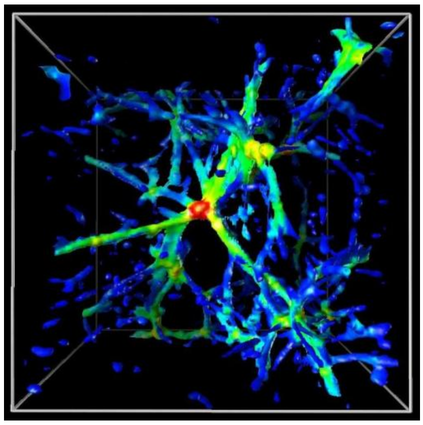
- Where is the rest? Our current idea is that it is in warm/hot intergalactic medium. This is diffuse, ionised filamentary gas that fills out the spaces between galaxies. In clusters of galaxies, the temperatures and densities are hot enough that it is possible to detect via its X-ray emission (see next section). In filaments, the gas is neither hot nor dense enough to emit much in X-rays, and instead must be constrained by detection of absorption line systems in the far-UV or X-ray (very highly ionised oxygen or nitrogen).

- A huge breakthrough in recent times has been the detection of 6-times-ionised oxygen and nitrogen from filaments of the IGM (see attached article by Nicastro 2004) from which $> 1/2$ of the baryonic density of the Universe has been inferred. These lines are *so* faint that one has to wait until a bright flare from a blazar happens to take the spectra (otherwise one needs to integrate on a 'normal' bright X-ray source for $\gg$months).

- I want to give an idea of how extreme this extrapolation is. At typical column densities for detection of $\sim 10^{15} cm^{-2}$, and for an ~AU/pc-sized source, one estimates around 10^{42} or 10^{52} ions were along the line of sight that were detected, corresponding to ~ the mass of an asteroid / the mass of Jupiter. From this small amount of (more-or-less) detected matter, one has extrapolated more than 1/2 of the baryonic density of the Universe!



A schematic diagram of the warm-hot intergalactic medium; the bulk of the gas is in filaments which connect galaxies

6.3 Total mass in galaxies

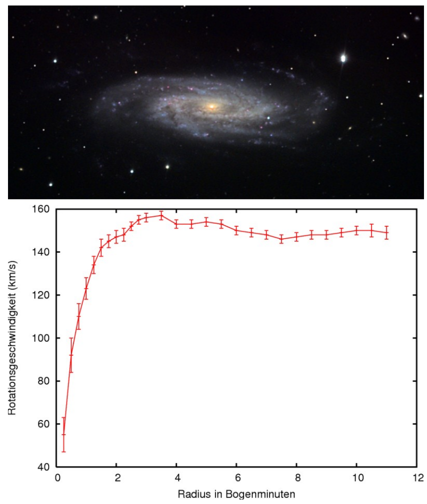
6.3.1 Galaxies

- the rotation velocities of stars orbiting in spiral galaxies are observed to rise quickly with radius and then to remain roughly constant; if measurements are continued with neutral hydrogen beyond the radii out to which stars can be seen, these *rotation curves* are observed to continue at an approximately constant level;

- in a spherically-symmetric mass distribution, test particles on circular orbits have orbital velocities of

$$v_{\rm rot}^2(r) = \frac{GM(r)}{r}\;; \qquad (6.6)$$

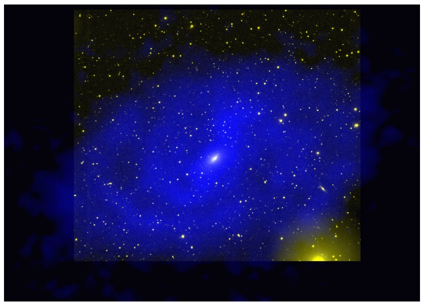
flat rotation curves thus imply that $M(r) \propto r$; based on the continuity equation $\mathrm{d}M = 4\pi r^2 \rho \mathrm{d}r$, this requires that the density falls off as $\rho(r) \propto r^{-2}$ (theory predicts a r^{-3} fall-off at large radii); this is *much* flatter than the light distribution, which shows that spiral galaxies are characterised by an increasing amount of dark matter as the radius increases;



After a quick rise, stellar velocities in spiral galaxies remain approximately constant with radius. (The galaxy shown is NGC 3198.)

