# Observational Cosmology

Scope:
To Provide you with the
Observational Basis for the
Modern View of Cosmology

Lecturer: Rychard Bouwens

February 5, 2024

# What do we know about the universe and how do we establish it from the observations?

- What does the Universe look like?!
- What is it made of?!
- Is it finite or infinite?!
- How old is it?!
- How will it end?!
- What does physics say about the Universe?!
- What does the Universe say about physics?!
- How did the structures we see form?!
- When did they form?!
- Where do we fit into this picture?!

# Lectures

Rychard Bouwens
Oort 459
bouwens@strw.leidenuniv.nl

Lecture Hours:
Mondays 9:00-10:45
Huygens 207

Course Website:
http://www.strw.leidenuniv.nl/~bouwens/obscosmo/

# Textbook?

Recommended Textbook for the course will be "Observational Cosmology" by Stephen Sarjeant

The textbook includes a useful discussion of the material, but the course will not be organized to follow the presentation in the book.

However, I will advise you as to where you can find the relevant material in the textbook.

Other material will regularly be made available in pdf format… especially from Matthias Bartelmann's Observational Cosmology notes

# Layout of the Course

This Week

Feb 5: Introduction / Overview / General Concepts
Feb 12: Age of Universe / Distance Ladder
Feb 19: Distance Ladder / Hubble Constant
Feb 26: Distant Measures / SNe science / Baryonic Content
Mar 4: Dark Matter Content of Universe / Cosmic Microwave Background
Mar 11: Cosmic Microwave Background
Mar 18: Cosmic Microwave Background / Large Scale Structure
Mar 25: Baryon Acoustic Oscillations / Dark Energy / Clusters
Apr 1: No Class
Apr 8: Clusters / Cosmic Shear
Apr 15: Dark Energy Missions / Review for Final Exam

May 13: Final Exam

# How will you be evaluated?

Final Exam (Written) -- 75%
Homework -- 25%

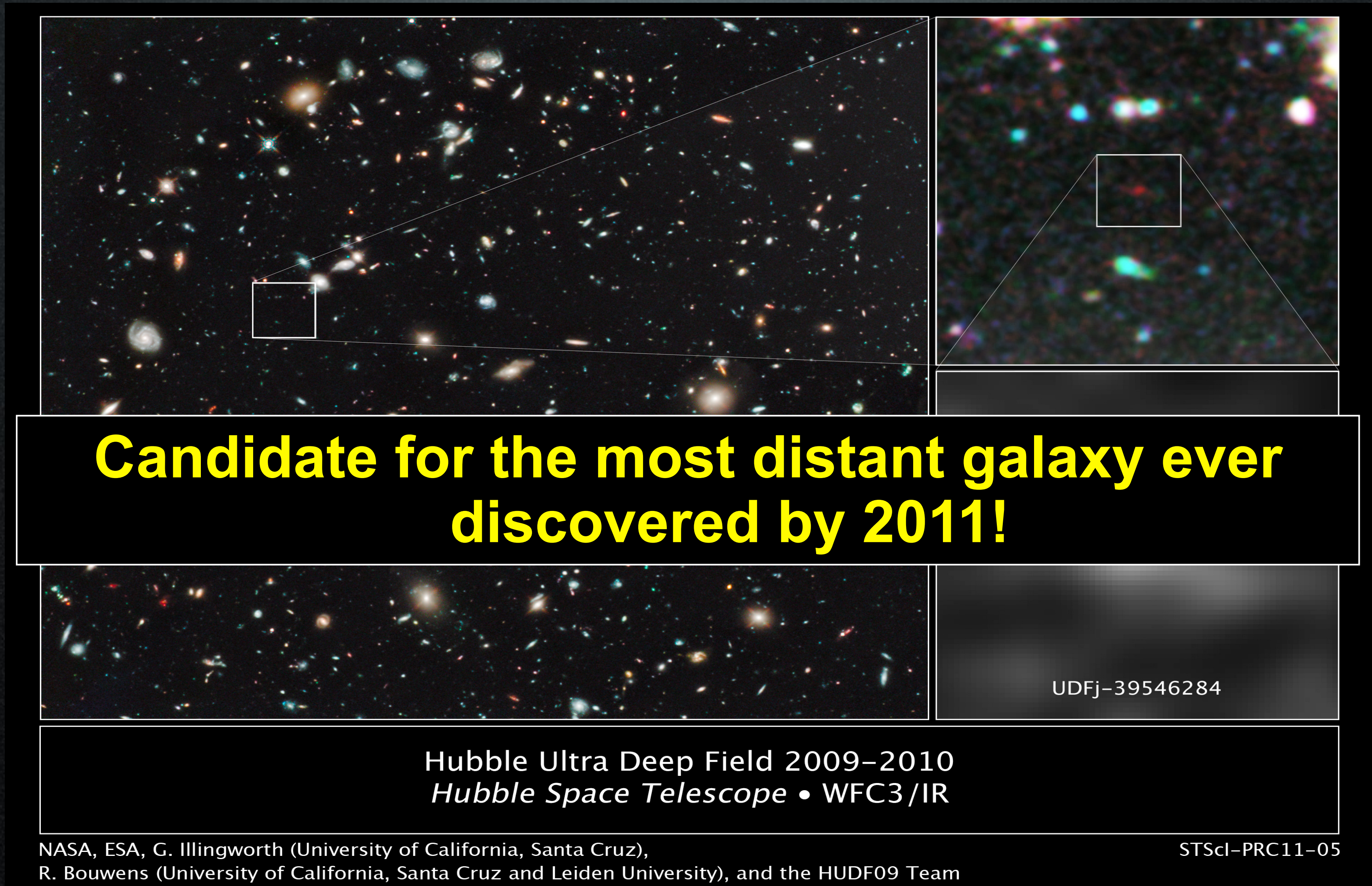
# Who am I?

My name is Rychard Bouwens
(studied in the United States: Berkeley & Santa Cruz)

I study the most distant galaxies in the universe

A good understanding of the cosmological model
is very important for my research

# Discovery of Plausible Galaxy just ~400-450 Myr after Big Bang

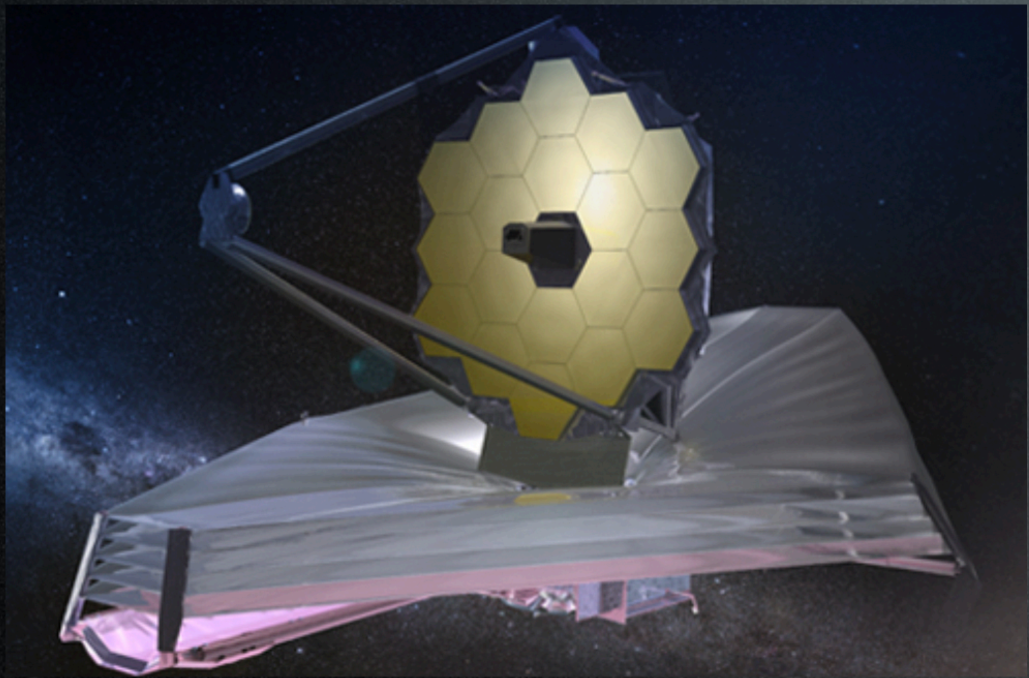



**Candidate for the most distant galaxy ever discovered by 2011!**

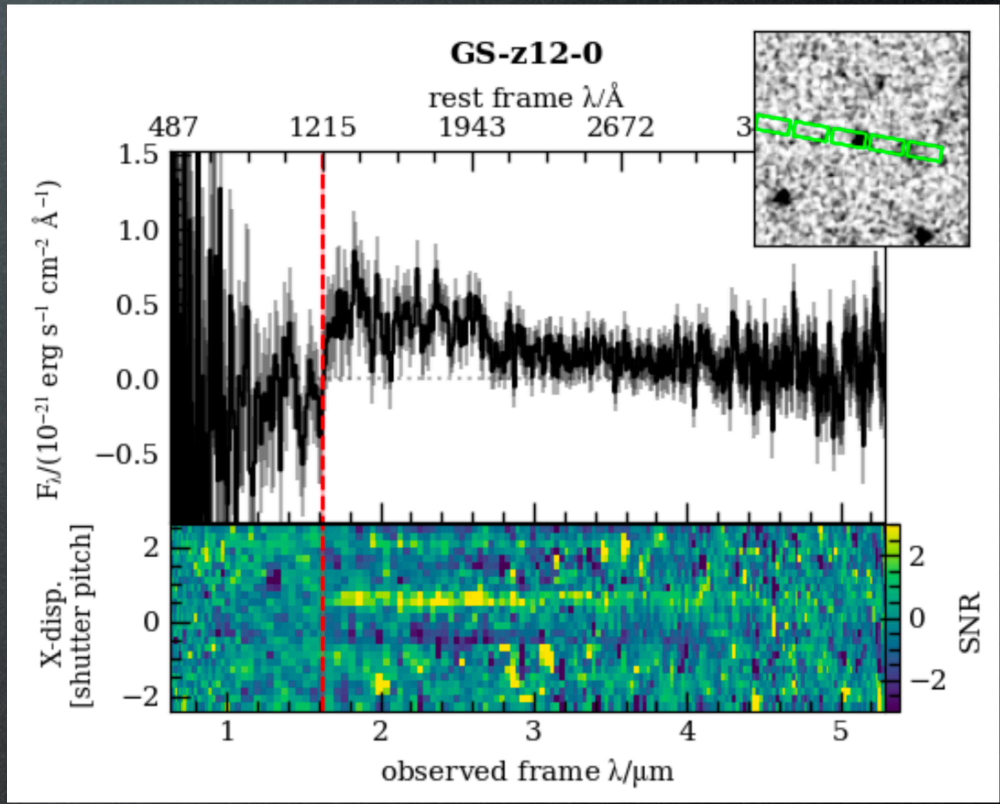
Bouwens et al. 2011 (Nature, January 27, 2011)

# What Happened with That Source?

According to a spectrum taken by JWST



z = 11.58
(390 Myr after the Big Bang)




Curtis-Lake+2023

Most Galaxy Ever Seen with HST !!!

# Teaching Assistants

Ivana van Leeuwen
vleeuwen@strw.leidenuniv.nl

Thomas Herard-Dimanche
herard@strw.leidenuniv.nl

Ivana and Thomas can also be available by appointment to answer your questions, and they also may hold office hours

# Who are you?

Why don't we go around the class and introduce ourselves briefly?

Name
Program -- Physics or Astronomy?
Master's Student?
First or Second Year?
Why interested in course?

# Please ask questions

This is *your* course.   It is your opportunity to learn.

By asking questions, you allow me to clarify issues

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- What does the Universe say about physics?!
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- When did they form?!
- Where do we fit into this picture?!

What is the age of the universe?  and how do we know?

How big is the universe?  how far are things away?
and how do we know?

Does the universe look the same in every direction? Is it uniform and homogeneous? Or does it look different every place you look?

Is the universe static and unmoving?  Or is the universe dynamic and evolving?

What is the universe made of? Again how do we know?

What are these things -- dark energy or dark matter?

What sort of structures do we find in the universe? How do they form? And how can we determine this observationally?

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May 13: Final Exam

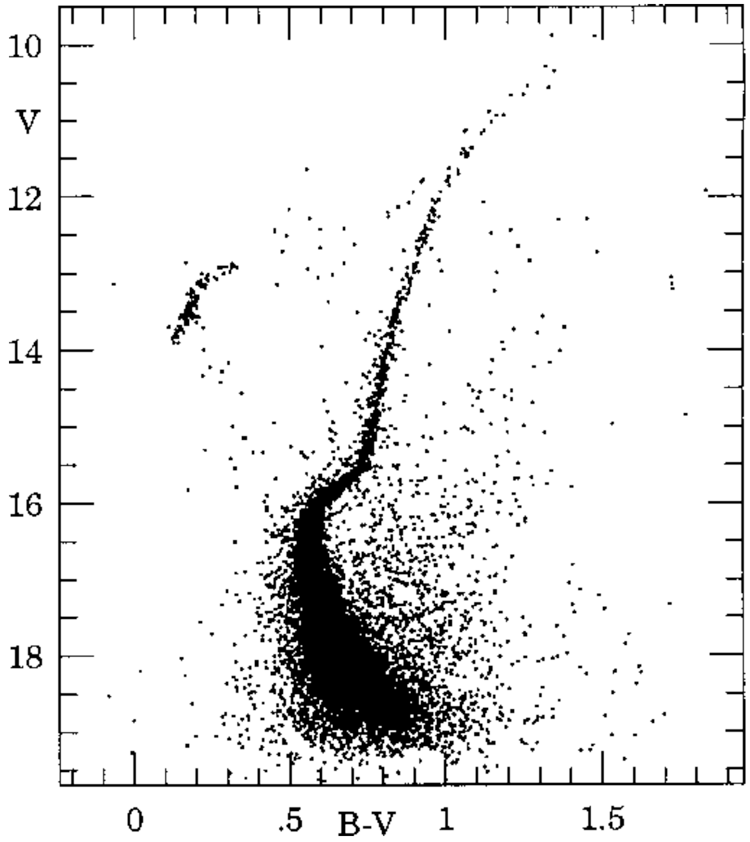
# So how do we construct a cosmological model?

i.e., a model for the universe, its evolution, its space/time structure...

We have lots of observations...

