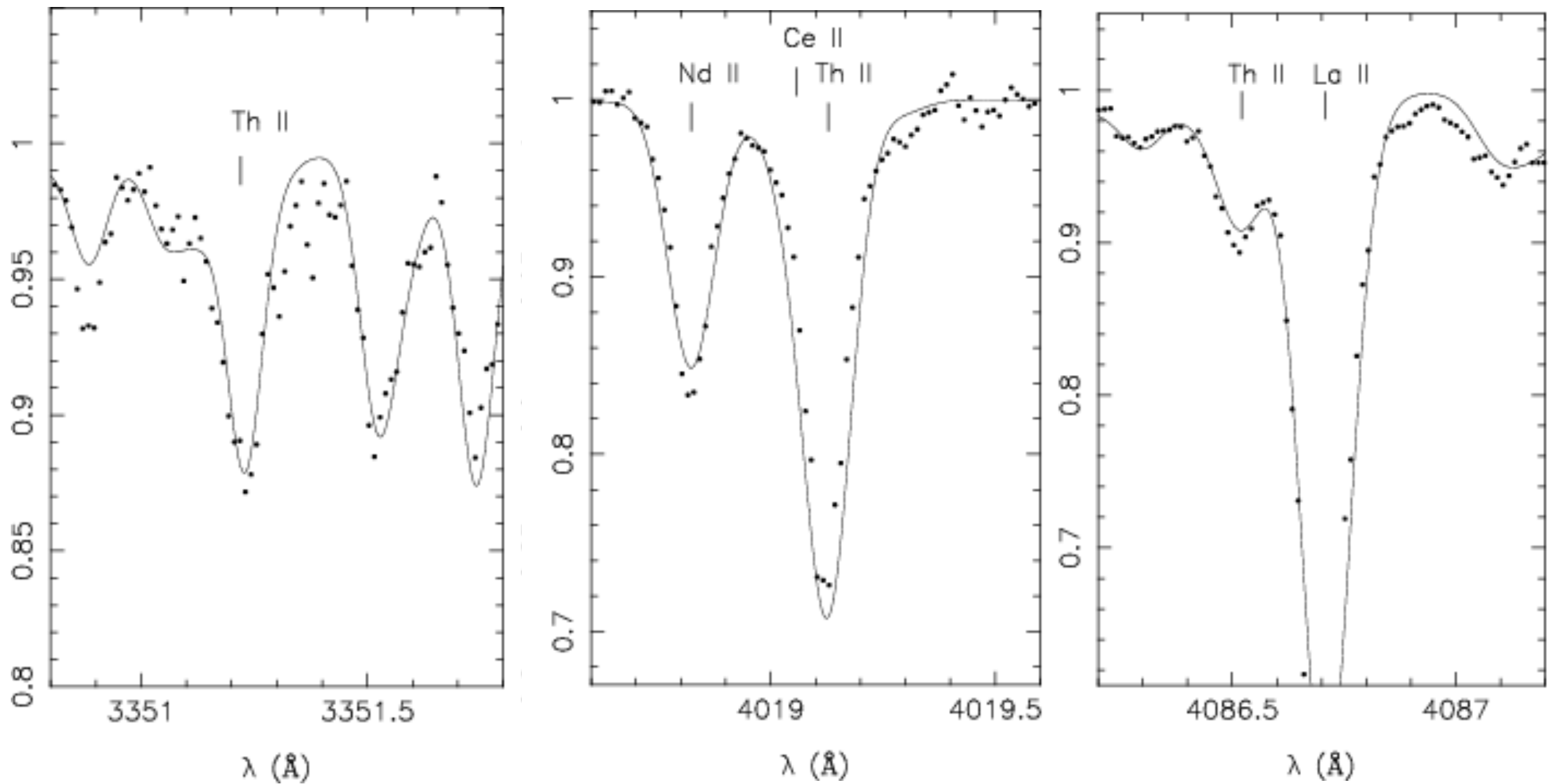


Age date earth

Age of the Universe (From Radioactive Decay)

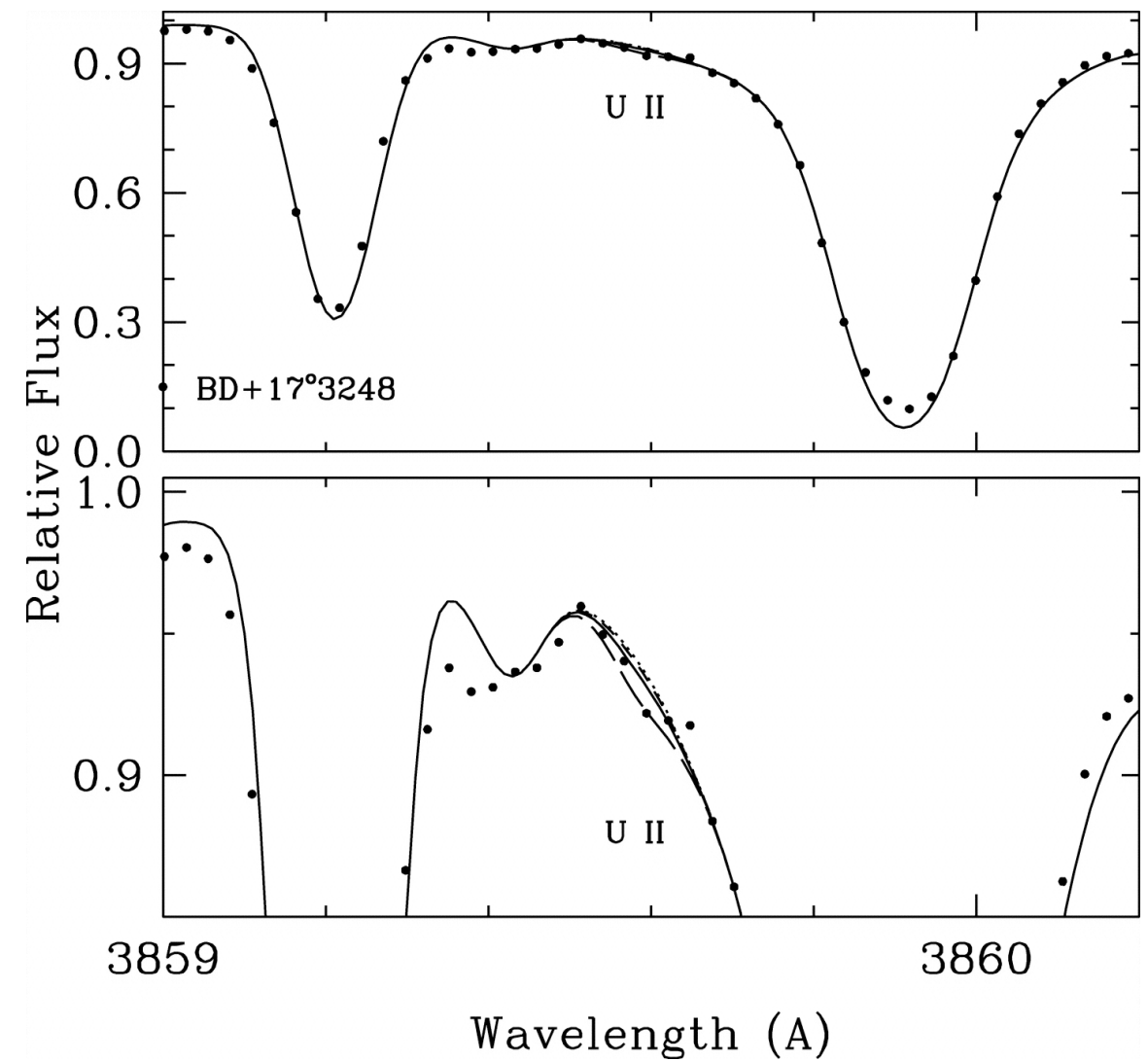
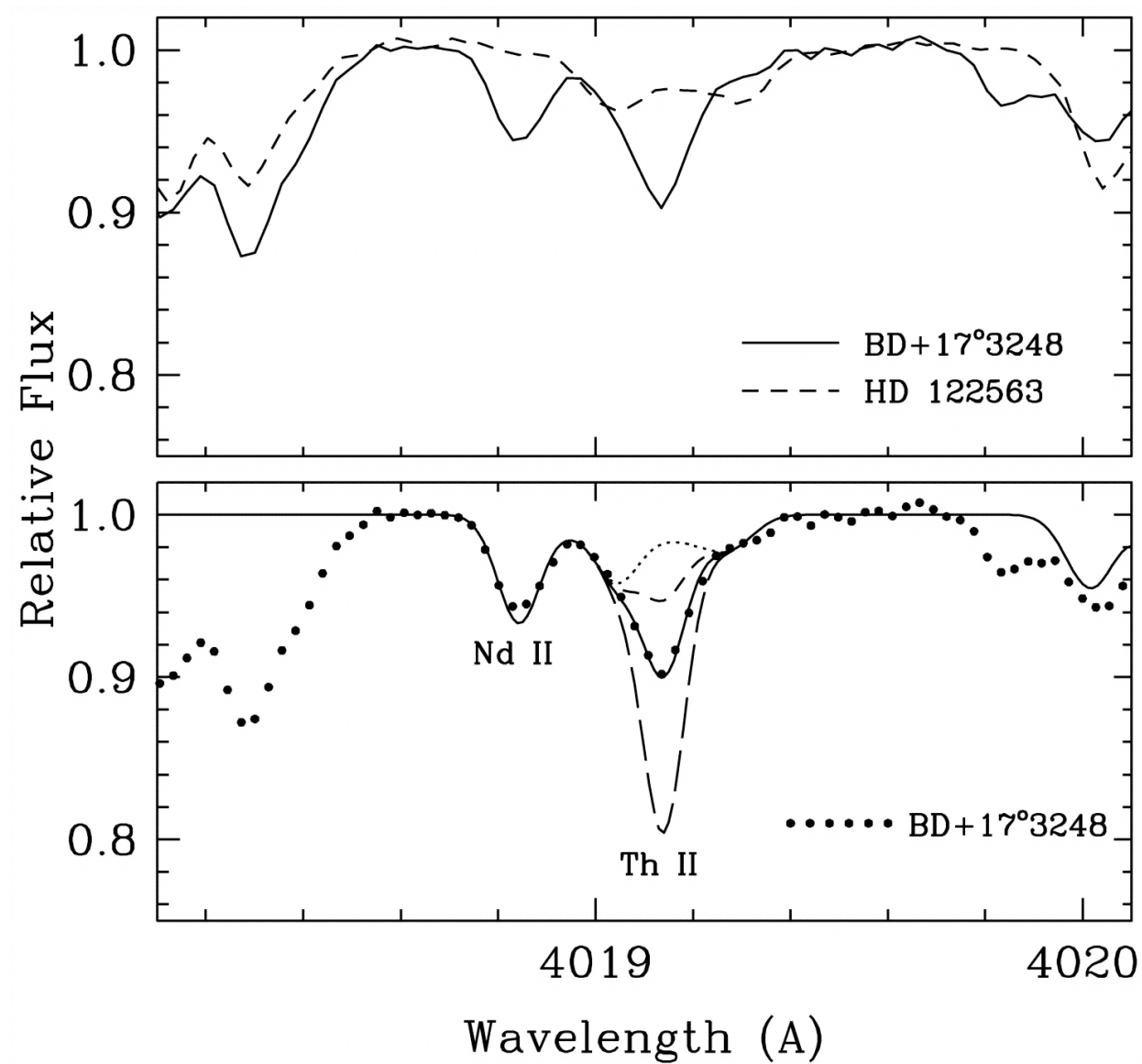
- Look for the signature of Uranium or Thorium in the spectra of old stars in our galaxy
- By measuring actual abundances, we have an estimate of the age -- given that we have theoretical predictions for the initial abundances of these elements.
- Spectral lines from Uranium and Thorium are relatively weak in general, so this is difficult
- In practice, one requires stars with extremely low metallicities, so the Thorium and Uranium lines are not buried under stronger metal lines
- Use of Thorium less ideal given its longer half life (14 Gyr). We expect overall abundance to drop by $\sim 2\times$ over age of universe.

Thorium Abundances of CS3 082-001 from Hill et al. (2002)



Sources like these have unusually weak Iron lines

Thorium and Uranium measurements of BD+17 3248 from Cowan et al. (2002)



Abundance ratios of BD+17 3248 from Cowan et al (2002)

TABLE 4
CHRONOMETRIC AGE ESTIMATES FOR BD +17°3248

Chronometer Pair	Predicted	Observed	Age (Gyr)	Solar ^a	Lower Limit (Gyr)
Th/Eu	0.507	0.309	10.0	0.4615	8.2
Th/Ir	0.0909	0.03113	21.7	0.0646	14.8
Th/Pt	0.0234	0.0141	10.3	0.0323	16.8
Th/U	1.805	7.413	≥ 13.4	2.32	11.0
U/Ir	0.05036	0.0045	≥ 15.5	0.0369	13.5
U/Pt.....	0.013	0.0019	≥ 12.4	0.01846	14.6

^a From Burris et al. 2001.

Age $\sim 13.8 \pm 4$ Gyr (but spread is large)

For CS31082-0018:

Age $\sim 12.5 \pm 3$ Gyr (Cayrel et al. 2001)

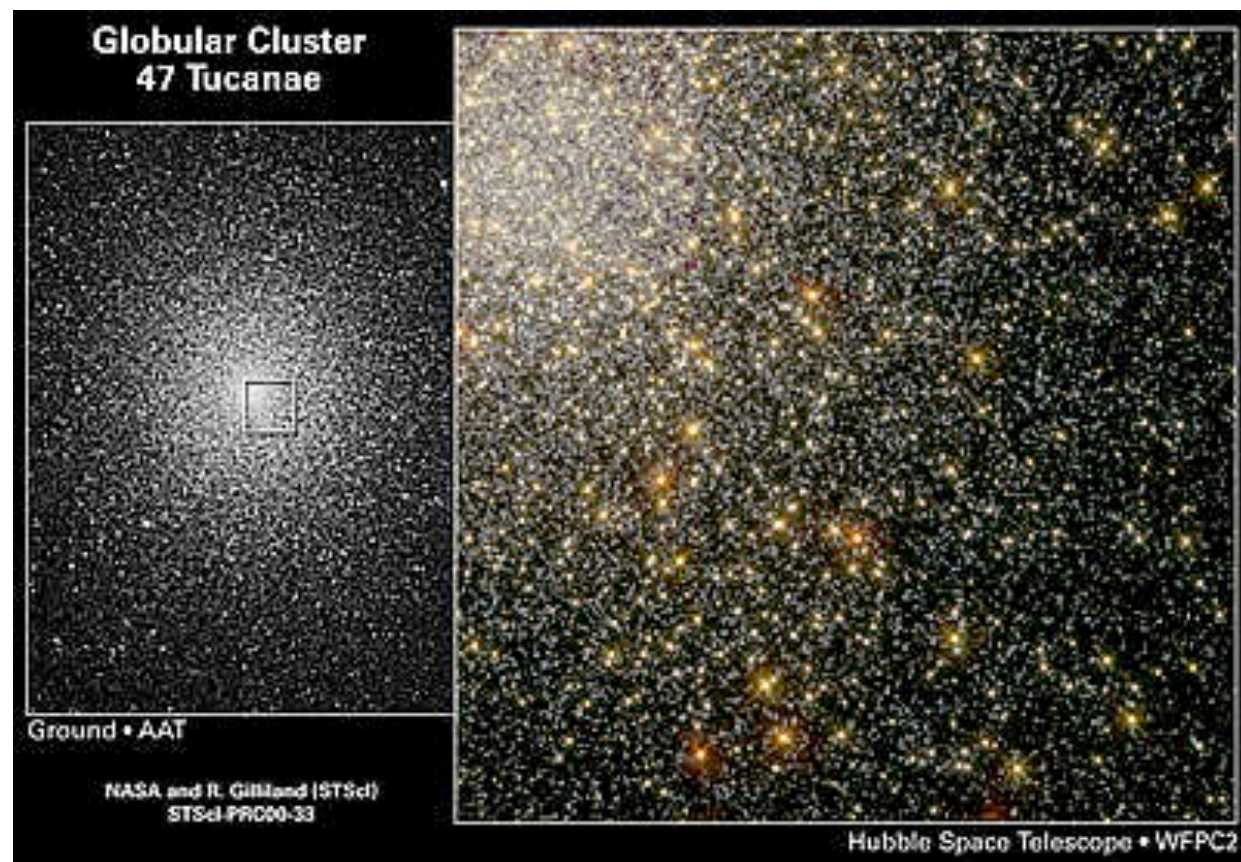
(Principal uncertainty is the production ratios)

For reference, radiometric dating to rocks we find on earth, moon, or in meteorites gives ages of ~ 4 -5 Gyrs. This is younger than the oldest stars in the Milky Way, but suggests that our solar system formed 4-5 Gyrs ago.

Age of the Universe (From Aging of Stars...)

Globular Clusters

- Very Compact, gravitationally bound star clusters
- Contain $\sim 10^4$ stars
- Generally very red, stars seem to have formed @ approximately the same time
- ~ 150 Globular Clusters in our own Milky Way



47 Tucanae

Globular Clusters

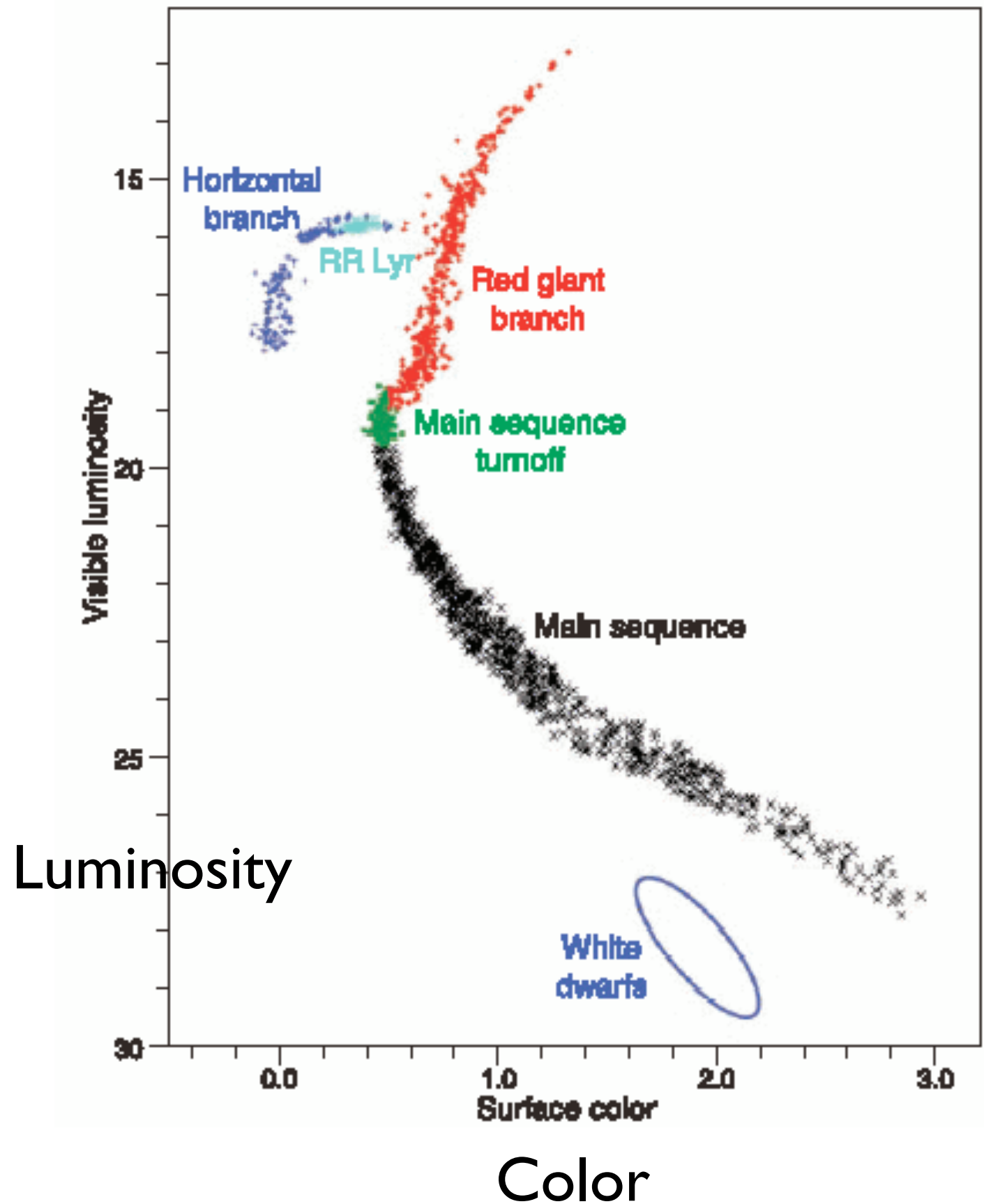
Since all stars in globular cluster seem to have formed at approximately same time....

Since evolution of Stars on the Main Sequence very well understood, we can use the position of the main sequence turn-off (MSTO) to age date a globular cluster

$$L \propto M^4 \Rightarrow t_{\text{(MSTO)}} \propto M^{-3} \propto L^{-3/4}$$

Most massive stars arrive at turn-off first:

Lower luminosity turn-offs \Rightarrow Older



Globular Clusters

Biggest uncertainty in age dating a globular cluster derives from its estimated distance

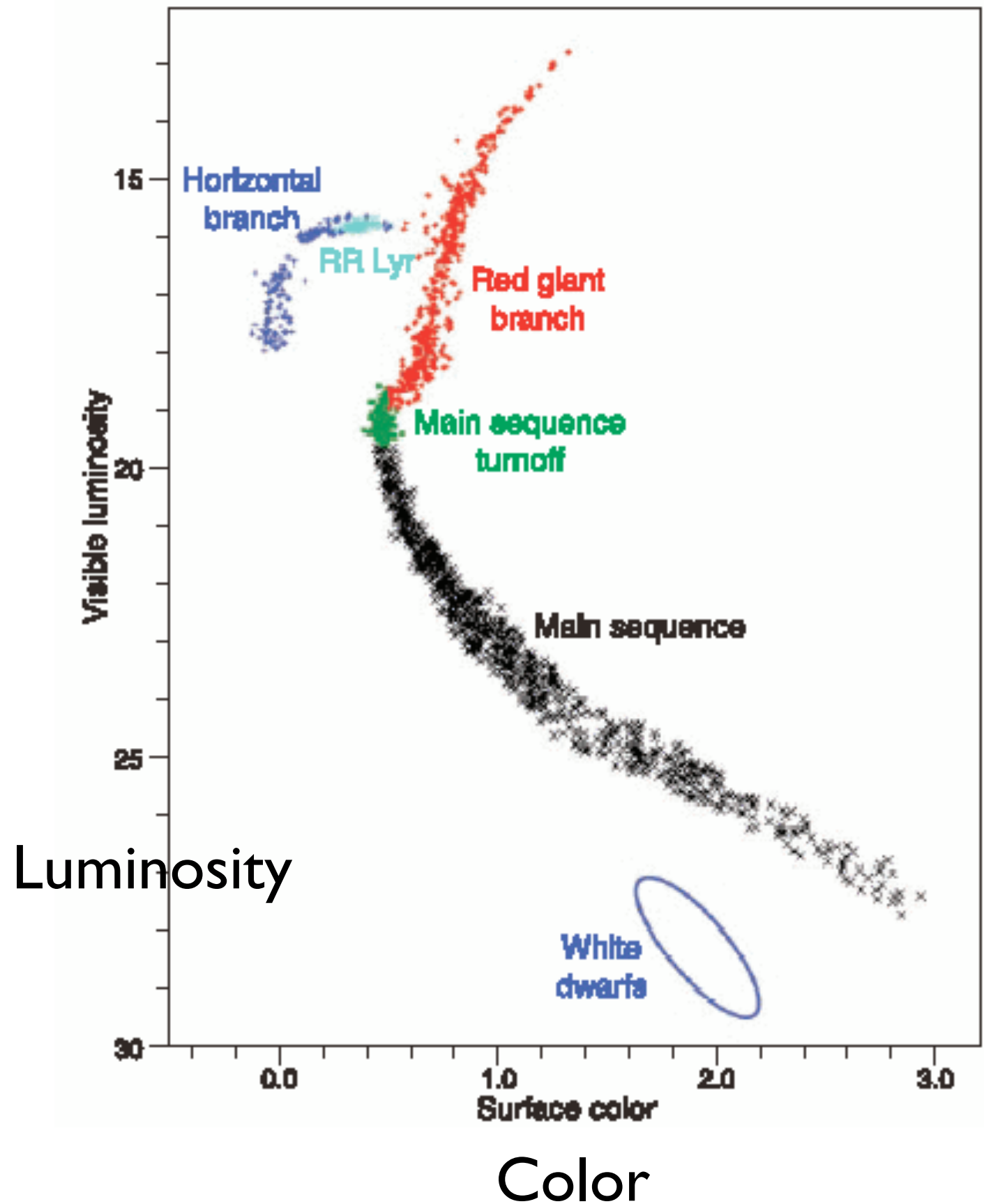
Use RR Lyrae stars to calculate distance to globular cluster

Typical ages found for oldest globular clusters

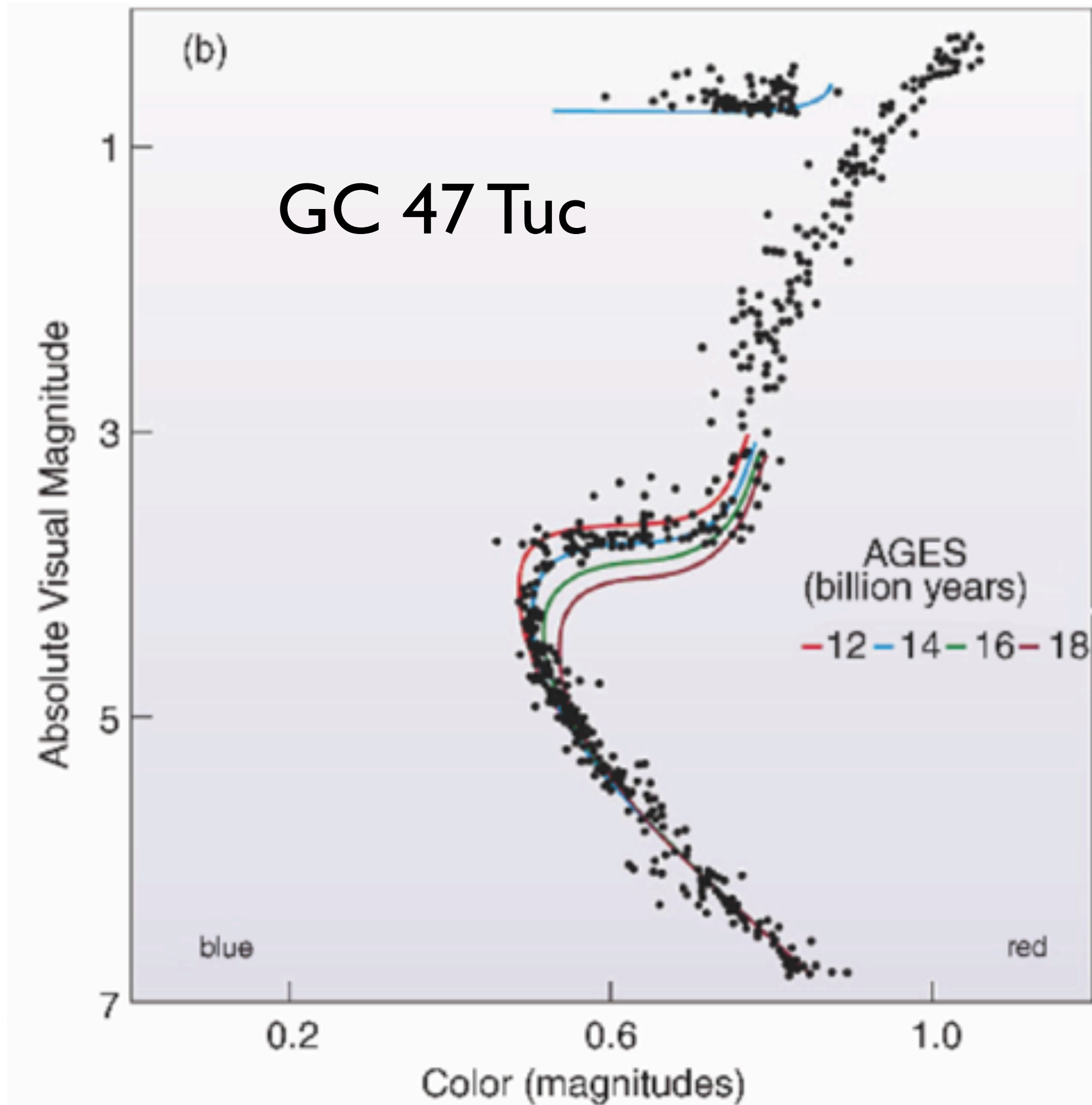
$\sim 12 \pm 1$ Gyr (disk GC)

or

$\sim 13 \pm 1$ Gyr (halo GC)



How does this work for real Globular Clusters?