- a mass distribution with $\rho \propto r^{-2}$ has formally infinite mass, which is physically implausible; however, at finite radius, the density of the galaxy falls below the mean density of the surrounding universe; the spherical collapse model often invoked in cosmology shows that a spherical mass distribution can be considered in dynamical equilibrium if its mean overdensity is approximately 200 times higher than the mean density $\bar{\rho}$;

- let R be the radius enclosing this overdensity, and M the mass enclosed, then
$$\frac{M}{V} = \frac{3M}{4\pi R^3} = 200\bar{\rho} \quad \Rightarrow \quad \frac{M}{R} = \frac{800\pi\bar{\rho}R^2}{3} \; ; \tag{6.7}$$
at the same time, (6.6) needs to be satisfied, hence
$$\frac{800\pi\bar{\rho}R^2}{3} = \frac{v_{\mathrm{rot}}^2}{G} \quad \Rightarrow \quad R = \left(\frac{3v_{\mathrm{rot}}^2}{800\pi G\bar{\rho}}\right)^{1/2} \; ; \tag{6.8}$$
inserting a typical numbers yields
$$R = 290\,\mathrm{kpc}\left(\frac{v_{\mathrm{rot}}}{200\,\mathrm{km\,s^{-1}}}\right) \; ; \tag{6.9}$$
with (6.6), this implies
$$M = \frac{Rv_{\mathrm{rot}}^2}{G} = 2.7\times10^{12}\,M_\odot\left(\frac{v_{\mathrm{rot}}}{200\,\mathrm{km\,s^{-1}}}\right)^3 \; ; \tag{6.10}$$
The actual normalisation constant is somewhat lower because of the r^{-3} fall-off (roughly 10^{12}) but this gives the flavour of the line of argument.



Far beyond the stars, flat rotation curves are inferred from the motion of neutral-hydrogen clouds (blue; the galaxy shown is NGC 2915).

- typical luminosities of spiral galaxies are given by the Tully-Fisher relation,
$$L = L_*\left(\frac{v_{\mathrm{rot}}}{220\,\mathrm{km\,s^{-1}}}\right)^{3\ldots4} \; , \tag{6.11}$$
with $L_* \approx 2.4\times10^{10}\,L_\odot$, or the normalising mass is roughly $M_* \approx 6\times10^{10}\,M_\odot$; thus, the mass-to-light ratio of a massive spiral galaxy is found to be
$$\frac{m}{l} \approx 60 \tag{6.12}$$
in solar units, or the mass-to-stellar mass ratio is
$$\frac{m}{m_*} \approx 25; \tag{6.13}$$
evidently, this exceeds the *stellar* mass-to-light ratio by far, but the details of the measurement depend on the maximum radius assumed...

- The same analysis can be run with elliptical galaxies (using other methods to estimate dynamical masses, using either velocity dispersions or weak lensing masses), typical values of $\frac{m}{m_*} \sim 45$ are typically found.

6.3.2 Mass in galaxy clusters: kinematic masses

- the next step upward in the cosmic hierarchy are galaxy clusters, which were first identified as significant galaxy overdensities in relatively small areas of the sky;

- rich galaxy clusters contain several hundred galaxies, which by themselves contain a total amount of stellar mass $sim10^{13}\,M_\odot$;

- Yet, the galaxies in rich galaxy clusters move with typical velocities of order $\lesssim 10^3\,\mathrm{km\,s^{-1}}$ which are measured based on redshifts in galaxy spectra; therefore, only one component of the galaxy velocity is observed; its distribution is characterised by the velocity dispersion σ_v;

- if these galaxies were not gravitationally bound to the clusters, the clusters would disperse within $\lesssim 1\,\mathrm{Gyr}$; since they exist over cosmological time scales, clusters need to be (at least in some sense) gravitationally stable;

