Chapter 3

The age of the Universe

A key constraint on cosmology is the 'observed' age of the Universe. We have no direct way to measure how long ago the Big Bang happened, but there are various ways to set lower limits to the age of the Universe. They are all based on the same principle: since the Universe cannot be younger than any of its parts, it must be older than the oldest objects it contains. Three methods for age determination have been developed. One is based on the radioactive decay of long-lived isotopes, another constrains the age of globular clusters, and the third is based on the age of white dwarfs. We shall discuss them in turn to find out how old the Universe should be at least.

It turns out that these arguments, because of astrophysical complications, no longer provide the strongest constraints on cosmological parameters. They are included here for three reasons: (1) historical importance - age of the Universe constraints played an important role historically, (2) it is an important consistency check, and (3) the fact that the age of the oldest constituents of the Universe that we can observe is finite (and not that old) is a fundamental argument for the Big Bang paradigm.

3.1 Nuclear cosmo-chronology

- nuclear cosmo-chronology compares the measured abundance of certain radioactive isotopes with their initial abundance.
- This method can in principle be a very powerful one. Since radioactive decay is described $\dot{N} = -\lambda N$, where N is the number of decaying nuclei in a closed sample and λ is the decay rate, integration gives
$$N(t) = N_0 \mathrm{e}^{-\lambda t} \tag{3.1}$$
- Since N and λ are measurable, the only barrier to measuring t is

knowing the initial abundance of nuclei N_0.

- In some well-posed cases (e.g., measuring the age of the Earth) the initial abundance can be estimated with some accuracy through the comparison of the abundances of different isotopes of both the decaying nuclei and the decay products.
- In other cases, the initial abundance can only be crudely estimated, and the constraints are considerably less robust (e.g., estimating the age of the Galactic disk).

3.1.1 The age of the Earth

- to give a specific example, consider the two uranium isotopes ^{235}U and ^{238}U; they both decay into stable lead isotopes, $^{235}\mathrm{U} \to {}^{207}\mathrm{Pb}$ through the actinium series and $^{238}\mathrm{U} \to {}^{206}\mathrm{Pb}$ through the radium series; the abundance of any of these two lead isotopes is the sum of the initial abundance, plus the amount produced by the uranium decay;
- since the radioactive decay is described $\dot{N} = -\lambda N$, where N is the number of decaying nuclei in a closed sample and λ is the decay rate, integration gives
$$N(t) = N_0 \mathrm{e}^{-\lambda t} \tag{3.2}$$
for the remaining number of initially N_0 radioactive nuclei, or
$$\bar{N} = N_0 \left(1 - \mathrm{e}^{-\lambda t}\right) = N(t)\left(\mathrm{e}^{\lambda t} - 1\right) \tag{3.3}$$
for the number of nuclei of the stable decay product;
- thus, the present abundance of ^{207}Pb nuclei is its primordial abundance $N_{207,0}$ plus the amount produced,
$$N_{207} = N_{207,0} + N_{235}\left(\mathrm{e}^{\lambda_{235} t} - 1\right) , \tag{3.4}$$
where N_{235} is the abundance of ^{235}U nuclei today; a similar equation with 235 replaced by 238 and 207 replaced by 206 holds for the decay of ^{238}U to ^{206}Pb; the decay constants for the two uranium isotopes are measured as
$$\lambda_{235} = (1.015\,\mathrm{Gyr})^{-1} , \quad \lambda_{238} = (6.45\,\mathrm{Gyr})^{-1} ; \tag{3.5}$$
- the idea is now that ores on Earth or meteorites formed during a period which was very short compared to the age of the Earth t_e, so that their abundances can be assumed to have been locked up instantaneously and simultaneously a time t_e ago; chemical fractionation has given different abundances to different samples, but could not distinguish between different isotopes of the same element; thus, we expect different samples to show different isotope *abundances*, but identical *abundance ratios* of different isotopes;

- the stable lead isotope ^{204}Pb has no long-lived parents and is therefore a measure for the primordial lead abundance; thus, the *abundance ratios* between ^{207}Pb and ^{208}Pb to ^{204}Pb calibrate the abundances in different samples;