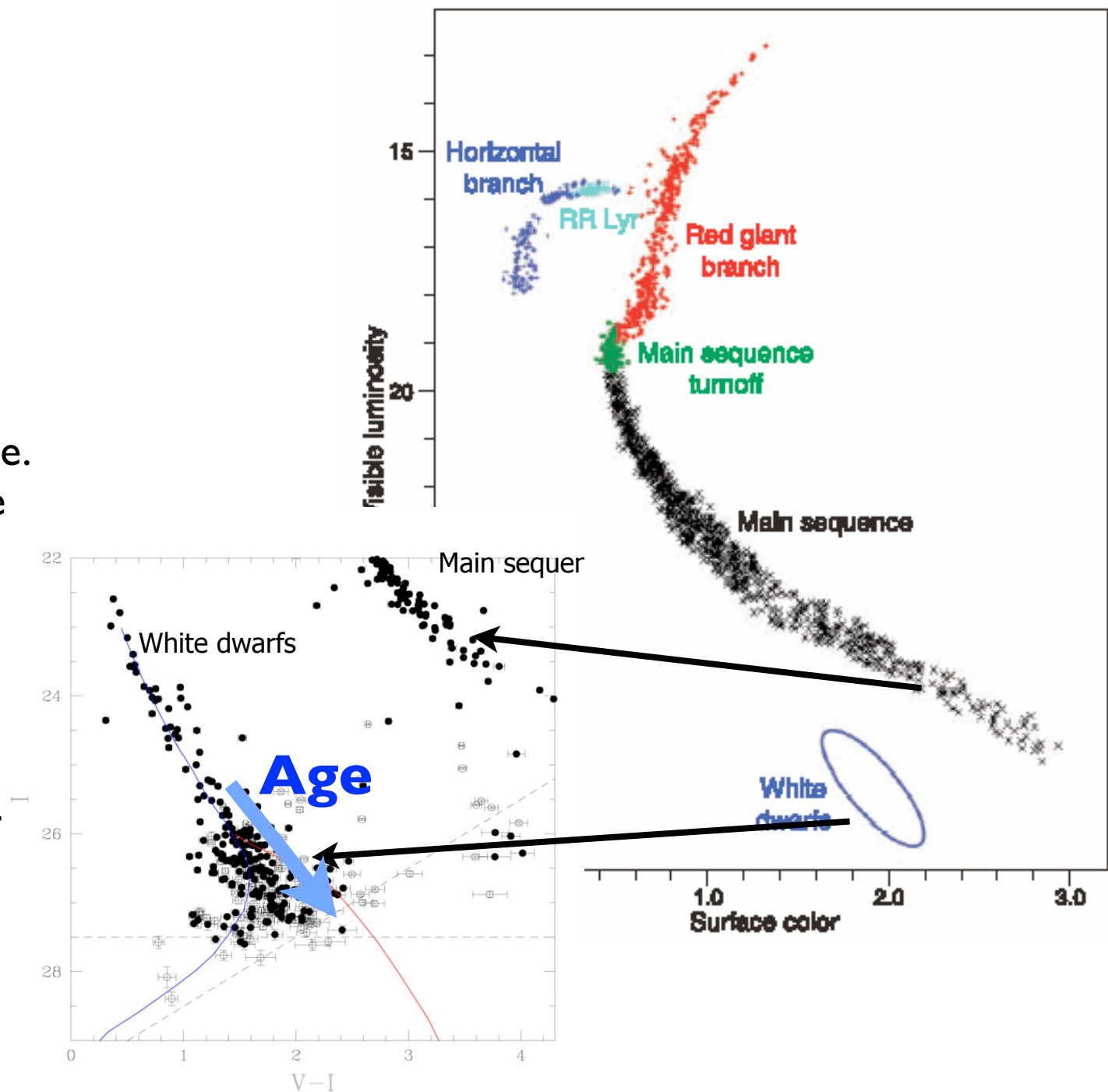
-- For those of you interested in the history, you should note that older estimates of the age of globular clusters were frequently in the range of $\sim 13\text{-}17$ Gyr (older than the age of the universe). However, current estimates are no longer so old. Why? What has changed?

- * Distances to globular clusters increased by $\sim 10\%$ based on Hipparcos calibration of the absolute magnitude of subdwarfs (lowers ages by $\sim 20\%$)

- * Inputs to Stellar Evolution models have been revised.

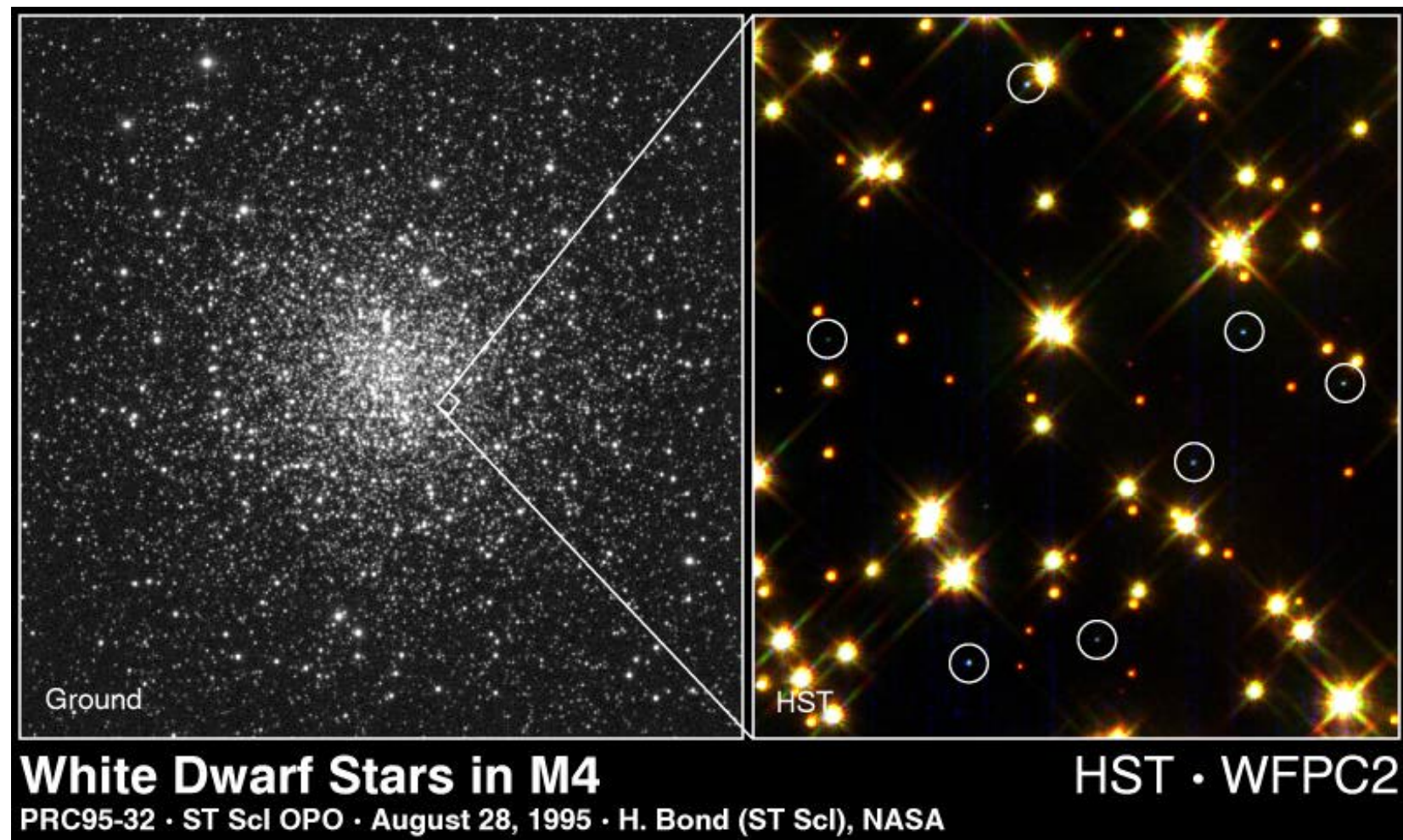
Age of the Universe (From Cooling of White Dwarfs)

- End stage for stellar evolution of stars with initial masses $< 8 M_{\text{sol}}$
- White dwarfs support themselves by electron degeneracy pressure (not fusion)
- White dwarfs radiate light, as they age. This causes them to cool and become fainter.
- Faintest white dwarfs are the oldest...
- By measuring the luminosity of the faintest white dwarfs, you can measure an age for the stars in a globular cluster



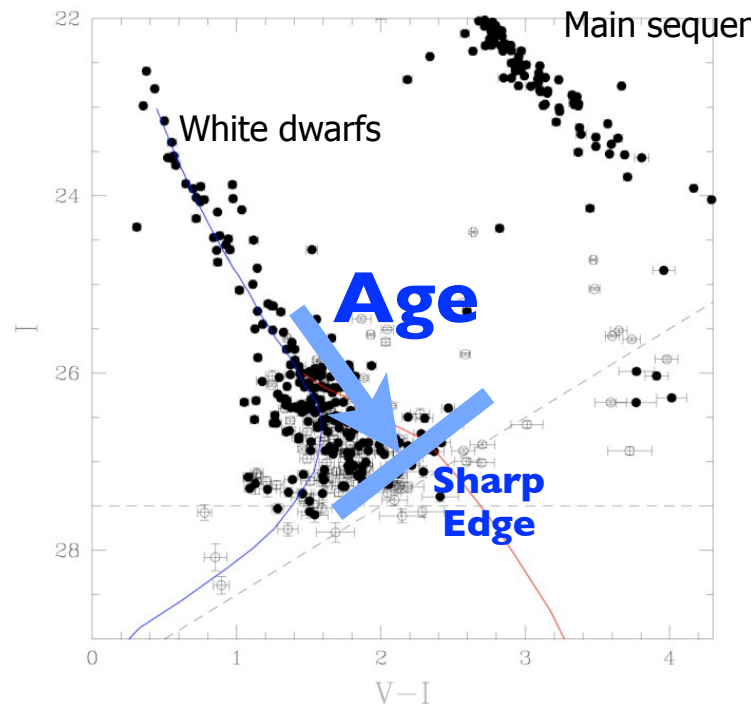
Age of the Universe (From Cooling of White Dwarfs)

- White dwarfs are faint, so need powerful telescopes like Hubble to construct these diagrams
- Observation shown at left required 8 days of HST observations... so very expensive in terms of telescope time
- Consequently this whole enterprise of age-dating white dwarfs hasn't been done with more than ~1-2 globular clusters



M4 (Messier 4) is 2.2 kpc (7200 light years away)
-- one of the two closest globular clusters to us.

Age of the Universe (From Cooling of White Dwarfs)



- Sharp edge gives the age for the oldest stars in the globular cluster
- For globular cluster shown to the right here, the measured ages are 12.7 ± 0.7 Gyr
- Physics relevant to the technique is very simple (since white dwarfs do not generate any energy from fusion, etc.)
- Key uncertainties include distance and reddening (again similar reasons as we encountered age dating the main sequence turn-off)

Age of the Universe

- Radiometric Dating $\Rightarrow \sim 13 \pm 4$ Gyr
- Aging of Stars (Main Sequence Turn-off) $\Rightarrow \sim 12 \pm 1$ Gyr
 $\sim 13 \pm 1$ Gyr
- White dwarf cooling $\Rightarrow 12.7 \pm 0.7$ Gyr

Evidence for Big Bang

I. Age of “Stuff” in Universe $\sim 1/H_0$

Radioactive Decay,
White Dwarf Cooling,
Globular Clusters

$\sim 13 \pm 1$ Gyr

“Expansion Rate
of Universe”

~ 13.8 Gyr

Other Evidence Universe isn't
Infinitely Old....

Cosmic Microwave Background

-- CMB discovered in 1964

-- Discovered

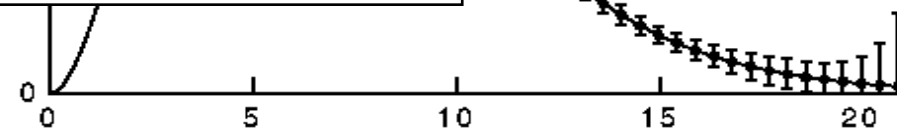
One only finds blackbody spectrum if the universe were in thermal equilibrium in the past...

and that is only possible if the universe were hotter and denser...

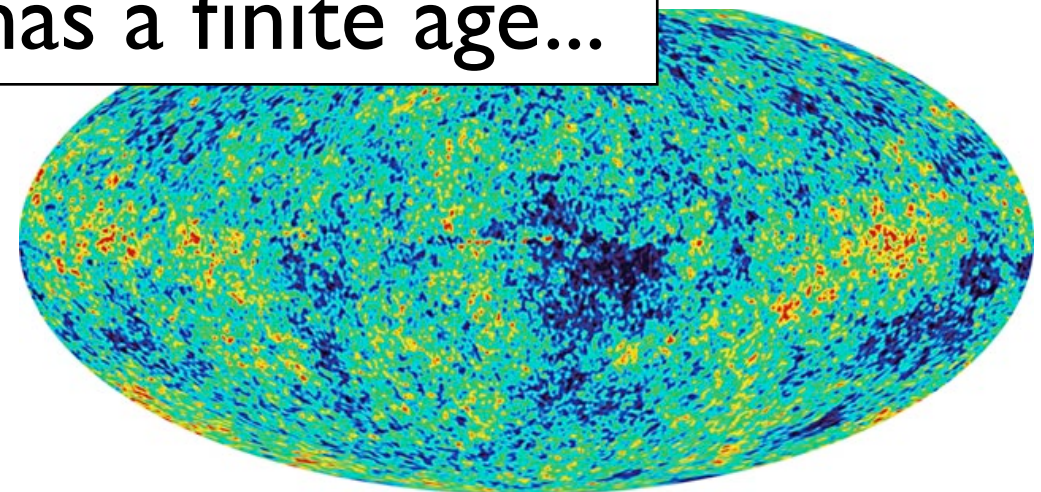
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Blackbody

data with 400 σ errorbars
— 2.728 K Blackbody



Argues the universe has a finite age...



Olber's Paradox...

(Henrich Olbers 1826)

“Why is the night sky dark?”

“If space and time are infinite, then every sightline must connect with a star somewhere”

Since surface brightness is preserved (one can demonstrate this with basic arguments) and every sightline connects with a star, then every spot on the sky must be as bright as a star...

Solution:

Universe is not infinitely old... and it is expanding...

How else we can tell time in the
universe?

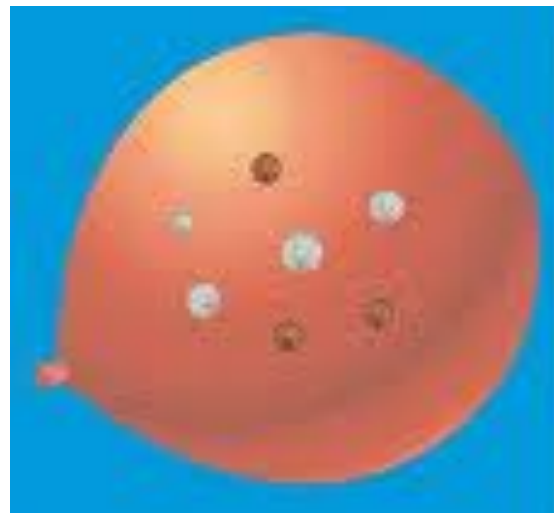
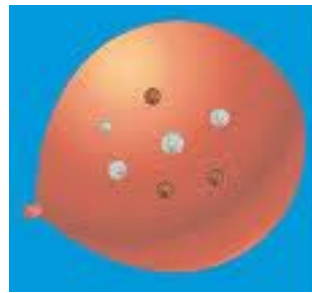
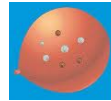
In cosmological setting, we monitor the passage of time in the universe both with a normal clock (seconds in proper time)

and also using the scale factor of the universe R .

Age of the Universe

(from the expansion rate of the universe)

beginning of
universe



Time



Now

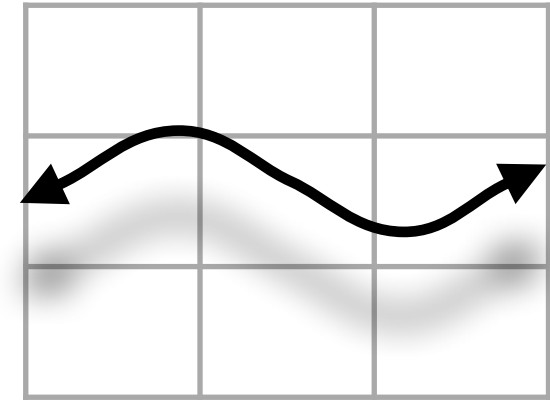
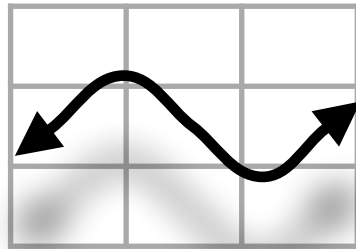
but instead of referring to some past epoch in terms of the scale factor of the universe then (i.e., R),

we do it in terms of the relative changes in the size of the universe...

How do we measure these relative changes?

By looking at changes to the wavelength of light which stretches by the same factor as the universe has expanded

The expansion of the universe causes light from distant galaxies to be redshifted...



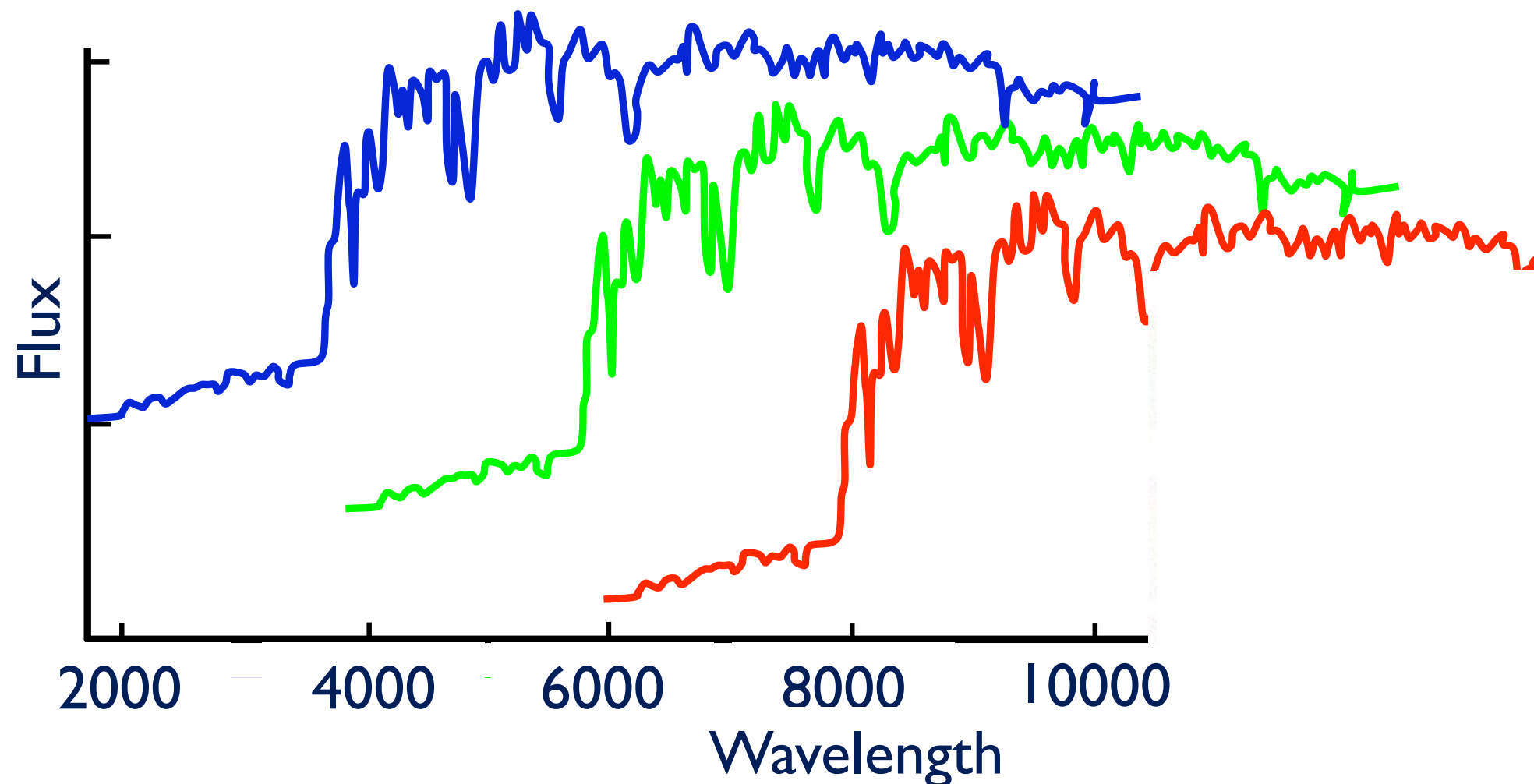
$$\frac{\lambda_0}{\lambda_{emitted}} = \frac{R(t_0)}{R(t_{emitted})}$$

Note 0 subscript indicates current date universe...

The expansion factor is the same for light, as for the universe itself

By measuring the stretching of light from distant objects, we can calculate how much the universe has expanded since the light was emitted

How does the stretching of light from real galaxies look?

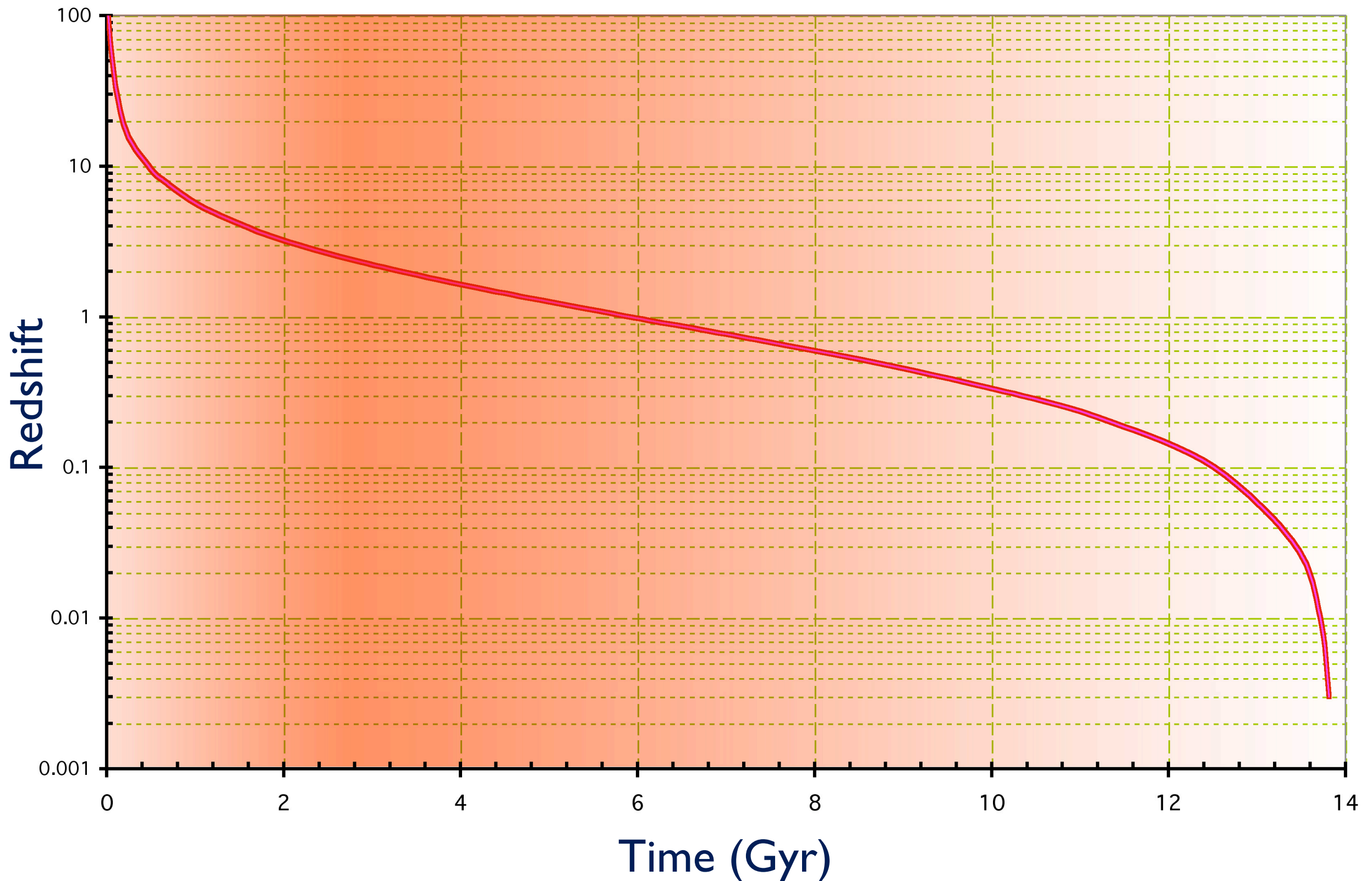


The quantity which encapsulates this measurement of the expansion of the universe since the photon was emitted is the redshift or cosmological redshift.

$$1 + \underset{\substack{\nearrow \\ \text{redshift}}}{z} = \frac{\lambda_0}{\lambda_{emitted}} = \frac{R(t_0)}{R(t_{emitted})}$$

Just like proper time, it provides us a way of ordering different events in the history of the universe.

What is the relation between redshift and time?



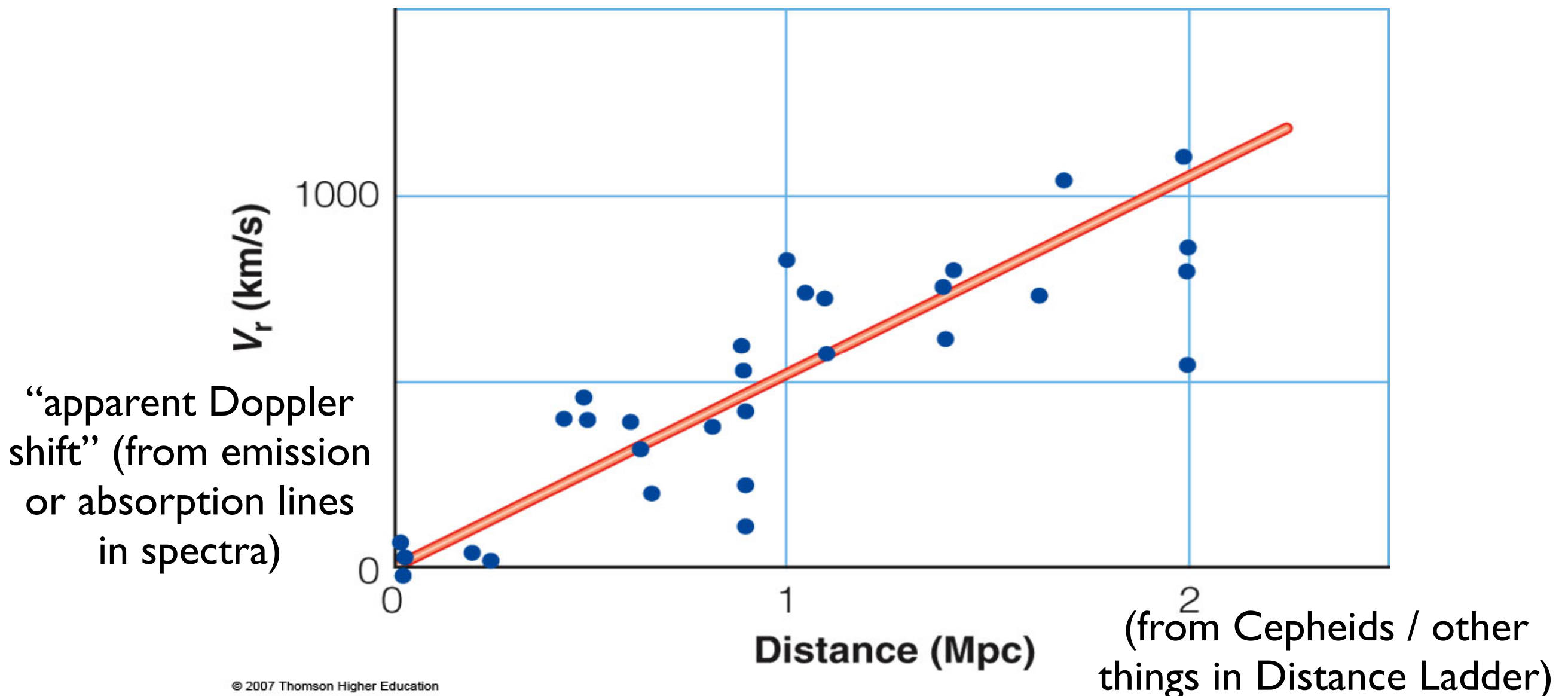
What is the Hubble constant?