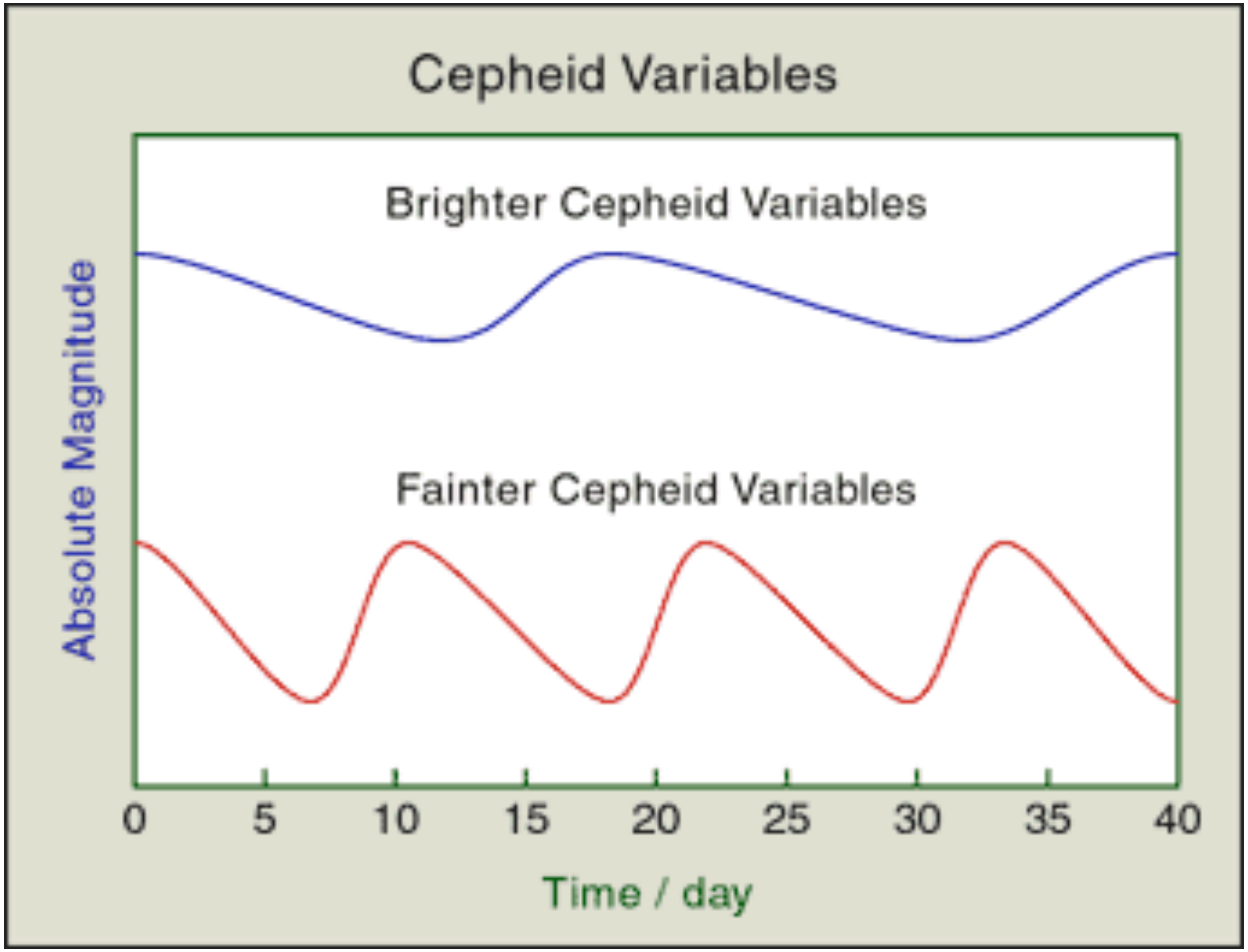
# Observations of Stars in Globular Clusters (Age of Universe)

## Hertzsprung-Russell Diagram

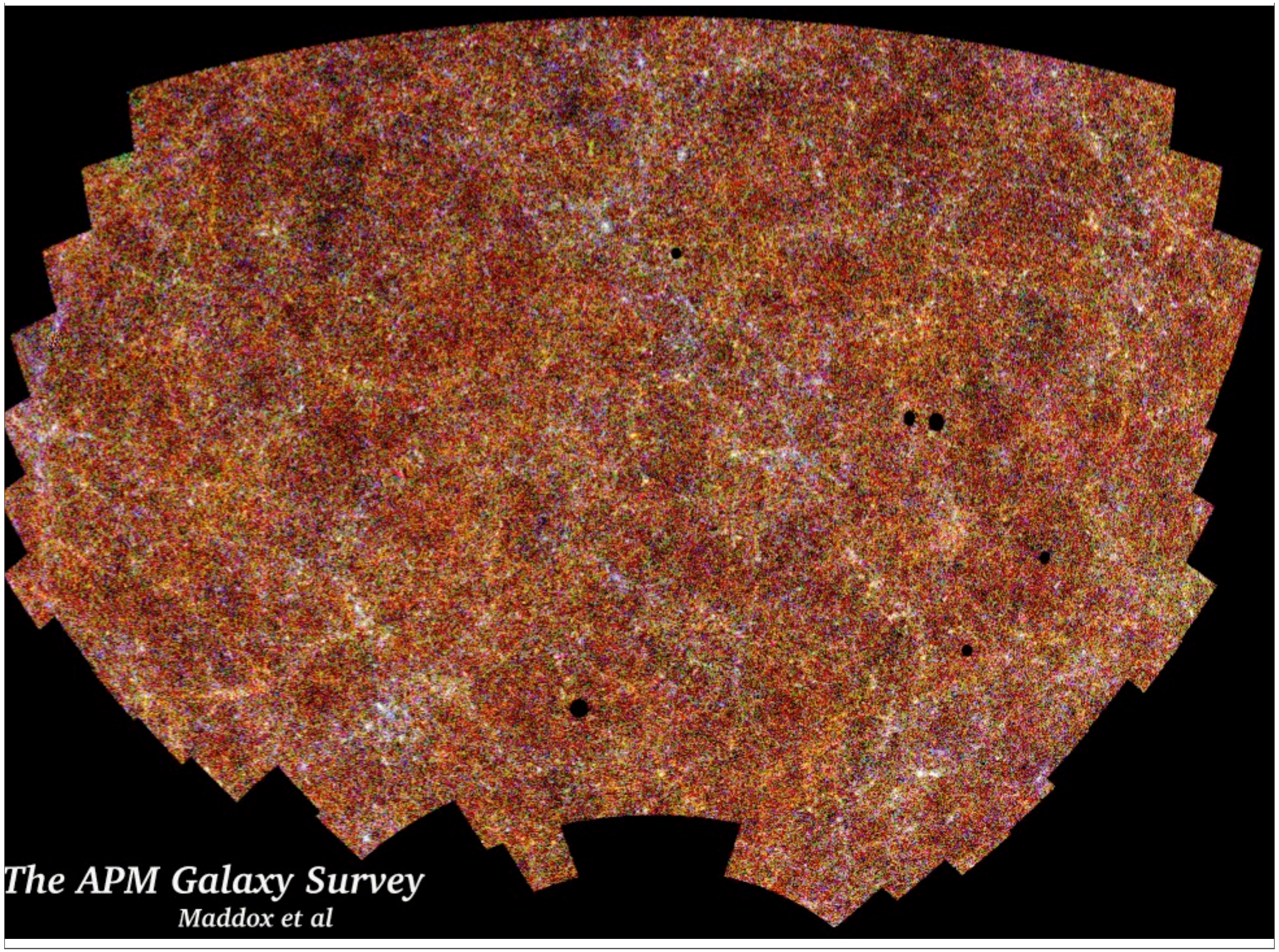



NGC 6397, Kaluzny (1997)

# Observations of Cepheid Variable Stars (Determining Distance Scale)

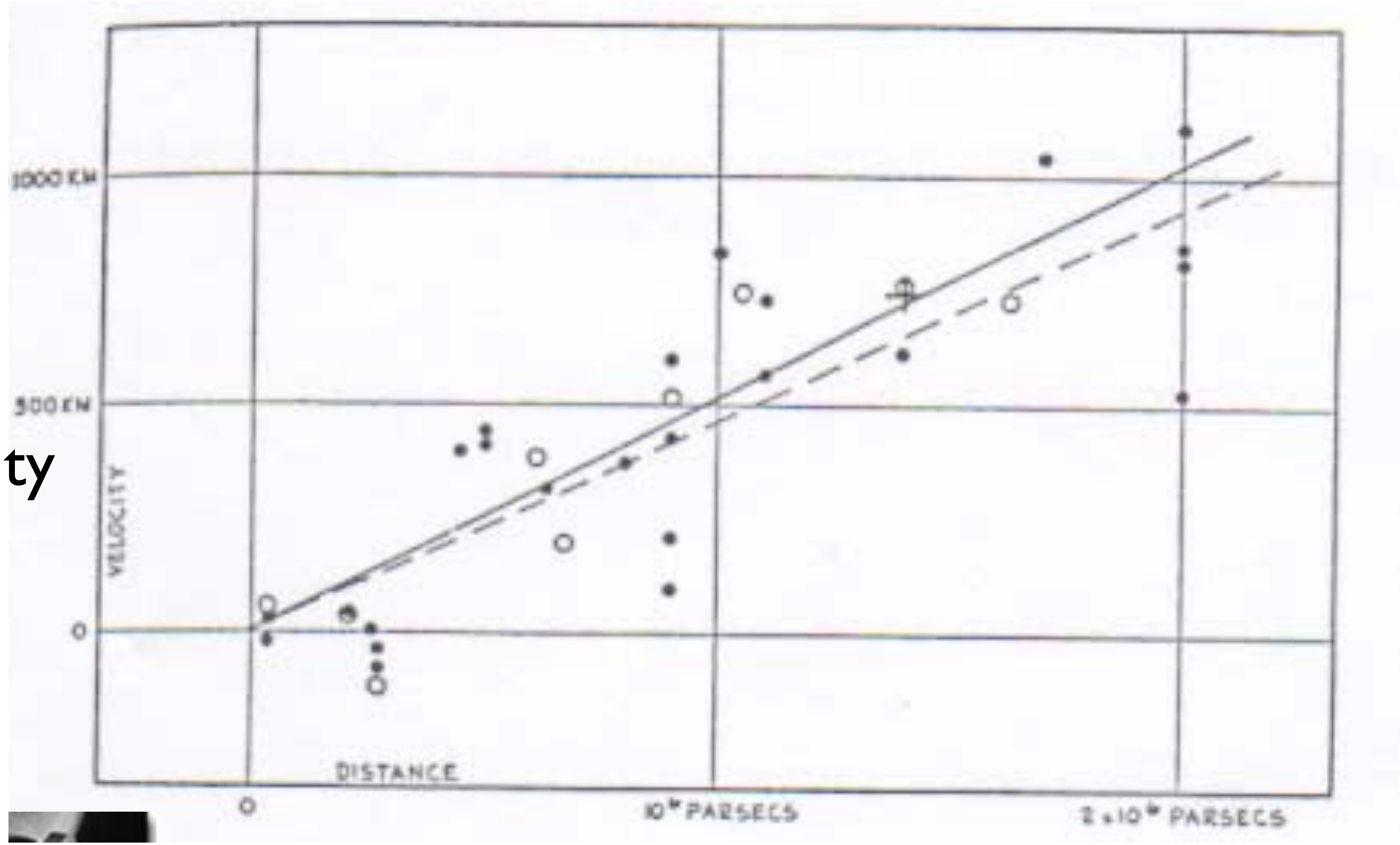


# Deep Optical Images over Wide Areas of Sky
# (to assess homogeneity and structure in universe)



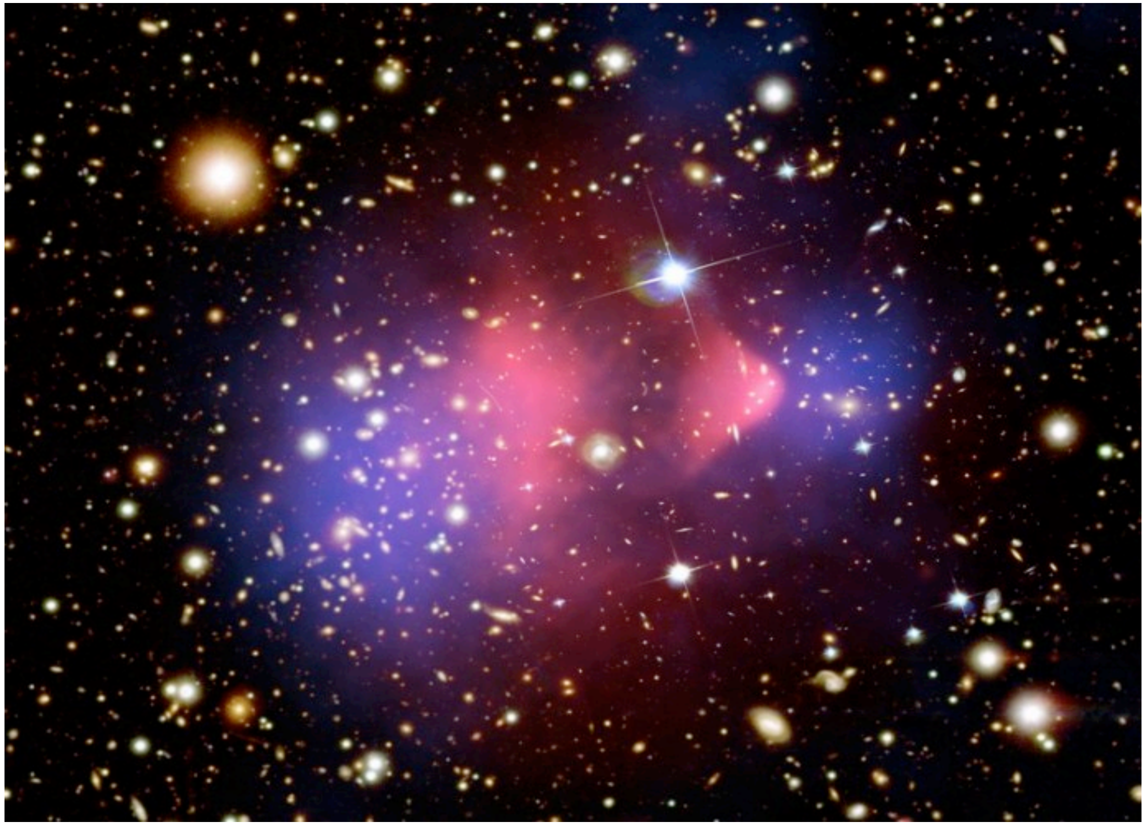
# Observations of Velocities of Nearby Galaxies
# (to establish expansion rate of universe)

Velocity




Distance

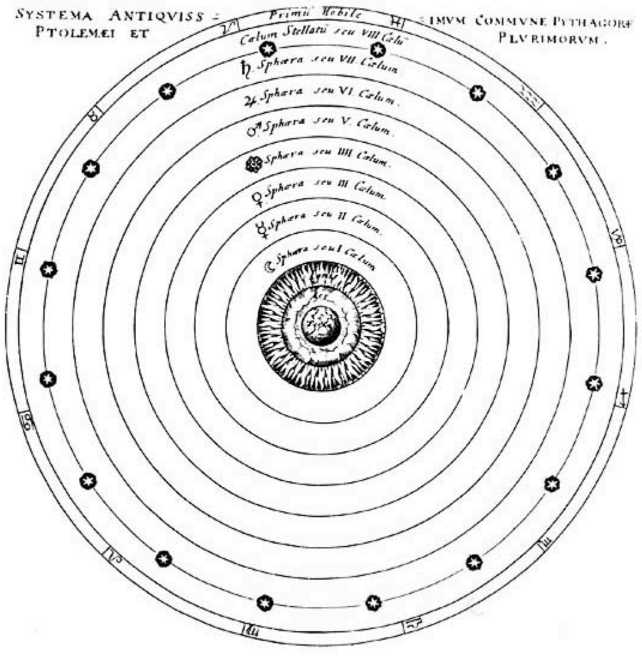
Observations of Galaxies Clusters in both baryons (from x-ray) and dark matter (Determining what the composition of the universe is)



To make sense of the many different observations we have of the universe, we need a framework or model from which to understand them!

There are millions of different models we could consider! Unfortunately, very few of these models actually prove to be useful for understanding in a simple way the diverse observations!

Aristotelian view of the Universe (55 concentric spheres grouped in 8 heavens, with Earth at center)



# We need a starting place to begin putting together a theory for the universe, for cosmology...

We start with

1)Einstein's general theory of relativity...

2) general cosmological principle (assumption)...

what an ambitious idea it is to put together such a simple theory for the universe!