- assuming virial equilibrium, the potential energy of the cluster galaxies should be minus two times the kinetic energy, or

$$\frac{GM}{R} \approx 3\sigma_v^2 \,, \tag{6.14}$$

where M and R are the total mass and the virial radius of the cluster, and the factor three arises because the velocity dispersion represents only one of three velocity components;

- we combine (6.14) with (6.7) to find

$$R = \left(\frac{9\sigma_v^2}{800\pi G\bar\rho}\right)^{1/2} \approx 2.5\,\mathrm{Mpc}\,, \tag{6.15}$$

and, with (6.14),

$$M \approx 2\times 10^{15}\,\mathrm{M}_\odot \,; \tag{6.16}$$

hence, the mass required to keep cluster galaxies bound despite their high velocities exceeds the mass in galaxies by 1-2 orders of magnitude;

Galaxies move so fast in galaxy clusters (here the Coma cluster) that much more than the visible mass is needed to keep them gravitationally bound; this was the first argument for dark matter, as put forward by Zwicky in the 1930s.

Galaxy clusters are the most luminous emitters of diffuse X-ray radiation. The figure shows the X-ray emission of the Coma cluster observed with the Rosat satellite.

6.3.3 Mass in galaxy clusters: the hot intracluster gas

- galaxy clusters are diffuse sources of thermal X-ray emission; their X-ray continuum is caused by thermal *bremsstrahlung*, whose bolometric volume emissivity is

$$j_\mathrm{X} = Z^2 g_\mathrm{ff} C_\mathrm{X}\, n^2\, \sqrt{T} \tag{6.17}$$

in cgs units, where Z is the ion charge, $g_{\rm ff}$ is the Gaunt factor, n is the ion number density, T is the gas temperature, and

$$C_{\rm X} = 2.68 \times 10^{-24} \tag{6.18}$$

in cgs units, if T is measured in keV;

- a common simple, axisymmetric model for the gas-density distribution in clusters is

$$n(x) = \frac{n_0}{(1+x^2)^{3\beta/2}} \;, \quad x \equiv \frac{r}{r_{\rm c}} \;, \tag{6.19}$$

  where $r_{\rm c}$ is the core radius;

- the line-of-sight projection of the X-ray emissivity yields the X-ray surface brightness as a function of the projected radius ρ,

$$S_{\rm X}(\rho) = \int_{-\infty}^{\infty} j_{\rm X}{\rm d}z = \frac{\sqrt{\pi}\Gamma(3\beta-1/2)}{\Gamma(3\beta)}\frac{Z^2 g_{\rm ff} C_{\rm X}\sqrt{T}n_0^2}{(1+\rho^2)^{3\beta-1/2}} \;, \tag{6.20}$$

  where we have assumed for simplicity that the cluster is isothermal, so T does not change with radius;

- the latter equation shows that two parameters of the density profile (6.19), namely the slope β and the core radius $r_{\rm c}$, can be read off the observable surface-brightness profile;

- the missing normalisation constant can then be obtained from the X-ray luminosity,

$$L_{\rm X} = 4\pi r_{\rm c}^3 \int_0^{\infty} j_{\rm X} x^2{\rm d}x = 4\pi r_{\rm c}^3 Z^2 g_{\rm ff} C_{\rm X}\sqrt{T}n_0^2 \frac{\sqrt{\pi}\Gamma(3\beta-3/2)}{4\Gamma(3\beta)} \;, \tag{6.21}$$

  and a spectral determination of the temperature T;

- finally, the total mass of the X-ray gas enclosed in spheres of radius R is

$$M_{\rm X}(R) = 4\pi r_{\rm c}^3 \int_0^{R/r_{\rm c}} n(x)x^2{\rm d}x \;, \tag{6.22}$$

  which is a complicated function for general β; for $\beta = 2/3$, which is a commonly measured value,

$$M_{\rm X}(R) = 4\pi r_{\rm c}^3 n_0 \left(\frac{R}{r_{\rm c}} - \arctan\frac{R}{r_{\rm c}}\right) \;, \tag{6.23}$$

  which is of course formally divergent for $R \to \infty$ because the density falls off $\propto r^{-2}$ for $\beta = 2/3$ and $r \to \infty$;