From the expansion rate of the universe

In 1929, Hubble showed that the velocities and distances are linearly correlated, and satisfy

$$v = H_0 d$$

where v is the recessional velocity (km/s) and d is the distance (Mpc). H_0 is a constant, “Hubble’s Constant” and has units of $\text{km s}^{-1} \text{ Mpc}^{-1}$.



Why important?

Establishes basic measures of distance and time in all extragalactic cosmological measurements

$$\text{Age of Universe} \sim 1/H_0$$

$$\text{Distance to Faraway Galaxy} \sim cz/H_0 \propto 1/H_0$$

$(z = v/c \rightarrow z = H_0 D/c \rightarrow D = cz/H_0)$

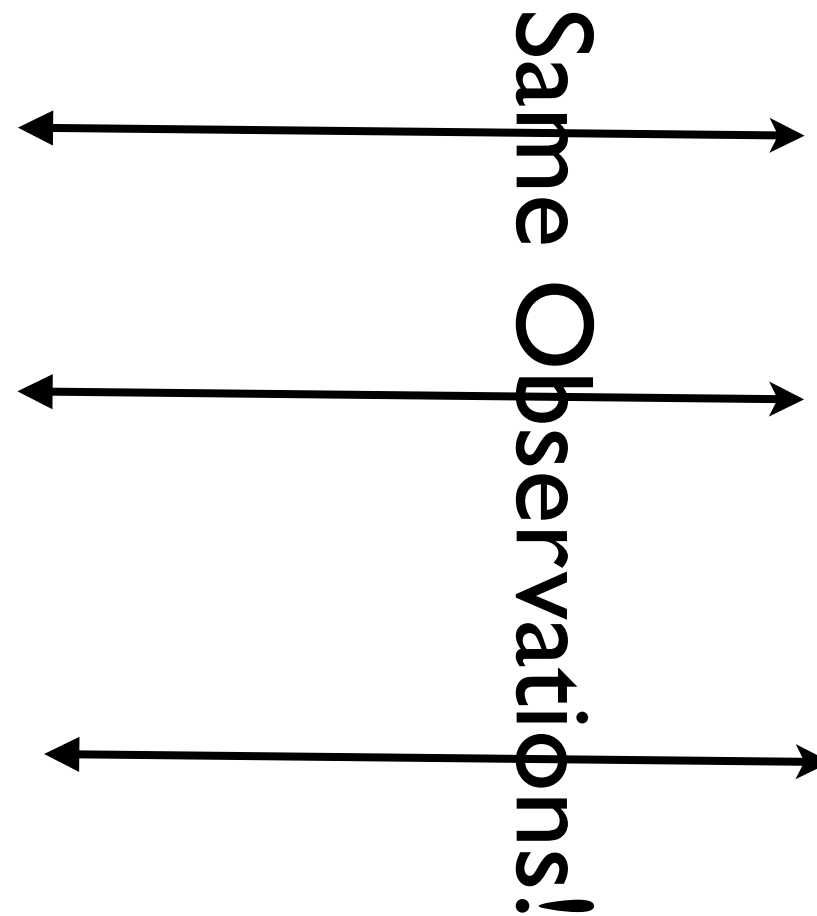
$$\begin{aligned} \text{Luminosity of Faraway Galaxy} &\sim \text{flux} \times (4\pi D^2) \\ &\sim \text{flux} \times 4\pi (cz/H_0)^2 \\ &\propto 1/H_0^2 \end{aligned}$$

Why important?

Hubble Parameter changes our interpretation substantially!

$$H_0 = 100 \text{ km/s/Mpc}$$

$$H_0 = 50 \text{ km/s/Mpc}$$



Universe twice as old!

Galaxies twice as far
away!

Galaxies four times
as luminous!

Why important?

For this reason, the Hubble constant is used to define some characteristic units of time and distance:

$$\text{Hubble Time } t_H = 1/H_0$$

$$\text{Hubble Distance } D_H = c/H_0$$

(distance light travels in Hubble time)

What is the Hubble constant?

Size of Universe: Taylor series expansion

$$R = R_0 + (dR/dt) (t - t_0)$$

$$R = R_0 + R_0(dR/dt)/R_0 (t - t_0)$$

$$R = R_0 + R_0 H_0 (t - t_0)$$

$$R = R_0 (1 + H_0 (t - t_0))$$



First order time derivative in
Taylor series expansion of universe

How do we determine the Hubble Constant?

It would appear simple

$$v = H_0 d$$

velocity of galaxies

“easy to determine”
(use wavelength of emission or
absorption lines)

but must observe at scales where effects
of inhomogeneities small

distance to galaxies

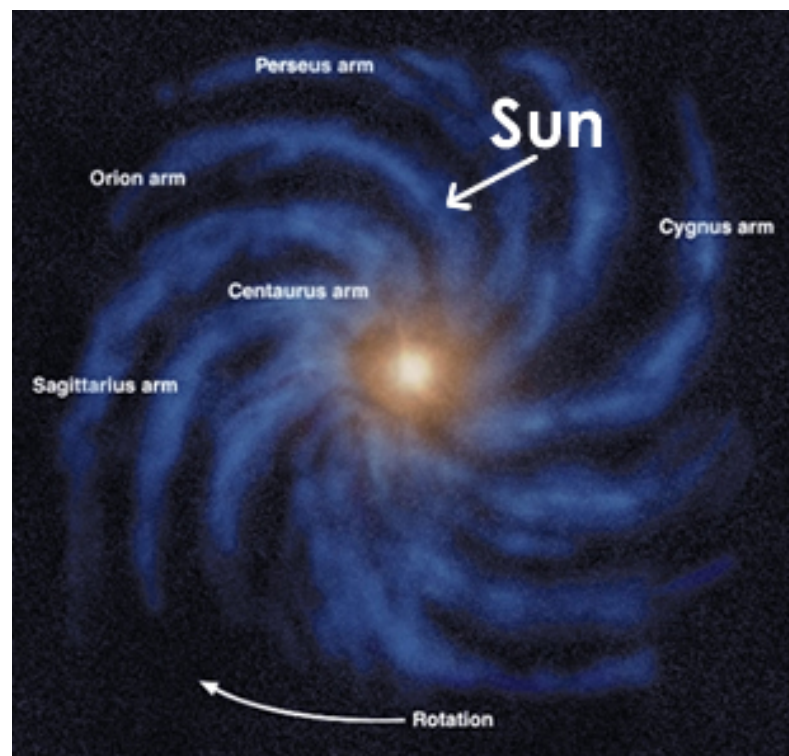
“difficult to determine
because of extreme distances”

Challenges:

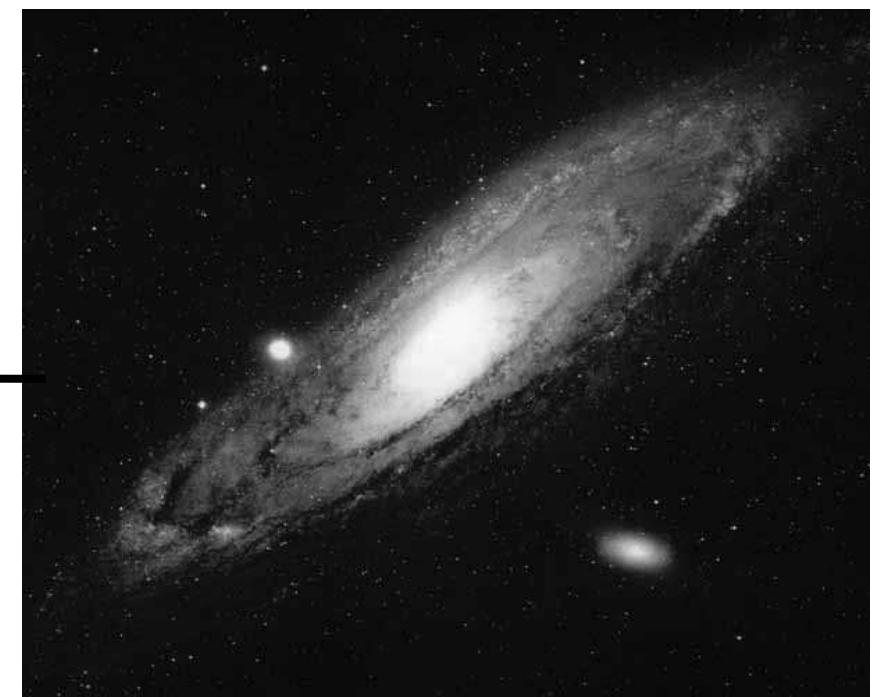
In addition to the underlying expansion of the universe, certain regions of the universe are more dense than others, and galaxies flow towards the overdensities

For example

Milky Way
Our Galaxy



M31
Andromeda



100 km/s

700 kpc

Hubble Law:

$$v = H_0 d$$

$$v_{\text{obs}} = H_0 d + v_{\text{peculiar}}$$

↑
cosmological expansion

↑
Doppler shift
(from peculiar velocities)

$$\sim \pm 500 \text{ km s}^{-1}$$

How does $H_0 d$ compare to v_{peculiar} ?

Relative Size of
 $H_0 d$ to v_{peculiar}

$$@ 15 \text{ Mpc} \Rightarrow (15 \text{ Mpc})(71 \text{ km/s/Mpc}) \sim 1065 \text{ km/s}$$

2x larger
Error $\sim 50\%$

$$@ 100 \text{ Mpc} \Rightarrow (100 \text{ Mpc})(71 \text{ km/s/Mpc}) \sim 7100 \text{ km/s}$$

14x larger
Error $\sim 7\%$

Challenges:

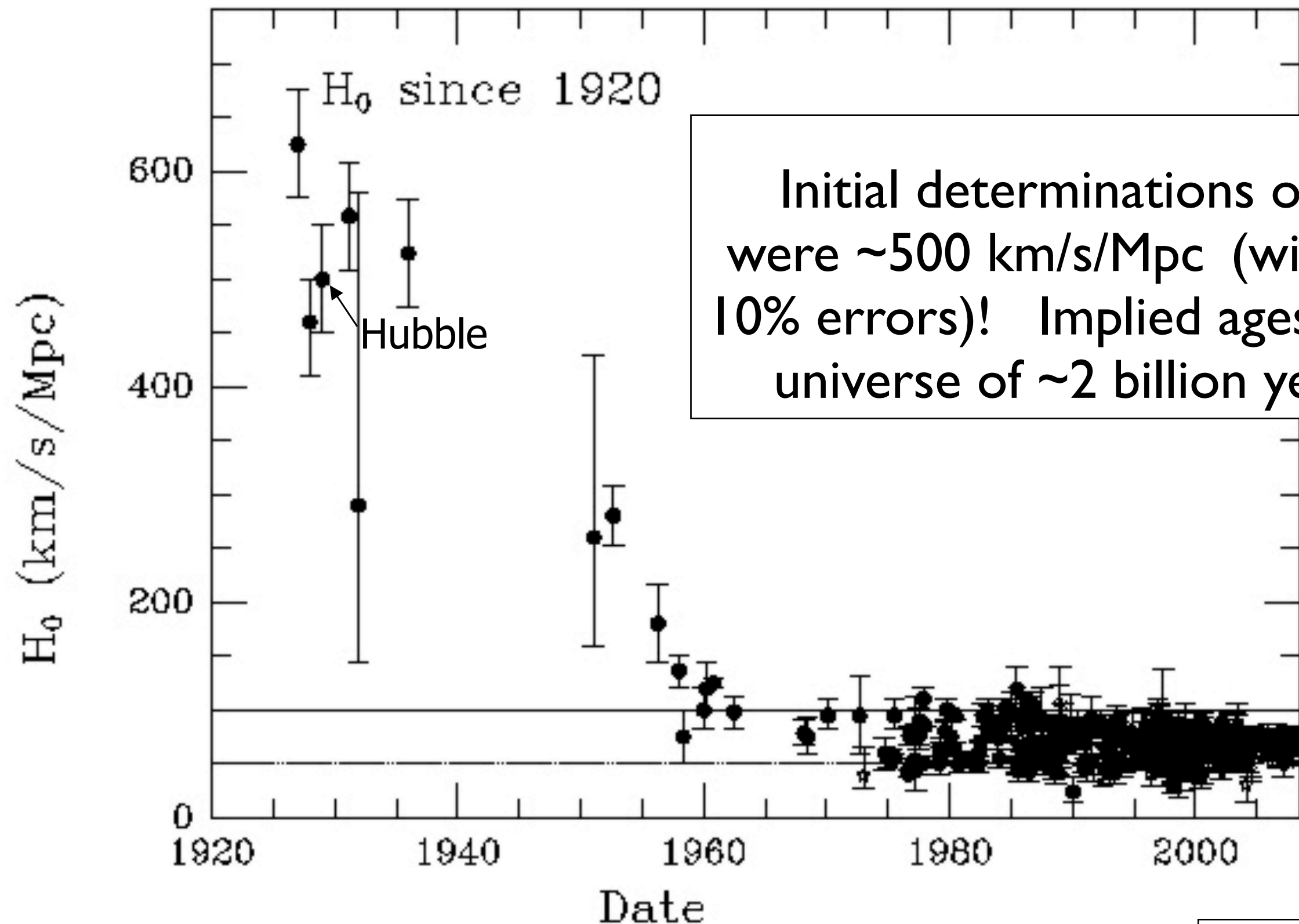
To quantify the Hubble constant, we

Want a model-independent geometric approach to
measures distances of sources at $>50\text{-}100\text{ Mpc}$

Unfortunately -- does not exist!

Brief Historical Background on the long road to measuring H_0

Hubble constant determinations vs. time



Credit: J. Huchra

Why were initial determinations of the Hubble constant $\sim 7\times$ too large?

- **Hubble used**

- ▶ **Cepheid variables as calibrated by Shapley (1930)**
- ▶ **brightest stars in galaxies as calibrated by Cepheids**

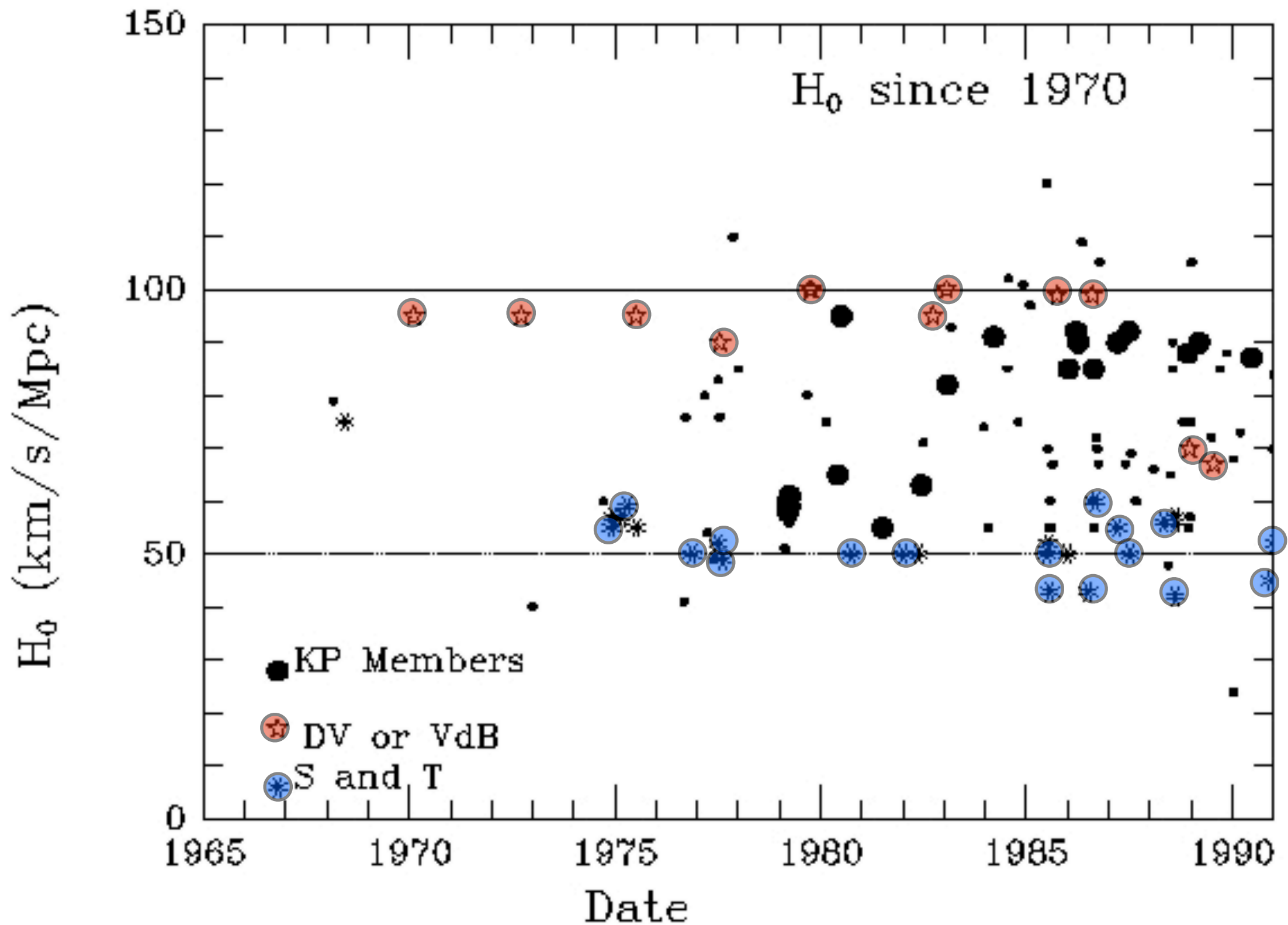
Wrong by factor of 2!

Wrong by factor of ~ 4 !

(Hubble had misidentified HII regions in other galaxies as single stars)

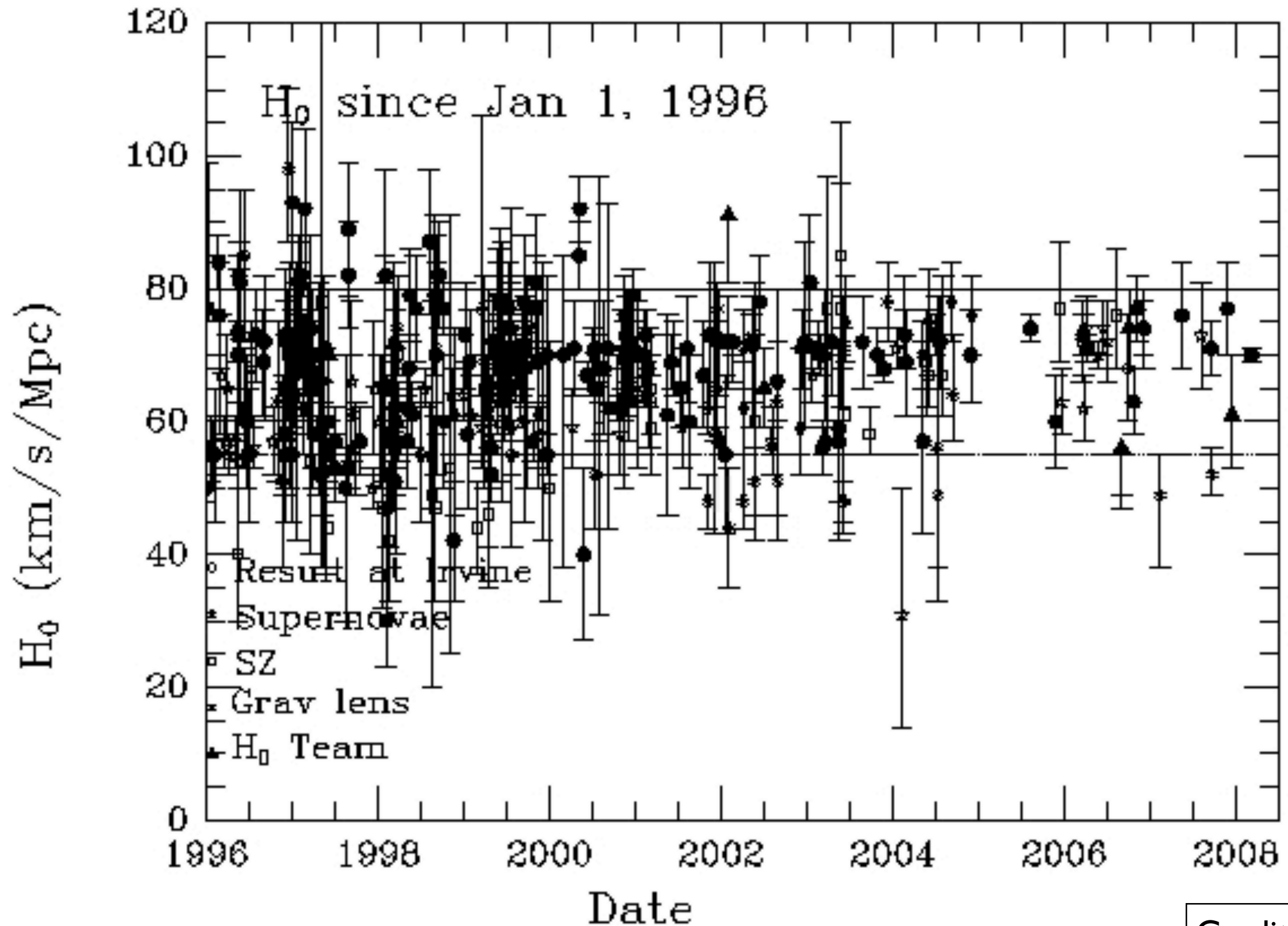
1918 Shapley establishes P-L relation for 11 galactic Cepheids (mostly Pop. I (class. Cepheids) $\Delta m \sim 1.5$ m zu weaker (half of this error due to extinction, other half comes from error in the parallax = distance)

Hubble constant determinations vs. time



Hubble constant determinations vs. time

(after observations began with Hubble Space Telescope)

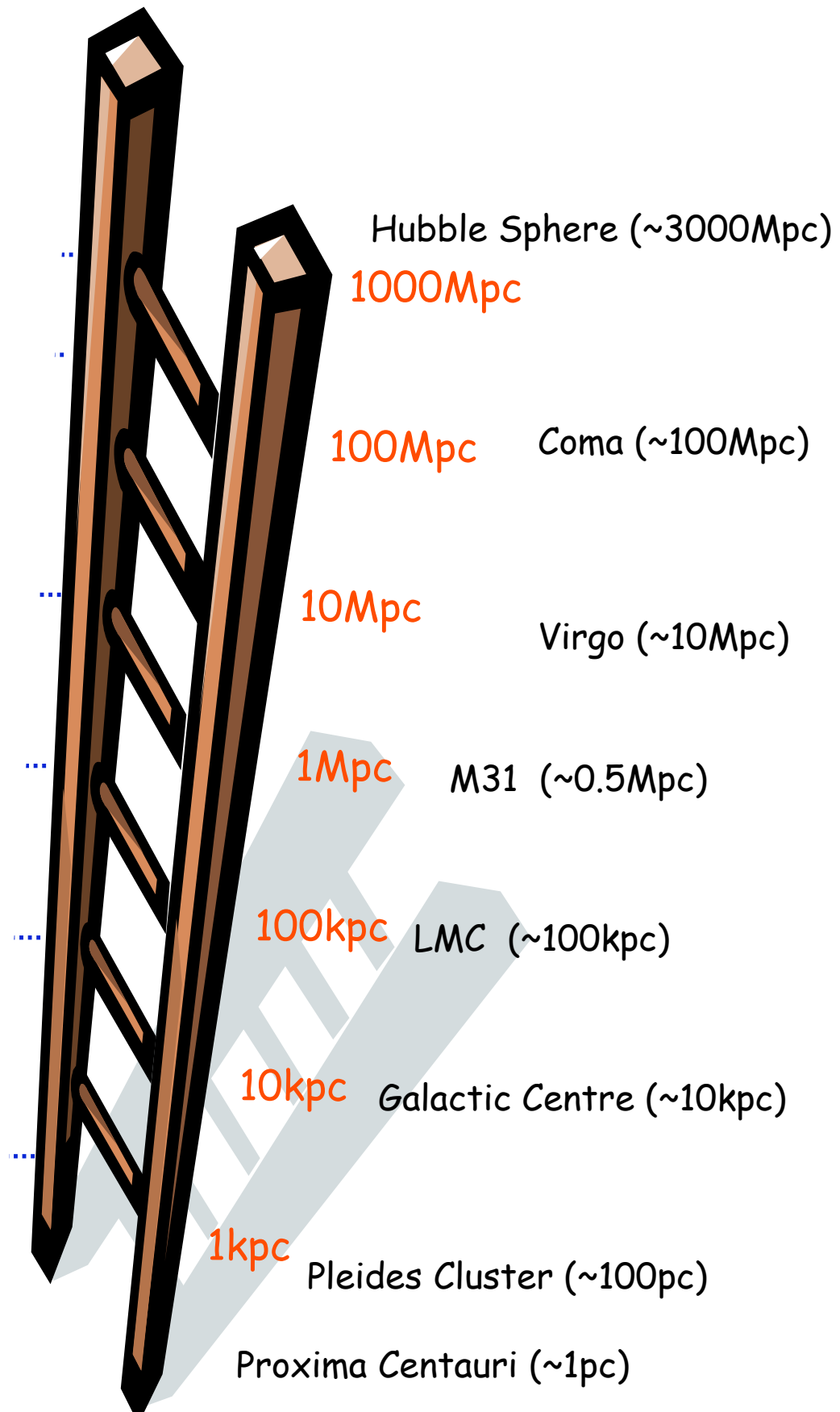


Credit: J. Huchra

Need to measure distances of galaxies where peculiar velocities don't have a big effect on cosmological flow....

How to measure distances to galaxies that are > 20 Mpc away?

We must use a multi-stage approach to determine distances!