- suppose we have two independent samples a and b. The key thing here is that we assume i) that the isotopic ratios in the initial samples were the same (i.e., the initial ^{235}U to ^{238}U ratios, and the initial ratios of ^{204}Pb to ^{206}Pb to ^{207}Pb were independent of sample), and ii) that the two samples had different initial U to Pb ratios. Then, the abundance ratios

$$R_{206} \equiv \frac{N_{206}}{N_{204}} \quad \text{and} \quad R_{207} \equiv \frac{N_{207}}{N_{204}} \tag{3.6}$$

are measured; according to (3.3), they are

$$\begin{aligned} R_{206} &= R_{206,0} + \frac{N_{238}}{N_{204}}\left(e^{\lambda_{238}t_e} - 1\right) , \\ R_{207} &= R_{207,0} + \frac{N_{235}}{N_{204}}\left(e^{\lambda_{235}t_e} - 1\right) ; \end{aligned} \tag{3.7}$$

the lead abundance ratios $R_{206,0}$ and $R_{207,0}$ should be the same in the two samples and cancel when the difference between the ratios in the two samples is taken; then, the ratio of differences can be written as

$$\frac{R^a_{207} - R^b_{207}}{R^a_{206} - R^b_{206}} = \frac{N_{235}}{N_{238}} \frac{e^{\lambda_{235}t_e} - 1}{e^{\lambda_{238}t_e} - 1} ; \tag{3.8}$$

once the lead abundance ratios have been measured in the two samples, and the present uranium isotope ratio

$$\frac{N_{235}}{N_{238}} = 0.00725 \tag{3.9}$$

is known, the age of the Earth t_e is the only unknown in (3.8); this method yields

$$t_e = 4.6 \pm 0.1\,\text{Gyr} ; \tag{3.10}$$

3.1.2 The age of the Galaxy

- a variant of this method can be used to estimate the age of the Galaxy, but this requires a model for how the radioactive elements were formed during the lifetime of the galaxy until they were locked up in samples where we can measure their abundances today;

- suppose there was an instantaneous burst of star formation and subsequent supernova explosions a time t_g ago and no further production thereafter; then, the radioactive elements found on Earth today decayed for the time $t_g - t_e$ until they were locked up when the Solar System formed; if we can infer from supernova theory what the primordial abundance ratio $^{235}U/^{238}U$ is, we can conclude from its present value (3.9) and the age of the Earth what the age of the Galaxy must be;

- the situation is slightly more complicated because element production did not stop after the initial burst; suppose that a fraction f of the heavy elements locked up in the Solar System was produced in a burst at $t = 0$, and the remaining fraction $1 - f$ was added at a steady rate until $t = t_g - t_e$ when the Earth was formed (i.e., we ignore all elements produced after the formation of the Earth);

- the differential equation we have to solve now is

$$\dot{N} = -\lambda N + p \,, \tag{3.11}$$

where p is the constant production rate; we solve it by variation of constants, starting from the *ansatz*

$$N = C(t)\mathrm{e}^{-\lambda t} \tag{3.12}$$

which solves (3.11) if

$$C = \frac{p}{\lambda}\mathrm{e}^{\lambda t} + D \tag{3.13}$$

with a constant D; thus, the abundance of a radioactive element with decay constant λ is

$$N = D\mathrm{e}^{-\lambda t} + \frac{p}{\lambda} \tag{3.14}$$

before $t_g - t_e$, and

$$N = N_0\mathrm{e}^{-\lambda[t-(t_g-t_e)]} \tag{3.15}$$

thereafter, where N_0 is the abundance of elements locked up in the Solar System, as before;

- now, let N_p be the total amount produced, then the initial conditions require that

$$N(0) = D + \frac{p}{\lambda} = fN_p \,, \tag{3.16}$$

and thus

$$N(t_g - t_e) = \mathrm{e}^{-\lambda(t_g-t_e)}\left[fN_p + \frac{p}{\lambda}\left(\mathrm{e}^{\lambda(t_g-t_e)} - 1\right)\right] \tag{3.17}$$

when the Earth formed, and

$$N(t_g) = \mathrm{e}^{-\lambda t_g}\left[fN_p + \frac{p}{\lambda}\left(\mathrm{e}^{\lambda(t_g-t_e)} - 1\right)\right] \tag{3.18}$$

now on the Earth.