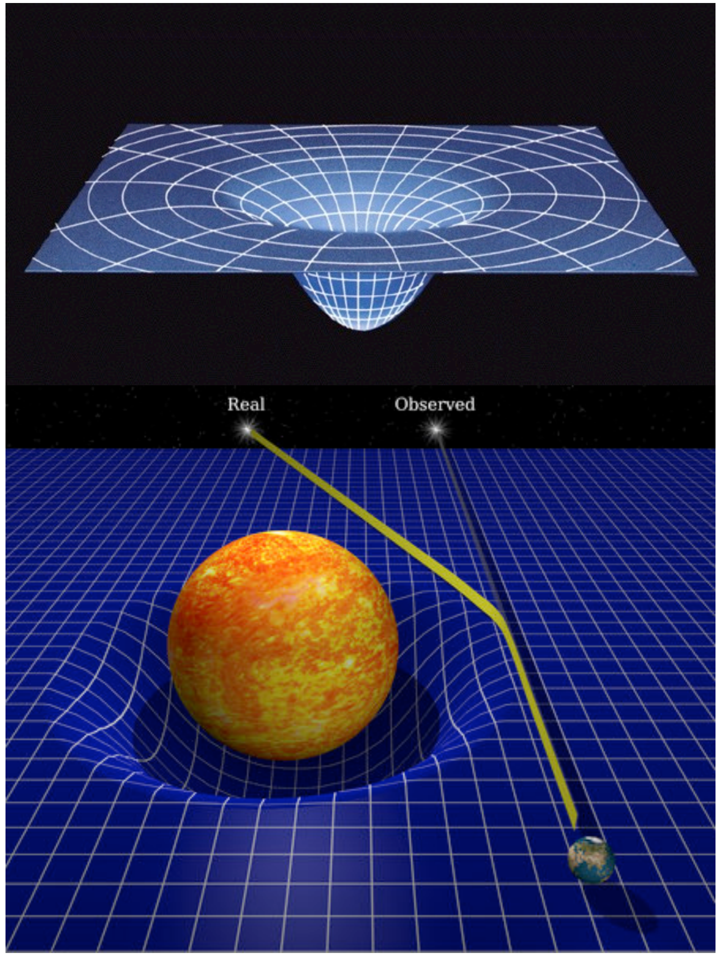
# Einstein's General Theory of Relativity

Einstein published the theory of general relativity in 1916

General Relativity explains gravity as occurring through through the curvature of space time

Gravity not explained through forces at a distance, but through geometry

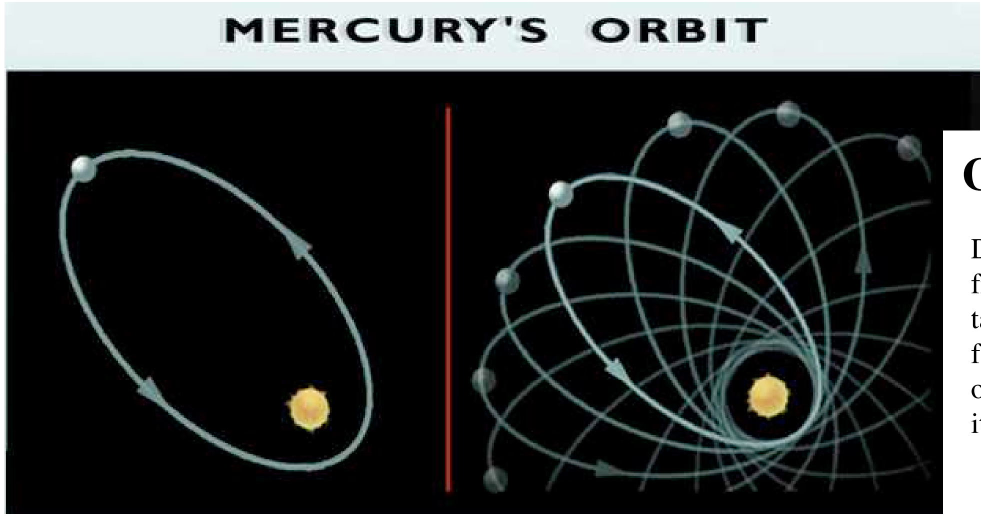
It has been extensively tested -- and explains many phenomena: Mercury's precession, deflection of light around sun, lensing of distant galaxies



# Einstein's General Theory of Relativity (Tests!)

## MERCURY's PRECESSION

The theory of General relativity was able to precisely account for the observed precession of Mercury's orbit.



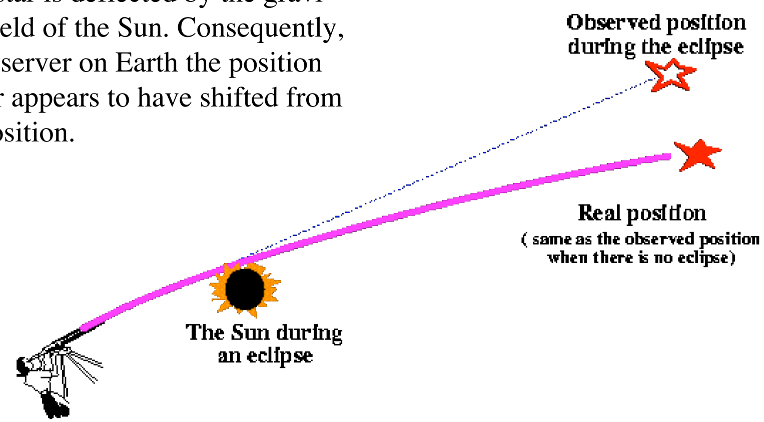

Newtonian gravity

General relativistic gravity
(amount of precession not on scale - artist's view of Mercury's precession)

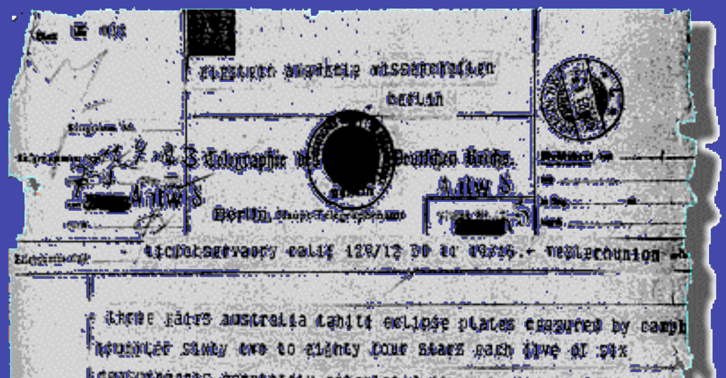
*[Image from www.gravitywarpdrive.com]*

## GRAVITATIONAL DEFLECTION OF LIGHT

During the eclipses, the beam of light from the star is deflected by the gravitational field of the Sun. Consequently, for the observer on Earth the position of the star appears to have shifted from its true position.




*Telegram from Sir Arthur Eddington (1919) announcing the observed apparent shift in the position of the star, due to the light deflection by the gravitational field of the Sun as predicted by GR.*



First observational test of the prediction of General Relativity

Credit: Perna

# Cosmological Principle
# (Assumption)

-- The universe looks the same everywhere

* Universe is homogeneous and isotropic

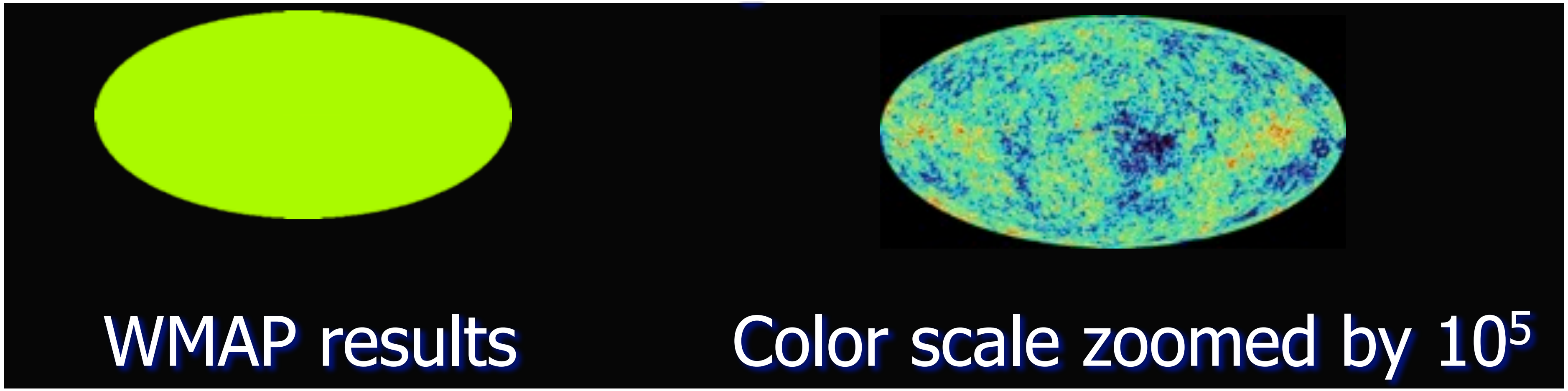
* We do not live in a special place in the universe

-- Expected to be true on the largest scales (statistically)

-- This assumption is supported by observations:

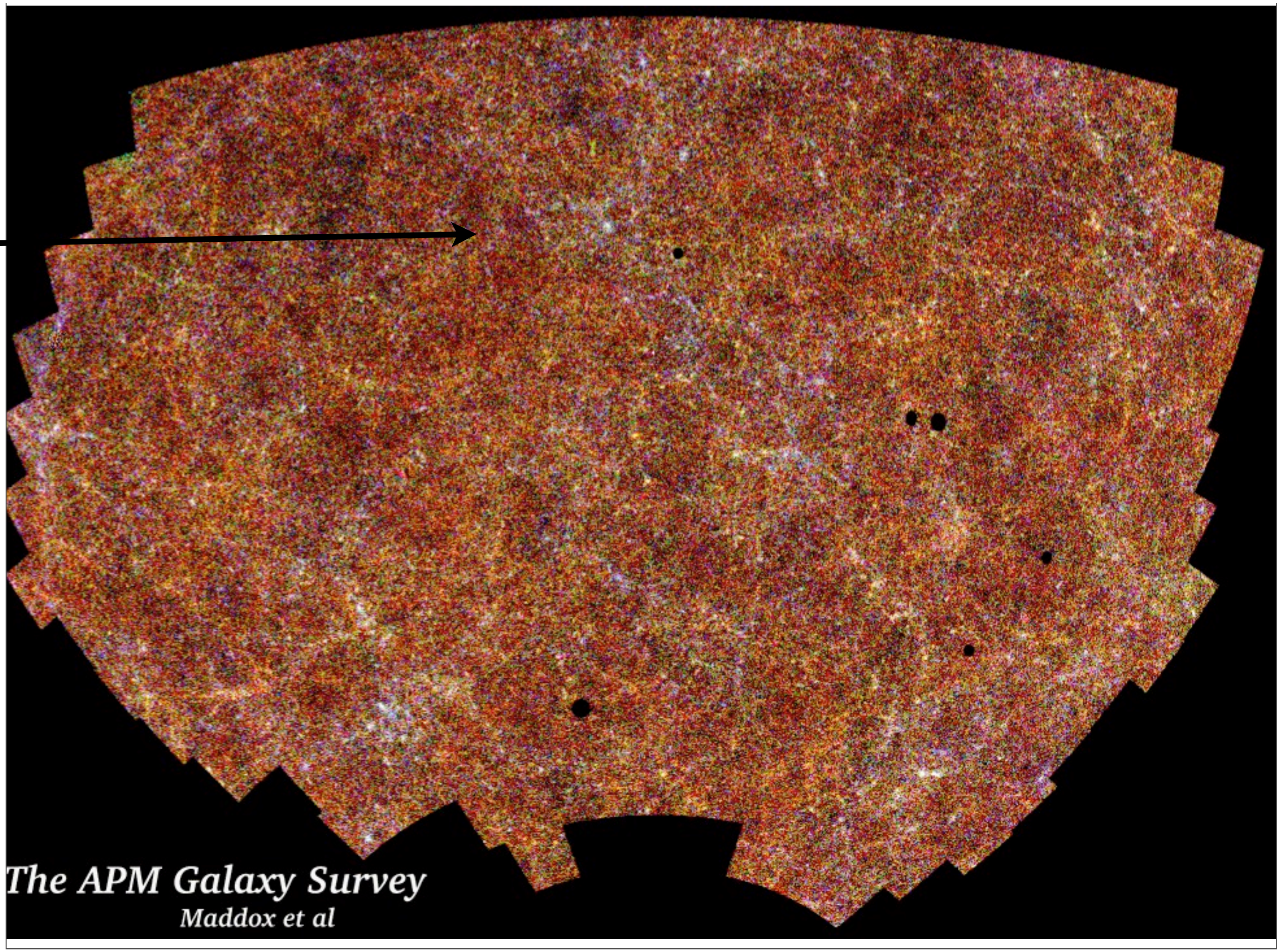
* cosmic microwave background

(perfect black-body T=2.73 Kelvin radiation seen in all directions in the sky, relic of Big Bang, from when universe was $3\times10^5$ yrs old)



-- The idea that the universe is homogeneous and isotropic is also supported by observations of different parts of the sky.

almost all white dots are galaxies!




The distribution of galaxies on the sky look almost identical in every direction we look!

## First Step In Constructing A Model for Evolution of the Universe: Create A Metric For Homogenous/Isotropic Universe

Before we can even begin to model the evolution of the universe itself, we need a coordinate system to describe time and space in the universe.

Otherwise, we have no way to even write down the equations to describe the evolution of the universe.

(This is really the first step in solving any physics problem -- one needs a coordinate system!)

# First Step In Constructing A Model for Evolution of the Universe: Create A Metric For Homogenous/Isotropic Universe

How to measure distance in space time?

In cartesian space,

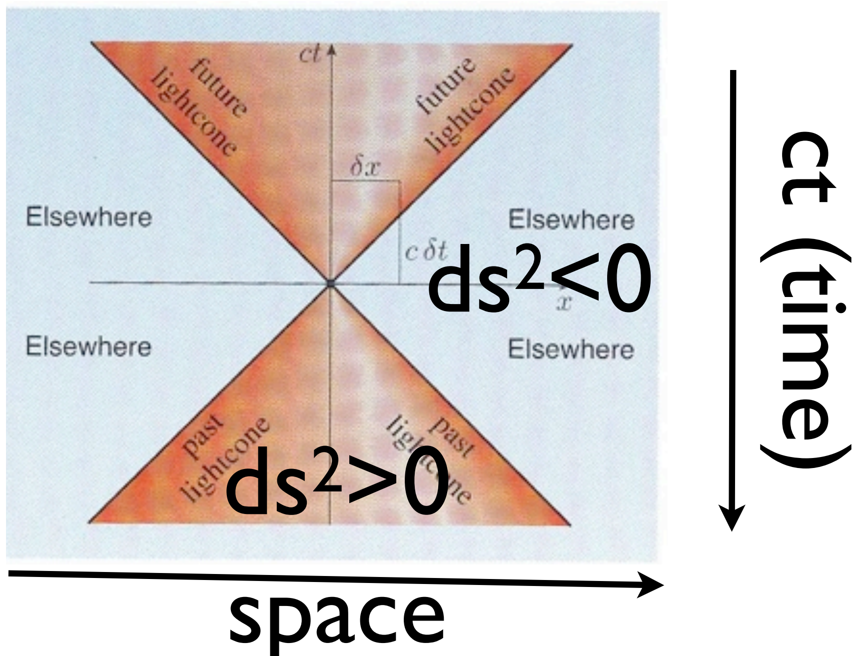
$$\text{spatial distance}^2 = dx^2 + dy^2 + dz^2$$

one can add time

$$ds^2 = c^2dt^2 - (dx^2 + dy^2 + dz^2)$$

$ds^2 > 0$: time-like interval
(causally connected)

$ds^2 < 0$: space-like interval
(not causally connected)



# First Step In Constructing A Model for Evolution of the Universe: Create A Metric For Homogenous/Isotropic Universe

How to measure distance in space time?