- inserting typical numbers, we first set $Z = 1 = g_{\rm ff}$ and $\beta = 2/3$ as above, then use $\Gamma(1/2) = \sqrt{\pi}$, $\Gamma(1) = 1 = \Gamma(2)$, and assume a hypothetic cluster with $L_{\rm X} = 10^{45}\,{\rm erg\,s^{-1}}$, a temperature of $kT = 10\,{\rm keV}$ and a core radius of $r_{\rm c} = 250\,{\rm kpc} = 7.75 \times 10^{23}\,{\rm cm}$;

- then, (6.21) yields the central ion density

$$n_0 = 5 \times 10^{-3}\,\mathrm{cm}^{-3} \tag{6.24}$$

  and thus the central gas mass density

$$\rho_0 = m_\mathrm{p} n_0 = 8.5 \times 10^{-27}\,\mathrm{g\,cm}^{-3}\ ; \tag{6.25}$$

- based on the virial radius (6.15) and on the mass (6.23), we find the total gas mass

$$M_\mathrm{X} = 1.0 \times 10^{14}\,M_\odot\ ; \tag{6.26}$$

  this is of the same order as the cluster mass in galaxies, and approximately one order of magnitude less than the total cluster mass;

- it is reasonable to believe that clusters are closed systems in the sense that there cannot have been much material exchange between their interior and their surroundings; if this is indeed the case, and the mixture between dark matter and baryons in clusters is typical for the entire universe, the density parameter in dark matter should be

$$\Omega_{\mathrm{dm},0} \approx \Omega_{\mathrm{b},0}\,\frac{M}{M_* + M_\mathrm{X}} \approx 10\Omega_{\mathrm{b},0} \approx 0.4\ ; \tag{6.27}$$

  more precise estimates based on detailed investigations of individual clusters yield

$$\Omega_{\mathrm{dm},0} \approx 0.3\ ; \tag{6.28}$$

6.3.4 Alternative cluster mass estimates

- cluster masses can be estimated in several other ways; one of them is directly related to the X-ray emission discussed above; the hydrostatic Euler equation for an isothermal gas sphere in equilibrium with the spherically-symmetric gravitational potential of a mass $M(r)$ requires

$$\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{GM(r)}{r^2}\ , \tag{6.29}$$

  where ρ and p are the gas density and pressure, respectively; assuming an ideal gas, the equation of state is $p = nkT$, where $n = \rho/m_\mathrm{p}$ is the particle density of the gas and T is its temperature; if we further simplify the problem assuming an isothermal gas distribution, we can write

$$\frac{kT}{m_\mathrm{p}\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r} = -\frac{GM(r)}{r^2} \tag{6.30}$$

or, solving for the mass

$$M(r) = -\frac{rkT}{Gm_\mathrm{p}}\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r} \; ; \tag{6.31}$$

- for the β model introduced in (6.19), the logarithmic density slope is

$$\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r} = \frac{\mathrm{d}\ln n}{\mathrm{d}\ln r} = -3\beta\frac{r^2}{1+r^2} \; , \tag{6.32}$$

thus the cluster mass is determined from the slope of the X-ray surface brightness and the cluster temperature,

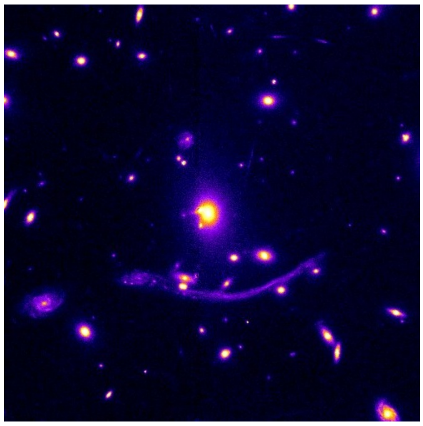
$$M(r) = \frac{3\beta rkT}{Gm_\mathrm{p}}\frac{r^2}{1+r^2} \; ; \tag{6.33}$$