- the production rate must be

$$p = \frac{(1-f)N_\mathrm{p}}{t_\mathrm{g} - t_\mathrm{e}}\ , \tag{3.19}$$

which gives the present abundance

$$N = N_\mathrm{p}\mathrm{e}^{-\lambda t_\mathrm{g}}\left[f + \frac{(1-f)}{\lambda(t_\mathrm{g}-t_\mathrm{e})}\left(\mathrm{e}^{\lambda(t_\mathrm{g}-t_\mathrm{e})} - 1\right)\right] \tag{3.20}$$

in terms of the produced abundance N_p;

- supernova theory says that the *produced* abundance ratio of the isotopes ^{235}U and ^{238}U is

$$\frac{N_{235,\mathrm{p}}}{N_{238,\mathrm{p}}} = 1.4 \pm 0.2\ ; \tag{3.21}$$

taking the ratio of (3.20) for the present abundances of ^{235}U and ^{238}U, inserting the decay constants from (3.5), the abundance ratios from (3.9) and (3.21), and the age of the Earth t_e from (3.10) yields an equation which contains only the age of the galaxy t_g in terms of the assumed fraction f; this gives

$$t_\mathrm{g} = \begin{cases} 6.3 \pm 0.2\,\mathrm{Gyr} & f = 1(\text{all in burst}) \\ 8.0 \pm 0.6\,\mathrm{Gyr} & f = 0.5 \\ 12 \pm 2\,\mathrm{Gyr} & f = 0(\text{constant}) \end{cases} \tag{3.22}$$

- of course, the Universe must be older than the Galaxy; common assumptions and results from galaxy-formation theory assert that there at least 1 Gyr is necessary before galactic disks could have been assembled; therefore, nuclear cosmochronology constrains the age of the Universe to fall within

$$7\,\mathrm{Gyr} \lesssim t_0 \lesssim 13\,\mathrm{Gyr}\ ; \tag{3.23}$$

3.2 Stellar ages

- another method for measuring the age of the Universe caused much trouble for cosmologists for a long time; it is based on stellar evolution and exploits the fact that the time spent by stars on the main sequence of the Hertzsprung-Russell diagram depends sensitively on their mass and thus on their color;

- Using this, if one can find collections of stars which are relatively ancient, it offers a chance to put a stringent lower limit on the age of the Universe. Globular clusters offer access to such populations (ancient and reasonably metal poor). In what follows, we will explore age-dating globular cluster stellar populations.

- stars are described by the stellar-structure equations, which relate the mass M, the density ρ and the pressure P to the radius r and specify the temperature T and the luminosity L; they read

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM\rho}{r^2} , \quad \frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2\rho , \tag{3.24}$$

  which simply state hydrostatic equilibrium and mass conservation, and

$$\frac{\mathrm{d}T}{\mathrm{d}r} = \frac{3L\kappa\rho}{4\pi r^2 acT^3} , \quad \frac{\mathrm{d}L}{\mathrm{d}r} = 4\pi r^2\rho\epsilon , \tag{3.25}$$

  which describe energy transport and production; κ is the opacity of the stellar material, ϵ is the energy production rate per mass, and a is the Stefan-Boltzmann constant;

- The goal is to understand how luminosity and temperature depend on lifetime and opacity (i.e., age and metallicity).

- What we'll do to get a flavor for the problem is to do a dimensional analysis (i.e., drop physical constants and equate $dP/dr \sim P/R$, etc.) We will explicitly keep track of the opacity κ in what follows. Using the equation for hydrostatic equilibrium and the ideal gas law $P = \frac{\rho kT}{\mu m_H}$ one finds $M \propto TR/\mu$, where μ is the mean molecular mass in units of the hydrogen atom mass.