Use a scale factor R to rewrite the distance formula:

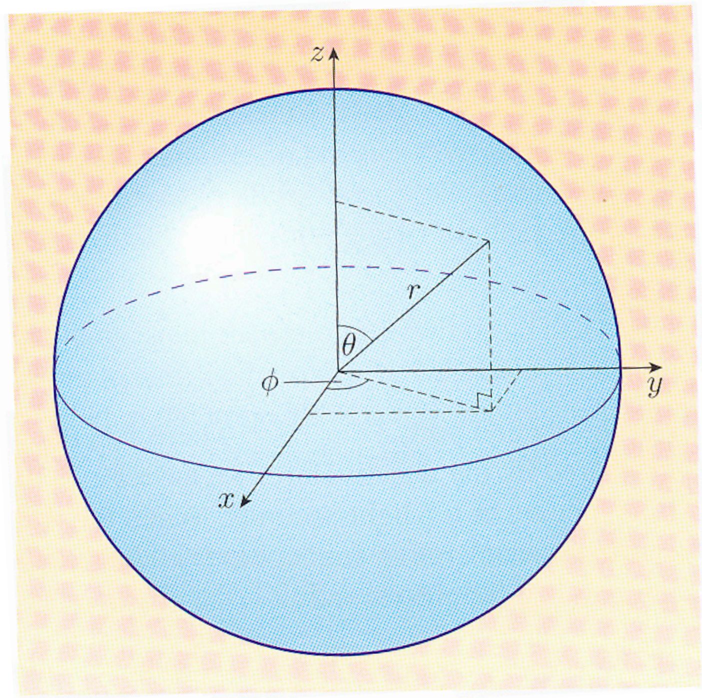
$$ds^2 = c^2dt^2 - R^2(dx^2 + dy^2 + dz^2)/R^2 = c^2dt^2 - R^2du^2$$

this is a useful parameterization because of our assumption of homogeneity and isotropy

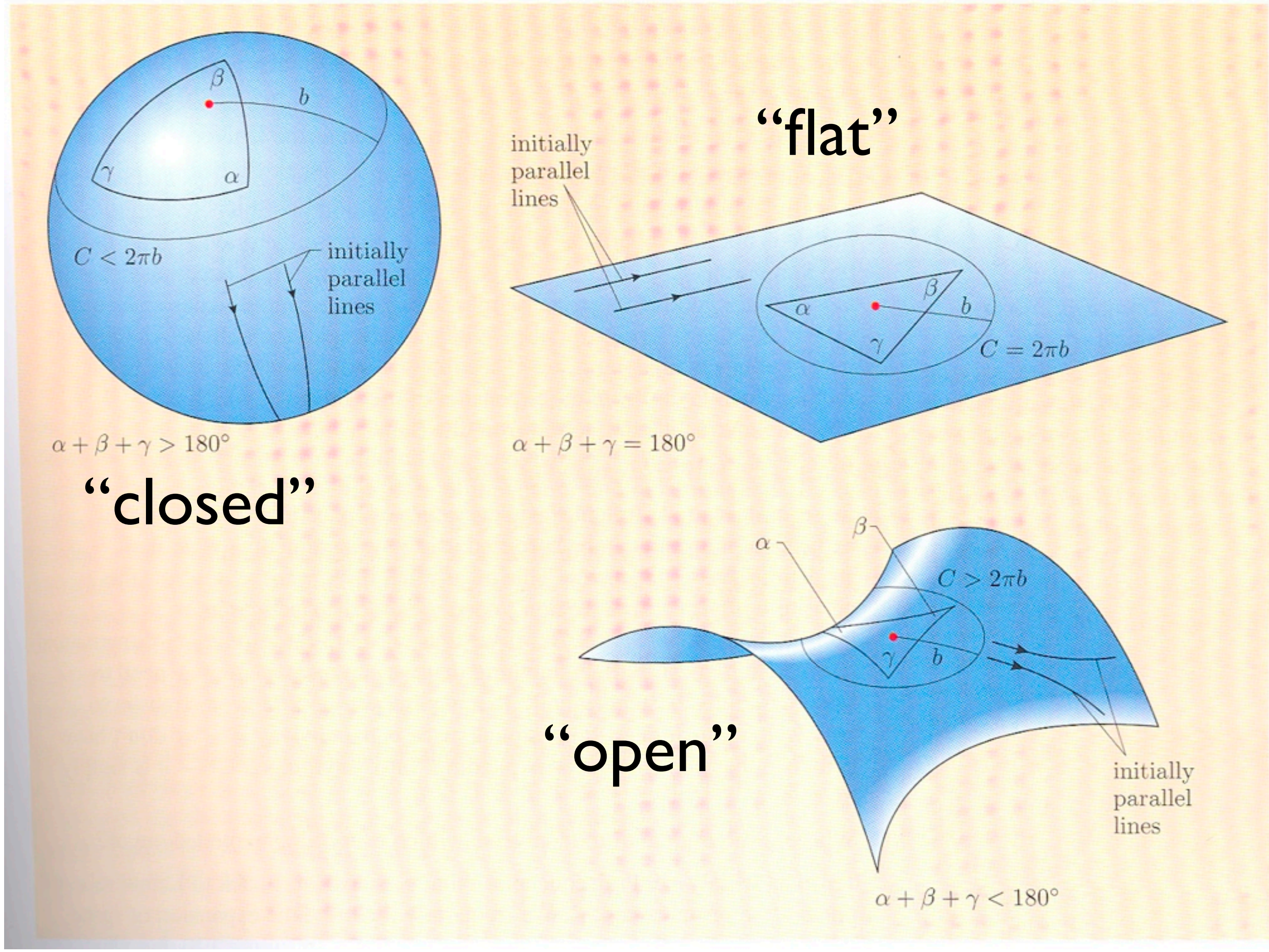
Then write it in generalized spherical coordinates...

$$ds^2 = c^2dt^2 - R^2\left\{\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right\}$$

Friedmann-Robertson-Walker metric



There are three generic space time structures that satisfy this general form:



# Assume universe is homogenous and isotropic with following structure...

How to measure distance in space time?

$$ds^2 = c^2 dt^2 - R^2 \left\{ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right\}$$

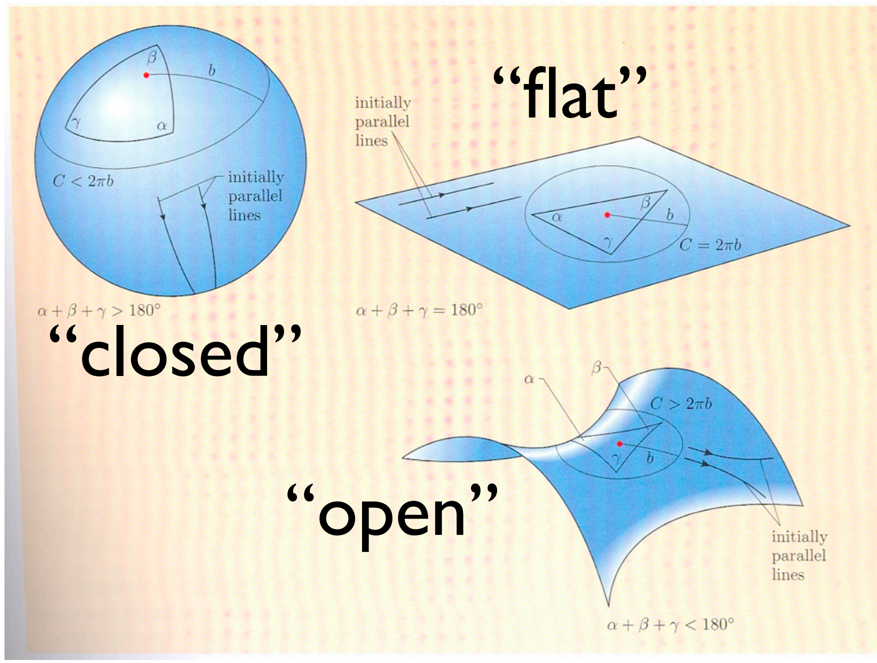
Friedmann-Robertson-Walker metric

Different topologies:

k = −1 (open)

k = 0 (flat)

k = +1 (closed)



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Friedmann-Robertson-Walker metric

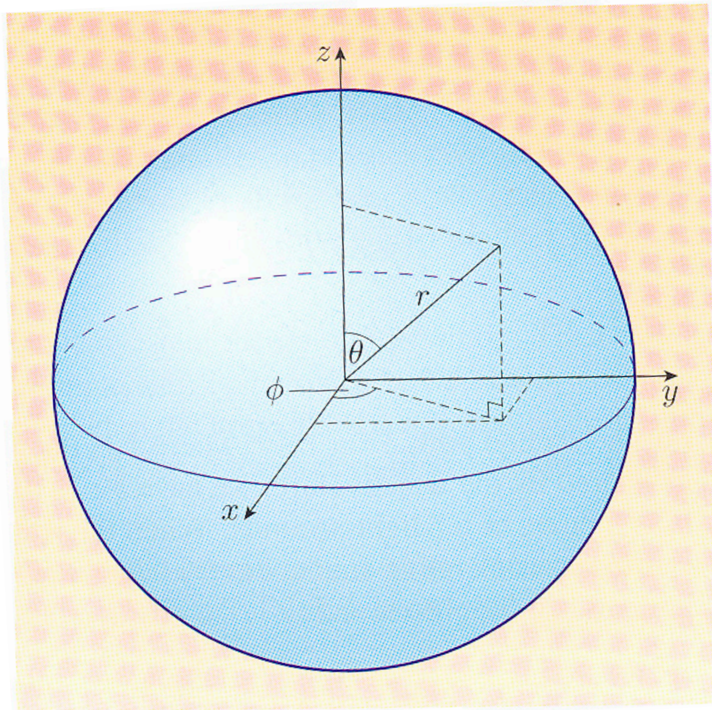
Important here is the R term!

R ∝ size of universe

R = "scale factor"

Note that R is a scale factor -- we do not know how large the universe is!

However, this scale factor R also often denoted by the parameter "a"



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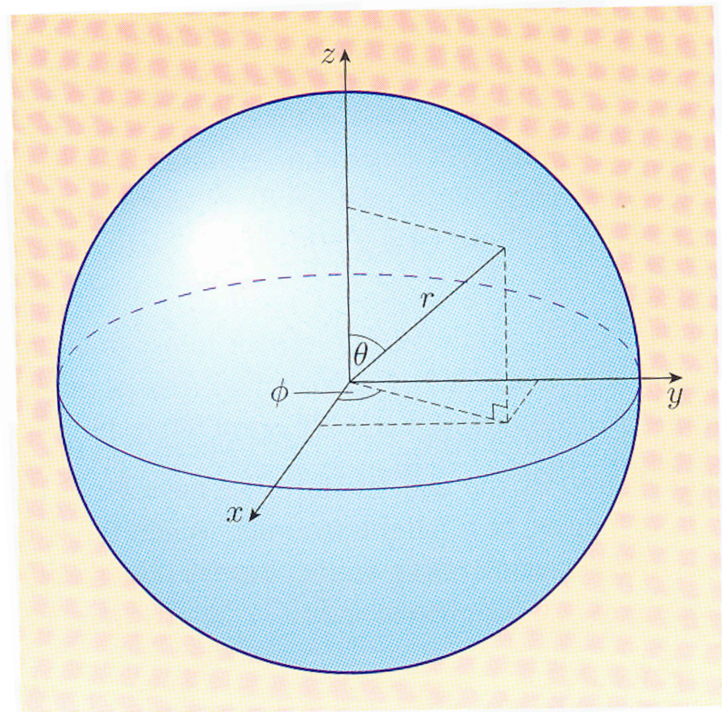
Friedmann-Robertson-Walker metric

Coordinates apart from scale factor R are "comoving"



**Using this space-time structure, what do Einstein's equations imply?**

Take Einstein's field equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} - g_{\mu\nu}\Lambda$$

"Curvature" = "Mass+Energy"

$R_{\mu\nu}$ $g_{\mu\nu}$ $T_{\mu\nu}$

Plug in the Friedmann-Robertson-Walker metric

# Equations for the expansion of the Universe

Take Einstein's field equation

Plug in the Friedmann-Robertson-Walker metric

This gives the following Friedmann equations....

$$\boxed{\dot{R}^2 = \frac{8\pi G(\rho_m + \rho_r)R^2}{3} - kc^2 + \frac{\Lambda c^2 R^2}{3}}$$

$$\boxed{\ddot{R} = -4\pi G\left(\rho_m + \rho_r + \frac{3p}{c^2}\right)\frac{R}{3} + \frac{\Lambda c^2 R}{3}}$$

# Equations for the expansion of the Universe

Let's interpret these equations:

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These equations give formulas for the time derivatives of R with respect with time

They tell us how the size of the universe (=R) evolves

# Equations for the expansion of the Universe

These equations imply that the overall size of the universe is changing with cosmic time, i.e., expanding or contracting.

Is it? What observational evidence do we have?

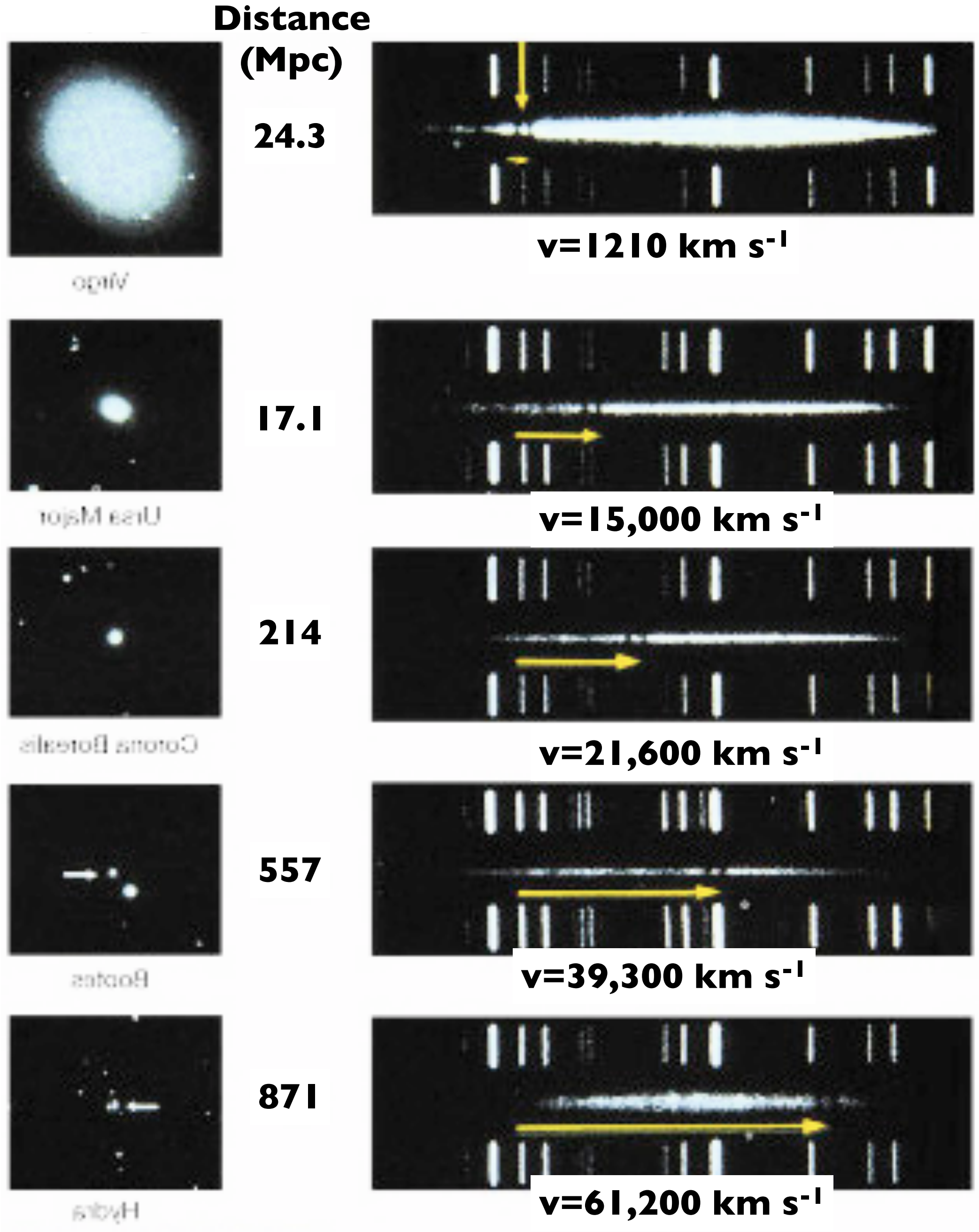
With the exception of our bright nearby galaxy Andromeda, nearly all galaxies are moving away from us, with a recessional velocity much greater than the Milky Way's escape velocity. Recessional velocity scales with distance (Hubble's law).

# What evidence do we have the universe is expanding, i.e. its size is changing?

In 1925 Edwin Hubble discovered Cepheid Variables in M31 (Andromeda "Nebula"). Hubble continued his search for Cepheids, and determined the distances to 18 galaxies.

At the same time, V. M. Slipher at Lowell Observatory looked at velocity shifts of extragalactic "nebulae" using the Calcium "HK" lines (Ca II, like in the Sun).