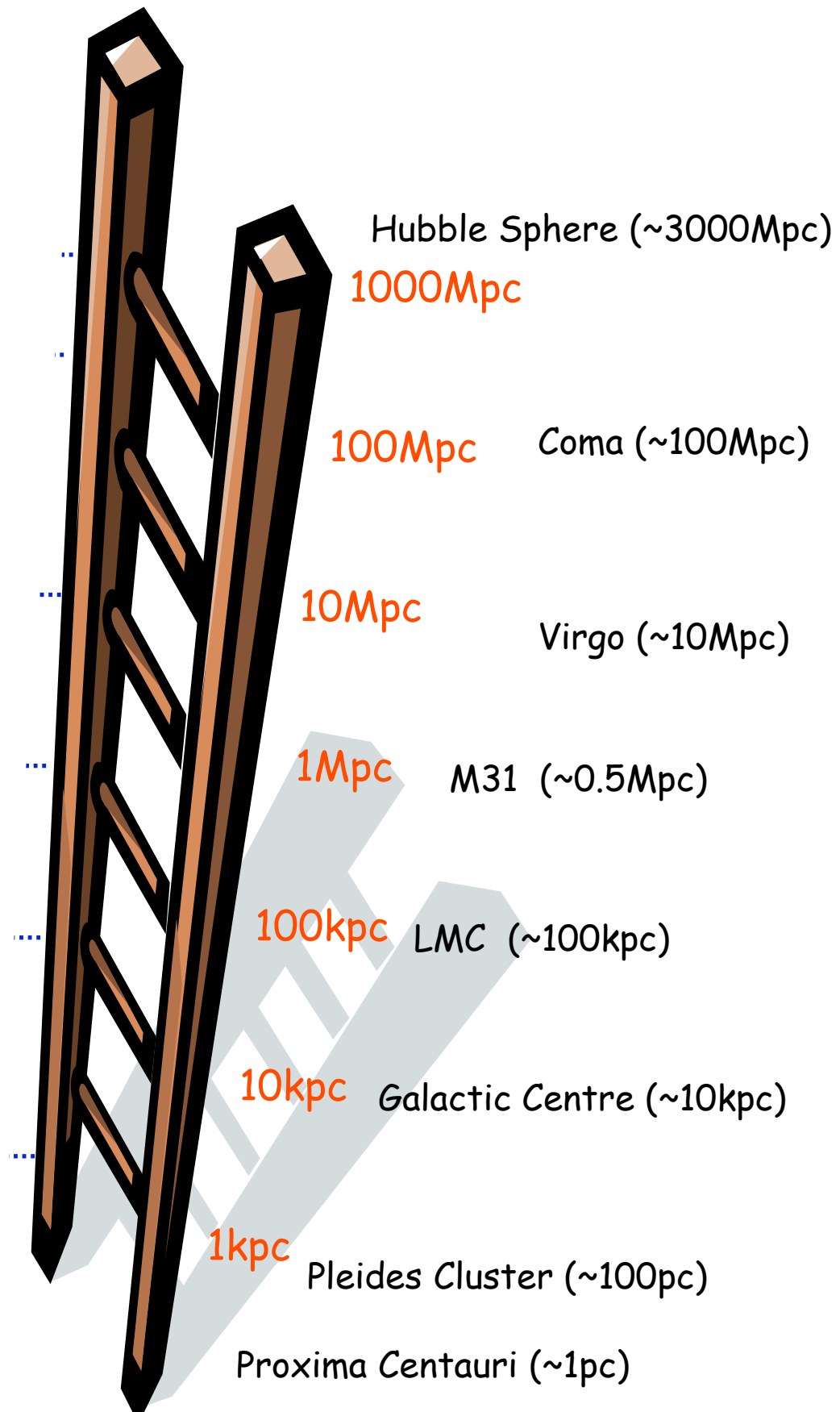
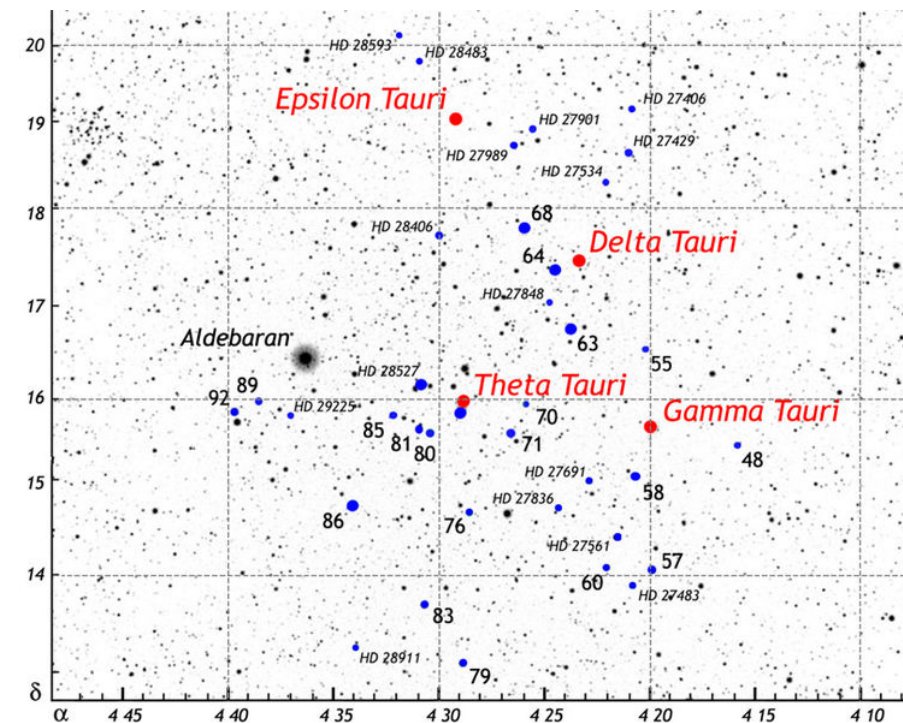
Each stage builds on previous stage!

Or using distance ladder analogy,
each rung of ladder builds on previous
rung!

We must use a multi-stage approach to determine distances!

Steps in Ladder

Hyades (~46 pc)

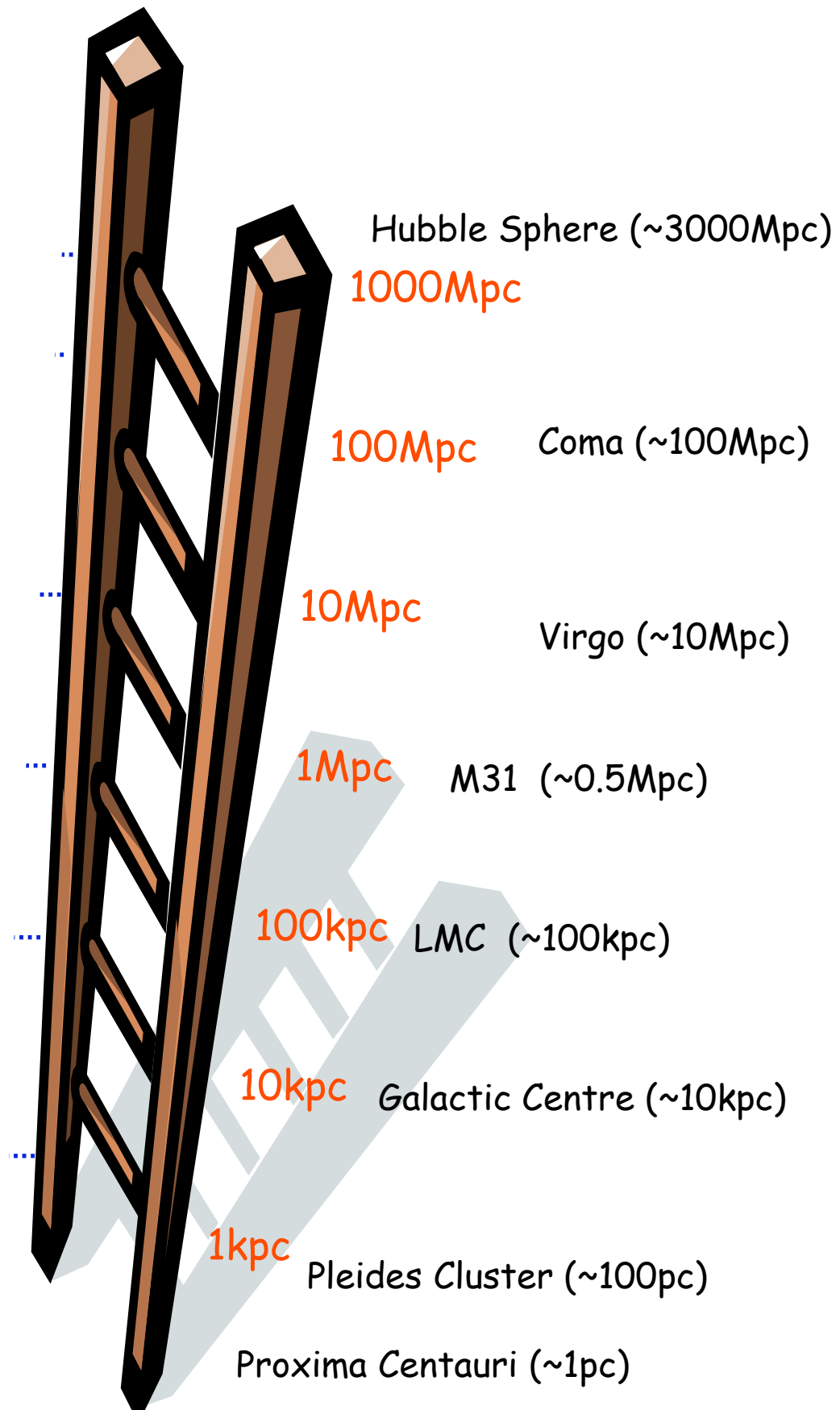


We must use a multi-stage approach to determine distances!

Steps in Ladder

Large Magellanic Cloud (~50 kpc)

Dwarf Galaxy (next to Milky Way)

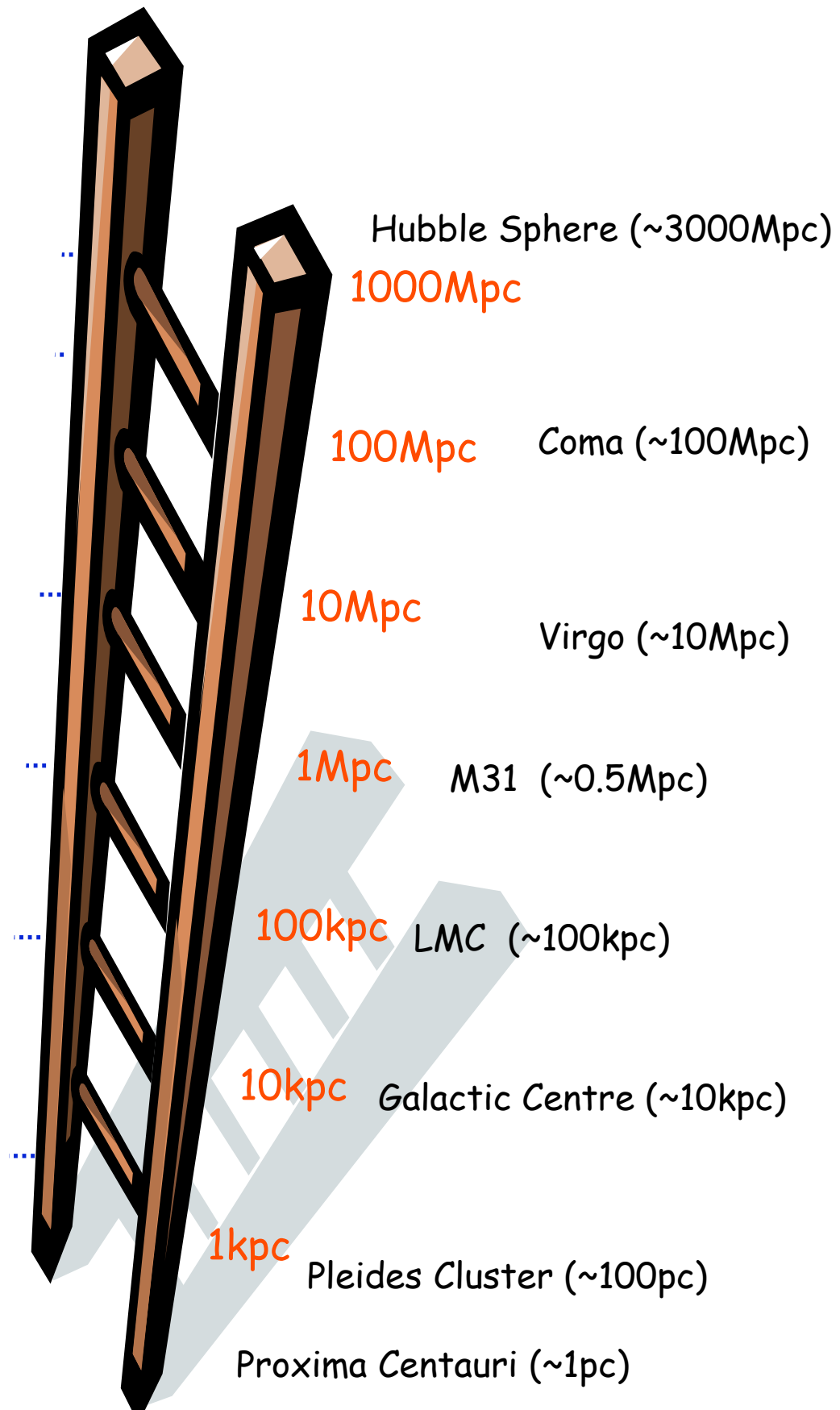
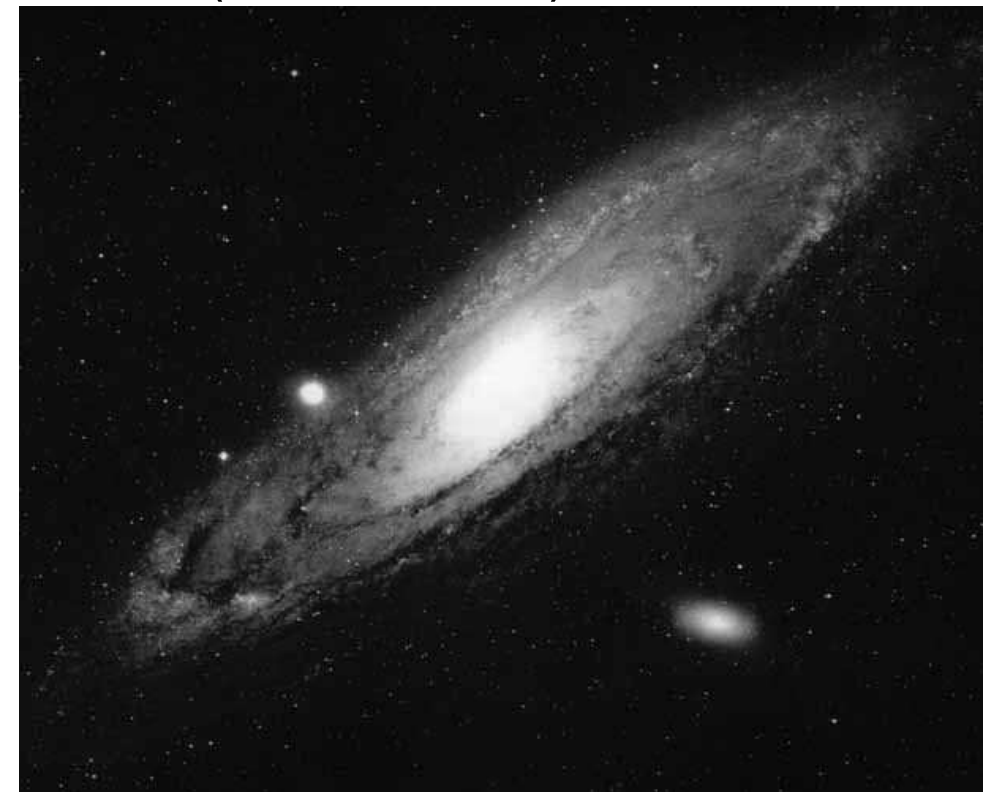


We must use a multi-stage approach to determine distances!

Steps in Ladder

M31 (~700 kpc)

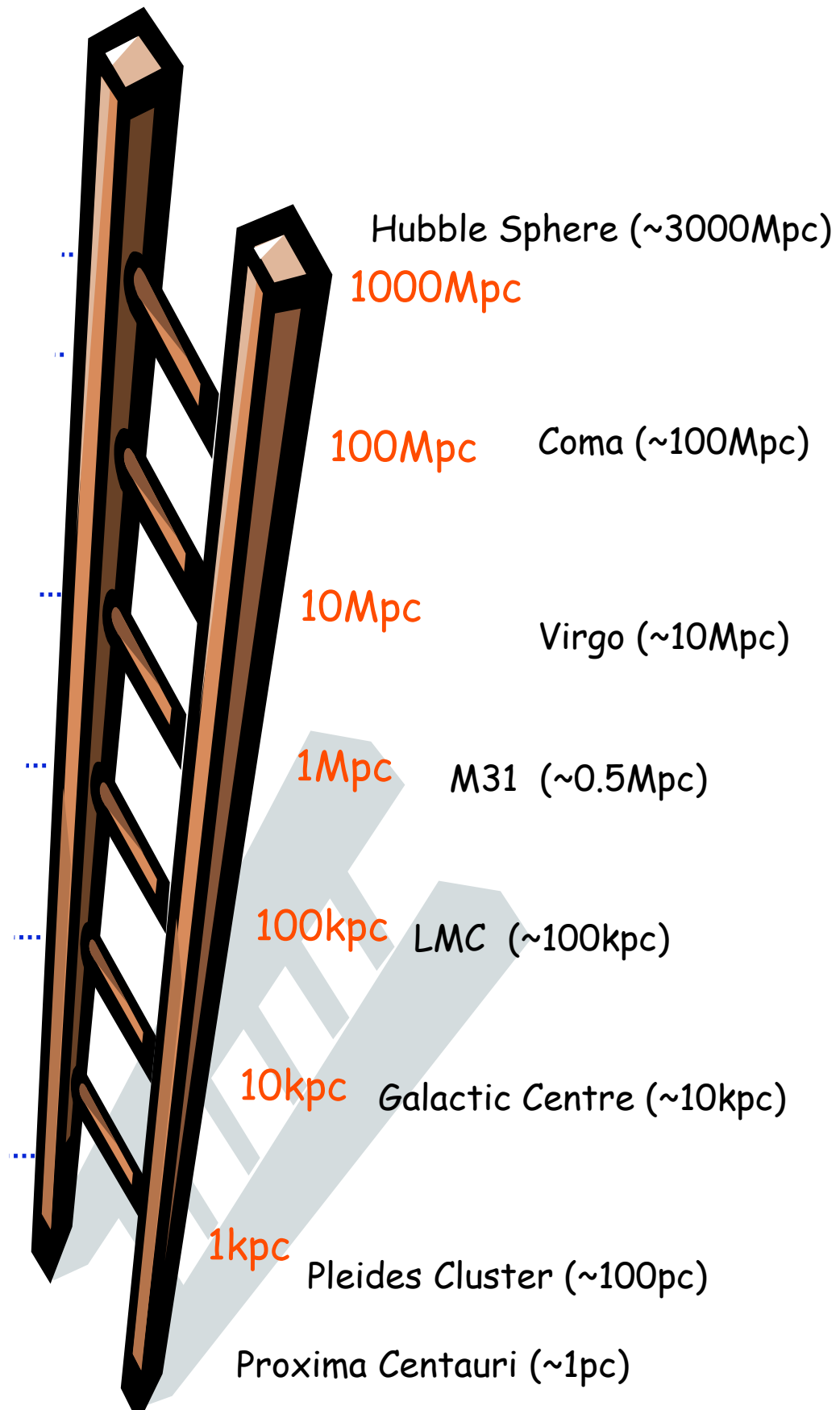
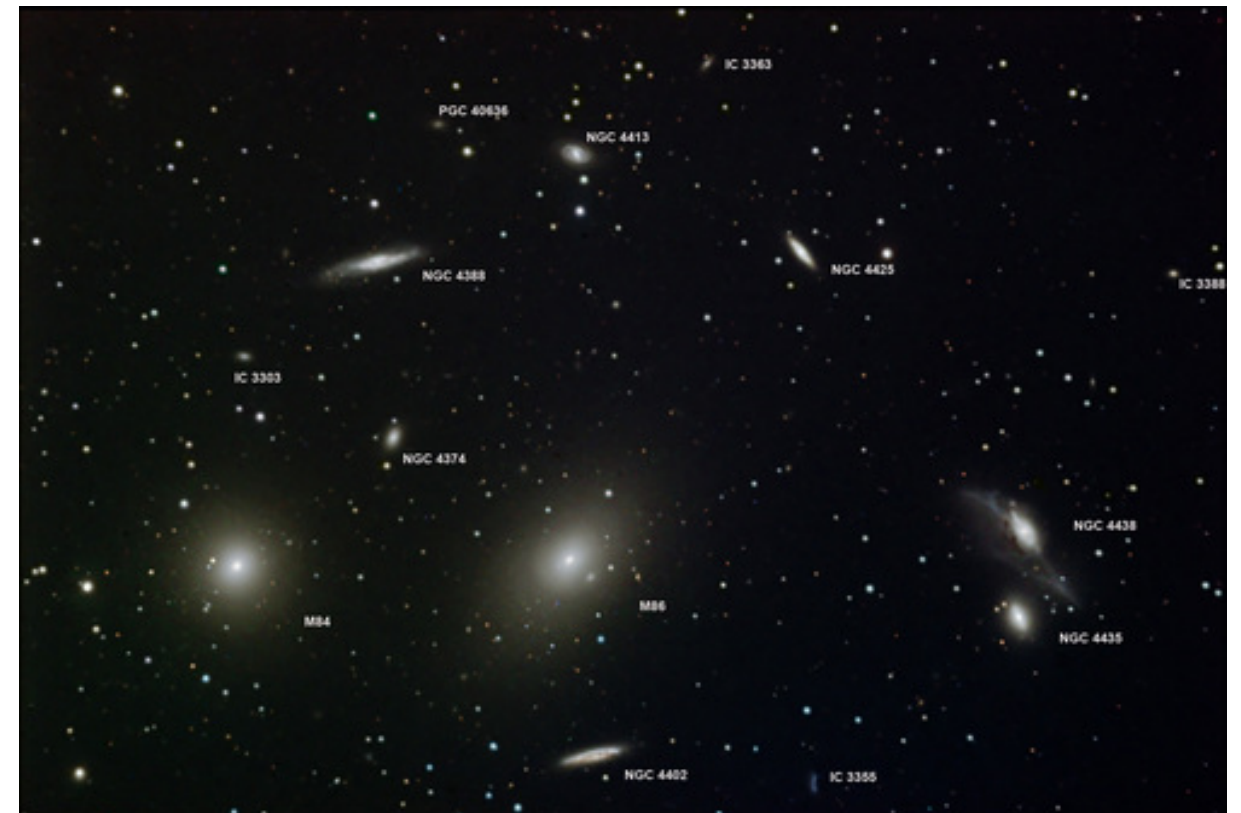
Luminous galaxy in local group of galaxies



We must use a multi-stage approach to determine distances!

Steps in Ladder

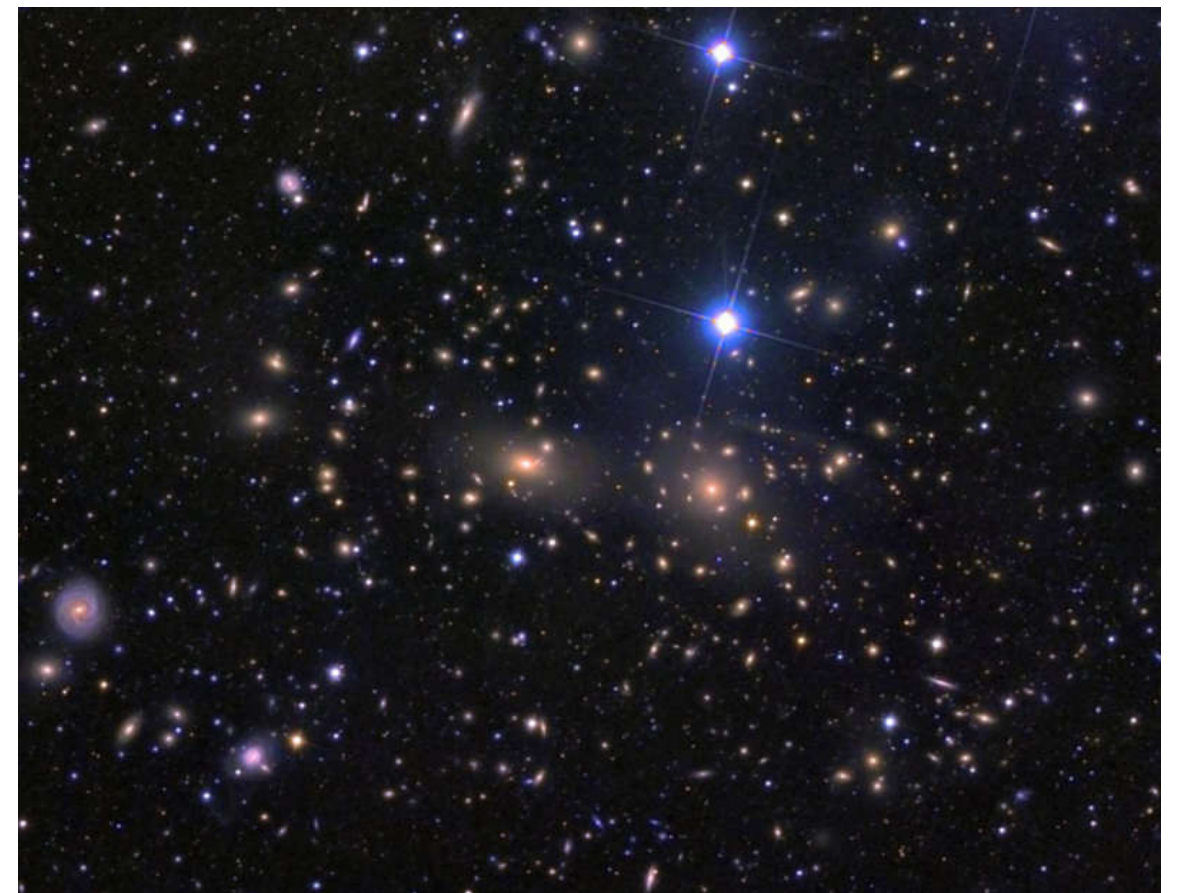
Virgo Cluster (~15-20 Mpc) Most Massive Nearby Cluster of Galaxies



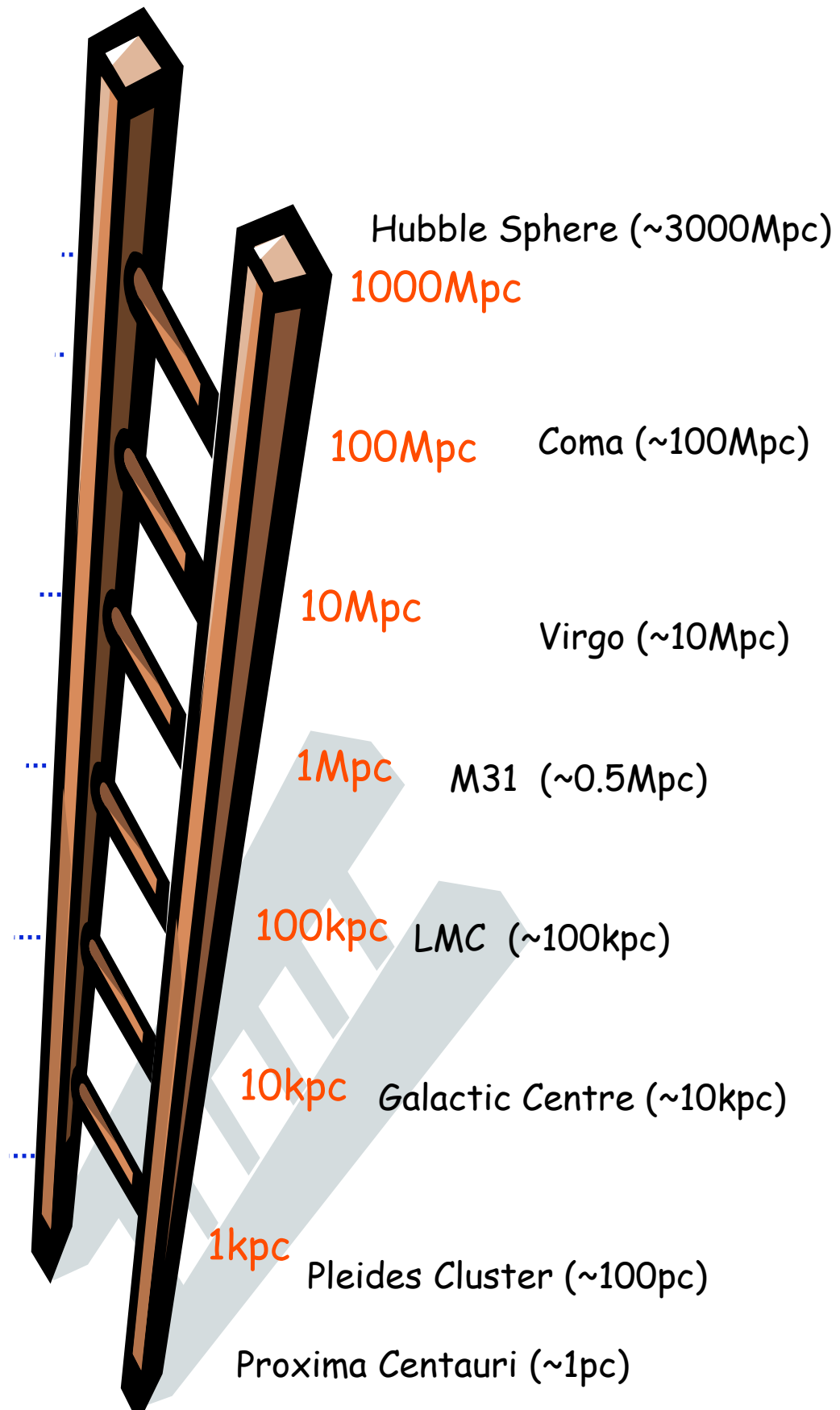
We must use a multi-stage approach to determine distances!