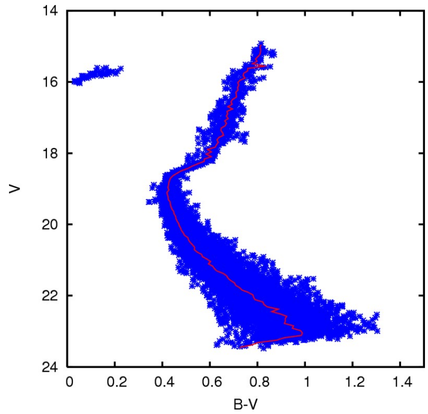
- Using then the equation for temperature change as a function of luminosity, density, radius, temperature and opacity, one derives $L \propto \mu^4 M^3 \kappa^{-1}$. Thus, given that the lifetime $\tau \sim M/L$, $\tau \propto M^{-2}$, $\tau \propto T^{-4}$, and $L \propto \kappa^{1/2}\tau^{-3/2}$. (Please work through this derivation; it is informative).

- Using then the Stefan-Boltzmann equation $L \propto R^2T^4$, one can use the above to determine $T \propto (\tau\kappa)^{-1/4}$.

- There are a few key results here. $L \propto M^3$ and $L \propto T^4$ describe (in a *very* approximate fashion) the stellar main sequence. Furthermore, lifetime is a very strong function of L and T; hot, luminous high-mass stars have short lifetimes whereas lower mass stars have very long lifetimes (in the case of stars considerably less massive than the Sun, the lifetime exceeds the Hubble time)[1].

- The key point for the purpose of age-dating the globular cluster population is that as a coeval stellar population ages, the point in its Hertzsprung-Russell diagram up to which the main sequence remains populated moves towards lower luminosities and temperatures as $(L, T) \simeq (\kappa^{1/2}\tau^{-3/2}, [\kappa\tau]^{-1/4})$;

[1]In a crude sense, this is used by astronomers to age-date light from integrated stellar populations: blue light is from young populations whereas red light is from older populations.



Colour-magnitude diagram of a globular cluster. The turn-off point in the main sequence is clearly visible, but not very well defined.

- Thus, the main sequence turn-off points of the populations in globular clusters can be used to derive lower limits to the age of the Galaxy and the Universe;
- in practice, such age determinations proceed by adapting simulated stellar-evolution tracks to the Hertzsprung-Russell diagrams of globular clusters and assigning the age of the best-fitting stellar-evolution model to the cluster;
- The key uncertainty in this game is distance. Since observations cannot tell the luminosity of the turn-off point on the main sequence, but only its apparent brightness, age determinations from globular clusters require that the cluster distances be known; there are several ways for estimating cluster distances; one uses the period-luminosity relation of certain classes of variable stars; another method uses that the horizontal branch has a typical luminosity and can thus be used to calibrate the cluster distance;
- therefore, uncertainties in the distance determinations directly translate to uncertainties in age the determinations; if the distance is overestimated, so is the luminosity, which implies that the age is underestimated, and vice versa;
- Another major uncertainty is reddening: this causes the observed Hertzsprung-Russell diagram to shift along a well-known vector towards lower luminosities and lower temperatures ("redder" colours); it can be corrected for to a certain extent using other information (typically foreground thermal IR emission or HI column density, assuming a dust-to-IR or dust-to-gas ratio), and using other well-defined features of the diagram like the red giant or horizontal branches as a consistency check;
- several other difficulties are typically met: the simulated stellar-evolution tracks depend on the assumed metallicity of the stellar material, which changes the opacity and thus the energy transport through the stars (the dependence on opacity can be seen above); the light from the clusters is reddened and attenuated by interstellar absorption; the stellar population tracks are uncertain (because of opacity uncertainties, convection uncertainties, etc); and other difficulties...
- globular clusters typically gave age determinations which were well above estimates based on the cosmological parameters assumed; in the past decade or so, this has changed because improvements in stellar-evolution theory have lowered the globular-cluster ages, while recently determined cosmological parameters now yield a higher age for the Universe as assumed before; now, globular-cluster ages imply

$$t \gtrsim 12\,\mathrm{Gyr} \tag{3.26}$$

for the age of the Universe;