Vesto Slipher
(1875-1969)

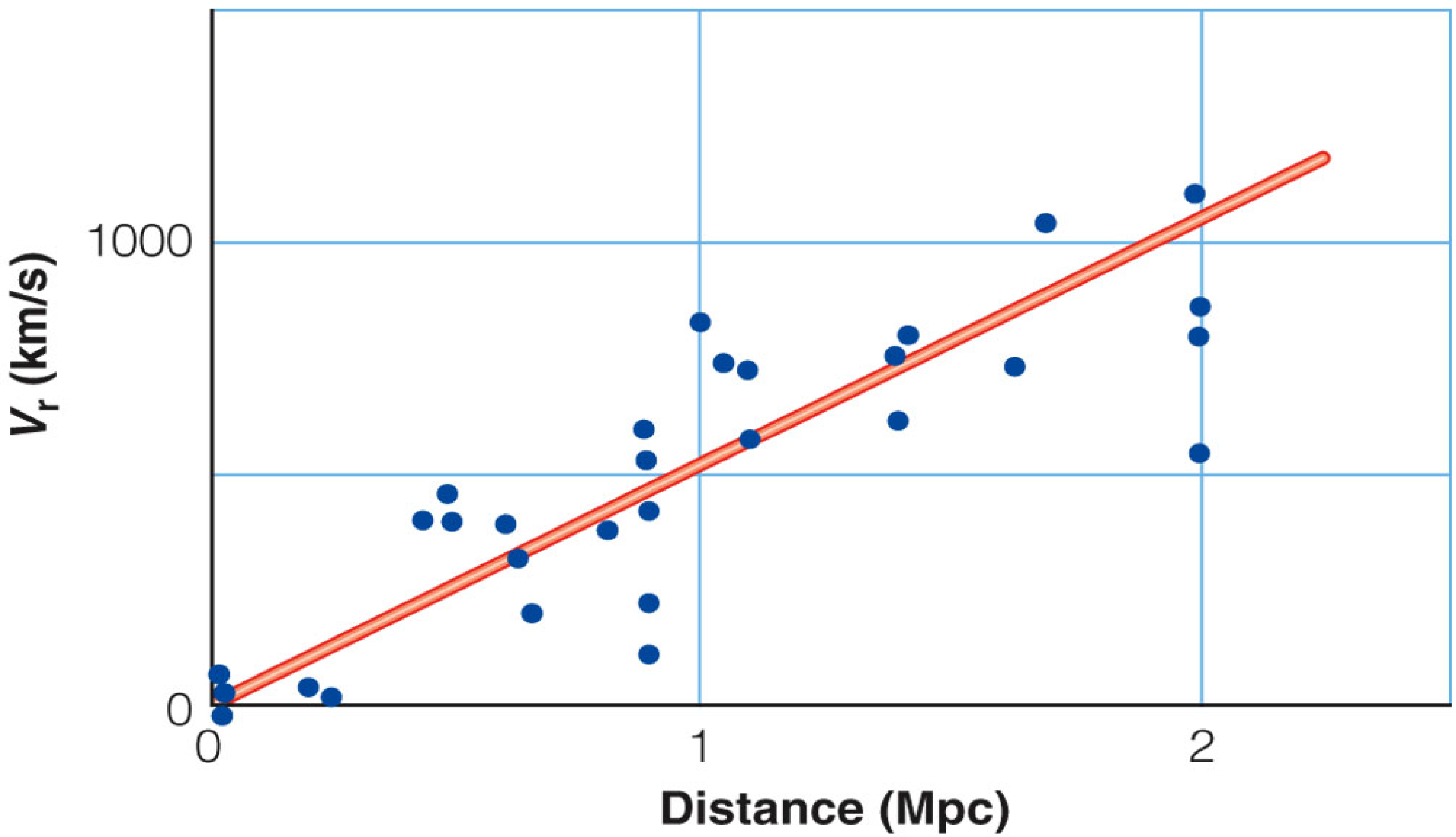


# What evidence do we have the universe is expanding, i.e. its size is changing?

In 1929, Hubble showed that the velocities and distances are linearly correlated, and satisfy

$$v = H_0 \, d$$

where v is the recessional velocity (km/s) and d is the distance (Mpc). $H_0$ is a constant, "Hubble's Constant" and has units of km $s^{-1}$ $Mpc^{-1}$.



# Understanding Explosions

**First picture:**

**How long ago did explosion happen?**

$$v \times t = d \quad \text{or} \quad v = \frac{1}{t} \times d$$

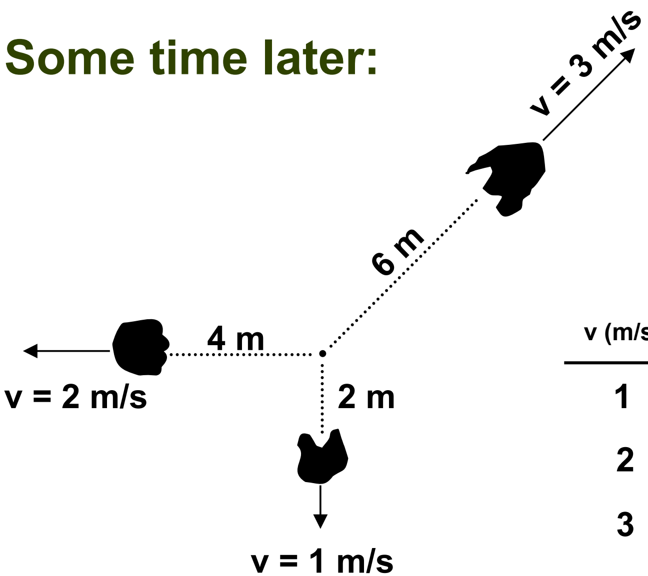
$$3\ \text{m/s} \times t = 6\ \text{m}$$

$$t = 2\ \text{s}$$

$\frac{1}{t}$ ↑ slope

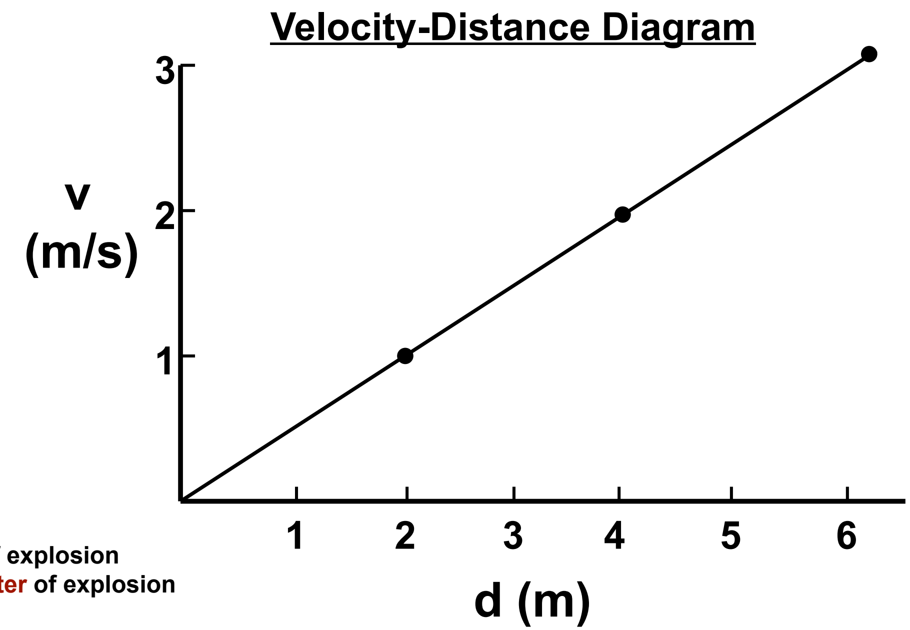
**The explosion occurred 2 seconds ago!**

**Some time later:**




| v (m/s) | d (m) |
|---|---|
| 1 | 2 |
| 2 | 4 |
| 3 | 6 |

d = distance from center of explosion
v = velocity away from center of explosion




**A linear relation between distance and velocity (faster objects are further away) indicates that an explosion has taken place at a distinct moment of time in the past.**

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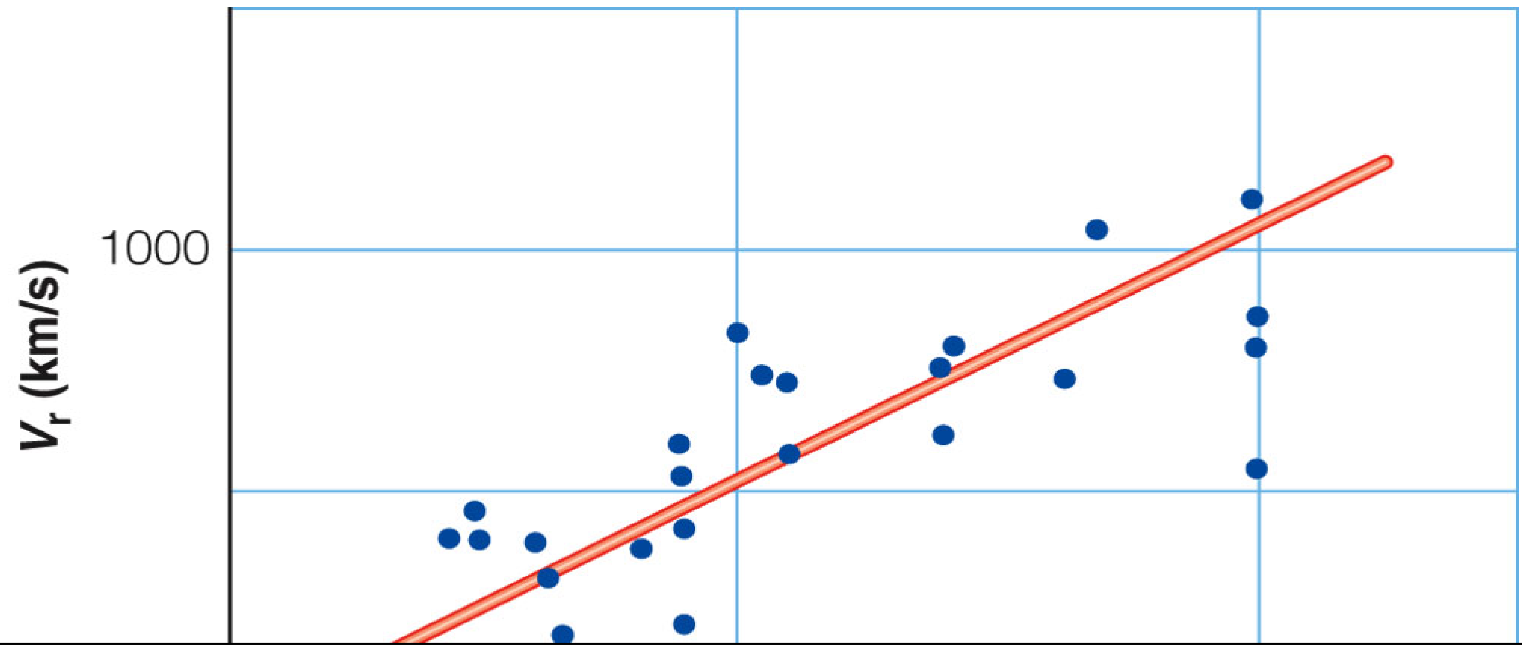


If all the galaxies receding away from us started this process at the same time, you would expect those sources that are travelling the fastest to be the furthest away!

# Equations for the expansion of the Universe

Let's interpret these equations:

$$\boxed{\dot{R}^2 = \frac{8\pi G(\rho_m + \rho_r)R^2}{3} - kc^2 + \frac{\Lambda c^2 R^2}{3}}$$

$$\boxed{\ddot{R} = -4\pi G\left(\rho_m + \rho_r + \frac{3p}{c^2}\right)\frac{R}{3} + \frac{\Lambda c^2 R}{3}}$$

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what are the variables?

R = size/scale of the universe
$\rho_m$ = mass density in matter (dark matter + baryons)
$\rho_r$ = mass density in radiation (photons)
$\Lambda$ = density in "so called" dark energy
G = Newton's gravitational constant
p = pressure
k = curvature (+1, 0, −1)

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key variables:

$\rho_m$ = mass density in matter

* Includes baryons + dark matter
* Opposes the expansion of the universe

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$$\boxed{\ddot{R} = -4\pi G\left(\rho_m + \rho_r + \frac{3p}{c^2}\right)\frac{R}{3} + \frac{\Lambda c^2 R}{3}}$$

key variables:

$\dot{R}$/R = Rate at which the universe is expanding

= Hubble Constant

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key variables:

$\Lambda$ = dark energy density

* Mysterious "vacuum energy" (physical origin not well understood)

* Accelerates expansion of the universe

# Equations for the expansion of the Universe

Let's interpret these equations:

$$\boxed{\dot{R}^2 = \frac{8\pi G(\rho_m + \rho_r)R^2}{3} - kc^2 + \frac{\Lambda c^2 R^2}{3}}$$

$$\boxed{\ddot{R} = -4\pi G\left(\rho_m + \rho_r + \frac{3p}{c^2}\right)\frac{R}{3} + \frac{\Lambda c^2 R}{3}}$$

key variables:

$\rho_r$ = energy density in radiation (i.e., photons)

* Opposes expansion of the universe

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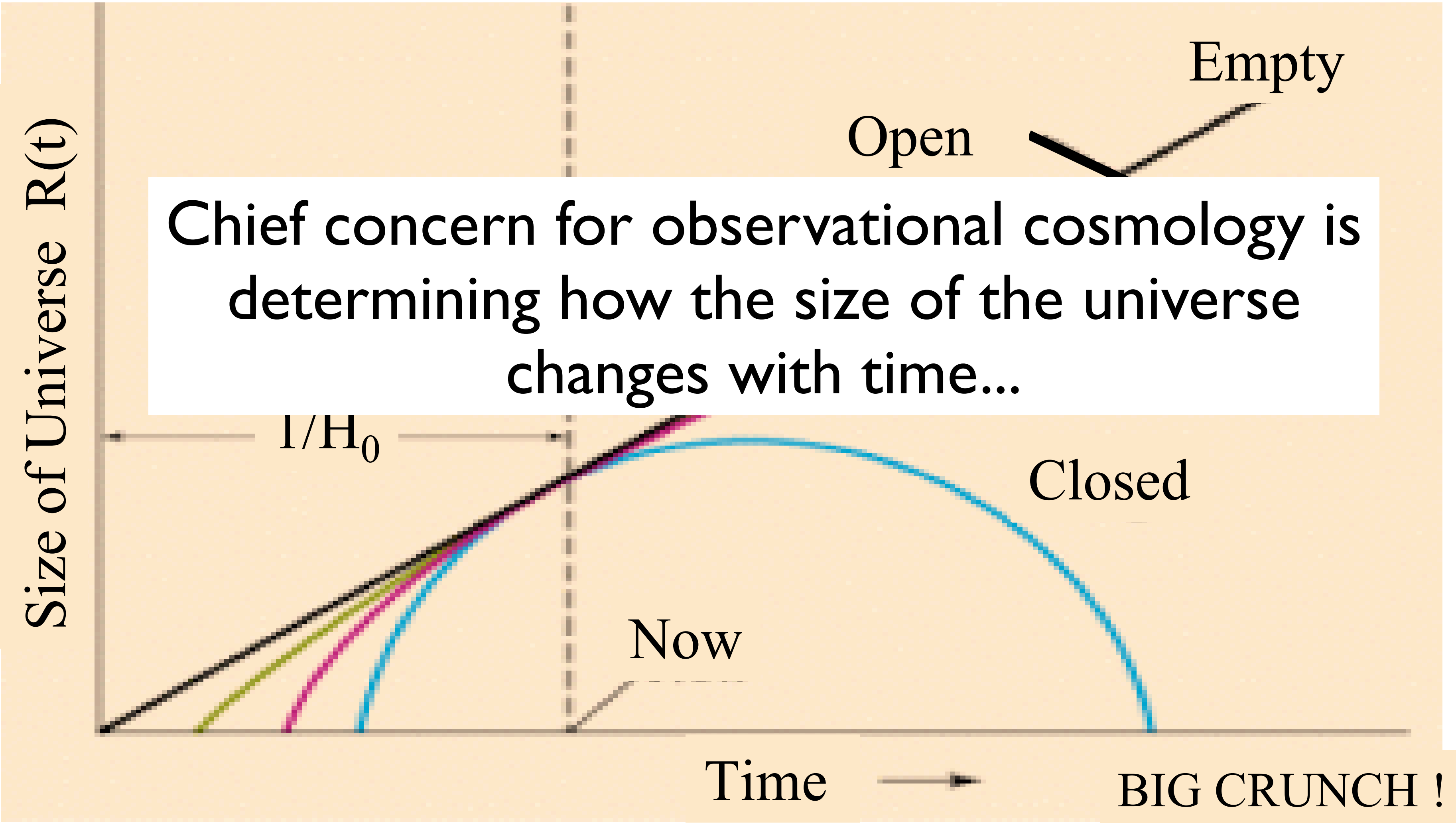
Acceleration

Mass Density
"Opposes Expansion"

Negative Energy
"Enhances Expansion"

Depending upon the mass density in the universe,
dark energy, and other properties,
the size of the universe can evolve in many different ways




Chief concern for observational cosmology is determining how the size of the universe changes with time...