Steps in Ladder

Coma Cluster (~100 Mpc) Very Massive Nearby Supercluster of Galaxies



credit: Dean Rowe



Primary Methods

(local, primarily geometric)

Radar Echo

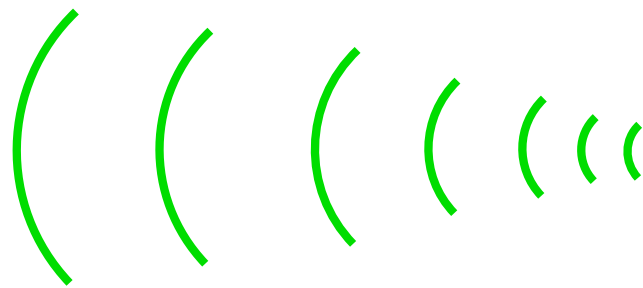
Parallax (Trigonometric, Secular,
Statistical)

Moving Cluster

Main Sequence Fitting to Star Clusters

Radio Echo

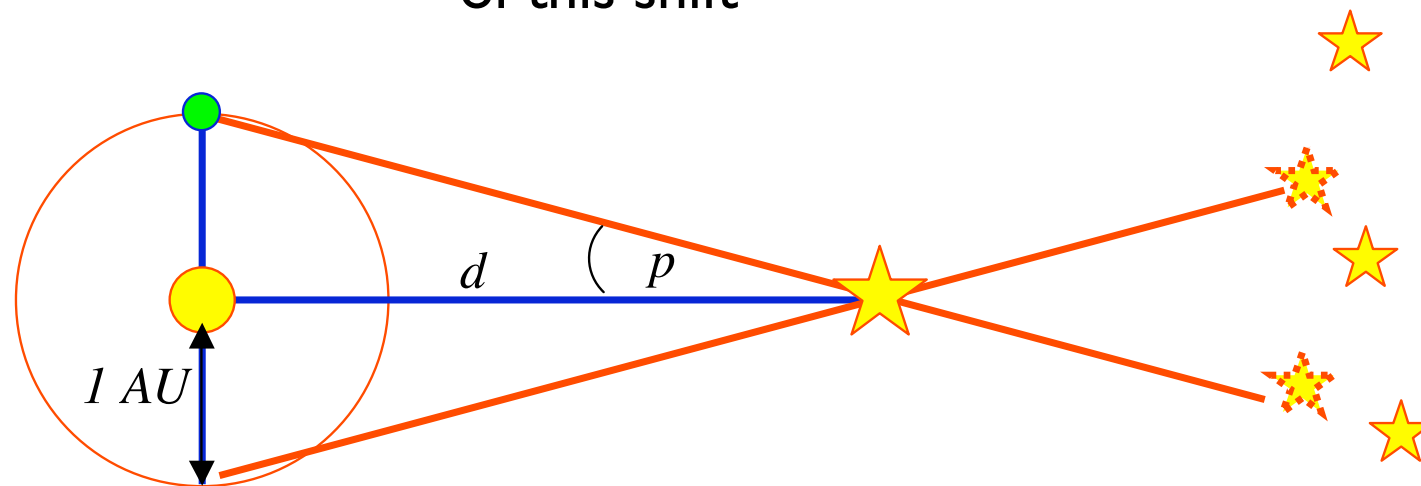
- Extremely accurate method to measure distances within solar system.
- Only useful out to ~10 astronomical units (for larger distances, the radio echo is too faint to detect!)



$$d = \frac{1}{2} c \Delta t$$

Trigonometric Parallax

- Observe same star 6 months apart
- Star will shift relative to background star field
- Measure the shift. Define parallax angle as half of this shift



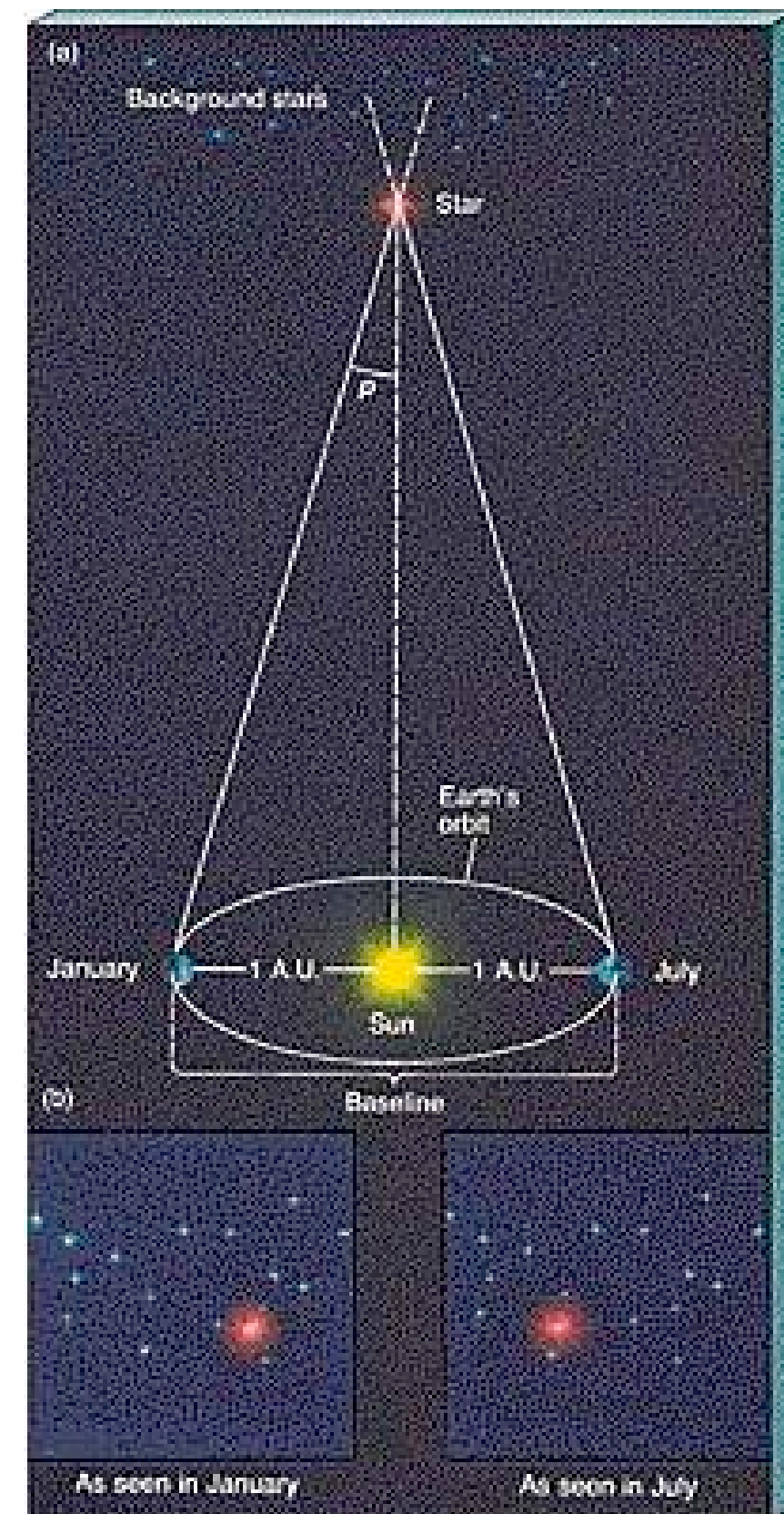
$$d = \frac{1 \text{ AU}}{\tan p_{\text{rads}}} \approx \frac{1}{p} \text{ AU}$$

- Stars with parallax angles of 1 arcsec have a distance of 1 parsec

$$1'' = 1 \text{ arcsec} = 1/60 \text{ arcmin} = 1/3600 \text{ degree} = 4.84814 \times 10^{-6} \text{ radians}$$

$$1 \text{ parsec} = 3.26 \text{ light years}$$

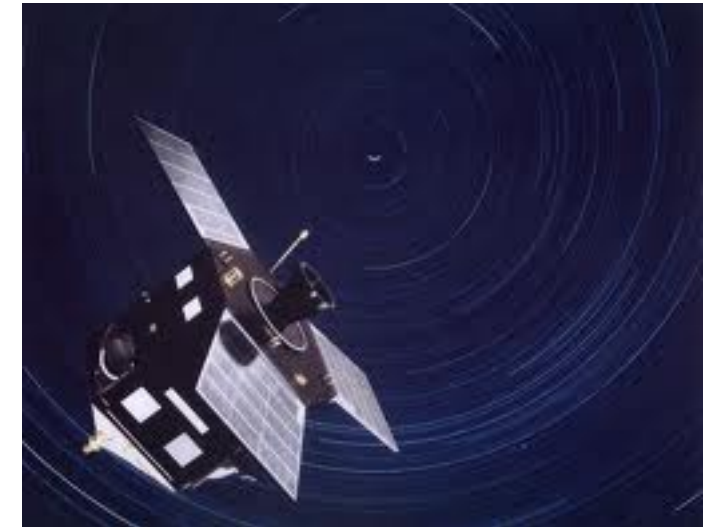
- Closest star Proxima Centauri has a parallax of 0.8'' and hence distance of 1.3 parsec = 4.3 light years



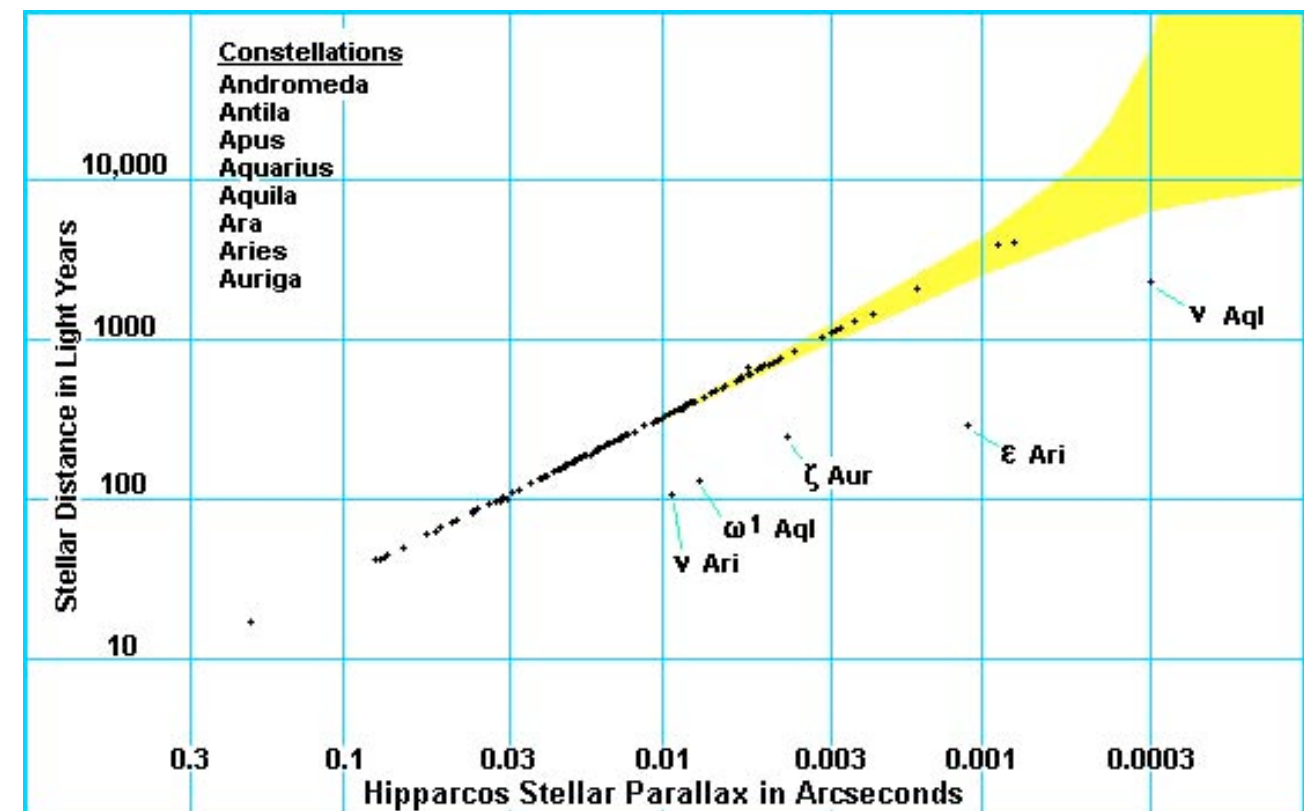
Trigonometric Parallax

Best spatial position, parallax, and proper motion measurements available thus far are from the Hipparcos Space Astrometry mission (1990-1993).

Hipparcos (1990-1993)



- 120,000 stars with ~ 0.001 arcsec astrometric precision, 0.001 arcsec/yr proper motion precision (Hipparcos catalog)
- $> 1,000,000$ stars with ~ 0.020 - 0.040 arcsec astrometric precision, other photometric information (Tycho catalogue)
- The Hipparcos Satellite Launched August 1989; 3-year Mission Finished August 1993
- Effective distance limit ~ 1000 pc ~ 1 kpc
- Complete to magnitude ~ 8 with stars out to magnitude 12



Trigonometric Parallax

The best work on high precision astrometry lies in the new GAIA mission (European Space Agency).

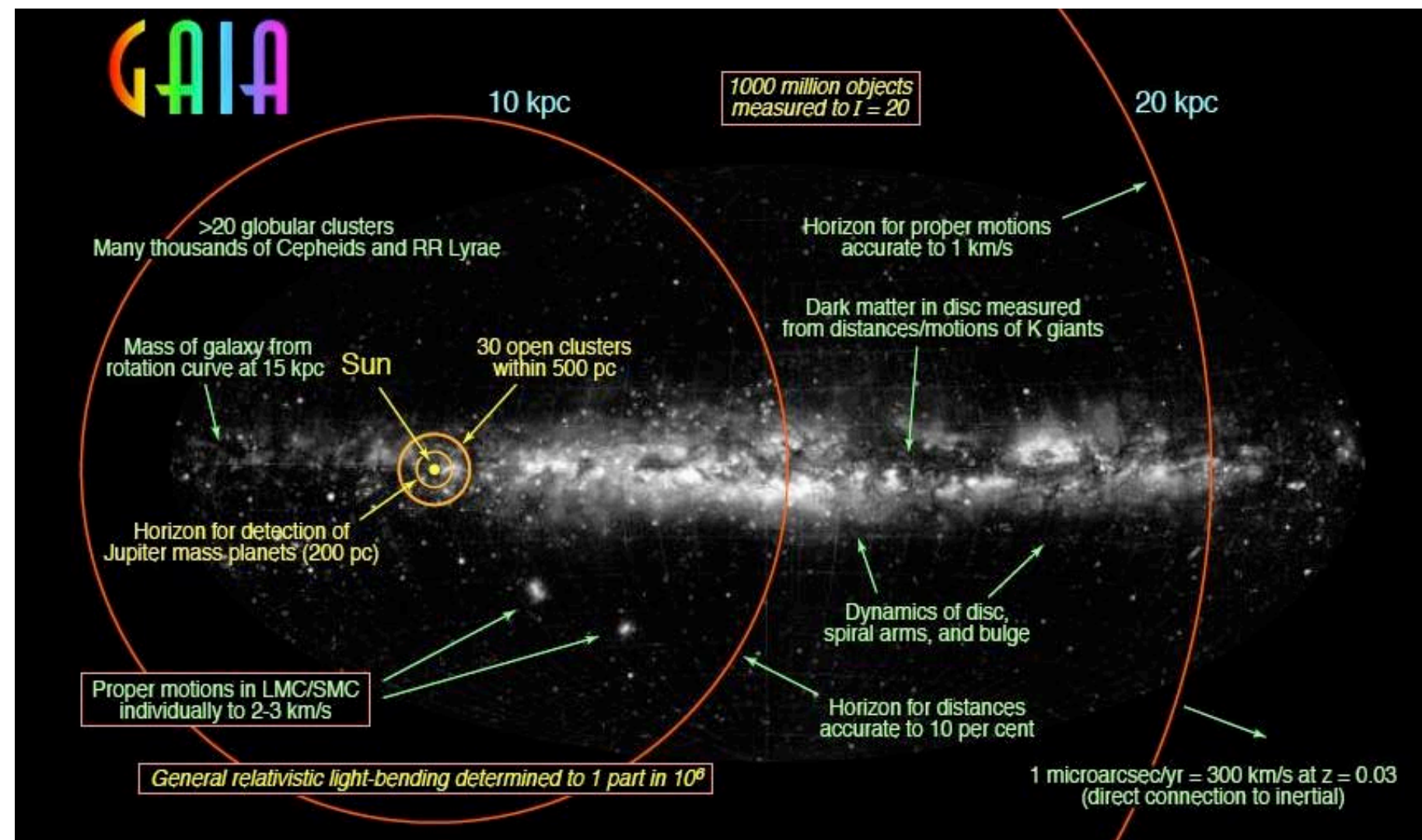
Positions, Proper Motions, and Characteristics of a billion stars to 20 magnitude

Astrometry accurate to 20 micro-arcsec at 15 mag,
200 micro-arcsec at 20 mag

Spatial motions and 3D view of stars in the Milky Way galaxy

Distances to 50 kpc away

GAIA (2013-2025?)



Moving Cluster Method

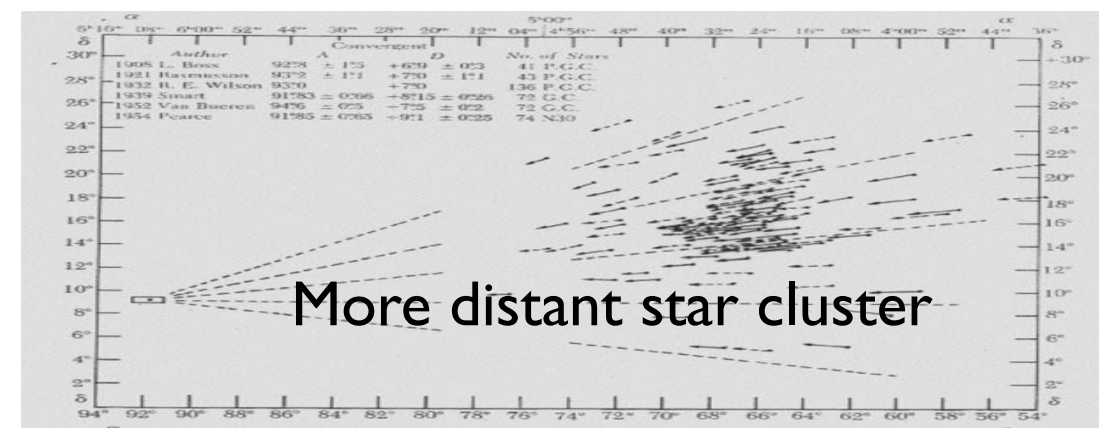
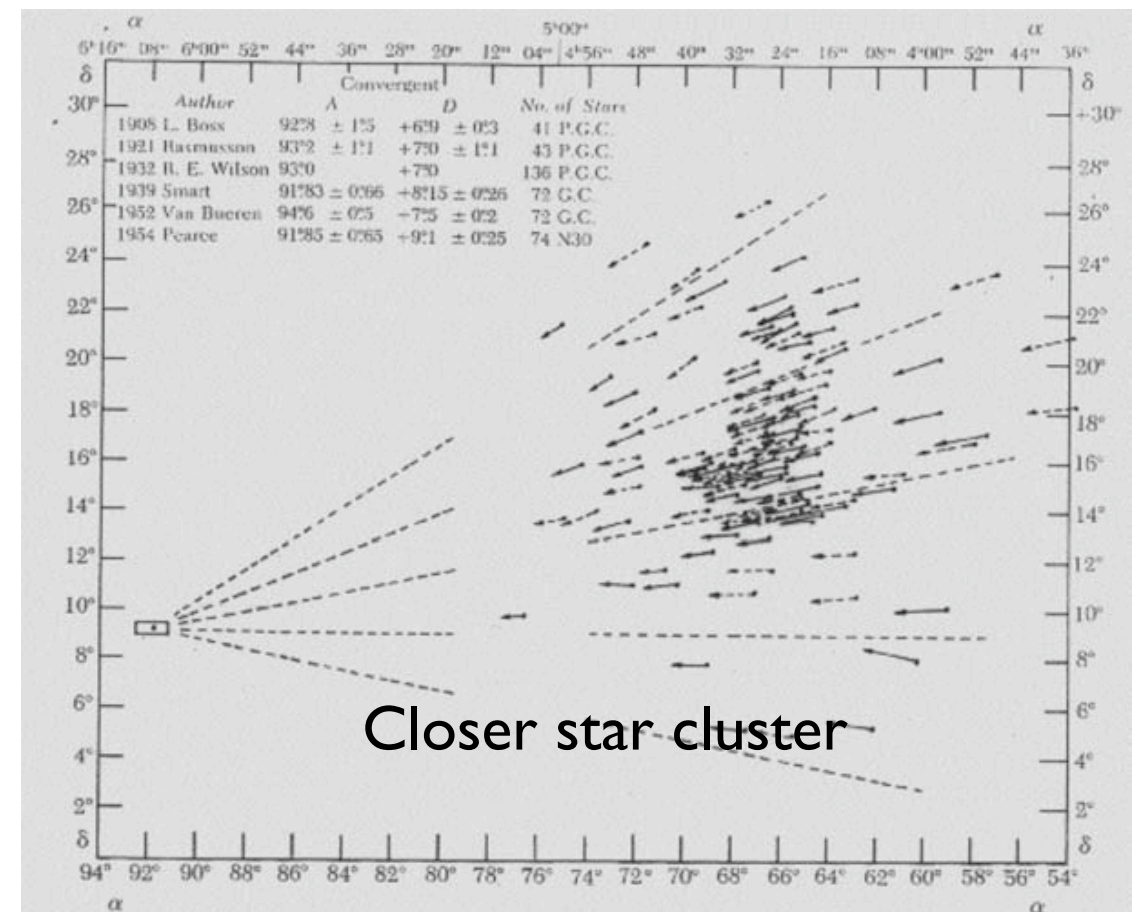
-- Powerful method of measuring distances to star clusters moving across our field of view.

-- The key concept with this approach to look for a convergence point in the motions of stars

-- If the stars seem to be converging only very gradually to some point in the far distance, they are likely very far away

-- If the stars seem to be converging at a sharp angle to some point very close by, the stars are much closer

Illustration of a group of stars moving across our field of view, converging to some point in the distance



Moving Cluster Method

So how far away is the star cluster?

We can determine it by comparing the proper angular motion

μ is the “proper angular motion”

-- perpendicular to our sightline

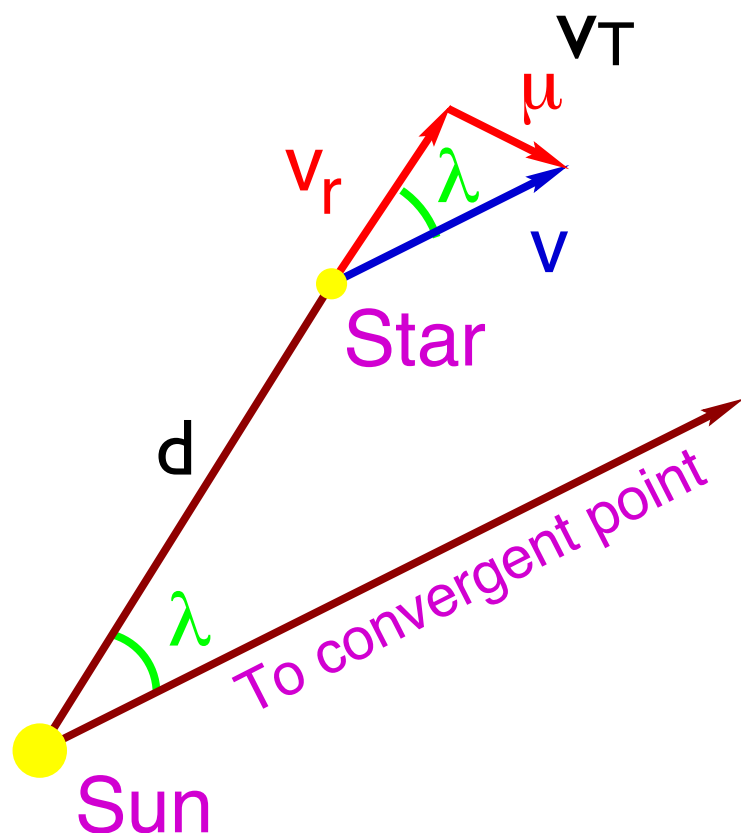
with v_T is the tangential velocity of the stars

We can compute v_T from λ and v_r :

v_r is the radial motion of stars

-- we can measure by measuring Doppler shift from spectral lines

λ is angle from star cluster to convergence point

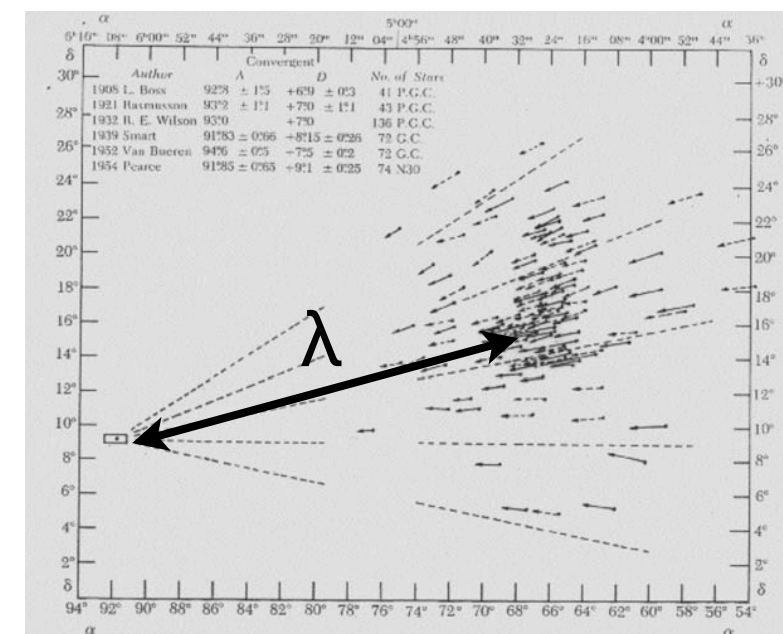


velocity of stars tangential to line of sight

$$v_T = 4.74 \mu d, \quad \mu (" / \text{yr}), \quad v_T (\text{km/s}), \quad d (\text{pc})$$

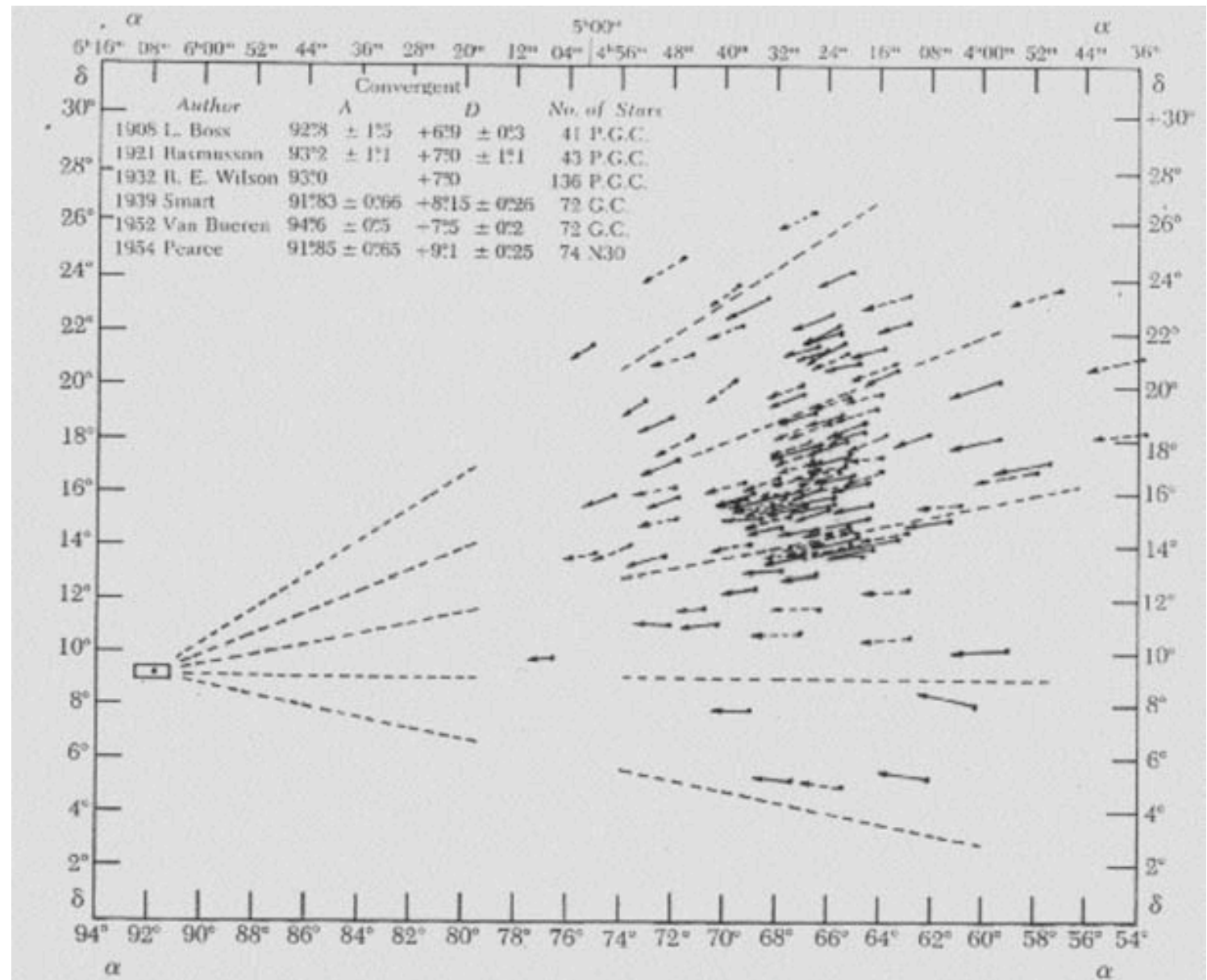
$$v_T = v_r \tan \lambda$$

$$d = v_T / 4.74 \mu = v_r (\tan \lambda) / 4.74 \mu$$



Moving Cluster Method

- Used in 1920 to determine the distance to Hyades (40 pc)
- Improved in 1960 to 46 pc
- Compare with Hipparcos measurement of 46.3 pc
- Such measurements were useful for calibrating many techniques such as main sequence fitting technique useful to measure distance to the Large Magellanic Cloud
- Distances to the Hyades and other star clusters have now been determined to great accuracy by Trigonometric parallax. So this method is primarily of historical interest