3.3 Cooling of white dwarfs

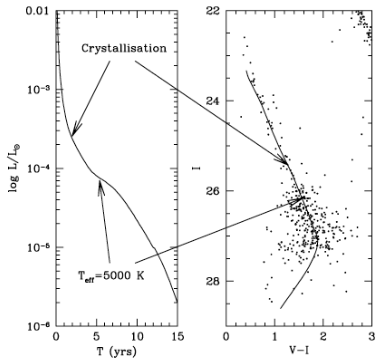


FIG. 12.—Left-hand panel shows the cooling curve for a 0.55 $M_\odot$ model. Right-hand panel shows the same cooling sequence in the color-magnitude diagram, compared to the observed cooling sequence. Arrows indicate the corresponding positions of two important epochs. The first is the onset of core crystallization, which contributes latent heat. The second is the point at which the effective temperature reaches 5000 K. This corresponds to a drop in photospheric hydrogen opacity and a change in the boundary condition of the cooling models. The result is a flattening of the cooling curve, which causes a corresponding jump in the observed luminosity function.

From Hansen et al. 2004

- a key method for cosmic age determinations is based on the cooling of white dwarfs. White dwarfs offer a number of advantages: no energy generation (except for some latent heat lost as the white dwarf rearranges its internal structure during some phases of cooling); the gas is degenerate making the interior approximately isothermal (because electrons have essentially infinite mean free path) and with a simple equation of state.

- White dwarfs have $L \propto 1/t$ approximately, with some mass dependence.

- What are white dwarf masses? – if there is mass dependence, we need to know the masses! Hydrostatic equilibrium gives $P \propto M^2/r^4$, and the equation of state of a non-relativitsic degenerate gas is $P \propto \rho^{5/3} \propto M^{5/3}/r^5$, giving then $r \propto M^{-1/3}$ (i.e., as mass increases, radius decreases).

- The surface gravity $g \propto M/r^2 \propto M^{5/3}$, i.e., surface gravity measurements (from the gravitational redshift of spectral lines) gives then mass estimates. **Result:** remarkably, white dwarfs almost all have masses between 0.55 $M_\odot$ and 0.6 $M_\odot$ (that's just the way mass loss during stellar evolution works).

- Thus, given M, we can use the T and L of white dwarf cooling sequences to get an age. The key uncertainties are distance and reddening (for the same reasons given above for main sequence-derived ages), along with uncertainties in white dwarf mass and composition (through heat capacities).

- Results so far
$$t_{\rm globularclusters} \sim 12\,Gyr \tag{3.27}$$
for the age of Galactic globular clusters (with a limit of 10Gyr), and ages of $>$ 7 Gyr for the galactic disk. See Hansen et al. 2004, ApJS, 155, 551 for a beautiful discussion of the method and sources of uncertainty.

3.4 Summary

- combining results, we see that the age of the Universe, as measured by its supposedly oldest parts, is at least $\gtrsim$ 11 Gyr, and this places serious cosmological constraints; in the framework of the

Friedmann-Lemaître models, this can be interpreted as limits on the cosmological parameters;

- suppose we live in an Einstein-de Sitter universe with $\Omega_{m,0} = 1$ and $\Omega_\Lambda = 0$; then, we know from (2.12) that

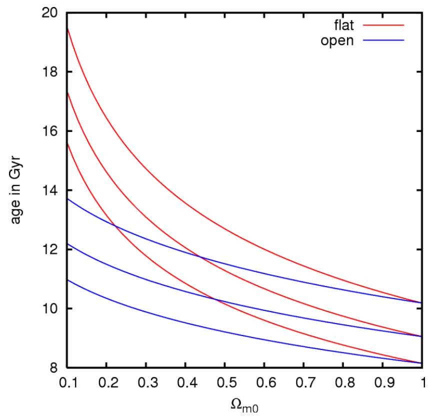
$$t_0 = \frac{2}{3H_0} \gtrsim 11\,\mathrm{Gyr} \quad \Rightarrow \quad H_0 \lesssim 2 \times 10^{-18}\,\mathrm{s}\,, \tag{3.28}$$

which reads

$$H_0 \lesssim 61\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{3.29}$$

in conventional units;

- as we shall see in the next chapter, the Hubble constant is measured to be larger than this, which can immediately be interpreted as an indication that we are *not* living in an Einstein-de Sitter universe;



Constraints on the cosmic age have meaningful implications on the cosmological parameters, in particular on the cosmic density parameter. The three curves for each cosmological model are obtained assuming $H_0 = (64, 72, 80)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.