There are many densities here in the Friedmann equations:

$\rho_m$, $\rho_r$, $\rho_\Lambda$

What difference do these densities make to the evolution of the universe?

An important number is the critical density:

$$\boxed{\rho_{crit} = \frac{3H^2}{8\pi G}}$$

$\rho_m > \rho_{crit}$ $\Rightarrow$ Universe eventually recollapses

$\rho_m = \rho_{crit}$ $\Rightarrow$ Universe expands forever

$\rho_m < \rho_{crit}$ $\Rightarrow$ Universe expands forever

Can we derive $$\boxed{\rho_{crit} = \frac{3H^2}{8\pi G}}$$ ?

It is similar to an escape velocity in Newtonian dynamics.

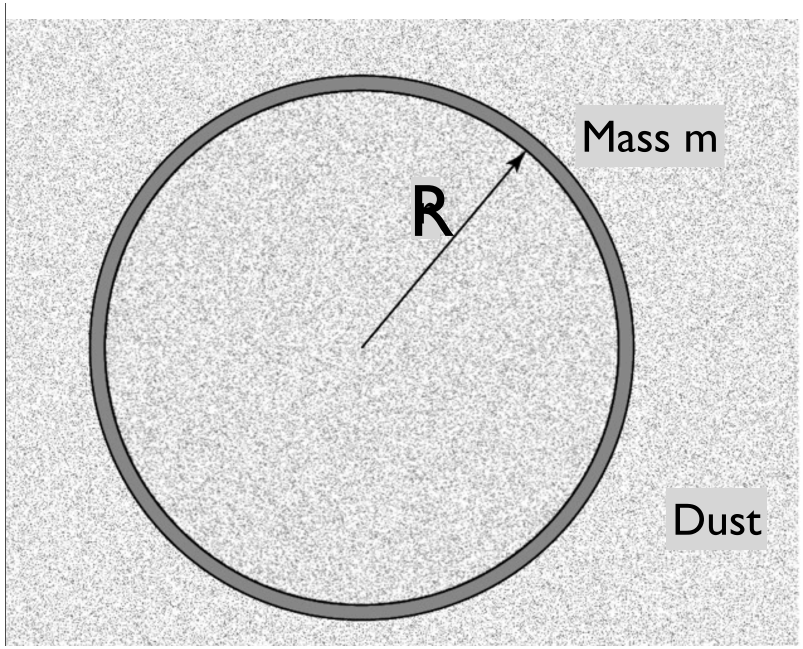
Imagine we cut out a sphere of matter from an expanding universe

$$\text{Kinetic Energy} \stackrel{?}{=} -\text{Potential Energy}$$

$$(1/2)mV_{esc}^2 \stackrel{?}{=} GmM/R$$

$$V_{esc}^2 \stackrel{?}{=} 2GM/R$$

$$V_{esc}^2 \stackrel{?}{=} 2G(((4/3)\pi\rho R^3)/R$$



Can we derive $\boxed{\rho_{crit} = \frac{3H^2}{8\pi G}}$ ?

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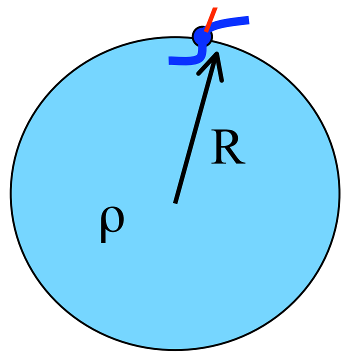


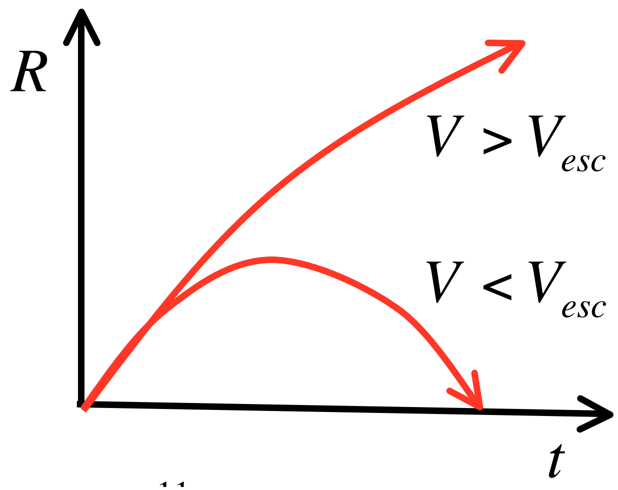

$$V_{esc}^2 = \frac{2GM}{R} = \frac{2G}{R}\left(\frac{4\pi R^3 \rho}{3}\right) = \frac{8\pi G R^2 \rho}{3}$$

Divide this equation by the square of the the Hubble Expansion equation

$$V = H_0 R$$

yields

$$\left(\frac{V_{esc}}{V}\right)^2 = \frac{8\pi G \rho}{3H_0^2} = 1$$




For the case that $V = V_{esc}$, $\rho$ must equal $\rho_{crit}$
which implies

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} \sim 10^{-26} kg\ m^{-3} \sim \frac{1.4 \times 10^{11} M_\odot}{Mpc^3}$$

However for convenience, astronomers introduce a dimensionless quantity to describe these densities $\rho_m$, $\rho_r$, $\rho_\Lambda$ relative to the critical density:

$$\boxed{\Omega_m = \frac{\rho_m}{\rho_{crit}}} \qquad \boxed{\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{crit}}}$$

$$\boxed{\Omega_r = \frac{\rho_r}{\rho_{crit}}}$$

If we assume no dark energy ($\Omega_\Lambda = 0$) and the radiation energy density small ($\Omega r \sim 0$),

$\Omega_m > 1 \Rightarrow$ Universe eventually recollapses

$\Omega_m = 1 \Rightarrow$ Universe expands forever

$\Omega_m < 1 \Rightarrow$ Universe expands forever

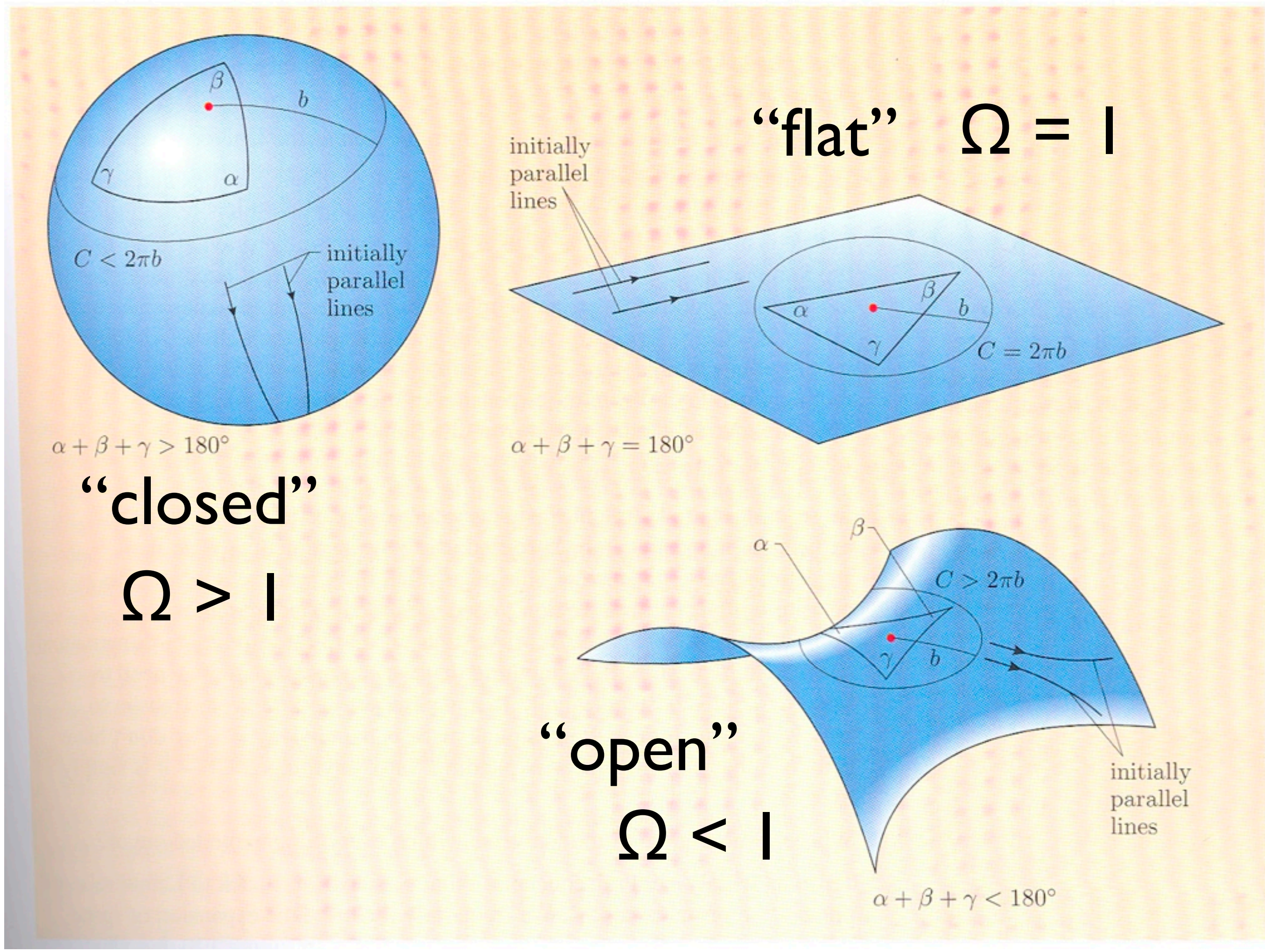
But for $\Omega_\Lambda \neq 0$, discussion is more complicated

$\Omega = \Omega m + \Omega r + \Omega_\Lambda$ determines geometry of universe

$\Omega > 1 \Rightarrow$ Universe is closed ($k = +1$)

$\Omega = 1 \Rightarrow$ Universe is flat ($k=0$)

$\Omega < 1 \Rightarrow$ Universe is open ($k=-1$)



We have just seen that the contents of the universe have an effect in speeding or slowing the expansion of the universe.

However, the expansion of the universe itself has an effect on the importance of the role or importance of each of the contents of the universe...

As the scale of the universe R expands, the energy density in matter $\rho_m$ scales as $R^{-3}$ (one factor of R for each of the three dimensions)

As the scale of the universe R expands, the energy density in radiation $\rho_r$ scales as $R^{-4}$ (one factor of R for each of the three dimensions and one factor for the effect of the expansion of the universe of the wavelength of photons in the universe)

We have just seen that the contents of the universe have an effect in speeding or slowing the expansion of the universe.

However, the expansion of the universe itself has an effect on the importance of the role or importance of each of the contents of the universe...

Finally, as the scale of the universe R expands, the energy density in dark energy remains the same -- since it is associated with the vacuum in space-time itself.

In summary,

$$\rho_m \propto R^{-3}$$

$$\rho_r \propto R^{-4}$$

$$\Lambda = \text{const}$$

Of the three categorically different components of the universe, which is likely to dominate at early times?

$$\rho_m \propto R^{-3}$$

$$\rho_r \propto R^{-4}$$

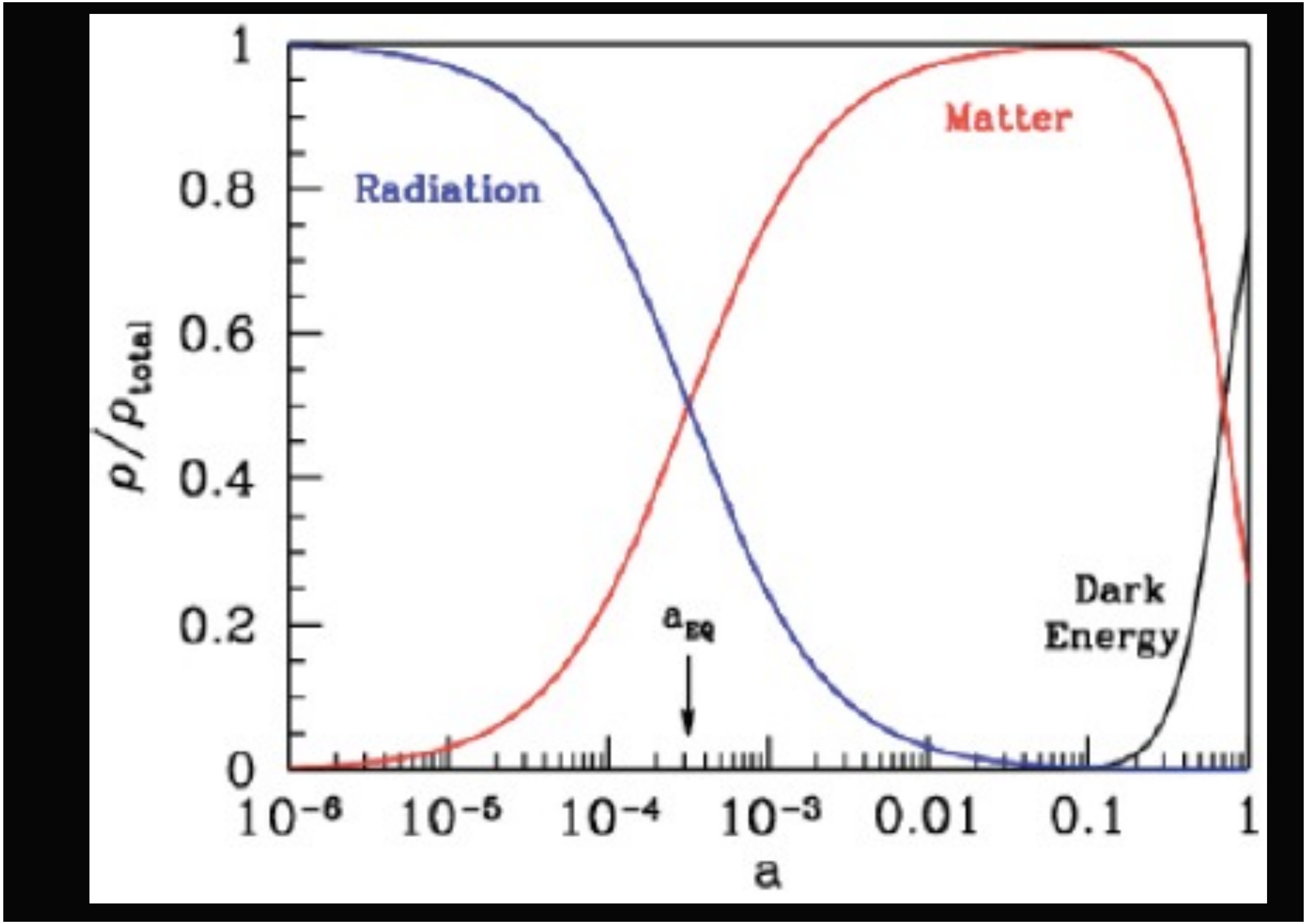
$$\Lambda = \text{const}$$

It is the one that scales as $R^{-4}$ (i.e. the radiation density) since in the limit of small R, this one will dominate

Which on will likely dominate at late times?

It is the one that scales as $R^0$ (i.e. the dark energy) since in the limit of large R, the others will go to zero

# How does the situation actually look?




Size of Universe

(note that in many contexts people use a instead of R)

The expansion rate of the universe shows a different time dependence depending on which component of the universe dominates the energy density...