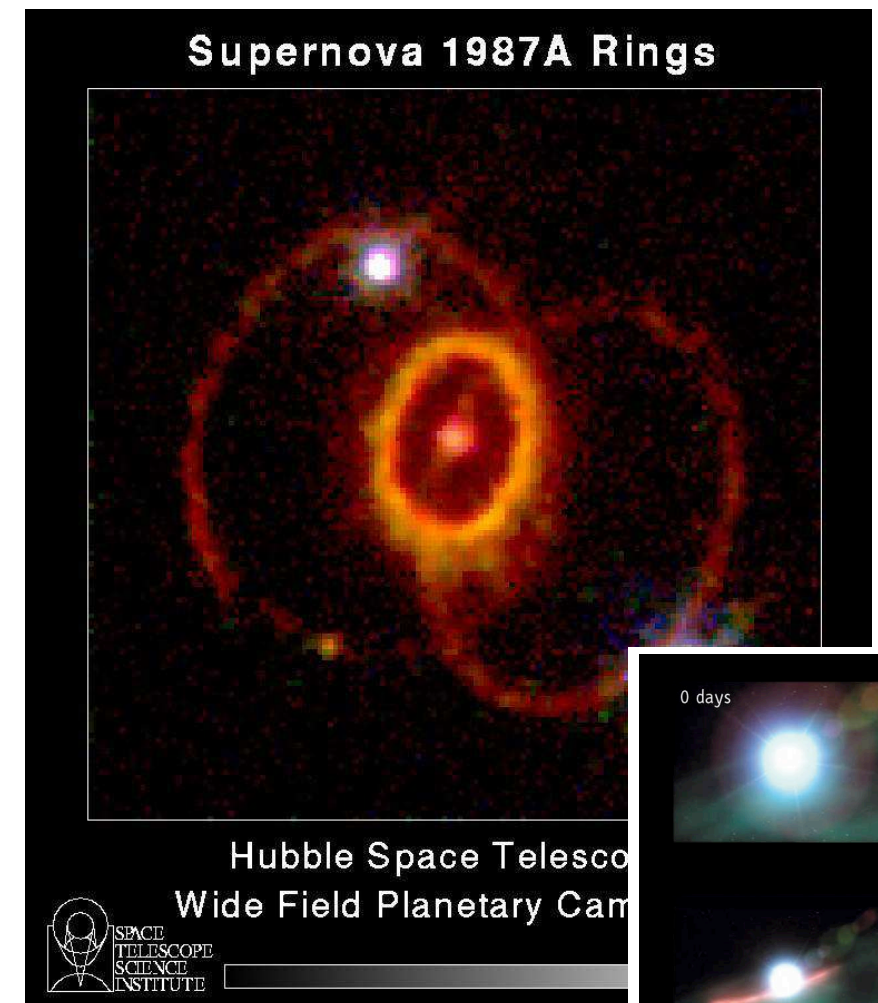


Also important to
determine distances to

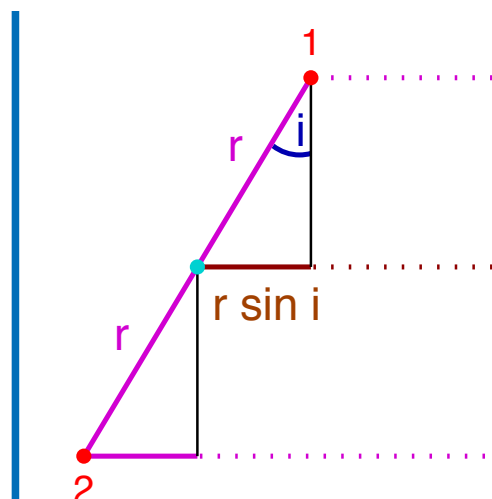
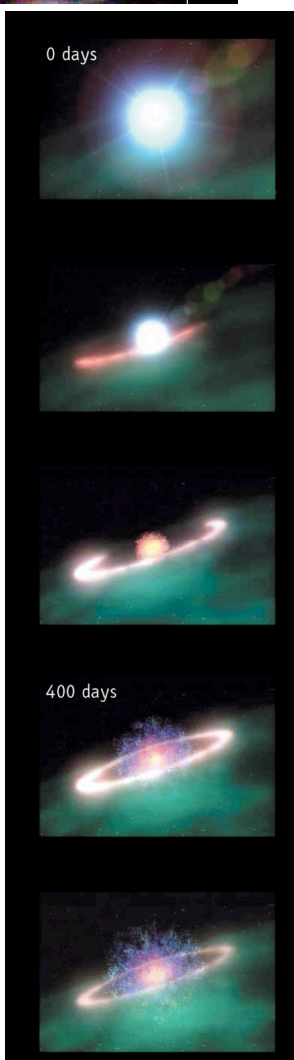
Ursa Major 23.9 pc
Scorpius-Centarus cluster 172 pc
Pleiades 126 pc

Light Echos

- Direct geometric determinations of distance are possible using light echos
- Such a situation occurred in 1987 with a supernovae in the Large Magellanic Cloud
- Energy from supernovae excited C and N in ring-like structure
- Time delay between detection of SNe explosion and echos gives physical size of ring-like structure
- By comparing the inferred physical size of the structure with the angular size of the structure ($1.66''$), we have the distance to the Large Magellanic Cloud



Time
Sequence



Thus from angular size of ring:

$$1.66'' = \frac{r \cos i}{d} \implies d = 52 \pm 3 \text{ kpc}$$

Other Primary Methods

- Eclipsing Binary Stars: From Photometric and Radial Velocity Measurements, the Fundamental Properties (Radii, Luminosity, Temperature) of the Stars Can Be Determined. By comparing the luminosity of the stars with their brightness, one can solve for the distance to the binary star system.
- Baade-Wesselink Method: For a Variable Star (With a Time-Varying Radius), one can determine the luminosity of the star from the available observations.

We can infer the temperature of a star from its spectrum (assuming the star is roughly a blackbody emitter). If we could infer its radius R , then we could derive its luminosity as

$$L = 4\pi R^2 \sigma_b T^4$$

Fortunately, we can measure the velocity at which a variable star is expanding and/or contracting from how much its light is Doppler-shifted and hence infer the change in radius.

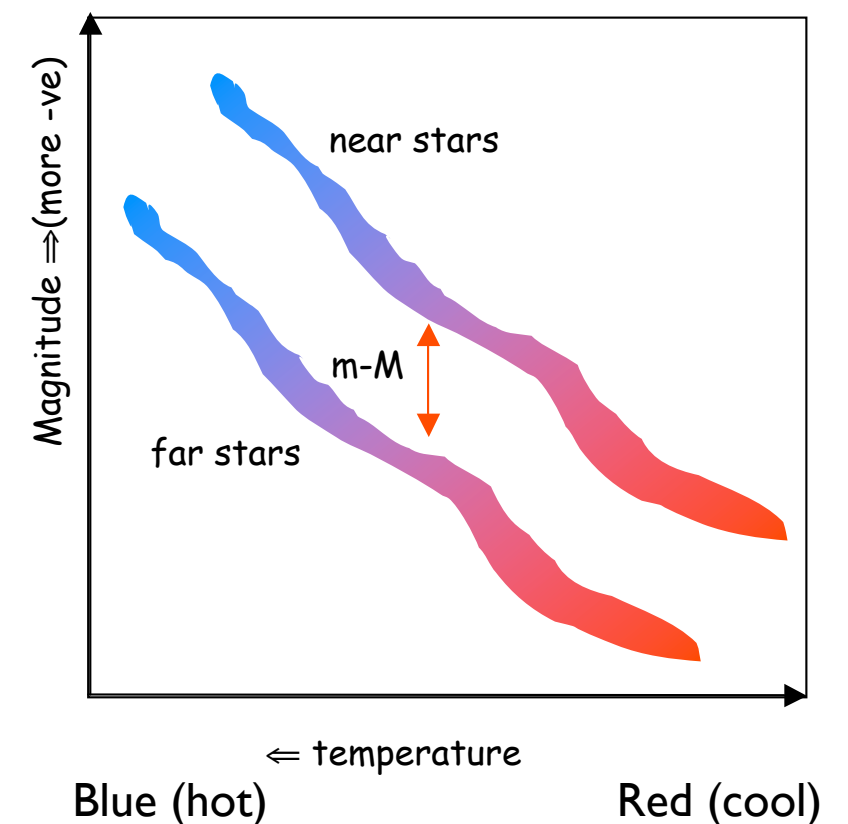
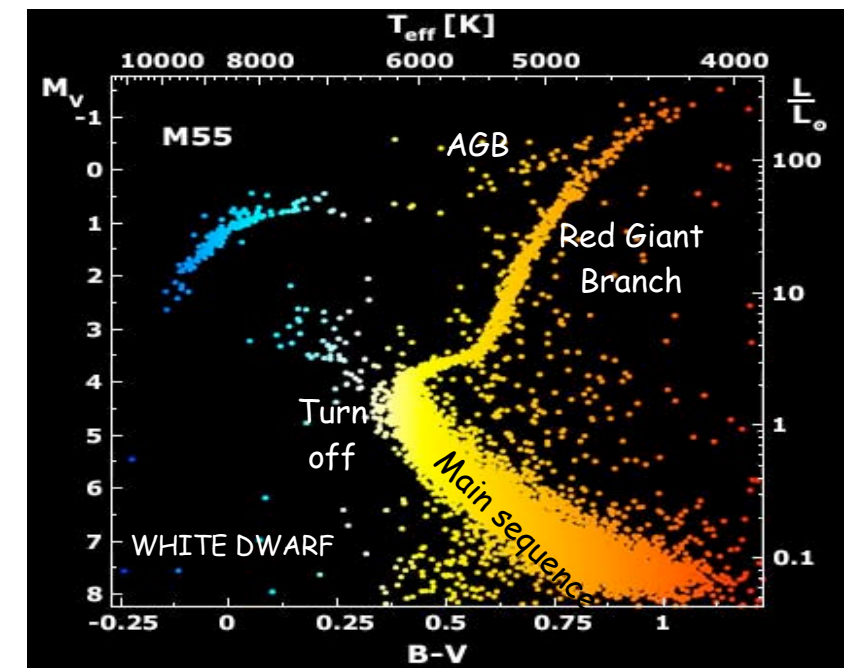
$$\text{Then } R_2 = R_1 + \int v(t) dt$$

where R_1 and R_2 are the size of the star at its minimum and maximum size. Putting this constraint together with the fact we can measure its relative brightness at minimum and maximum size, we have two equations and two unknowns (R_1 and R_2). We can then determine the luminosity of a variable star directly and also its distance.

Main-Sequence Fitting

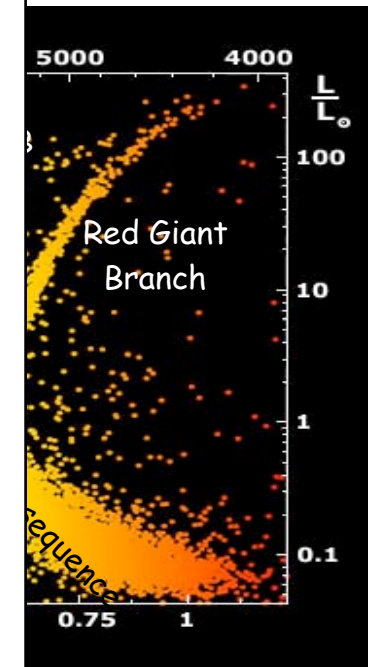
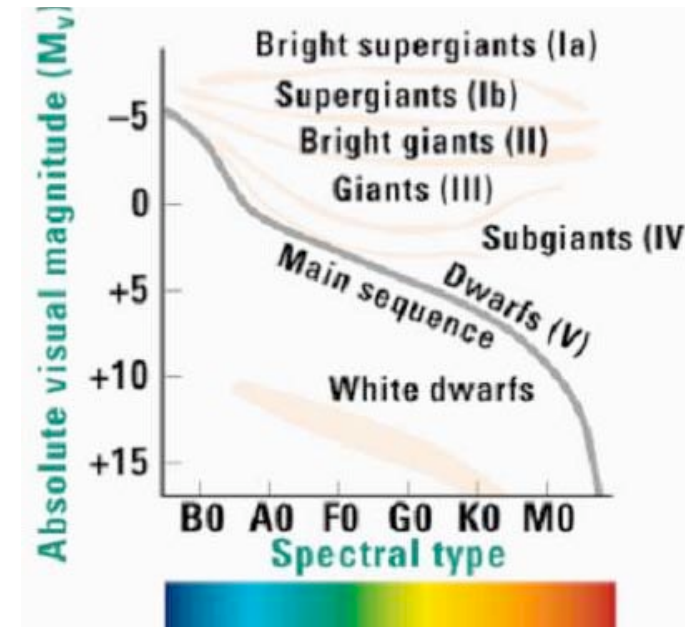
- Stars on the Hydrogen-burning Main Sequence are effectively standard candles, exhibiting consistent luminosities and colors
- Magnitudes for color-magnitude sequence calibrated using parallax distances from Hipparcos (Hyades closest star cluster to use as calibrator)
- Magnitude shift gives difference in distances:
distance modulus = $m_1 - m_2 = 5 \log (D_1/D_2)$
- Useful out to a few 10s of kpc (beyond this distance, main sequence becomes too faint)...
- Need to correct for dust extinction
- Can be affected by metallicity of stars

HR Diagram



Main-Sequence Fitting

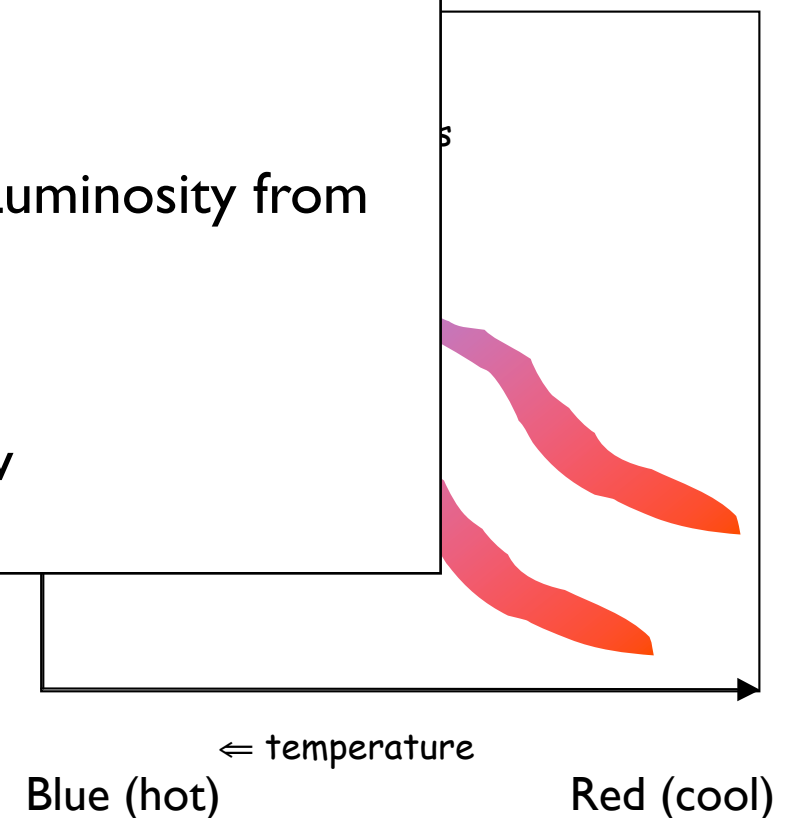
- Stars on effectively
 - Magnitude using parallax
- Somewhat similar technique is that of spectroscopic parallax technique: **H-R diagram**
- Spectral Types (Prominent Lines):
- O - He I, HeII
 - B - He
 - A - H
 - F-G - Metals
 - K-M - Molecular Lines



- Magnitude distance
 - Useful on
 - Need
 - Can be
- Surface Gravity (from pressure causing line broadening):
- Class I (Supergiants)
 - Class V (Dwarfs)
- Estimate Temperature from Spectral Type and Mass/Luminosity from surface gravity

$$L = 4\pi\sigma T^4 R^2$$

Distance then from inverse square law



Secondary methods
(standard candles -- calibrated
based on primary methods)

Cepheids and RR Lyraes
“often treated as primary methods”

Cepheids

-- Very Luminous, pulsating variable stars that lie on the instability strip

-- Henrietta Leavitt showed that there was a tight period-luminosity (P-L) relationship for these stars

$$L \sim P^{1.3}$$

-- Advantages

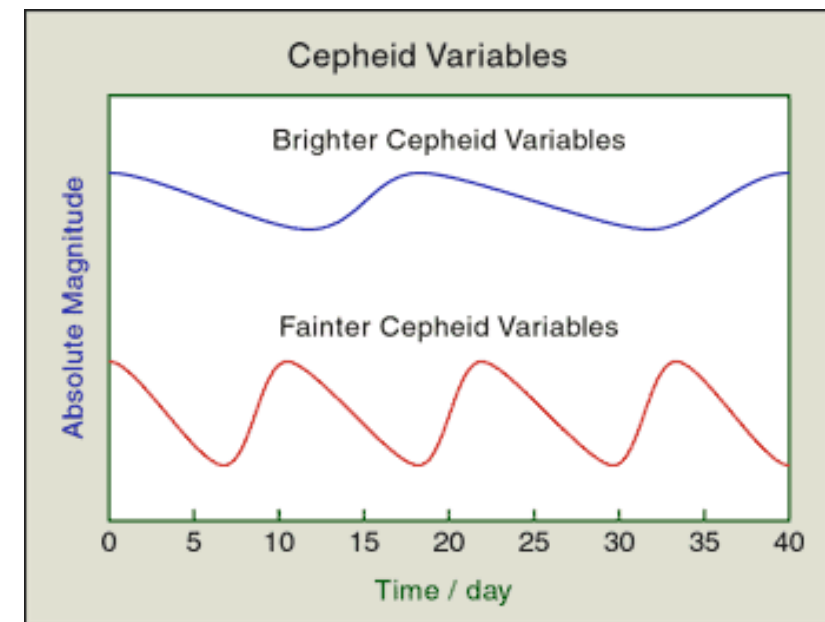
- * Very luminous stars that can be seen from great distance in nearby galaxies
- * physical origin of the period-luminosity relation well understood

-- Disadvantages

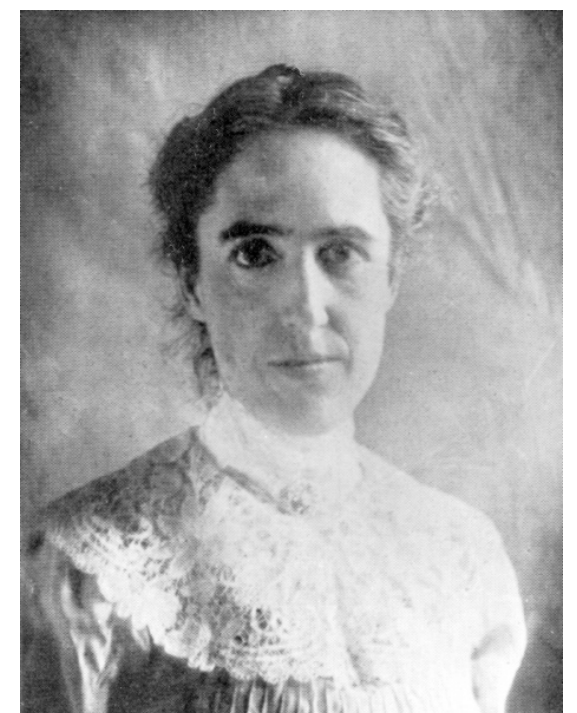
- * Relative Rare
- * Multi-epoch observations are necessary -- since need to determine period of cepheids
- * Light from these galaxies can be affected by dust extinction and so corrections need to be made.

-- One of the most important standard candles for setting up the entire distance ladder

-- More Luminous Cepheids have longer periods



Henrietta Leavitt (1868-1921)



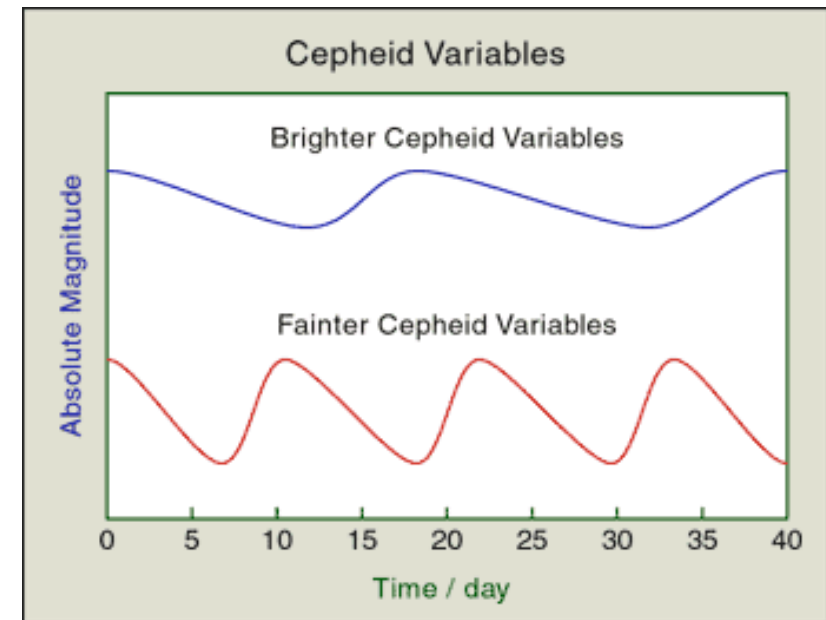
Cepheids

-- Cepheid Luminosities are typically calibrated using those in the Large Magellanic Clouds

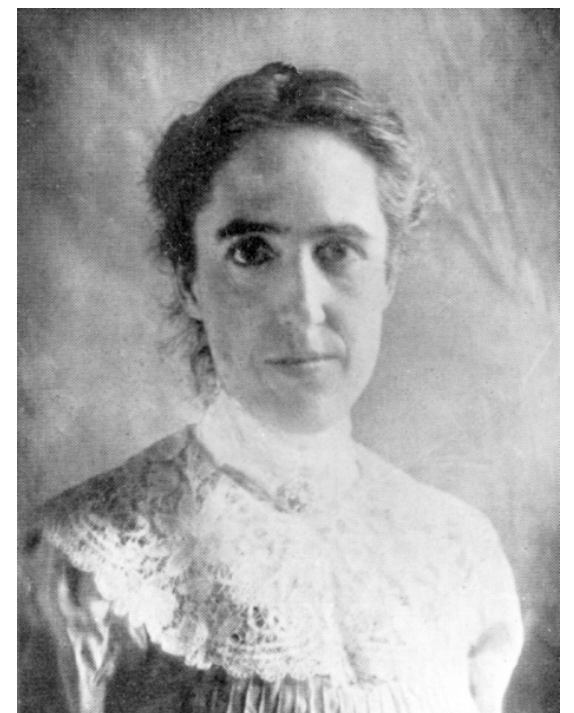
-- Uncertainties:

- * Luminosity of the brightest Cepheids may deviate from P-L relationship
- * Luminosity exhibits slight dependence on metallicity of star

-- More Luminous Cepheids have longer periods



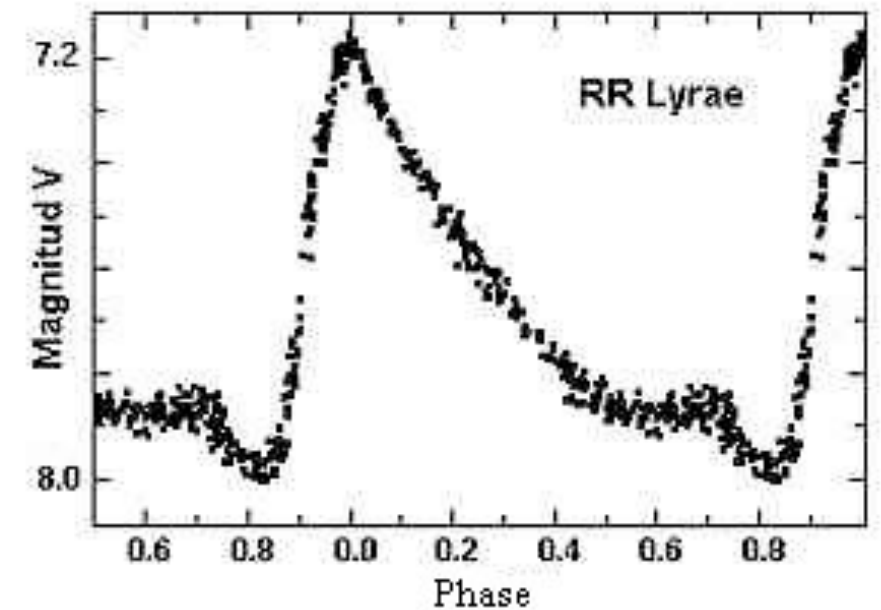
Henrietta Leavitt (1868-1921)



RR-Lyraes

- Less luminous, pulsating variable stars that lie on the instability strip
- Pulsation periods much shorter, so require less observing time to find
- Found predominantly amongst older stellar populations
- Therefore useful to derive distances to globular clusters
- Because of their low luminosities (factor of >6 fainter than Cepheids), only useful to measure distances to M31 (~ 1 Mpc away).