- with the typical numbers used before, i.e. $R \approx 2.5\,\mathrm{Mpc}$, $\beta \approx 2/3$ and $kT = 10\,\mathrm{keV}$, the X-ray mass estimate gives

$$M(R) \approx 1.1 \times 10^{15}\, M_\odot \; , \tag{6.34}$$

in reassuring agreement with the mass estimate (6.16) from galaxy kinematics;

- a third, completely independent way of measuring cluster masses is provided by gravitational lensing; without going into any detail on the theory of light deflection, we mention here that it can generate image distortions from which the projected lensing mass distribution can be reconstructed; mass estimates obtained in this way by and large confirm those from X-ray emission and galaxy kinematics, although interesting discrepancies exist in detail;



Strong gravitational lensing in galaxy clusters can cause strong distortions of background galaxies into arcs (shown is the large arc in the cluster Abell 370). They allow independent cluster-mass estimates.

- none of the cluster mass estimates is particularly reliable because they are all to some degree based on stability and symmetry assumptions; for mass estimates based on galaxy kinematics, for instance, assumptions have to be made on the shape of the galaxy orbits, the symmetry of the gravitational potential and the mechanical equilibrium between orbiting galaxies and the body of the cluster; numerous assumptions also enter X-ray based mass determinations, such as hydrostatic equilibrium, spherical symmetry and, in some cases, isothermality of the intracluster gas; gravitational lensing does not need any equilibrium assumption, but inferences from strongly distorted images depend very sensitively on the assumed symmetry of the mass distribution;

6.4 Mass density from cluster evolution

- a very interesting constraint on the cosmic mass density is based on the evolution of cosmic structures; Abell's cluster catalog covers the redshift range $0.02 \lesssim z \lesssim 0.2$, which encloses a volume of

$\approx 9 \times 10^8\,\mathrm{Mpc}^3$; of the 2712 clusters in the catalog, 818 fall into (the poorest) richness class 0; excluding those, there are 1894 clusters with richness class ≥ 1 in that volume, which yields an estimate for the spatial cluster density of

$$n_\mathrm{c} \approx 2 \times 10^{-6}\,\mathrm{Mpc}^{-3} \; ; \tag{6.35}$$

- it is a central assumption in cosmology that structures formed from Gaussian random density fluctuations; the spherical collapse model then says that gravitationally bound objects form where the linear density contrast exceeds a critical threshold of $\delta_\mathrm{c} \approx 1.686$, quite independent of cosmology; the probability for this to happen in a Gaussian random field with a (suitably chosen) standard deviation $\sigma(z)$ is

$$p_\mathrm{c}(z) = \frac{1}{2}\mathrm{erfc}\left(\frac{\delta_\mathrm{c}}{\sqrt{2}\sigma(z)}\right) \; , \tag{6.36}$$

  where

$$\sigma(z) = \sigma_0 D_+(z) \tag{6.37}$$

  because the linear growth of the density contrast is determined by the growth factor, a fitting formula for which was given in (2.20);

- now, the present-day standard deviation σ_0 must be chosen such as to reproduce the observed number density of clusters given in (6.35); the measured probability for finding a cluster is approximated by

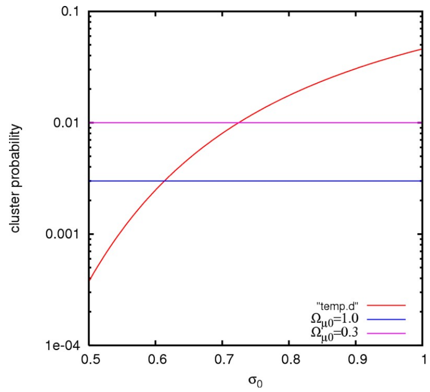
$$p'_\mathrm{c} = \frac{Mn_\mathrm{c}}{\rho_\mathrm{c}\Omega_\mathrm{m}} \approx 3 \times 10^{-3}\Omega_\mathrm{m0}^{-1} \; ; \tag{6.38}$$

  the standard deviation σ in (6.36) must now be chosen such that this number is reproduced, which yields