From Friedmann equations,

$$\boxed{\dot{R}^2 = \frac{8\pi G(\cancelto{0}{\rho_m} + \rho_r)R^2}{3} - \cancelto{0}{k}c^2 + \frac{\cancelto{0}{\Lambda} c^2 R^2}{3}}$$

For example, let's assume the universe is radiation dominated

$$(dR/dt)^2 = 8\pi G(\rho_{r,0}R^{-4})R^2/3$$

$$(dR/dt)^2 = (8\pi G\rho_{r,0}/3)R^{-2}$$

$$dR/dt = (8\pi G\rho_{r,0}/3)^{1/2} / R$$

$$R\, dR = (8\pi G\rho_{r,0}/3)^{1/2}\, dt$$

$$R^2 / 2 = (8\pi G\rho_{r,0}/3)^{1/2}\, t$$

$$R = (32\pi G\rho_{r,0}/3)^{1/4}\, t^{1/2}$$

The expansion rate of the universe shows a different time dependence depending on which component of the universe dominates the energy density...

From Friedmann equations,

$$\boxed{\dot{R}^2 = \frac{8\pi G(\rho_m + \cancelto{0}{\rho_r})R^2}{3} - \cancelto{0}{k}c^2 + \frac{\cancelto{0}{\Lambda} c^2 R^2}{3}}$$

For example, let's assume the universe is matter dominated

$$(dR/dt)^2 = 8\pi G(\rho_{m,0}R^{-3})R^2/3$$

$$(dR/dt)^2 = (8\pi G\rho_{m,0}/3)R^{-1}$$

$$dR/dt = (8\pi G\rho_{m,0}/3)^{1/2} / R^{-0.5}$$

$$R^{0.5}\, dR = (8\pi G\rho_{m,0}/3)^{1/2}\, dt$$

$$R^{1.5} / 1.5 = (8\pi G\rho_{m,0}/3)^{1/2}\, t$$

$$R = (18\pi G\rho_{m,0}/3)^{1/3}\, t^{2/3}$$

The expansion rate of the universe shows a different time dependence depending on which component of the universe dominates the energy density...

From Friedmann equations,

$$\boxed{\dot{R}^2 = \frac{8\pi G(\cancelto{0}{\rho_m} + \cancelto{0}{\rho_r})R^2}{3} - \cancelto{0}{k}c^2 + \frac{\Lambda c^2 R^2}{3}}$$

For example, let's assume the universe is dark-energy dominated

$$(dR/dt)^2 = \Lambda c^2 R^2/3$$

$$(dR/dt)^2 = (\Lambda c^2/3)R^2$$

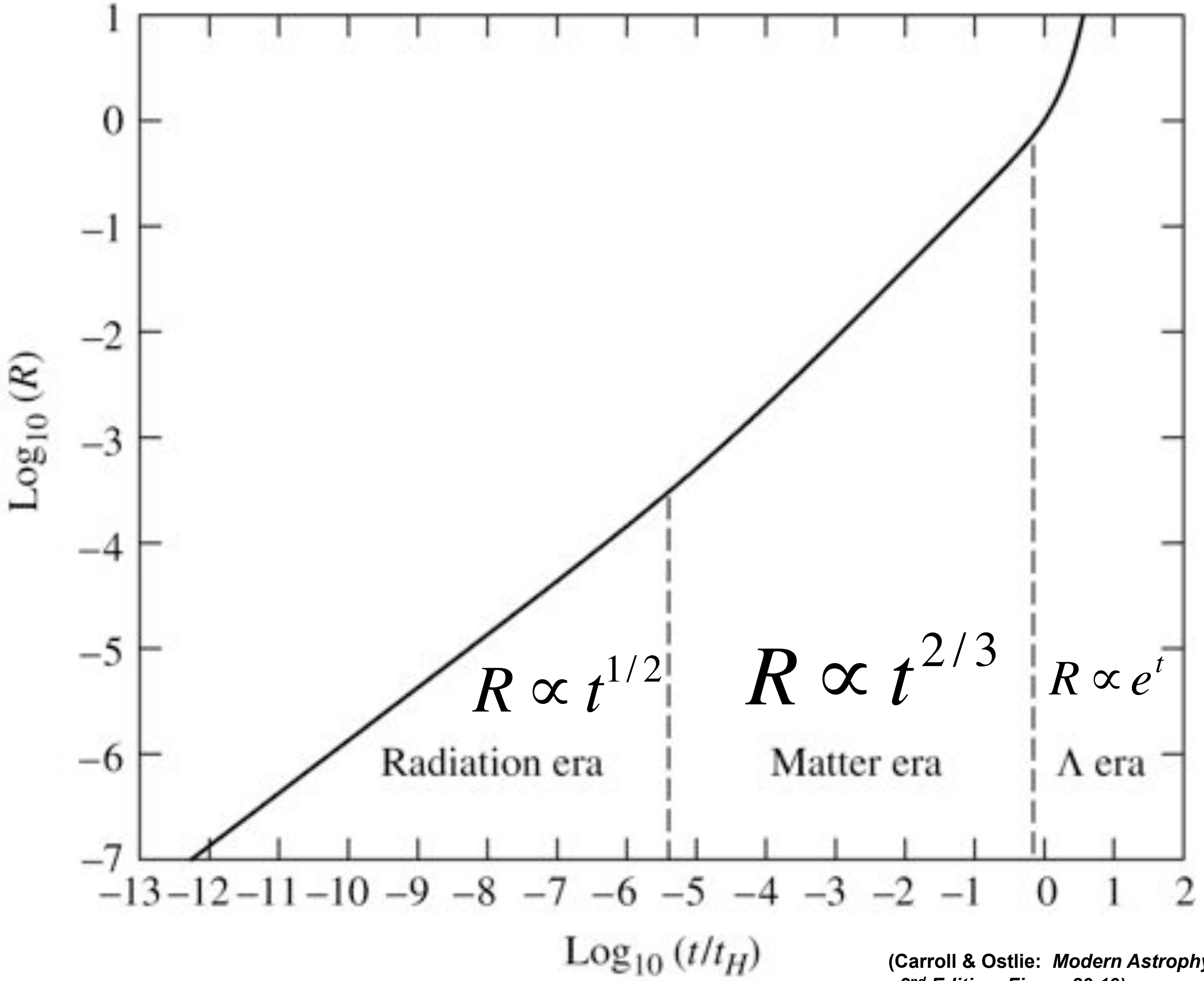
$$dR/dt = (\Lambda c^2/3)^{1/2}\, R$$

$$dR/R = (\Lambda c^2/3)^{1/2}\, dt$$

$$\log R = (\Lambda c^2/3)^{1/2}\, t$$

$$R = e^{(\mathrm{Constant})t}$$

The expansion rate of the universe shows a different time dependence depending on which component of the universe dominates the energy density...



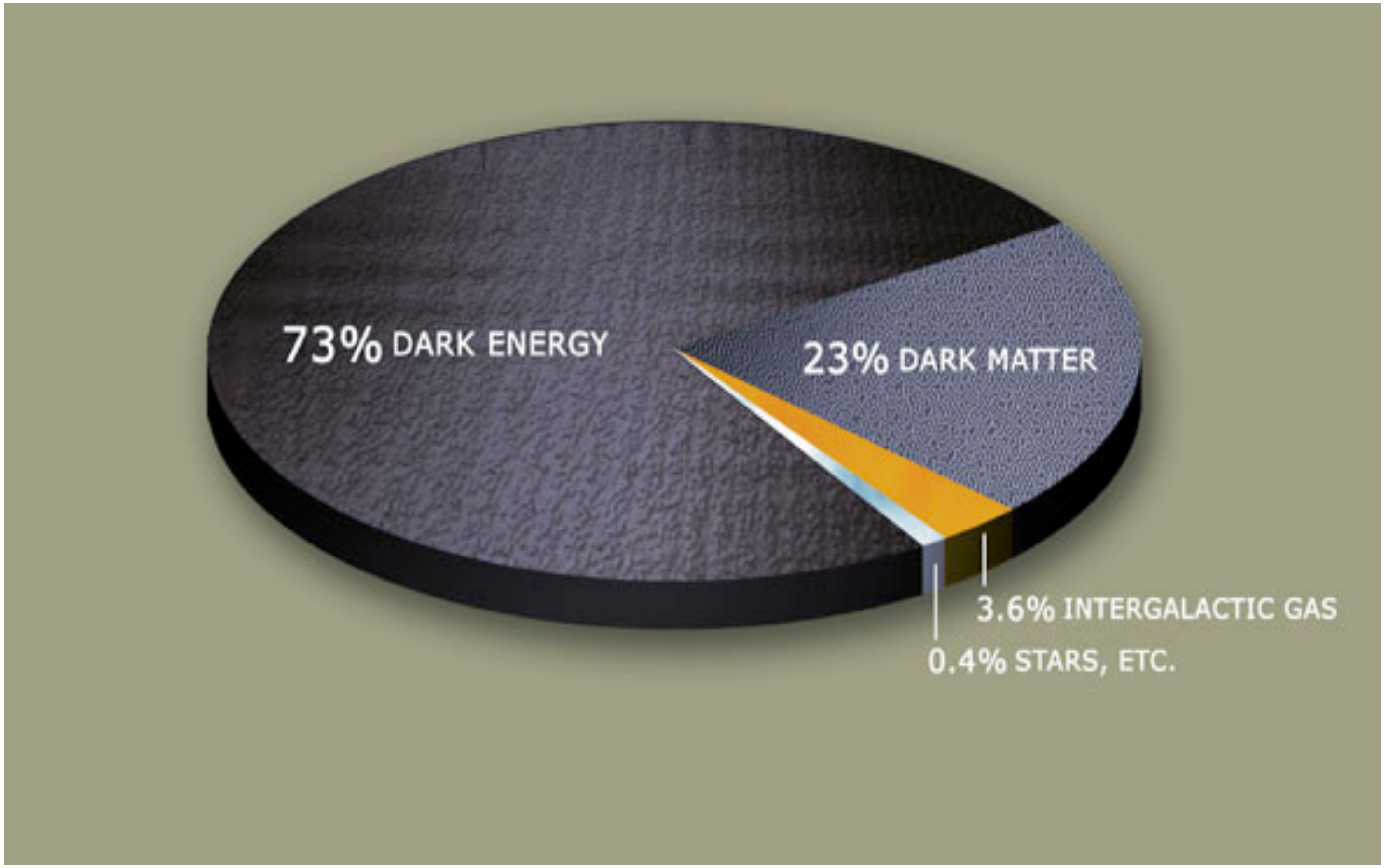

(Carroll & Ostlie: *Modern Astrophysics* 2nd Edition, Figure 29.19)

# What are the key parameters we hope to determine in observational cosmology?

Just Discussed!

| | |
|---|---|
| $H_0$ | Hubble Constant |
| $\Omega_M$ | Matter Density in Matter |
| $\Omega_\Lambda$ | Density of Dark Energy |
| $\Omega_b$ | Matter Density in Baryons |
| $\Omega_r$ | Energy Density in Radiation |
| $n_s$ | Slope of Primordial Power Spectrum |
| $\sigma_8$ | RMS fluctuations of the mass density in spheres of $8h^{-1}$ Mpc |

But all matter in the universe is not the same, there is dark matter, baryonic matter... and baryonic matter comes in many different forms....



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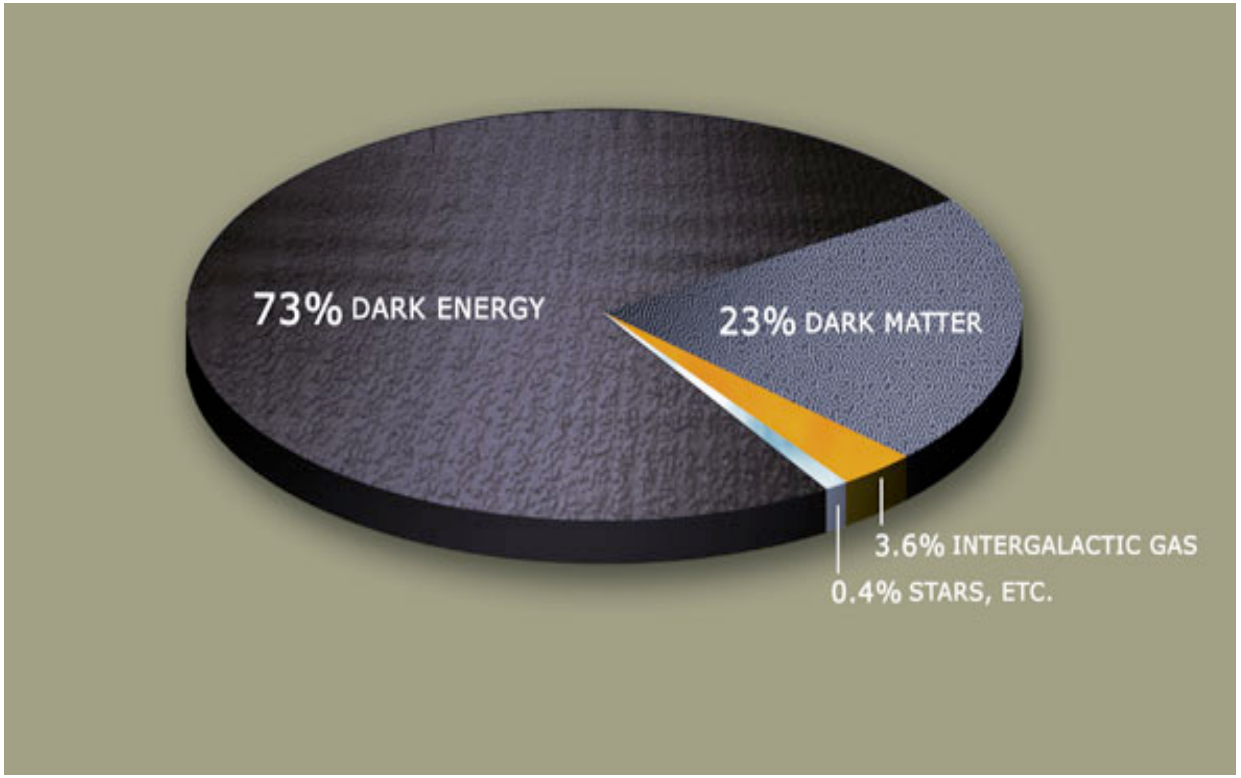
In addition to the contribution of baryons, dark matter, and dark energy, we also have the contribution of radiation!



# Key Parameters to determine in Observational Cosmology

Just Discussed!