Typical Oscillation Pattern for RR-Lyraes (0.6 days)



Large Magellanic Cloud

-- Large Magellanic Cloud is an extremely important rung in the distance ladder

“anchor point”

-- Calibrated by Light Echos

-- Calibrated by Main Sequence Fitting

-- Calibrated by RR Lyrae

Large Magellanic Cloud (~50 kpc)
Dwarf Galaxy (next to Milky Way)

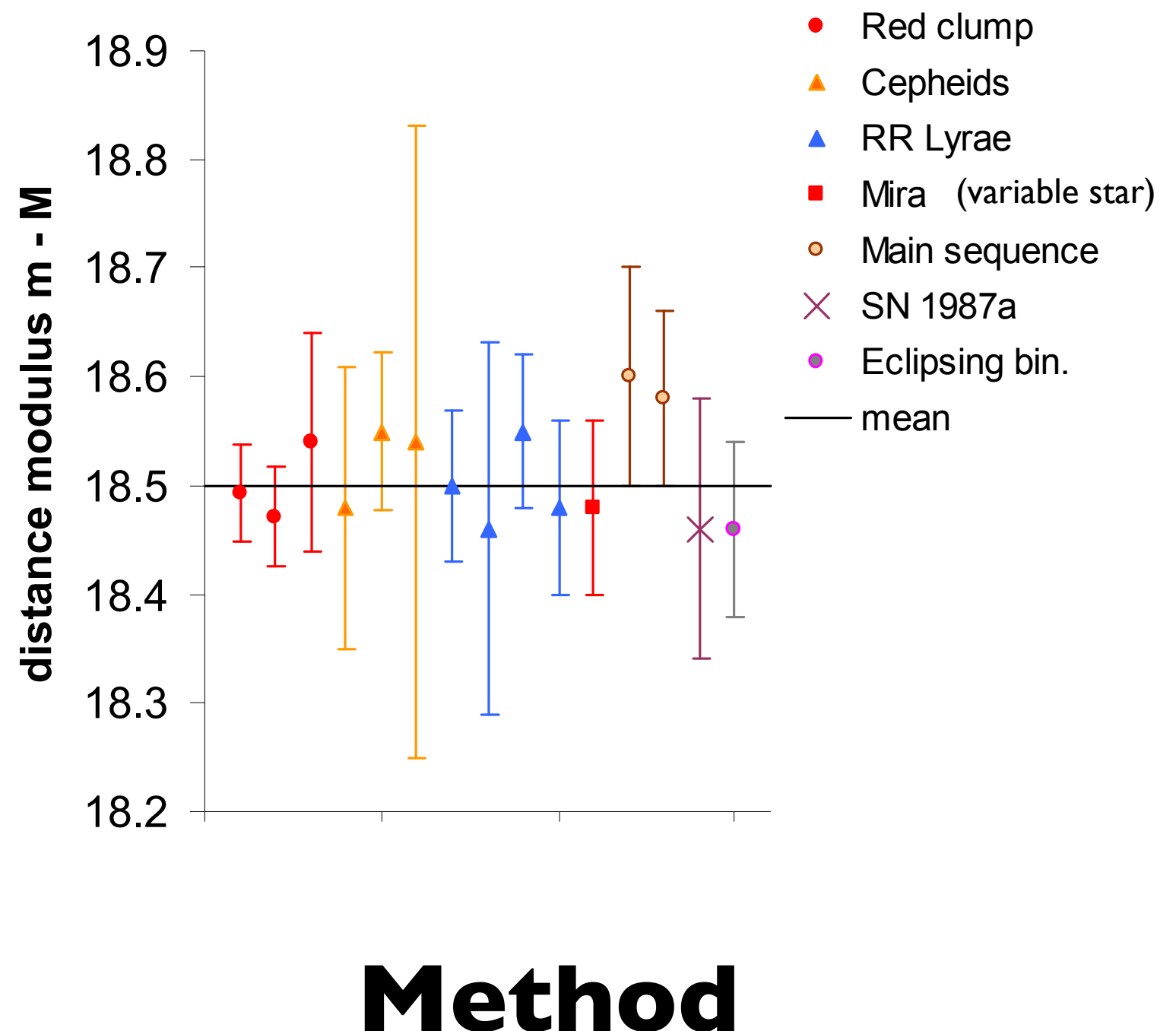


Large Magellanic Cloud

- Recent measurements of LMC distance show excellent consistency

- ▶ weighted mean distance modulus 18.50 ± 0.02 mag
- ▶ 50.1 ± 0.5 kpc
- ▶ Cepheid distances consistent with mean

Alves, *New AR 48* (2004) 659,
compilation of measurements
since 2002



Secondary methods (calibrated based on primary methods)

Cepheids and RR Lyraes

Planetary Nebula LF

Globular Cluster LF

Brightest Stars

Type Ia Supernovae

Tully-Fisher and Faber Jackson

$D_n - \sigma$

Surface Brightness Fluctuations

Secondary methods

(calibrated based on primary methods)

Some techniques
are particularly
relevant to
systems that are
actively forming
stars

Cepheids and RR Lyraes

Planetary Nebula LF

Globular Cluster LF

Brightest Stars

Type Ia Supernovae

For other
techniques are
useful for systems
with either old or
young stars

Other techniques
are particularly
relevant to
systems with lots
of old stars

Tully-Fisher and Faber Jackson

$D_n - \sigma$

Surface Brightness Fluctuations

Secondary methods

(calibrated based on primary methods)

Some techniques
are based on
standard candles

Cepheids and RR Lyraes

Planetary Nebula LF

Globular Cluster LF

Brightest Stars

Type Ia Supernovae

Other techniques
are based on the
idea of a standard
rod

Tully-Fisher and Faber Jackson

$D_n - \sigma$

Surface Brightness Fluctuations

For a standard candle

$$\text{Distance} = (L_{\text{candle}} / (4\pi f))^{1/2}$$

$f = \text{flux}$

For a standard rod

$$\text{Distance} = R_{\text{standard-rod}} / \theta$$

$\theta = \text{angle subtended by source}$
 on sky

For the standard candle approaches

Some are based on the luminosity function (volume density as a function of luminosity) of bright sources:

Planetary Nebula LF

Globular Cluster LF

Brightest Stars

For each population, there is some clear feature in the luminosity function that one can pick out in distant objects

For the standard candle approaches

Some are based on the luminosity function (volume density as a function of luminosity) of bright sources:

All of the objects below are visible at great distances

Planetary Nebula LF

Late stages of stellar evolution

Globular Cluster LF

Compact Old Star Clusters

Brightest Stars

Most luminous red giant stars

For the standard candle approaches

Some are based on the luminosity function (volume density as a function of luminosity) of bright sources:

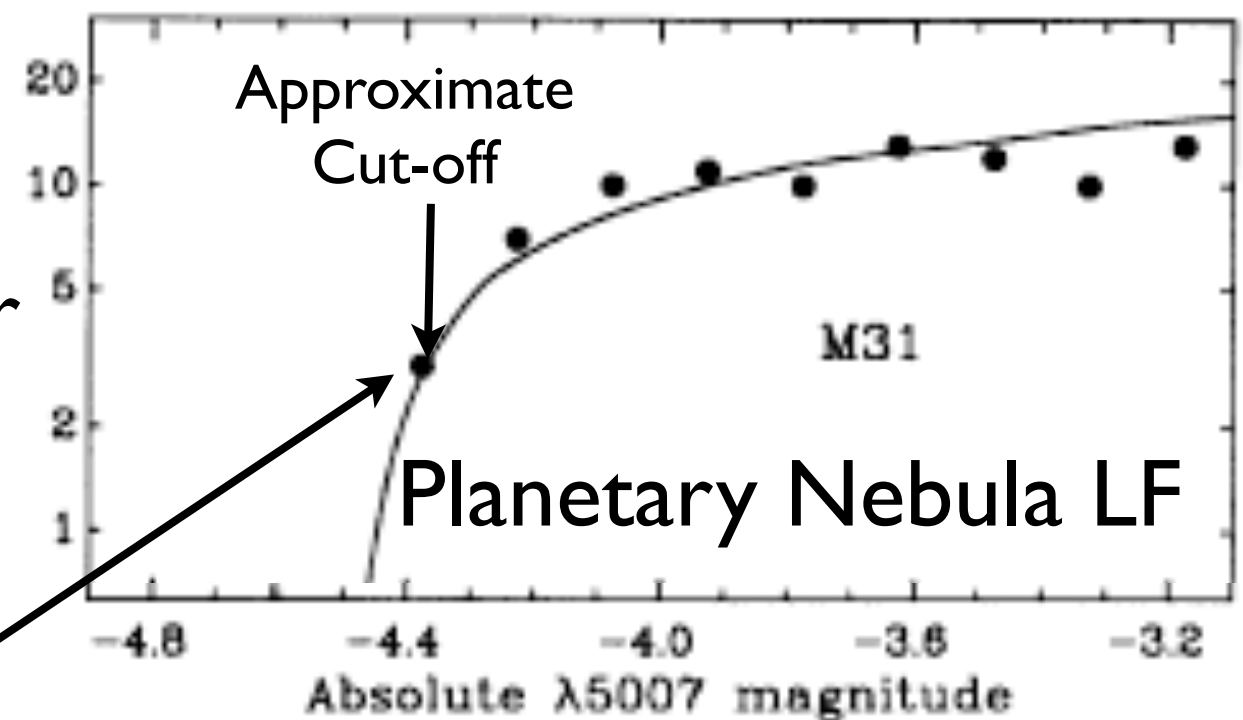
Planetary Nebula LF

Globular Cluster LF

Brightest Stars

Look for characteristic feature

Number



For the standard candle approaches

Some are based on the luminosity function (volume density as a function of luminosity) of bright sources:

What do we look for in LF?

Planetary Nebula LF \rightarrow \sim maximum luminosity

Globular Cluster LF \rightarrow \sim central luminosity

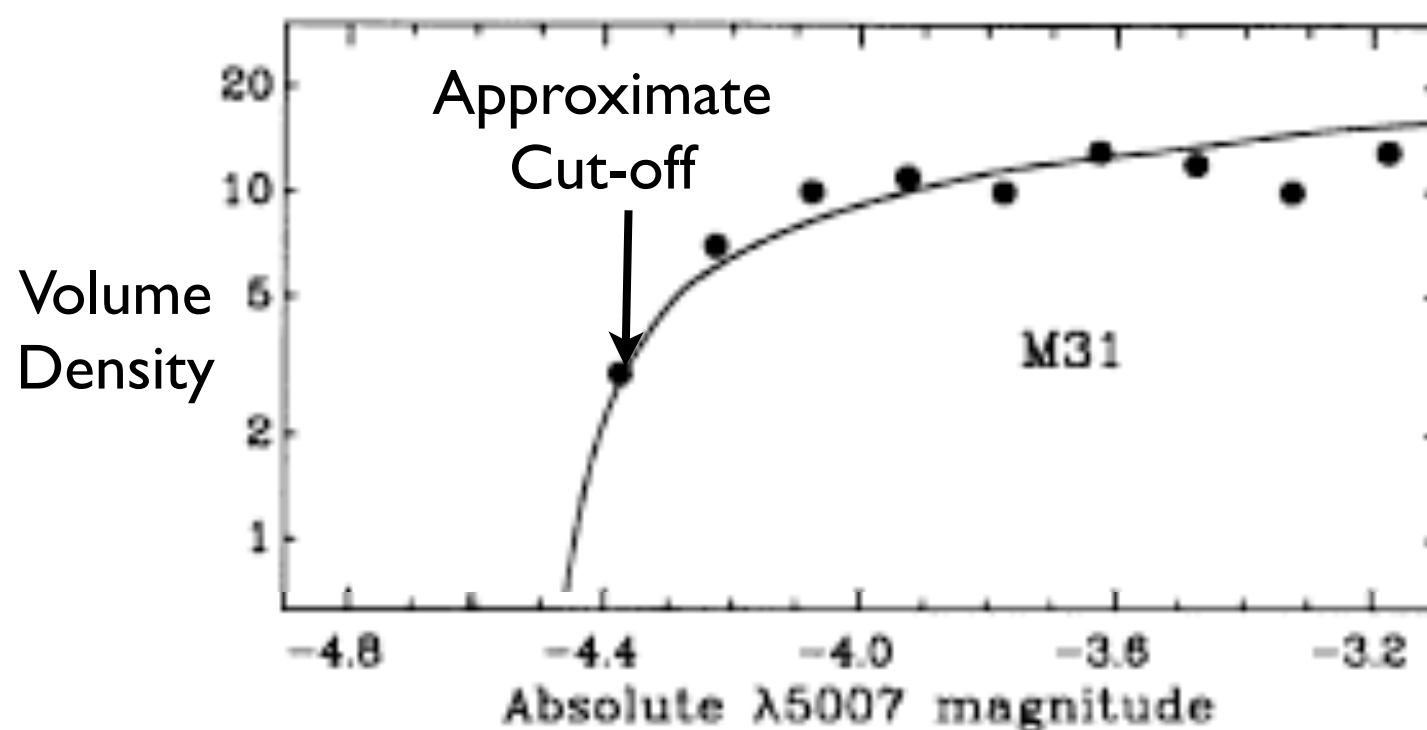
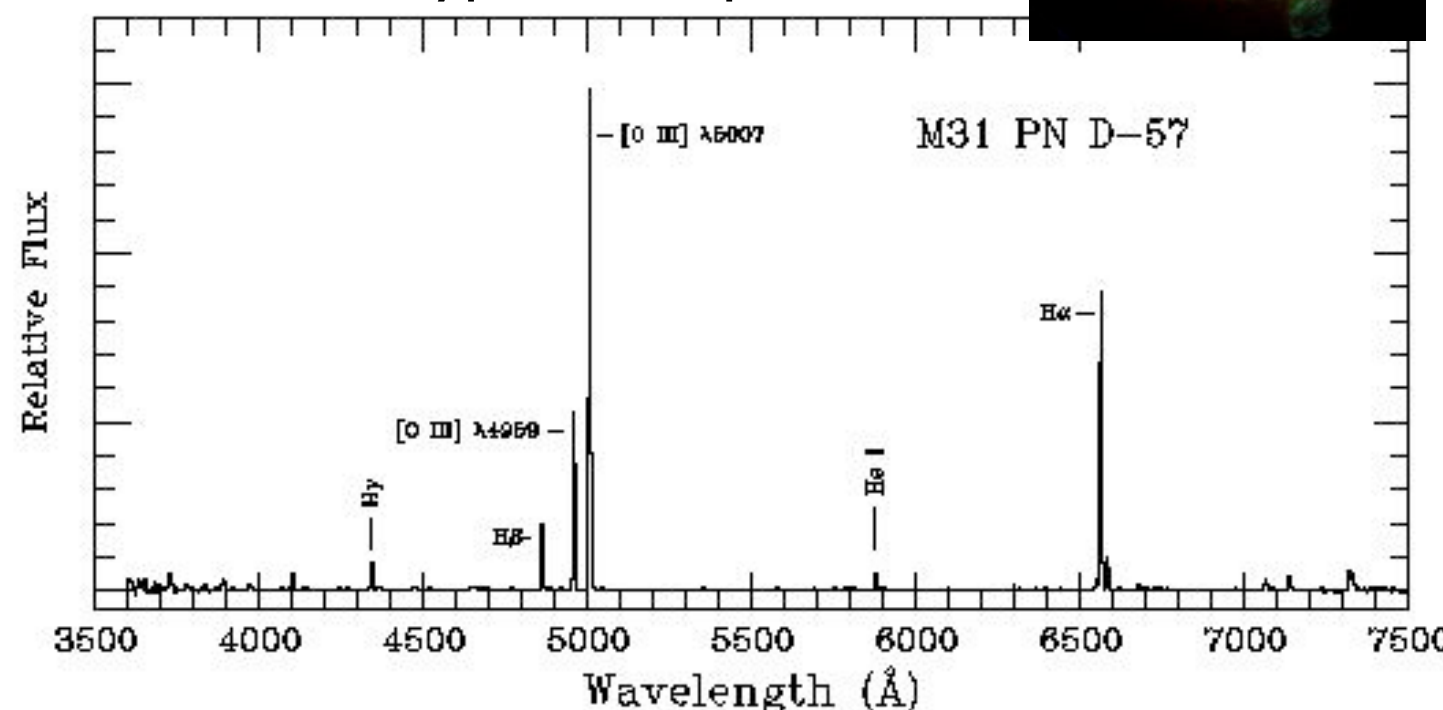
Brightest Stars \rightarrow \sim maximum luminosity

Planetary Nebula



- Near end stages of stellar evolution for stars with initial mass $0.8\text{-}8.0 M_{\text{sun}}$
- Easily detectable to great distances due to existence of strong emission lines in spectrum
- Identified using [OIII] 5007 narrow band filter
- Luminosity Function (LF) shows a sharp cut-off at some characteristic luminosity
- Cut-off can show some dependence on metallicity
- PN LF calibrated using PN in M31
- Useful out to 40 Mpc (>Virgo)
- Physical Basis Fairly Well Understood from stellar evolution

Typical PN spectrum



Globular Cluster Luminosity Function

-- The luminosity function of global clusters is approximately gaussian with a well-defined peak

-- Advantages:

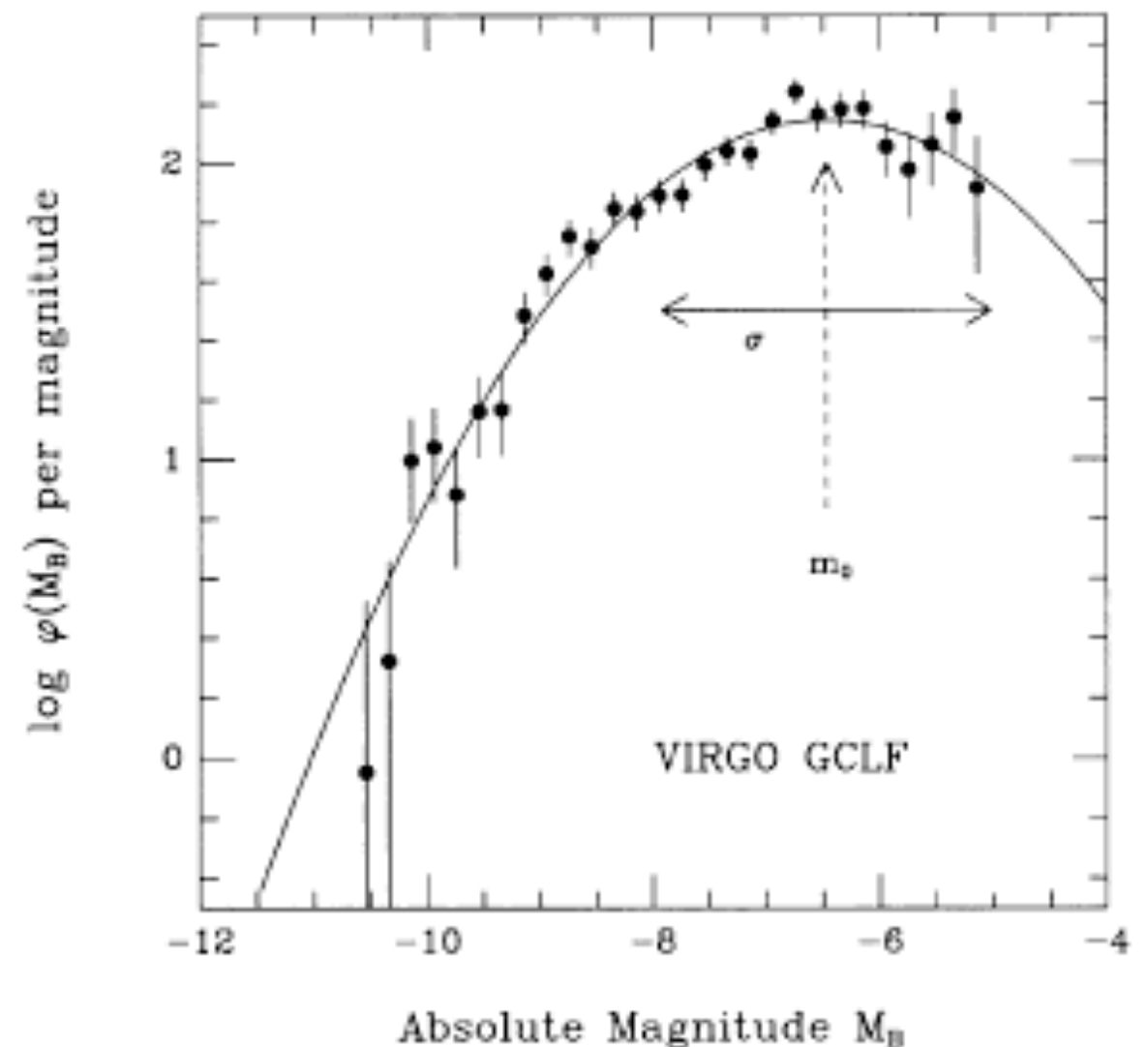
- * Luminous, easy to find
- * No dust

-- Disadvantages:

- * Later Galaxies on Hubble Sequence have many fewer globular clusters, so not practical for these systems
- * Physical Basis Not Well Understood
- * Can depend on luminosity, galaxy type
- * Peak Luminosity shows moderate scatter (~ 0.3 mag), not best technique

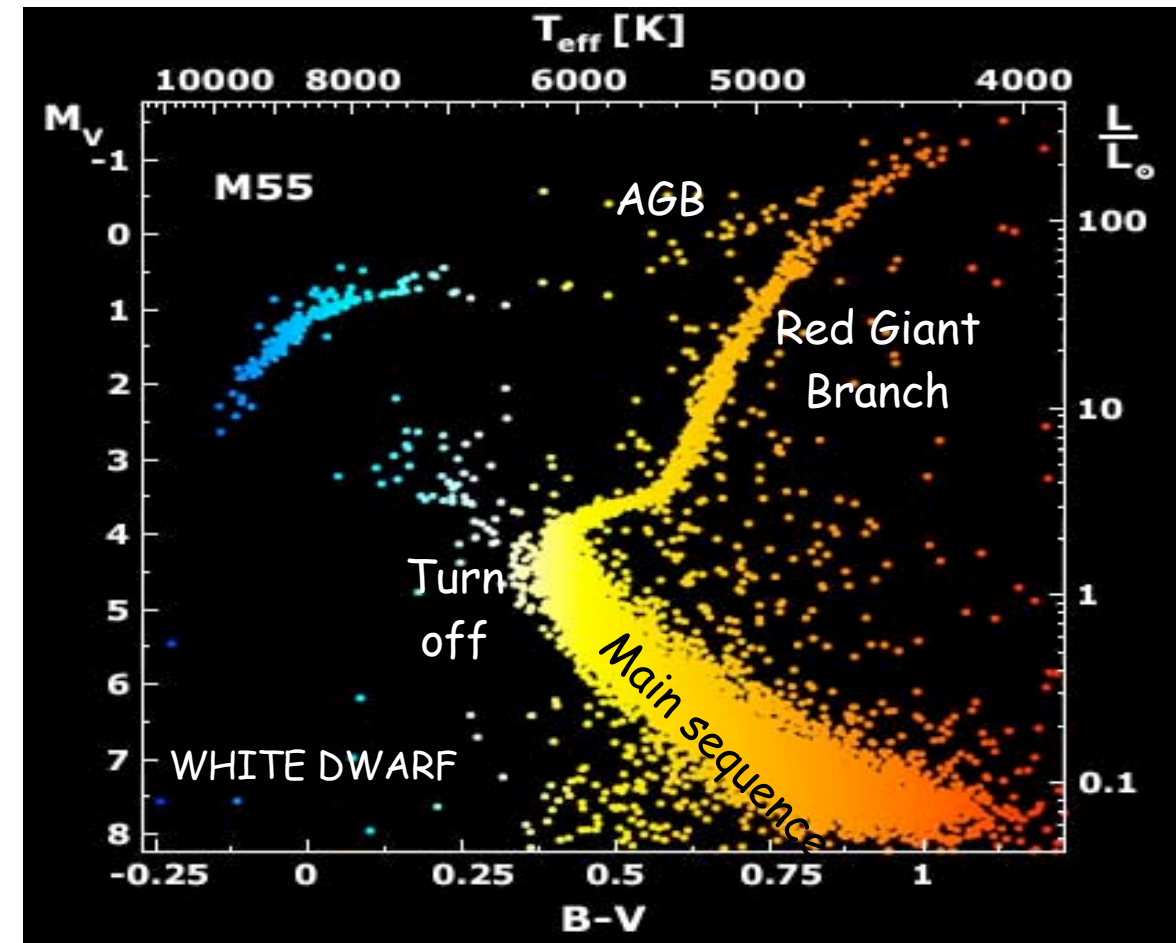
-- Useful for measuring distances to ~ 200 Mpc

GCLF for Virgo cluster, Jacoby et al. 1992



Brightest Stars

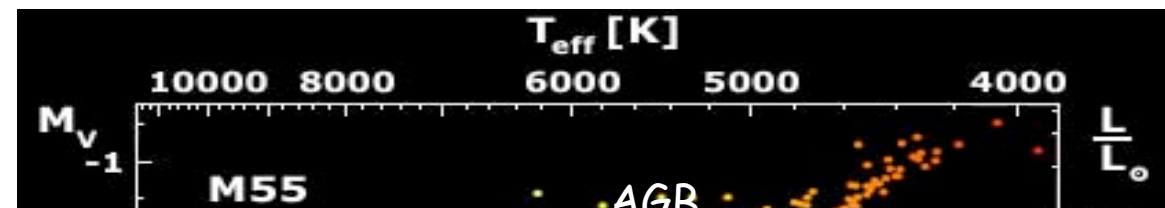
- Brightest stars in galaxies are red giants
- The luminosity of these stars do not vary substantially from galaxy to galaxy, if stars are old and metal poor
- Stars are reasonably bright and are plentiful in older galaxies, so it is easy to apply to such systems
- Because of the limited brightness of red giant stars, cannot be used beyond Virgo cluster at 15 Mpc
- Only useful for measuring distances to older galaxies
- Potential contamination of nearby galaxy by chance alignment with stars in our own galaxy (in the halo)



Brightest Stars

-- Brightest stars in galaxies are red giants

-- The luminosity of these stars do not vary substantially

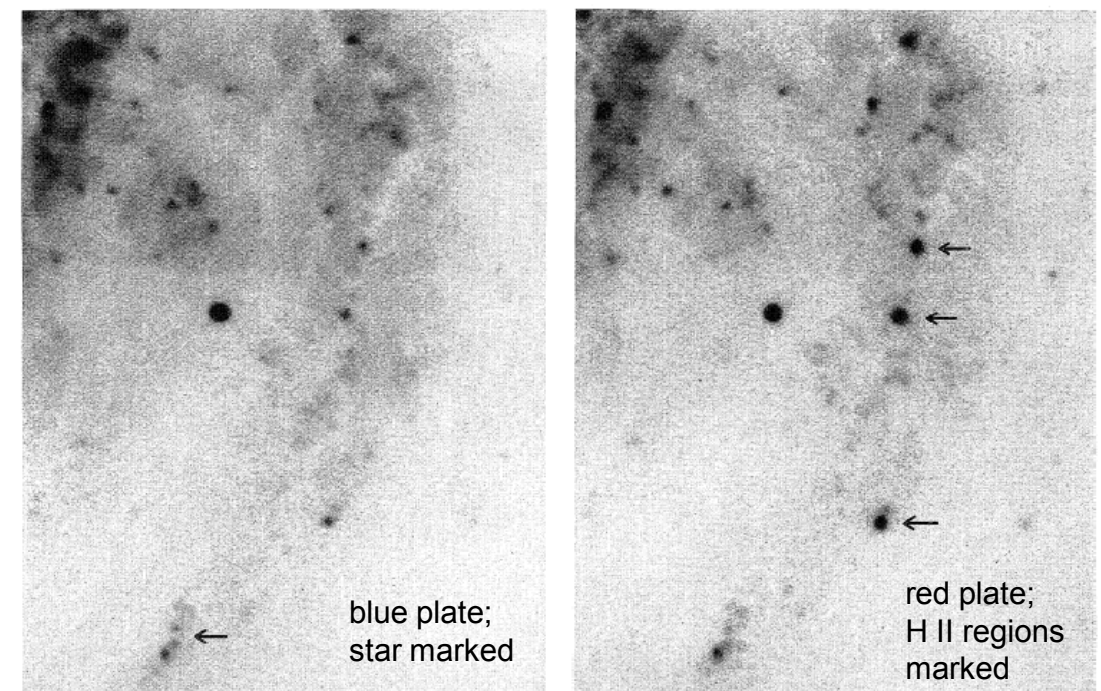


The Brightest Star Technique Was Even Used by Edwin Hubble.

However, it was not used correctly -- since Hubble was not looking at individual stars!

Stars and H II regions

M100 spiral arm



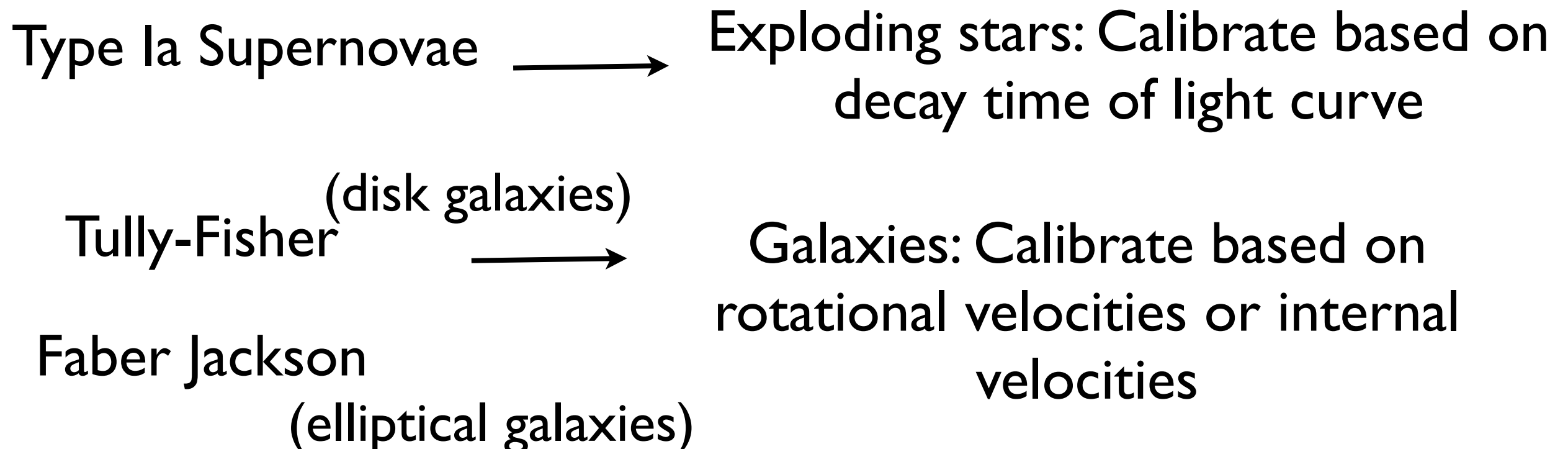
Hubble was seeing HII regions (ionised gas around young stars)!