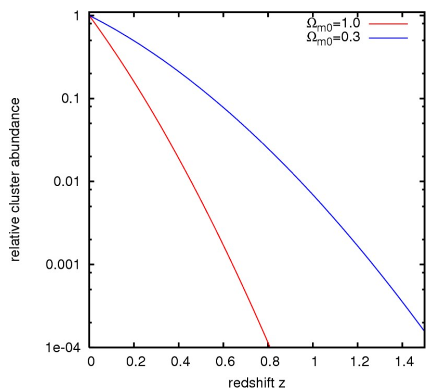
$$\sigma_0 \approx \begin{cases} 0.61 & \Omega_\mathrm{m0} = 1.0 \\ 0.72 & \Omega_\mathrm{m0} = 0.3 \end{cases} \; ; \tag{6.39}$$

- equations (6.36) and (6.37) can now be used to estimate how the cluster abundance should change with redshift; simple evaluation reveals that the cluster abundance is expected to drop very rapidly with increasing redshift if Ω_m0 is high, and much more slowly if Ω_m0 is low;

- qualitatively, this behaviour is easily understood; if, in a low-density universe, cluster do not form early, they cannot form at all because the rapid expansion due to the low matter density prevents them from growing late in the cosmic evolution;



Cluster probability as a function of σ for two different values of Ω_m0.



Evolution of the cluster abundance, depending on the density parameter Ω_m0.

- from the observed slow evolution of the cluster population as a whole, it can be concluded that the matter density must be low; estimates arrive at
$$\Omega_{m0} \approx 0.3 \;, \tag{6.40}$$
in good agreement with the preceding determinations;

6.5 Musings on the nature of the dark matter

- The preceding discussion should have demonstrated that the matter density in the Universe is I) considerably less than its critical value, approximately one third of it. However, II) only a small fraction of this matter is visible; thus we call the remaining invisible majority *dark matter*.

- What is this dark matter composed of? Can it be baryons? Tight limits are set by primordial nucleosynthesis, which predicts that the matter density in baryonic matter should be $\Omega_B \approx 0.04$, cf. (5.27). In the framework of the Friedmann-Lemaître models, the baryon density in the Universe can be higher than this only if baryons are locked up in some way before nucleosynthesis commences. They could form black holes before, but their mass is limited by the mass enclosed within the horizon at, say, up to one minute after the Big Bang. According to (2.6), the scale factor at this time was $a \approx 10^{-10}$, and thus the matter density was of order $\rho_m \approx 10^{30}\rho_{cr} \approx 10\,\mathrm{g\,cm^{-3}}$. The horizon size is $r_H \approx ct \approx 1.8 \times 10^{12}\,\mathrm{cm}$, thus the mass enclosed by the horizon is $\approx 3 \times 10^4 M_\odot$, which limits possible black-hole masses from above.

- It is expected that quantum effects cause black holes to radiate, thus to convert their mass to radiation energy and to "evaporate". The estimated time scale for complete evaporation is
$$\tau_{bh} \approx 4 \times 10^{70}\,\mathrm{s}\left(\frac{M}{M_\odot}\right)^3 \;, \tag{6.41}$$
which is shorter than the Hubble time (4.35) if
$$M \lesssim 4 \times 10^{15}\,\mathrm{g} \;. \tag{6.42}$$
Black holes formed very early in the Universe should thus have disappeared by now.

- Gravitational microlensing was used to constrain the amount of dark, compact objects in the halo of the Milky Way. Although they were found to contribute part of the mass, they cannot account for all of it. In particular, black holes with masses of the order $10^{3...4}\,M_\odot$ should have been found through their microlensing effect.

- We are thus guided to the conclusion that the dark matter is most probably not baryonic and not composed of compact dark objects. We shall see later that and why the most favoured hypothesis now holds that it is composed of weakly interacting massive particles. Neutrinos, however, are ruled out because their total mass has been measured to be way too low.