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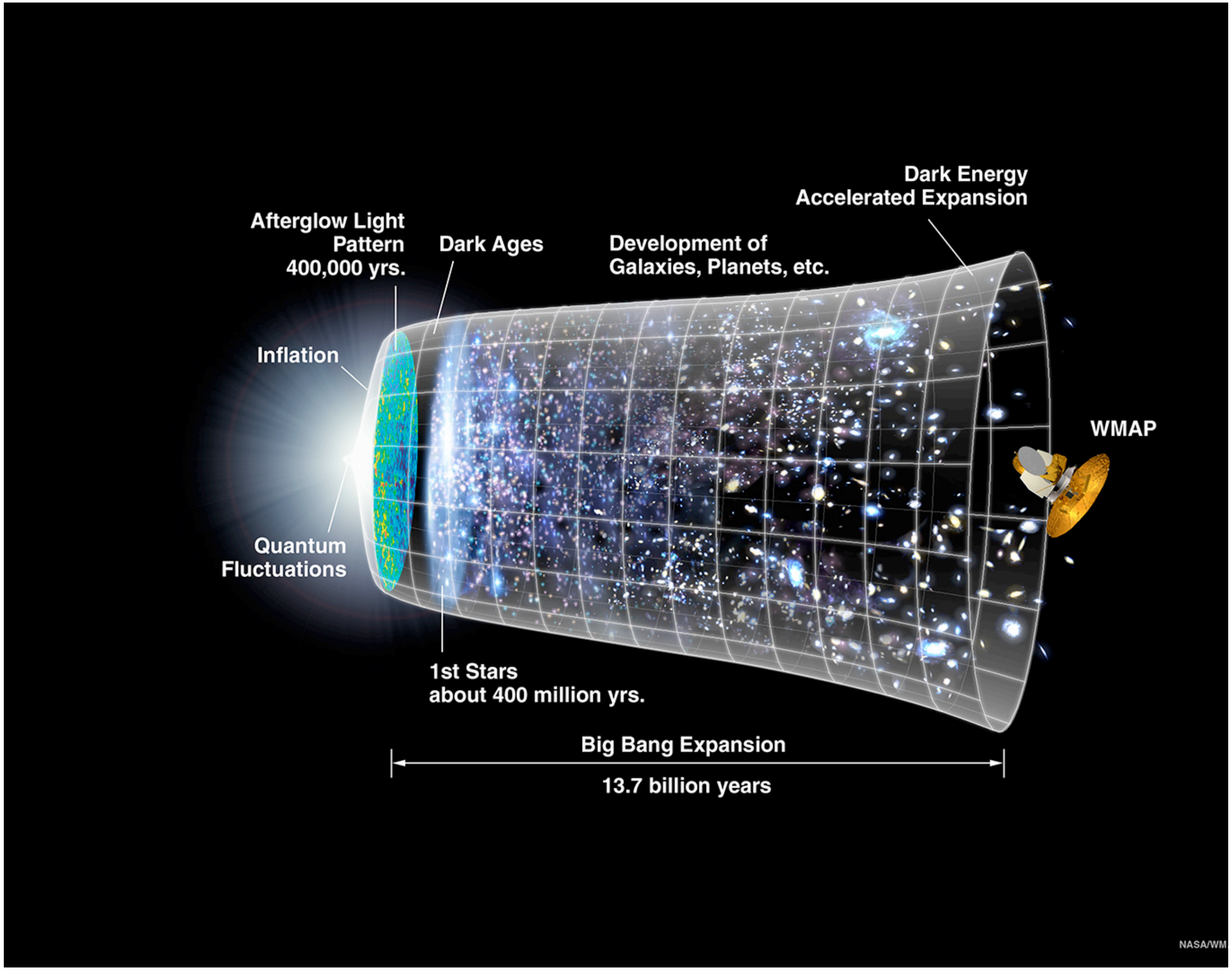
| $\sigma_8$ | RMS fluctuations of the mass density in spheres of $8h^{-1}$ Mpc |
| w | 'w' parameter |

# How do we determine what the Age of Universe is?

# You all probably have a rough idea about the approximate history of the universe



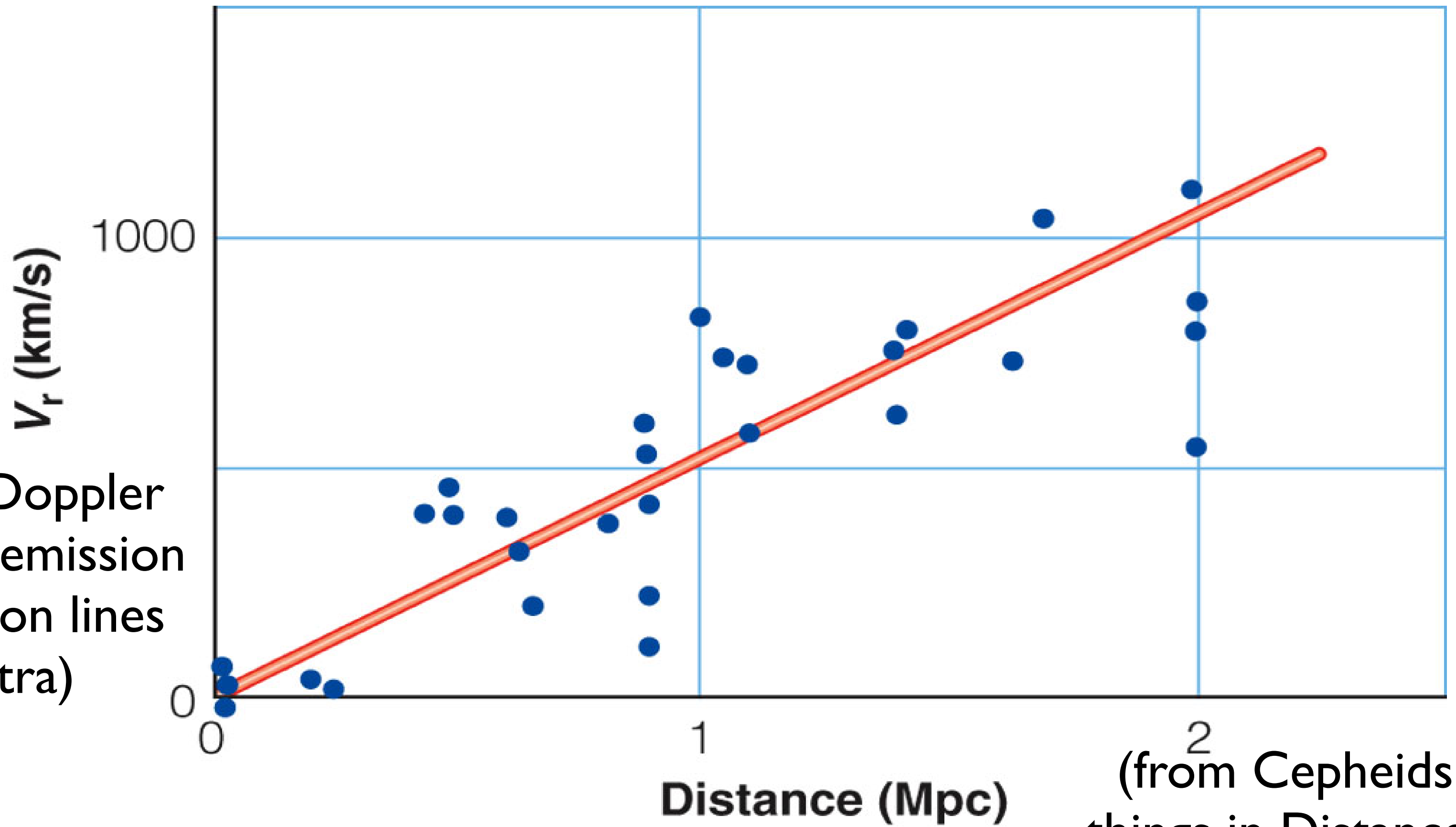
Let’s first estimate the age of the universe assuming our basic cosmological framework is correct....

# From the expansion rate of the universe

In 1929, Hubble showed that the velocities and distances are linearly correlated, and satisfy

$$v = H_0\, d$$

where v is the recessional velocity (km/s) and d is the distance (Mpc). $H_0$ is a constant, "Hubble's Constant" and has units of km $s^{-1}$ $Mpc^{-1}$.




"apparent Doppler shift" (from emission or absorption lines in spectra)

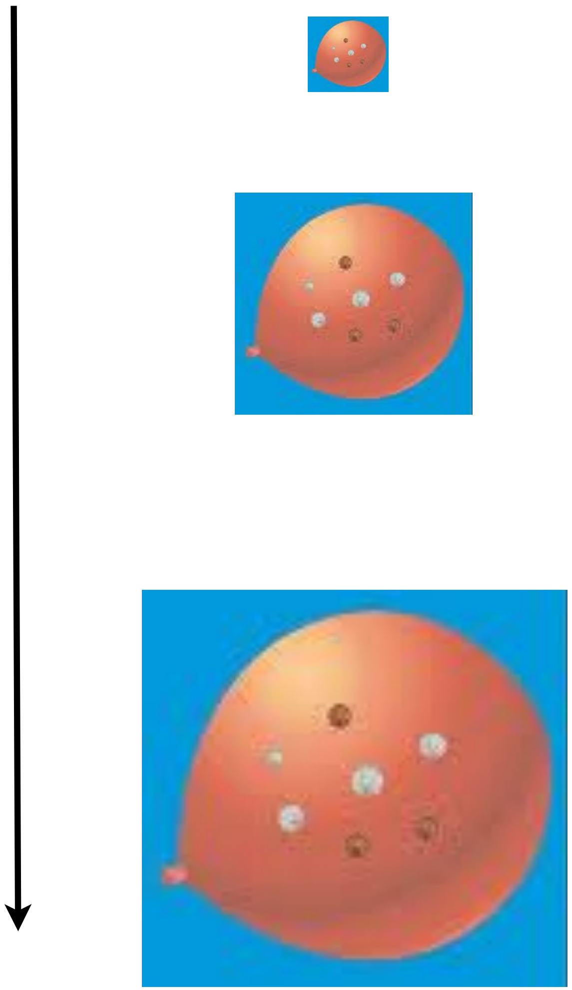
(from Cepheids / other things in Distance Ladder)

# Age of the Universe

(from the expansion rate of the universe)

beginning of universe

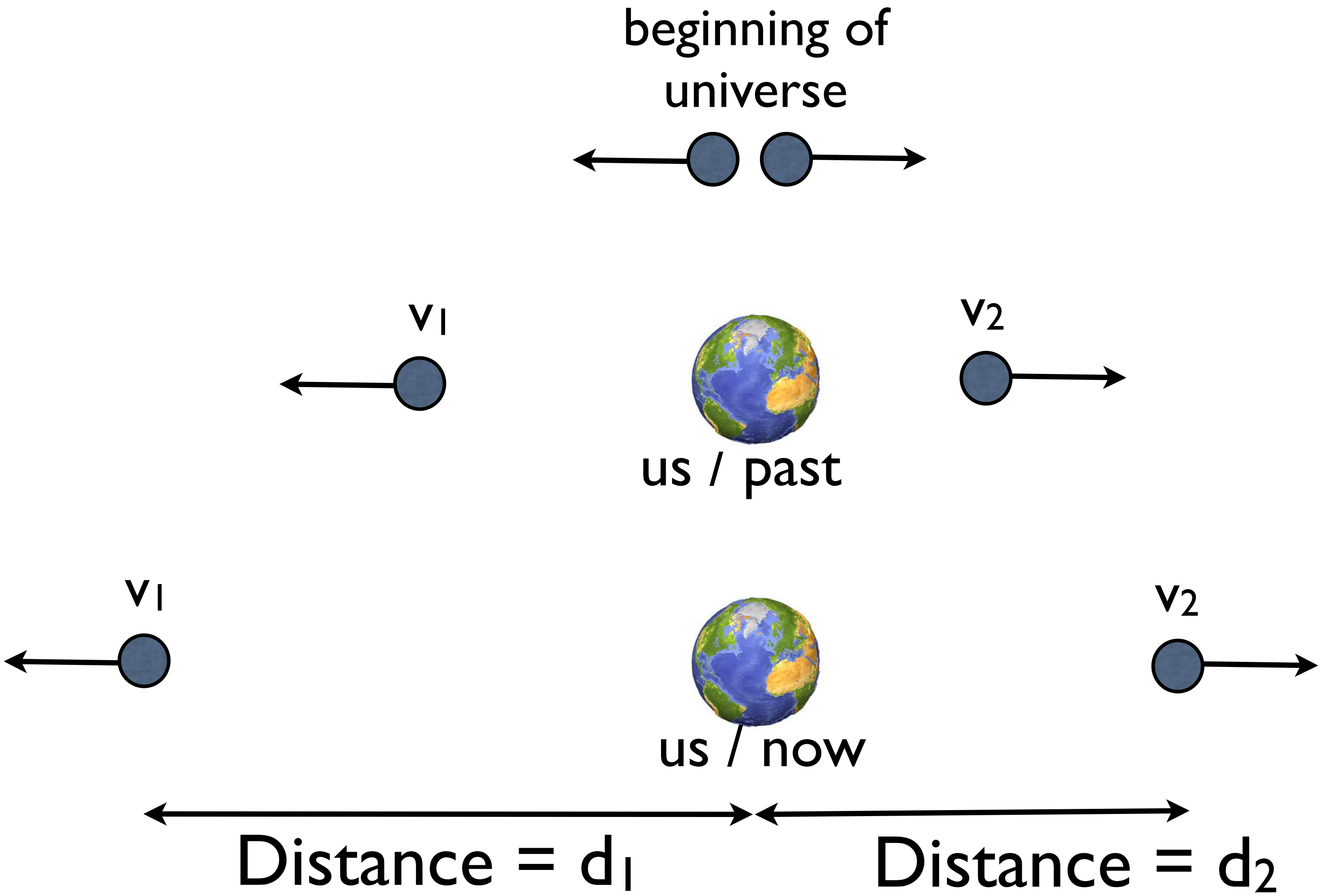
Time



Now

# Age of the Universe

(from the expansion rate of the universe)




$$\text{Age of Universe} \sim d_1 / v_1 \sim d_2 / v_2$$

# Age of the Universe

(from the expansion rate of the universe)

(ignoring gravity or dark energy)

$$\text{Age of Universe} \sim d_1 / v_1$$
$$\sim d_1 / (H_0 d_1)$$
$$\sim 1 / H_0$$

$$v = H_0 d$$

Compute using current $H_0$

$H_0 = 71$ km/s/Mpc

Age of Universe ~ $1/H_0$ ~ 13.8 billion years

Somewhat fortituous that this is close to current best estimate ~13.7 billion years

# Age of Universe

(using solutions to Friedmann's equations)

$$
\begin{aligned}
t &= \int_0^t dt' \\
&= \int_0^{R_0} \frac{dR}{\dot{R}} \\
&= \int_0^{R_0} \frac{dR}{R(\dot{R}/R)} \\
&= \int_0^{R_0} \frac{dR}{RH(R)}
\end{aligned}
$$

R = size of universe

H(R) = Hubble "constant" for the universe with size R

# In search of an equation for H(R):

One of the two Friedmann's equations:

$$\boxed{\dot{R}^2 = \frac{8\pi G(\rho_m + \rho_r)R^2}{3} - kc^2 + \frac{\Lambda c^2 R^2}{3}}$$

Divide by $R^2$

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G(\rho_m + \rho_r)}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{R^2}$$

Factor out $H_0^2$

$$H^2 = H_0^2 \left[\frac{8\pi G\rho_m}{3H_0^2} + \frac{8\pi G\rho_r}{3H_0^2} + \frac{\Lambda c^2}{3H_0^2} - \frac{kc^2}{R^2 H_0^2}\right]$$

Express $\rho_m = \rho_{m,0} (R_0/R)^3$,
$\rho_r = \rho_{r,0} (R_0/R)^4$

$$H^2 = H_0^2 \left[\frac{8\pi G\rho_{m,0}}{3H_0^2}\frac{R_0^3}{R^3} + \frac{8\pi G\rho_{r,0}}{3H_0^2}\frac{R_0^4}{R^4} + \frac{\Lambda c^2}{3H_0^2} - \frac{kc^2}{R^2 H_0^2}\right]$$

# In search of an equation for H(R):

$$H^2 = H_0^2 \left[ \frac{8\pi G \rho_{m,0}}{3H_0^2} \frac{R_0^3}{R^3} + \frac{8\pi G \rho_{r,0}}{3H_0^2} \frac{R_0^4}{R^4} + \frac{\Lambda c^2}{3H_0^2} - \frac{kc^2}{R^2 H_0^2} \right]$$

Express using $\Omega_m$, $\Omega_r$, $\Omega_\Lambda$, $\Omega_k$

$$\Omega_m = \frac{8\pi G \rho_m}{3H^2} \qquad \Omega_\Lambda = \frac{\Lambda c^2}{3H^2}$$

$$\Omega_r = \frac{8\pi G \rho_r}{3H^2} \qquad \Omega_k = \frac{-kc^2}{H^2}$$

$$H(R)^2 = H_0^2 \left[ \Omega_{m,0}(R/R_0)^{-3} + \Omega_{r,0}(R/R_0)^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0}(R/R_0)^{-2} \right]$$

$H(R)^2 = H_0^2 E^2(R)$ **where** $E^2(R) = [\Omega_{m,0}(R/R_0)^{-3} + \Omega_{r,0}(R/R_0)^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0}(R/R_0)^{-2}]$

$H(R) = H_0 E(R)$ **where** $E(R) = [\Omega_{m,0}(R/R_0)^{-3} + \Omega_{r,0}(R/R_0)^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0}(R/R_0)^{-2}]^{1/2}$

# Age of Universe

(using solutions to Friedmann's equations)

$$
\begin{aligned}
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&= \int_0^{R_0} \frac{dR}{R(\dot{R}/R)} \\
&= \int_0^{R_0} \frac{dR}{RH(R)} \\
&= \frac{1}{H_0} \int_0^{R_0} \frac{dR}{RE(R)}
\end{aligned}
$$

$$H(R) = H_0 E(R)$$