These are much brighter than individual stars!

Difference = 2 mag

For the standard candle approaches

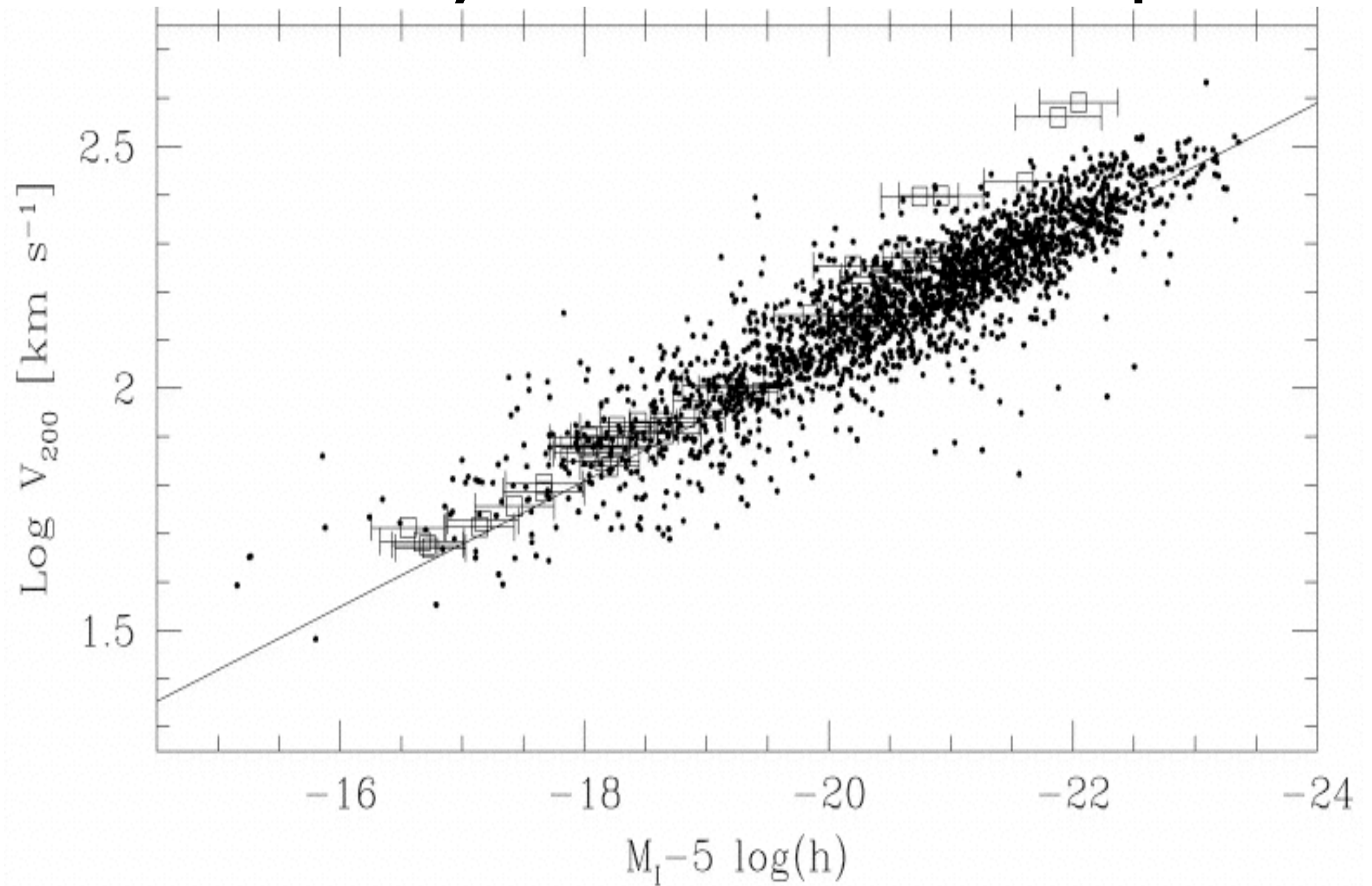
Some are based on sources having a given luminosity that we can calculate based on its properties



Intrinsic correlations for spiral galaxies

Tully-Fisher Relationship

Rotation
Velocity

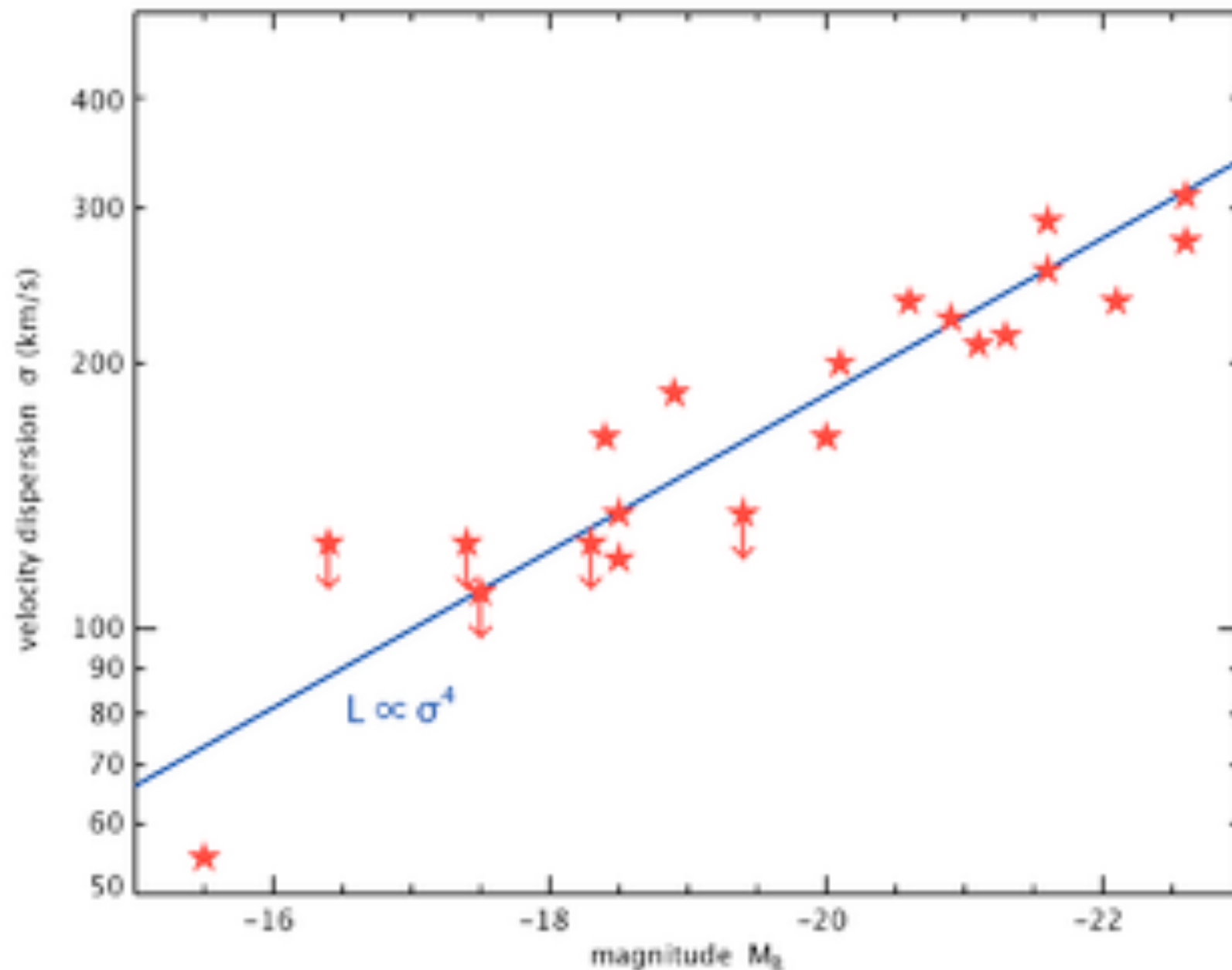


Absolute Magnitude = Luminosity

Intrinsic correlations for elliptical galaxies

Faber-Jackson Relationship

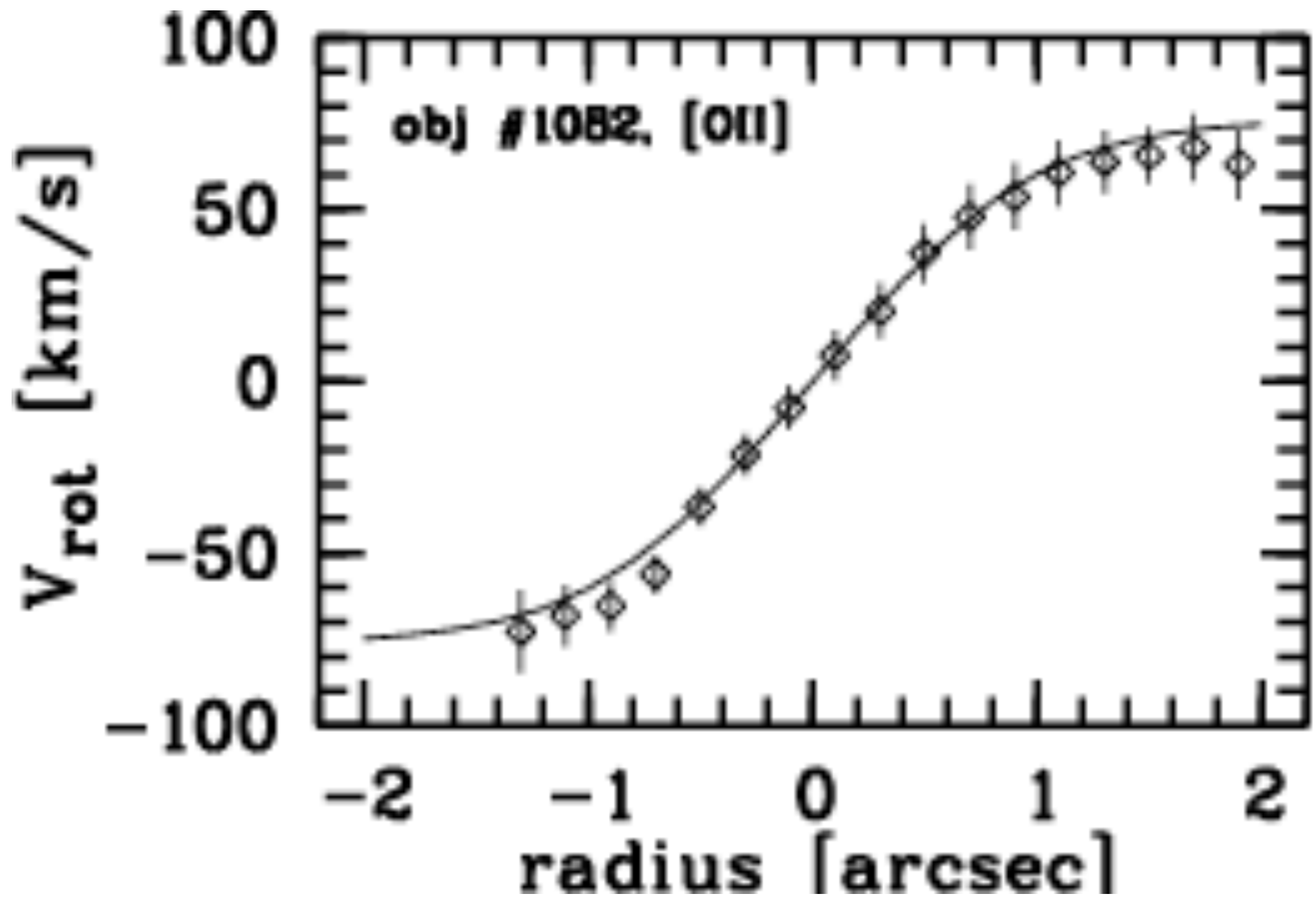
Velocity
Dispersion



Absolute Magnitude = Luminosity

Rotational Velocity Measurements

Rotation
Velocity



Velocity Dispersion Measurements

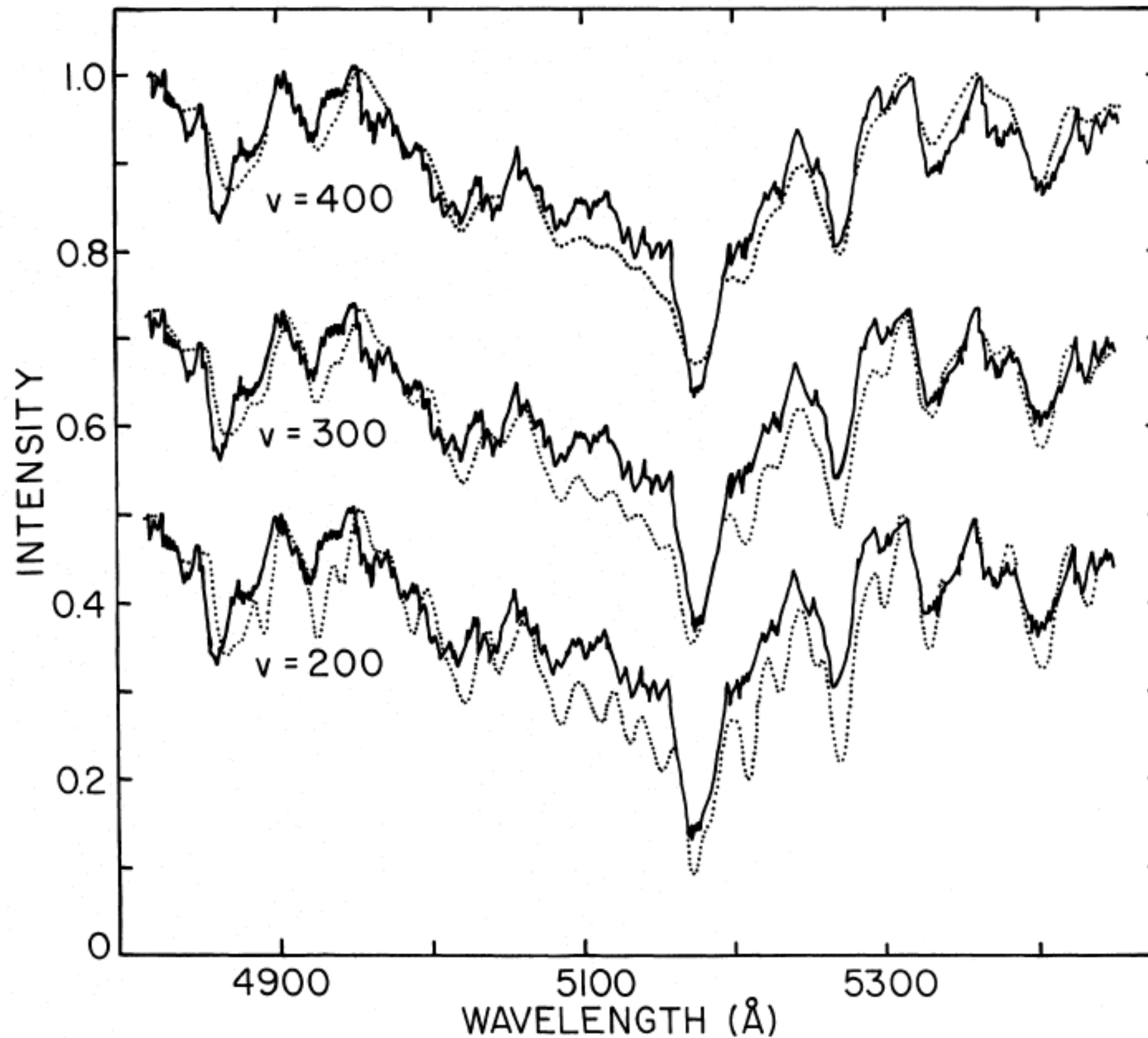
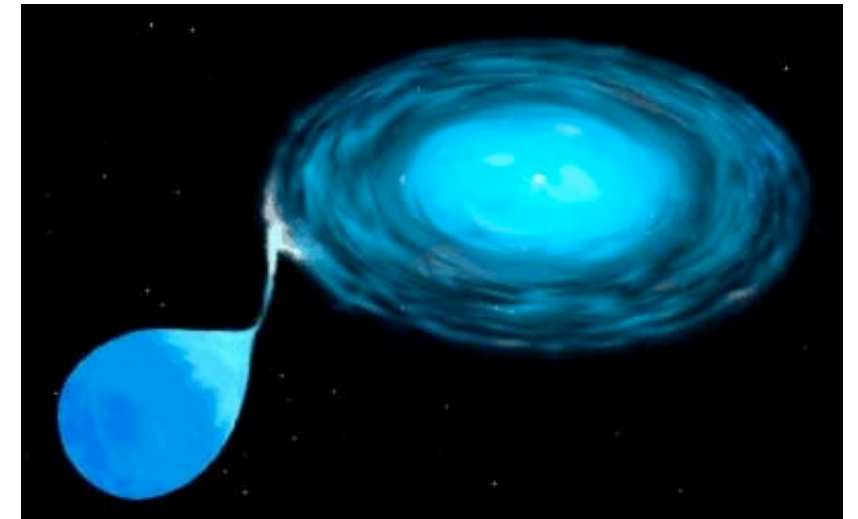


FIG. 3.—NGC 4472 compared with standard star HR 1805 (K3 III), broadened by various line-of-sight velocities (*dotted line*)

Supernovae Ia

- Likely occurs when a white dwarf is pushed over the Chandrasekhar limit of $>1.4 M_{\text{solar}}$ by accretion from a nearby companion
- All SNIa explosions have very similar luminosity. Therefore, it is useful as a standard candle
- Advantages: Since SNIa are among the brightest standard candles, it can be employed out to great distances. In fact, it is now being used to measure distances to galaxies with redshifts of $z \sim 2$ (>3 Gpc away).
- Disadvantages: SNIa explosions are somewhat rare and therefore it is difficult to use to measure the distance to a given galaxy
- Will discuss in much more detail in ~ 1 -2 weeks

Accretion of matter from a nearby companion onto a white dwarf



For the standard rod approaches

Based on the empirical relationship of the size to the internal properties of galaxies

(due to common formation mechanism for galaxies)

$D_n - \sigma$



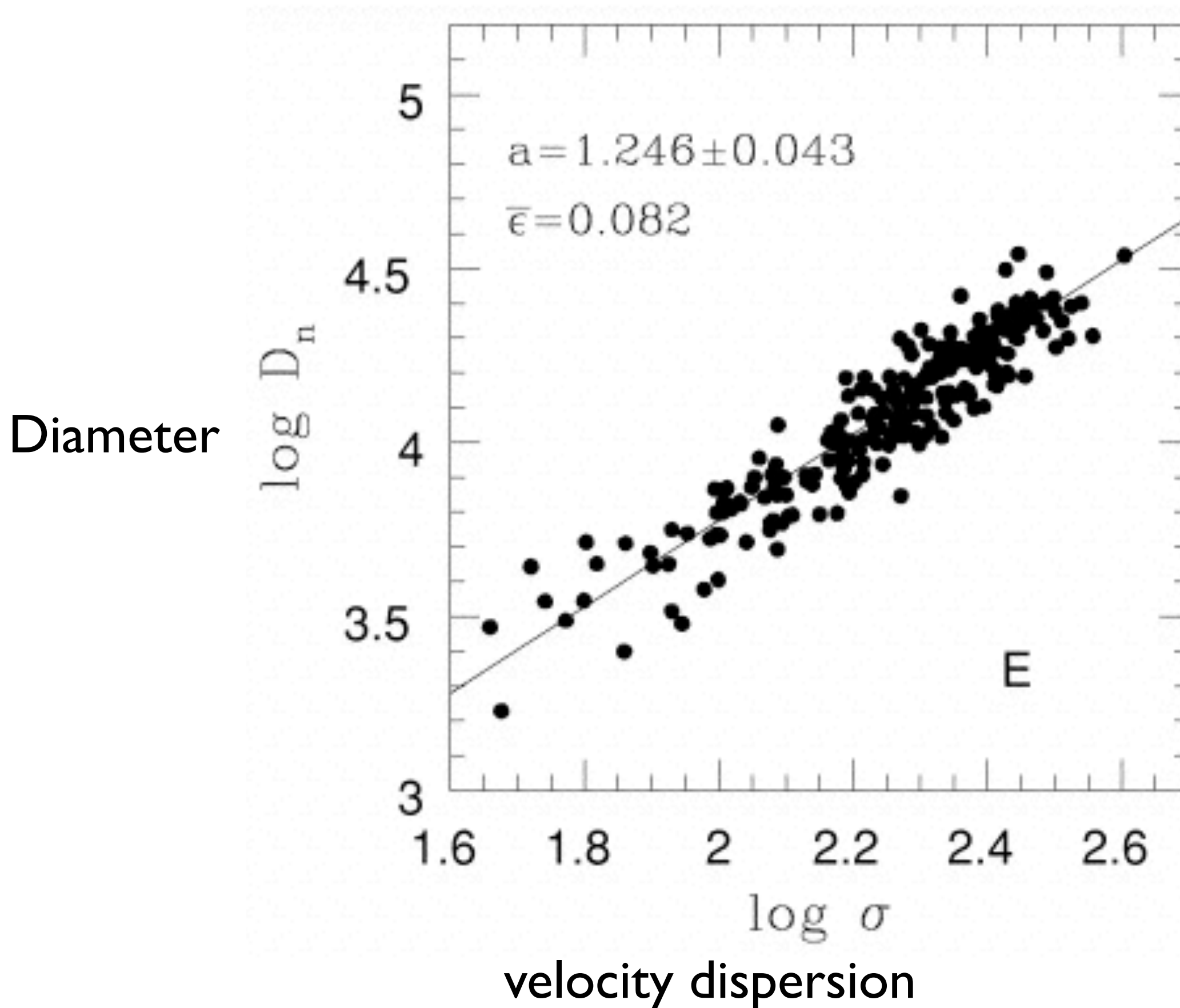
Relation of Galaxy Diameter D_n to the scatter in its internal velocities

Surface
Brightness
Fluctuations



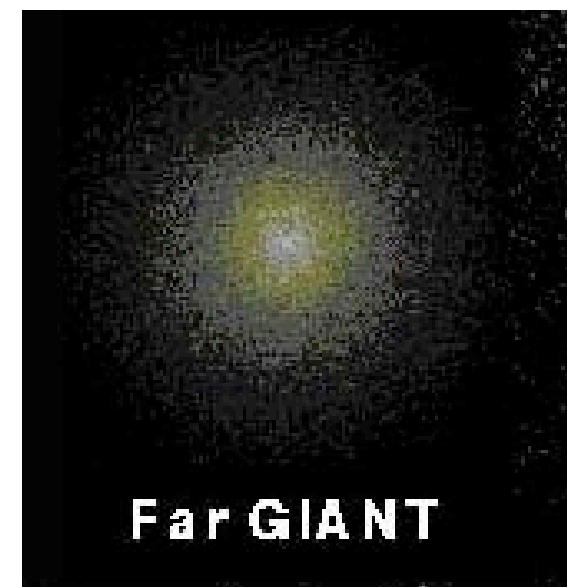
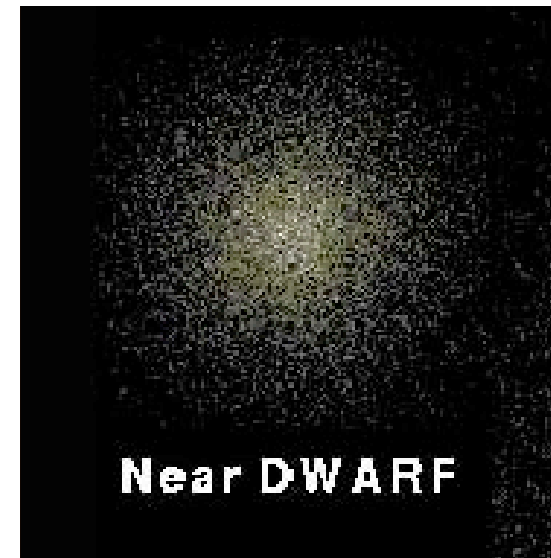
Approximately fixed surface density of bright stars in galaxy. Fluctuations in this density tell us how many bright stars there are per pixel

Intrinsic correlations for elliptical galaxies



Surface Brightness Fluctuations

- Surface brightness of old galaxies dominated by ultra-bright red giant stars
- Physical Surface density of such ultra-bright red giant stars is relatively constant
- If you are close to such a galaxy, you would expect it to look grainy with discrete bright stars
- If you are far from such a galaxy, you would expect it to look smoother



Surface Brightness Fluctuations

- It is possible to quantify the smoothness of such objects by looking at the fluctuations in the pixel surface brightnesses

Larger Fluctuations \Rightarrow Nearby Galaxies

Smaller Fluctuations \Rightarrow More Distant Galaxies

- A useful way of modeling the fluctuations in the surface brightness is using the Poissonian distribution where r

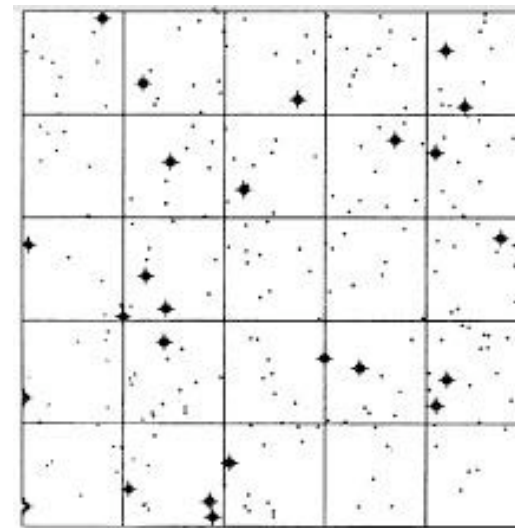
$$P(N) = e^{-\mu} (\mu^N / N!)$$

where μ is the mean number of stars per pixel

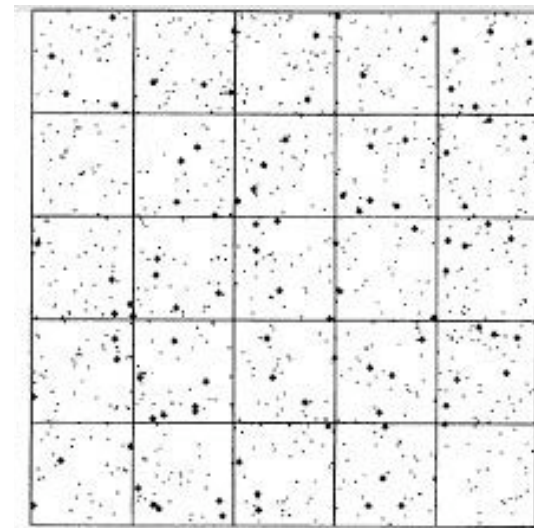
- The standard deviation σ is $\mu^{1/2}$

- The relative fluctuations are $\mu^{1/2}/\mu \sim \mu^{-1/2}$

Larger Fluctuations



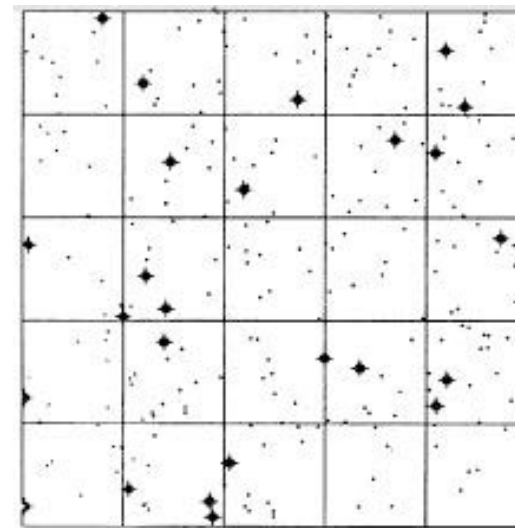
Smaller Fluctuations



Surface Brightness Fluctuations

- Strong Wavelength and Color Dependence (ultra-bright stars are red), so calibration is necessary
- Can Measure Distance Reliably to ~ 70 Mpc with the Hubble Space Telescope

Larger Fluctuations



Smaller Fluctuations

